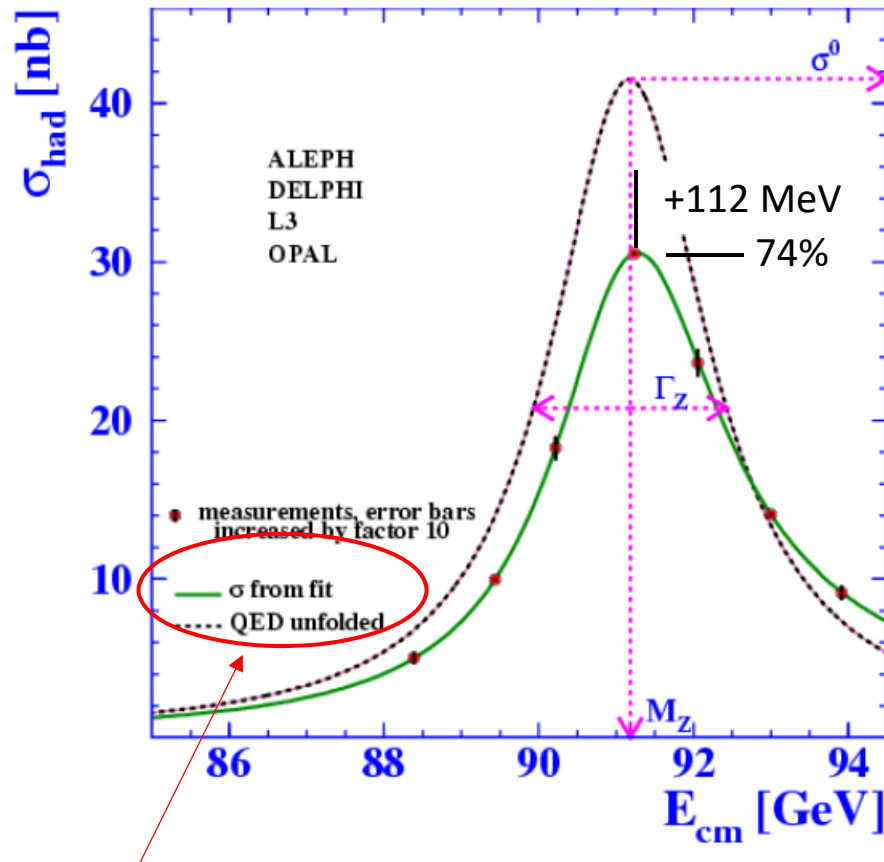


# Measurement of Z-lineshape

Strongly influenced by Bremsstrahlung



$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

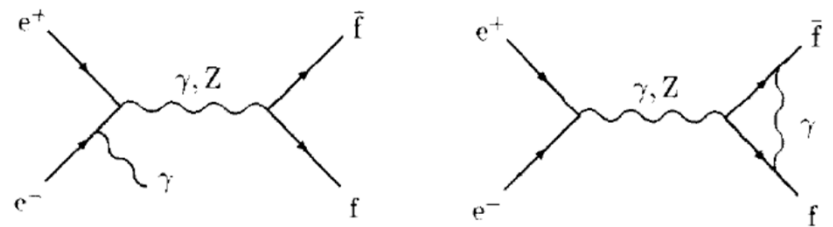
Peak:  $\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$

- Resonance position  $\rightarrow M_Z$
- Height  $\rightarrow \Gamma_e \Gamma_\mu$
- Width  $\rightarrow \Gamma_Z$

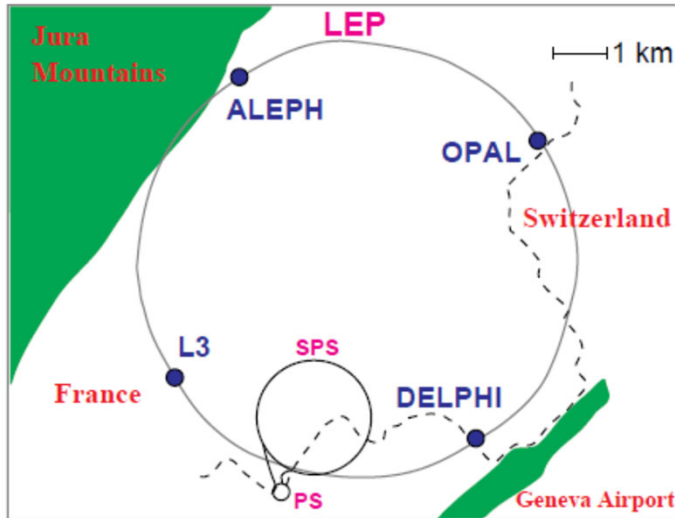
Bremsstrahlung corrections

$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^1 G(z,s) \sigma_{ff}^0(zs) dz \quad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

