

# Cosmic Structure Formation

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Heidelberg University  
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## Problem Sheet 11

Discussion in the tutorial group on Feb. 1st, 2023

1. **Conditional probability in a Gaussian random field.** Consider a statistically homogeneous and isotropic Gaussian random density field. We wish to derive the probability distribution  $P(\vec{q}_1, \vec{q}_2)$  for find a particle at position  $\vec{q}_1$  and another at position  $\vec{q}_2$ .
  - (a) Set up the characteristic function for this problem, including the components of the covariance matrix.
  - (b) Write down an equation for the distribution  $P(\vec{q}_1, \vec{q}_2)$  by introducing suitable conditional probabilities.
  - (c) Carry out the remaining operations.
2. **Mean force difference in a Gaussian random field.** Consider once more a statistically homogeneous and isotropic Gaussian random density field characterized by a power spectrum  $P_\delta(k)$ .
  - (a) Derive a probability distribution  $P(\vec{q}_1, \vec{q}_2, \delta_3)$  for finding a pair of particles at positions  $\vec{q}_1$  and  $\vec{q}_2$  and a density contrast  $\delta_3$  at a third position.
  - (b) Average the force difference  $\vec{f}_1 - \vec{f}_2$  over  $\delta_3$  first, and integrate over all positions  $\vec{y}$  of  $\delta_3$ .
  - (c) Using the previous result, find an expression for the mean force difference  $\langle \vec{f}_1 - \vec{f}_2 \rangle$ .