Cosmic Structure Formation

Heidelberg University
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Problem Sheet 1

Discussion in the tutorial group on Nov. 3rd, 2022

1. **Friedmann's first equation**

   Imagine a sphere filled with a gas of homogeneous density ρ, and a test mass m on its surface.

   a) Set up the Newtonian equation of motion for the test mass, extending the gravitational potential with the cosmological constant as appropriate.

   b) Integrate this equation of motion once and find the Newtonian counterpart of the spatial curvature K.

   c) Discuss and explain why this isolated sphere is an appropriate representative of a spatial section of a spatially homogeneous and isotropic universe.

2. **Derivation of the growth index**

   We have defined the logarithmic derivative

   $$f = \frac{\mathrm{d}\ln D_+}{\mathrm{d}\ln a} \tag{I}$$

   of the growth factor in the lecture.

   a) Use the linear growth equation to show that

   $$\frac{\mathrm{d}f}{\mathrm{d}\ln a} + \frac{1}{2}(1+\varepsilon)f + f^2 = \frac{3}{2}(1-\omega)\ , \tag{II}$$

   where

   $$\varepsilon = 3 + 2\frac{\mathrm{d}\ln E}{\mathrm{d}\ln a}\ , \quad \omega = 1 - \Omega_\mathrm{m}(a)\ . \tag{III}$$

   b) Transform $\mathrm{d}\ln f/\mathrm{d}\ln a$ to a derivative with respect to Ω_m, apply the ansatz

   $$f = \Omega_\mathrm{m}^\gamma(a) \tag{IV}$$

   and linearize in the small parameters ε and ω to find

   $$\frac{\mathrm{d}f}{\mathrm{d}\ln a} = -\varepsilon\gamma f\ . \tag{V}$$

   c) Insert this result into (II), solve for γ and show that

   $$\gamma = \frac{\varepsilon + 3\omega}{2\varepsilon + 5\omega}\ . \tag{VI}$$

   d) Evaluate this expression for the ΛCDM model and find

   $$\gamma = \frac{6}{11} \tag{VII}$$

   there.