Radiator function  $G(z,s)$



# Large Electron Positron Collider (LEP)

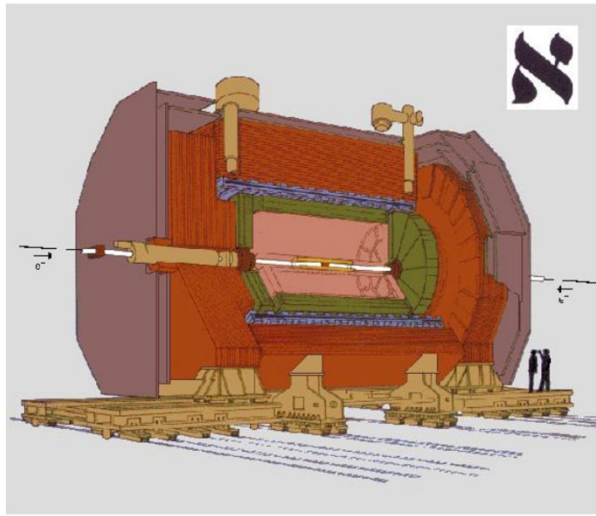


Circumference	~27 km
Centre-of-mass energy	92.1 GeV(LEP1) to 209 GeV(LEP 2)
Accelerating gradient	Up to 7 MV/m (SC cavities)
Number of bunches	4 x 4
Current per bunch	~ 750 $\mu$ A
Luminosity (at Z0)	~ $24 \times 10^{30} \text{cm}^{-2} \text{s}^{-1}$ (~1 Z0/sec)
Luminosity (at LEP2)	~ $50 \times 10^{30} \text{cm}^{-2} \text{s}^{-1}$ (3 WW/hour)
Interaction regions	4 (ALEPH,DELPHI,L3,OPAL)
Energy calibration	< 1 MeV (at Z0)

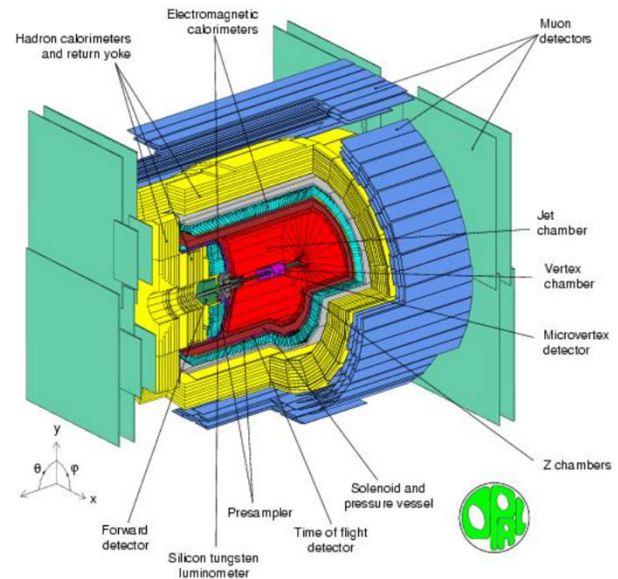
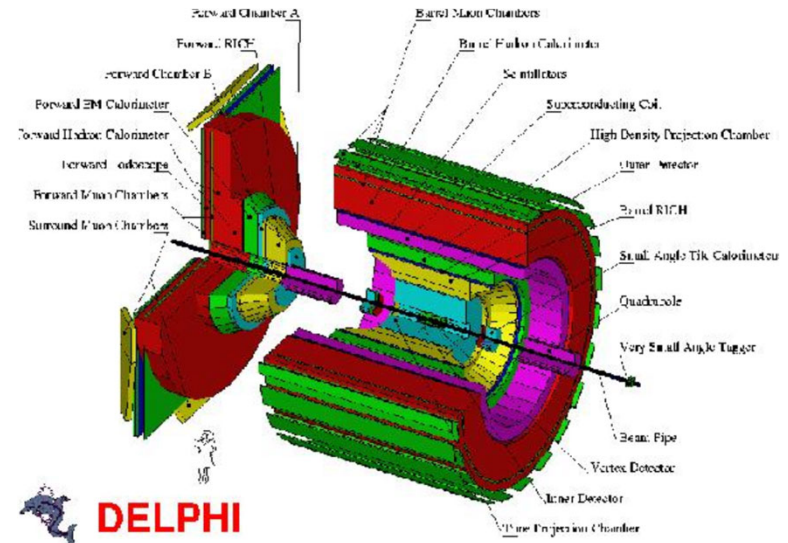
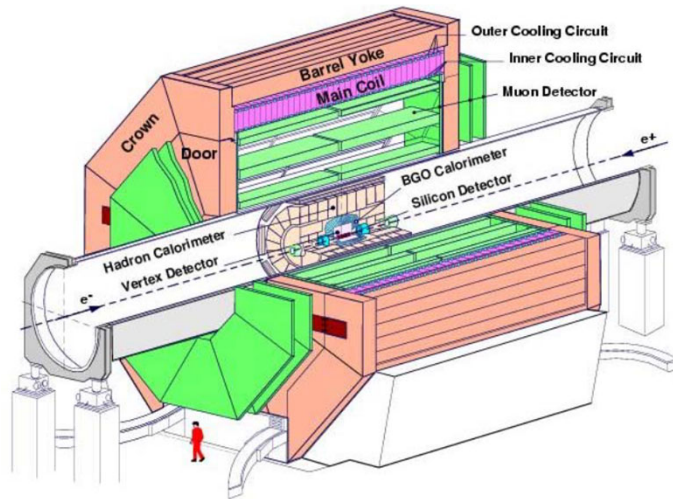
Number of Events										
Year	$Z \rightarrow q\bar{q}$					$Z \rightarrow \ell^+\ell^-$				
	A	D	L	O	LEP	A	D	L	O	LEP
1990/91	433	357	416	454	1660	53	36	39	58	186
1992	633	697	678	733	2741	77	70	59	88	294
1993	630	682	646	649	2607	78	75	64	79	296
1994	1640	1310	1359	1601	5910	202	137	127	191	657
1995	735	659	526	659	2579	90	66	54	81	291
Total	4071	3705	3625	4096	15497	500	384	343	497	1724

