Standard Model of Particle Physics

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Problem Sheet 01

to be handed in on: Monday 26.04.2020 and discussed on: 27/28.04.2020

Problem 1. Natural Units

This exercise is to recall the use(fulness) of natural units. Recall that in particle physics we set $\hbar = 1$ and $c = 1$. Use this to solve the following questions:

a) Find the connection of energy (MeV) with time (s), length (m), and mass (g).
Express a cross-section of 1 millibarn in units of GeV^{-1}.
Hint: $c = 299792458\,\mathrm{m/s}$, $\hbar = 6.582119 \times 10^{-22}\,\mathrm{MeV}\cdot\mathrm{s}$, $1\,\mathrm{eV} = 1.602176 \times 10^{-19}\,\mathrm{J}$ *and* $1\,\mathrm{barn} = 10^{-24}\,\mathrm{cm}^2$

b) One often also sets the Boltzmann constant to one: $k_B = 1$. In this case, find the connection between temperature (K) and energy (MeV) and between temperature (K) and time (s).
Hint: $k_B = 1.380649 \cdot 10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}$.

c) The Planck mass M_{Pl} is defined as $G = 1/M_{Pl}^2$ where G is Newton's constant. Starting from M_{Pl} construct one unit of time and one unit of length (these are called the Planck time and length respectively). Evaluate them in SI units (you will have to restore $\hbar$ and c for this). Use the Planck mass, time and length to construct one unit of speed and one unit of action. Evaluate them in SI units and comment.
Hint: $G = 6.67430 \times 10^{-11}\,\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$

Problem 2. $2 \to 2$ scattering

We consider a scattering experiment where we have 2 incoming and 2 outgoing particles ($1 + 2 \to 3 + 4$) of masses m_1, m_2, m_3 and m_4 respectively. In the laboratory frame the incoming particles have 4-momentum $P_1^\mu = (E_1, 0, 0, p_1)$ and $P_2^\mu = (E_2, 0, 0, -p_2)$.

a) Calculate, in the center-of-mass system, the energies and momenta of the individual particles, and give their asymptotic behaviour ($s \gg m_i^2$). *Remember:* $s = (P_1^\mu + P_2^\mu)^2$

b) Show that the scattering angle θ^* in the center-of-mass (COM) frame is given by

$$\cos\theta^* = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2)}}\,. \tag{1}$$

Remember: $s = (P_1^\mu + P_2^\mu)^2$, $t = (P_3^\mu - P_1^\mu)^2$ *and* $u = (P_4^\mu - P_1^\mu)^2$. *Finally* $\lambda(a,b,c) = a^2 - 2a(b+c) + (b-c)^2$ *and* θ^* *is the angle between particles 1 and 3* ($\vec{p_1}\cdot\vec{p_3} = |\vec{p_1}||\vec{p_3}|cos\theta^*$).

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c) Calculate the energies and momenta of the individual particles in the lab frame (as opposed to the COM frame).

d) Calculate the scattering angle θ in the lab frame.

Problem 3. Kinematics

a) Show that the decay $e^- \to e^-\gamma$ is kinematically forbidden.

b) In Grand Unified Theories the proton can decay into e^+ and π^0 ($p^+ \to e^+ + \pi^0$). Show that this decay is kinematically allowed.

c) Use a simple argument to show that electron-positron pair production from photon decay ($\gamma \to e^+e^-$) is forbidden as well.

d) Electron-positron pair production is one of the dominant modes of interaction of an incoming photon with matter. How is this reconcilable with the previous result?

e) A photon γ ($p^2 = 0$) with 4-momentum $p^\mu = (E, E, 0, 0)$ scatters with an electron at rest. After the process the photon has 4-momentum $p^\mu = (E', E' cos\theta, E' sin\theta, 0)$. Show that

$$E' = \frac{E}{1 + \frac{E}{m_e}(1 - \cos\theta)} \,. \tag{2}$$

Problem 4. 3-Body Phase Space

Consider an unpolarized 3-body decay $p \to p_1 + p_2 + p_3$, e.g. β-decay of a muon: $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. Show that when $m_\mu \gg m_{e,\nu}$ the phase space in the rest frame of the decaying particle is:

$$\int d^4p_1 d^4p_2 d^4p_3 \delta(p_1^2 - m_1^2)\delta(p_2^2 - m_2^2)\delta(p_3^2 - m_3^2)\Theta(E_1)\Theta(E_2)\Theta(E_3)\delta(p_1 + p_2 + p_3 - p)$$
$$= \pi^2 \int_0^{m_\mu/2} dE_1 \int_{m_\mu/2 - E_1}^{m_\mu/2} dE_3 \tag{3}$$

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