Cosmic Structure Formation

Heidelberg University
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Problem Sheet 7

Discussion in the tutorial group on Dec. 15th, 2022

1. **Density-momentum correlation function**. For the density-momentum correlation function,

$$C_{\delta p}(r) = \mathrm{i}\left\langle \int_k \int_{k'} k^2 k' \tilde{\psi}\tilde{\psi}' \mathrm{e}^{\mathrm{i}k\cdot q + \mathrm{i}k'\cdot(q+r)} \right\rangle$$
$$= -\mathrm{i} \int_k k^2 k P_\psi(k)\, \mathrm{e}^{-\mathrm{i}k\cdot r}\ ,$$

show that

(a) the correlation function vanishes at zero lag, i.e. $C_{\delta p}(0) = 0$ and

(b) the correlation function between any particle pair (i, j) changes sign if the particles are interchanged,

$$C_{\delta_i p_j} = -C_{\delta_j p_i}\ .$$

2. **Power spectrum with density-density and density-momentum correlations**. Consider now the free generating functional $Z_0[\mathbf{L}]$ after applying two density operators, and begin with expression (5.16) from the lecture notes.

(a) Convince yourself that

$$\mathbf{s}^\top C_{\delta\delta}\mathbf{s} = C_{\delta_i\delta_j} s_i s_j\ , \quad \mathbf{s}^\top C_{\delta p}\mathbf{L}_p = C_{\delta_i p_j} s_i L_{p_j}\ .$$

(b) Use these expressions to show that

$$Z_0[\mathbf{L}] = \int \mathrm{d}\mathbf{r}\, [1 + F(k_1, q, t_1)] \exp\left(-\frac{1}{2}\mathbf{L}_p^\top C_{pp}\mathbf{L}_p + \mathrm{i}\mathbf{L}_q \cdot \mathbf{r}\right)$$

with

$$F(k_1, q, t_1) = C_{\delta_1\delta_2} + 2\mathrm{i}C_{\delta_1 p_2} k_1 t_1 - \left(C_{\delta_1 p_2} k_1 t_1\right)^2\ .$$