Events per  
experiment

# LEP Detectors

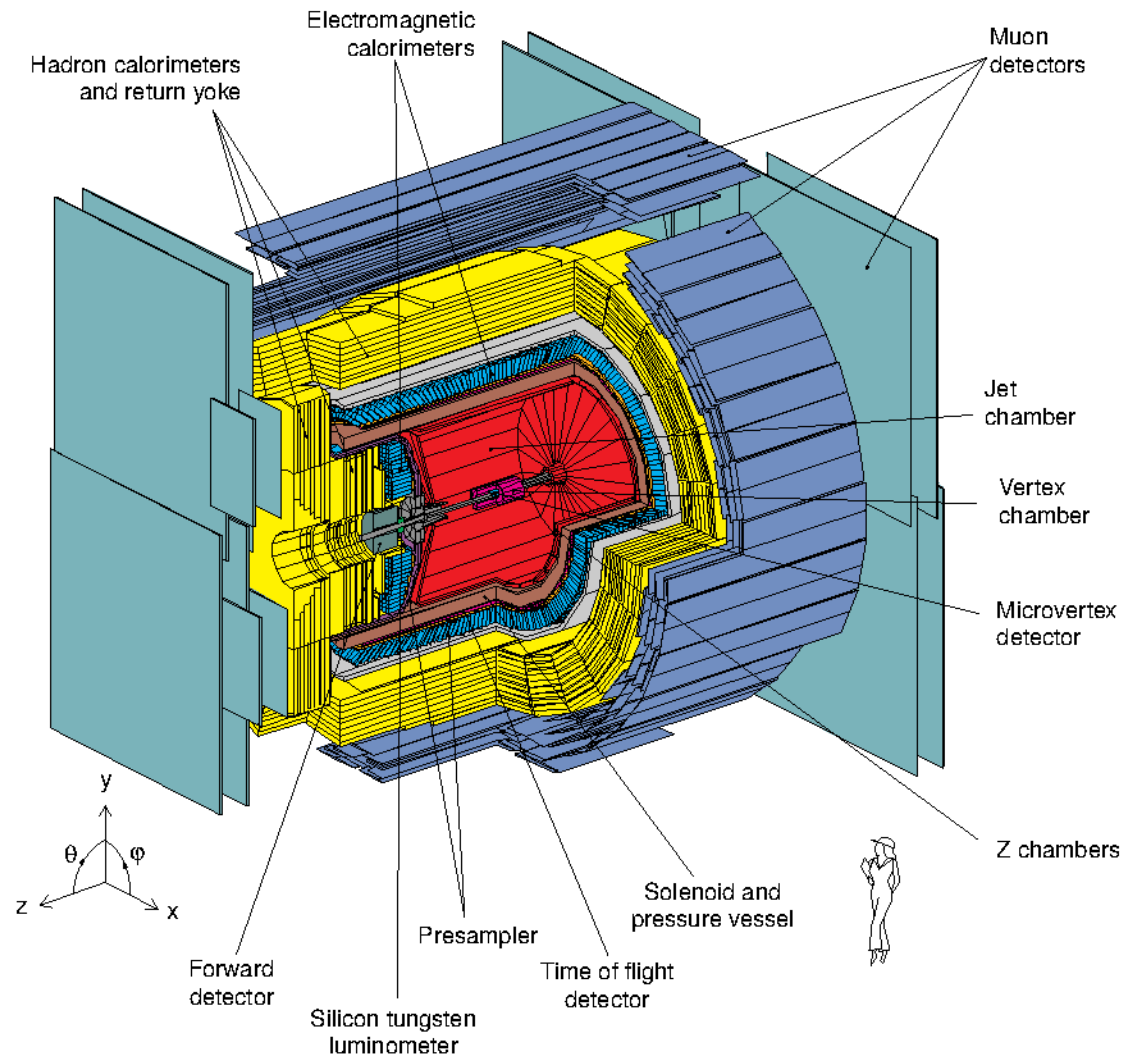
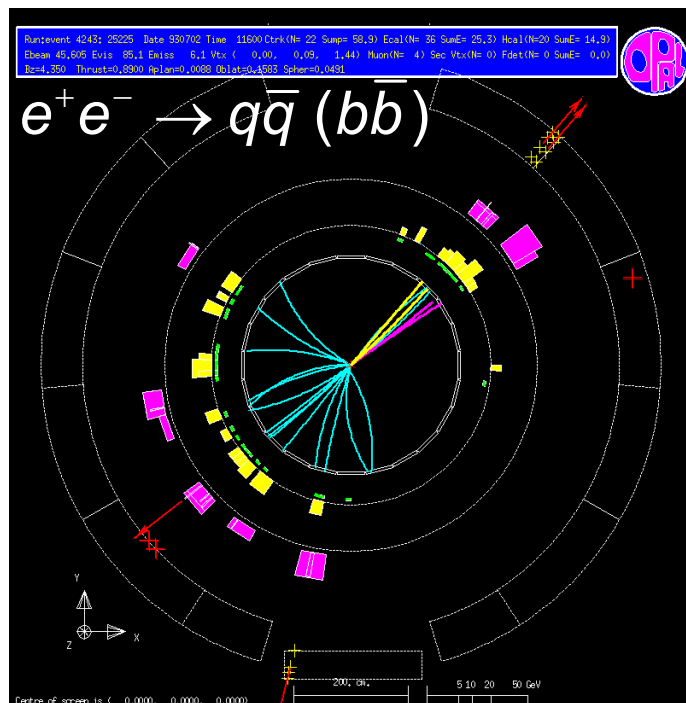


The ALEPH Detector



# Example: OPAL Detector

Heidelberg:  
Jet-Chamber



# Measurement of Cross Section $\sigma(\sqrt{s})$ :

$$\sigma(\sqrt{s}) = \frac{N_{events} (1 - b)}{\varepsilon A \cdot \mathcal{L}_{int}}$$

with

