Institute for Theoretical Physics — Heidelberg University
Prof. Dr. Ulrich Schwarz — Mike Brandt
Stochastic Dynamics — Winter term 2025/2026

Assignment 0

Handout 20/10 – Return 20/10 – Discussion 20/10

Exercise 0.1 [0 points]: The Geometric Distribution

Consider a probabilistic experiment with two outcomes, e.g. a coin where we assume the probabilities

$$P(\text{head}) = p, \qquad P(\text{tail}) = q = 1 - p.$$

1. What is the probability $P(n)$ that the event 'head' occurs for the first time *after* $(n+1)$ trials?
2. Write down the probability space $(\Omega, \mathcal{F}, P)$ for this experiment explicitly. Which kind of mathematical object is n?
3. The GENERATING FUNCTION defined by

$$f(z) := \sum_{n=0}^{\infty} P(n)\, z^n, \qquad z \in \mathbb{C}.$$

   is a practical tool for characterizing a discrete probability distribution P. In order to see this, find an analytical expression for the generating function for the geometric distribution. Evaluate $f(z)$, $f'(z)$ and $f''(z)$ at $z = 1$. Find the variance too.
4. MEMORYLESSNESS: Proof that the relation

$$P(n \geq k + \ell \mid n \geq k) = P(n \geq \ell) \qquad n, k, \ell \in \mathbb{N}$$

   is true for the geometric distribution. Describe this phenomenon in words.
   Fact: The geometric distribution is the one and only one discrete distribution with this property.

Exercise 0.2 [0 points]: Photon Gases

The geometric distribution and Poisson distribution can be used to describe gases of photons with different levels of coherence. The goal of this exercise is to understand the structure of the probability distribution for the number of photons in the gas.

1. THERMAL BLACK-BODY-RADIATION
   - What is a black body and why does is radiate? What is a mode?
   - Why can we assume, that the photons (or the modes) do not interact with each other?

- Why can we assume, that the photons in the gas have no fugacity? What follows for a change of the total number of photons?
- Suppose a new photon is generated. If the probability of climbing into mode ν is p_ν, what is then the probability for the mode, to have $(n+1)$ photons in it?

2. COHERENT LASER BEAM

   - A coherent beam of monochromatic laser light has the quantum-optical state

   $$|\alpha\rangle := \mathrm{e}^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \qquad \alpha \in \mathbb{C},$$

   where $|n\rangle$ are orthonormal eigenstates with n photons in that mode. Calculate the probability that there are m photons in the beam.
   - What is the defining property of those states $|\alpha\rangle$?
   - Which kind of probability distribution is it?
   - What are the two significant features of this distribution that are in concordance with the coherent laser beam?

Now, calculate the number of relative fluctuation i.e. $\frac{\sqrt{\mathrm{Var}(n_\nu)}}{\langle n_\nu \rangle}$ and compare them for the two cases.