Assignment 1

Handout 20/10 – Return 27/10 – Discussion 03/11

Please upload your solutions as a single PDF file to the Übungsgruppensystem at https://uebungen.physik.uni-heidelberg.de. You can hand in your solutions in pairs of two.

Exercise 1.1 [5 points]: General properties of probability spaces

A probability space is defined by a sample space Ω of elementary events, an event space $\mathcal{F}$ (typically the power set of Ω) and a probability measure $p(A)$ for all $A \in \mathcal{F}$. Use the axioms from the lecture to show the following statements:

1. $p(A) \leq p(B)$ for $A \subseteq B$.
2. For any $A_1, A_2, ... \in \mathcal{F}$ holds:
$$p\Big(\bigcup_{i=1}^{\infty} A_i\Big) \leq \sum_{i=1}^{\infty} p(A_i).$$
3. For $A_1, A_2, ...A_n \in \mathcal{F}$ holds:
$$p\Big(\bigcap_{i=1}^{n} A_i\Big) \geq 1 - n + \sum_{i=1}^{n} p(A_i).$$

Exercise 1.2 [6 points]: Card games

Consider a well-shuffled deck of n cards from which you draw $k \leq n$ at random at the beginning of a game.

1. Show that the number of possible starting hands is given by:
$$\binom{n}{k} = \frac{n!}{(n-k)!k!}.$$
2. Characterize the probability space $(\Omega, \mathcal{F}, p)$ corresponding to this random experiment.

Now consider a standard deck of cards (7-10, J, Q, K, A in four colors) from which you draw 4.

3. What is the probability that all of them are of the same color?
4. What is the probability to draw at least one ace?

Exercise 1.3 [4 points]: Probability weights

1. Consider $\Omega = \{x_1, x_2, x_3, ...\}$ and $w_1, w_2, ... \geq 0$ with $Z := \sum_i w_i > 0$. For $A \subseteq \Omega$, we define:
$$p(A) = \frac{1}{Z} \sum_{i=1}^{\infty} w_i \delta_{x_i}(A),$$
where:
$$\delta_{x_i}(A) = \begin{cases} 1 \text{ for } x_i \in A \\ 0 \text{ else} \end{cases} .$$
Show that p is a probability measure on Ω, i.e. satisfies the axioms A4-A6.

2. Consider a box whose volume is divided into n non-overlapping volumes $V_1, ...V_n$. You know that there is a particle inside the box but you do not know in which part of it. How can you characterize the random experiment of opening the box and finding the particle in one of its volumes?