Cosmic Structure Formation

Heidelberg University
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Problem Sheet 6

Discussion in the tutorial group on Dec. 8th, 2022

1. **Derivatives of correlation functions**. Given the correlation functions $a_{1,2}(q)$ defined in the lecture and the correlation function $\xi_\psi(q)$ of the velocity potential, show that

   (a) the first derivative of $\xi_\psi(q)$ is
   $$\xi'_\psi(q) = q a_1(q)$$

   (b) and its second derivative is
   $$\xi''_\psi(q) = a_1(q) + a_2(q) \; .$$

2. **Linearised power spectrum**. With these results, and defining

   $$I_n^\alpha(k) = 2\pi \int_0^\infty \mathrm{d}q\, q^n \xi_\psi^{(n)}(q) \int_{-1}^1 \mathrm{d}\mu\, \mu^\alpha \mathrm{e}^{\mathrm{i}kq\mu} \; ,$$

   (a) confirm that

   $$I_1^0(k) = 4\pi \int_0^\infty \mathrm{d}q\, \xi_\psi(q) \left[kq\, j_1(kq) - j_0(kq)\right] \; ,$$
   $$I_1^2(k) = 4\pi \int_0^\infty \mathrm{d}q\, \xi_\psi(q) \left[j_0(kq) + kq\left(1 - \frac{4}{k^2q^2}\right) j_1(kq)\right] \; ,$$
   $$I_2^2(k) = 4\pi \int_0^\infty \mathrm{d}q\, \xi_\psi(q) \left[\left(2 - k^2q^2\right) j_0(kq) - \frac{4}{kq} j_1(kq)\right] \; ,$$

   (b) show that the equation

   $$\int \mathrm{d}^3q \, \left(a_1(q) + \mu^2 a_2(q)\right) \mathrm{e}^{\mathrm{i}kq\mu} = I_1^0(k) + I_2^2(k) - I_1^2(k)$$

   holds and

   (c) use it to derive that the power spectrum

   $$\mathcal{P}(k) = \int_q \left[\mathrm{e}^{t^2k^2[a_1(q)+\mu^2 a_2(q)]} - 1\right] \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{q}}$$

   approximately equals
   $$\mathcal{P}(k) \approx t^2 P_\delta(k)$$

   at linear order.