Cosmic Structure Formation

Heidelberg University
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Problem Sheet 5

Discussion in the tutorial group on Dec. 1st, 2022

1. **Collapse time in Zel'dovich approximation**. We have seen in the lecture that cosmic particle trajectories can be well described by the Zel'dovich approximation

$$\vec{q}(t) = \vec{q}^{\,(\mathrm{i})} + t\vec{p}^{\,(\mathrm{i})} \;,$$

that the local deformation of the matter flow is given by the Jacobian matrix

$$J = \mathbb{1}_3 + t\Lambda \;, \quad \Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3) \;,$$

and that the probability distribution for the eigenvalues is proportional to

$$|(\lambda_1 - \lambda_2)\,(\lambda_1 - \lambda_3)\,(\lambda_2 - \lambda_3)| \;. \tag{9}$$

   (a) Set up the condition for the earliest and the latest time of matter collapse along a given particle trajectory.

   (b) Derive an equation for the probability of matter collapse at the latest by the time t_c.

   (c) What would be the probability for matter collapse to occur within $\mathrm{d}t$ of t?

2. **Density and momentum correlations**. For the correlation functions $C_{\delta\delta}(r)$, $C_{\delta p}(r)$, and $C_{pp}(r)$ derived in the lecture and given in the lecture notes,

   (a) derive the asymptotic behaviour for small distances r. For doing so, use the series expansion of the respective spherical Bessel functions. Express the results in terms of the moments

$$\sigma_n^2 = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\, k^{2n-2} P_\delta(k) \;.$$

   (b) For the three-dimensional power spectrum derived in Problem Sheet 4, calculate the moments σ_n^2 appearing in (a). Can you derive expressions for $C_{\delta p}$ and C_{pp}?

   (c) For this power spectrum, what do you expect for $C_{\delta\delta}$? Can you confirm this expectation?