$N_{events}$	=	number of selected events
$b$	=	background fraction in sample
$\varepsilon A$	=	efficiency · acceptance
$\mathcal{L}_{int}$	=	Integrated luminosity

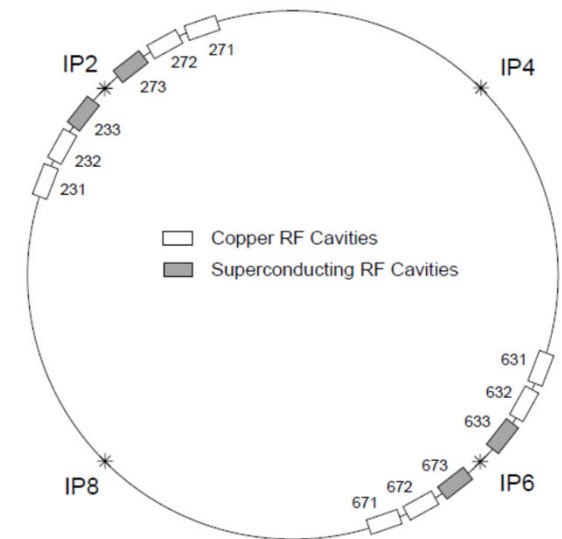
$= 2E_B$

Requires calibration of beam energy and experiment dependent correction (synchrotron loss). Uncertainties in the energy scale translates into an absolute error of  $m_Z$ .

