Theoretical Astrophysics

Heidelberg University
Winter Term 2023/24

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Problem Sheet 11

Discussion in the tutorial groups in the week of January 22, 2024.
The *final exam* is going to take place in room **Phil 12, gHS**, on
Thursday, **February 08, 2024**, between **10:00 and 12:00**.

1. **Comprehension Question**.
   (i) What are main differences between stellar dynamics and hydrodynamics, despite the similarity between the Jeans and the Euler equations?
   (ii) Summarise the assumptions underlying the Jeans equation.
   (iii) If collisions between stars become important, what needs to be changed in the equations of stellar dynamics?

2. **Gas sphere: Radial kinetic pressure and velocity dispersion**. Consider the radial number density profile for particles of equal mass m

$$n(r) = \begin{cases} \frac{n_0 R^2}{r^2} & \text{for } 0 \leq r \leq R \\ 0 & \text{else} \end{cases}$$

   and the anisotropy parameter $\beta(x) = \beta_0$ =const.
   (a) Fix n_0 by the total mass M_0 and calculate the potential gradient $\partial_r \Phi$.
   (b) Derive the radial kinetic pressure $n\sigma_r^2$ in the system.

3. **Gas sphere: Line-of-sight velocity dispersion**. We consider the **same** situation as in problem 2. Here, we want to calculate the line-of-sight velocity dispersion $\sigma_\parallel$.
   (a) Verify that

$$\int \frac{\mathrm{d}y}{y\sqrt{y^2-1}} = \arccos\frac{1}{y},$$
$$\int \frac{\mathrm{d}y}{y^3\sqrt{y^2-1}} = \frac{\sqrt{y^2-1}}{2y^2} + \frac{1}{2}\arccos\frac{1}{y}.$$

   (b) Calculate the projected density

$$I(s) = 2\int_s^\infty \frac{r\mathrm{d}r}{\sqrt{r^2-s^2}} n(r).$$

   (c) Calculate the line-of-sight velocity dispersion. To ease the calculation, you may use the simplified radial kinetic pressure

$$n\sigma_r^2(r \ll R) = \frac{GM_0^2}{8\pi(1-\beta_0)mR^2r^2}.$$

4. **Hernquist profile**. A convenient model density profile for different kinds of astrophysical objects is the Hernquist profile

$$\rho(x) = \frac{\rho_0}{x(1+x)^3}$$

proposed by L. Hernquist (1990). The dimension-less radius x is defined as $x = r/a$, with a scale or core radius a.

   (a) Write the density amplitude ρ_0 in terms of the total mass M contained in the profile.

   (b) Derive the Newtonian potential $\Phi(x)$ of objects with the Hernquist density profile.

   (c) Assuming that the number-density $n(x)$ of the stars in a Hernquist-like object follows the matter-density profile, and assuming that the anisotropy parameter $\beta = 1/2$ is independent of the radius, obtain the radial kinetic pressure $n\sigma_r^2$.

5. **Chandrasekhar's tensor (classroom assignment)**. For the Hernquist profile in Problem 4, calculate Chandrasekhar's tensor ${U^i}_j$ of the potential energy.