Problem Set 1

Exercises for course Fundamentals of Simulation Methods, WS 2021
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Hand in until Wednesday, 27.10.2021, 23:59
Tutorials times: 28.10.2021 - 29.10.2021
Group 1: Brooke | Thursday 11:00 - 13:00
Group 2: Glen | Thursday 14:00 - 16:00
Group 3, Jan | Friday 11:00 - 13:00

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1. Packing of numbers [2 pt]

Estimate how many numbers there are in the interval between 1.0 and 2.0, and in between the interval of 255.0 to 256.0, for IEEE-754

1. single precision
2. double precision

2. Pitfalls of integer and floating point arithmetic [6 pt]

1. Consider the following C/C++ code:

   ```
   int      i =  7;
   float    y = 2*(i/2);
   float    z = 2*(i/2.);
   printf("%e  %e \n", y,z);
   ```

   which prints out two float numbers. Explain why the numbers are not all equal.

2. Consider the following numbers:

   ```
   double  a =  1.0e17;
   double  b = -1.0e17;
   double  c = 1.0;
   double  x = (a +  b) + c;
   double  y =  a + (b  + c);
   ```

   Calculate the results for x and y. Which one is correct, if any? Explain, why the law of associativity is here broken.

3. Consider the following C/C++ code:

```
float   x =  1e20;
float   y;
y = x*x;
printf("%e %e\n", x,y/x);
```

   Explain what you see.

3. Machine epsilon [5 points]

Write a computer program in C, C++, or Python that experimentally determines the machine epsilon ϵ_m, i.e. the smallest number ϵ_m such that $1 + \epsilon_m$ still evaluates to something different from 1, for the following data types:

1. float
2. double
3. long double

Evaluate and print out $1 + \epsilon_m$. Do you see something strange?

4. Near-cancellation of numbers [7 pt]

Consider the following function:

$$f(x) = \frac{x + \exp(-x) - 1}{x^2} \tag{1}$$

Clearly for $x = 0$ this function is ill determined. However, for the limit $x \downarrow 0$ the function goes to a non-zero and non-infinite value.

1. Determine $\lim_{x\downarrow 0} f(x)$
2. Write a computer program that asks for a value of x from the user and then prints $f(x)$
3. For small (but positive non-zero) values of x this evaluation goes wrong. Determine experimentally at which values of x the formula goes wrong.
4. Explain why this happens.
5. Add an if-clause to the program such that for small values the function is evaluated in another way that does not break down, so that for all positive values of x the program produces a reasonable result.

Problem set solutions must be submitted to the course webpage. To pass the course, you are required to get at least half of the total amount of points given for the problems throughout the entire course. We expect everyone to hand in their own solution, even if it is identical to the solution of your teammate (max. 2 persons together; please specify your team partner, who should be in the same tutorial group).