# LEP Beam Energy Calibration

$$E_B = \frac{e c}{2\pi} \oint_s B(s) ds$$

Beam energy calibration requires precise measurements of the average B-field along the LEP ring. In addition interaction point dependent corrections are necessary to account for the energy loss by synchrotron radiation (260 MeV / turn) and the asymmetric position of the RF cavities during LEP-1.



Different measurement to determine B-field of dipole magnets have been used:

- Field display: NMR probes / rotating coil inside a reference magnet powered in series with the LEP dipole magnets. Problem: different position and different environment. Used to extrapolate from periods w/o other measurements
- Flux loop measurements: induction loops in all 8 octants → measure induction voltage when the B field is ramped.
- NMR probes inside the ring dipole magnets (installed only in 1995)

Good reproducibility but no absolute calibration.



## Measurements to calibrate flux-loops / NMR probes:

- Proton calibration: LEP ring was filled with 20 GeV protons.  
→ precise determination of proton velocity → proton momentum → B field  
Method reached absolute accuracy of  $10^{-4}$  at 20 GeV.
- Resonant depolarization (ultimate method, precision better than 1 MeV ( $10^{-5}$ )).  
Method is a “g-2 experiment” where the electron g-2 is known and the average B-field / average electron energy  $E_B$  is determined instead.

Spin-tune:

$$\Delta \nu = \frac{g - 2}{2} \frac{E_B}{m_e c^2}$$

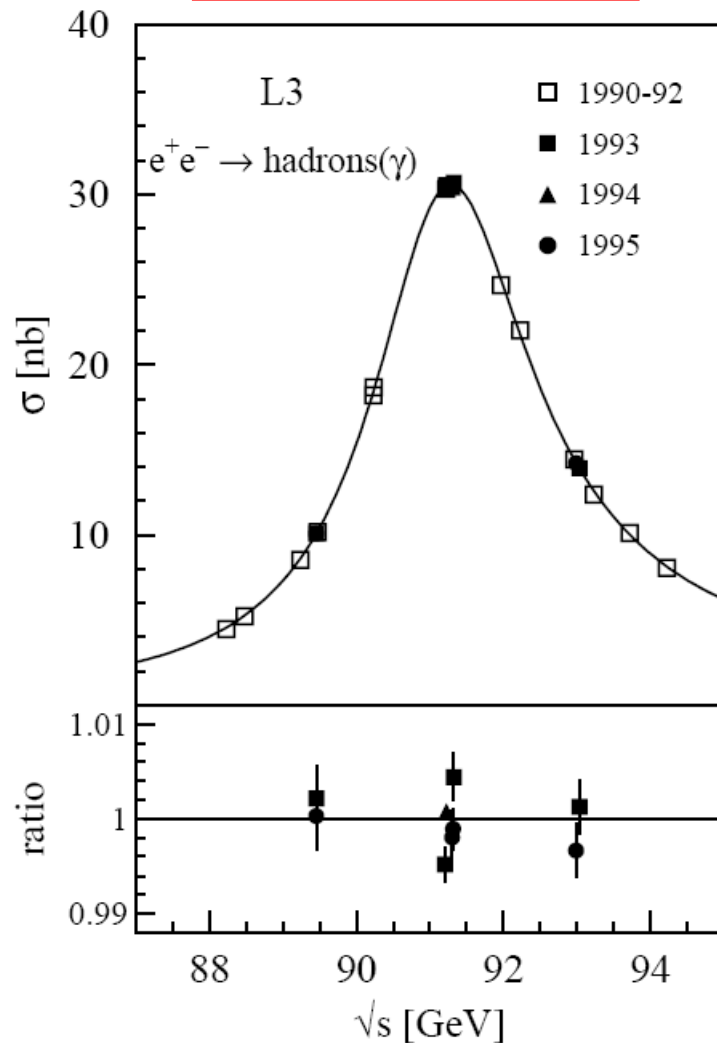
Absolute energy calibration at 17 ppm level has been achieved.

Requires the correction of many unexpected effects:

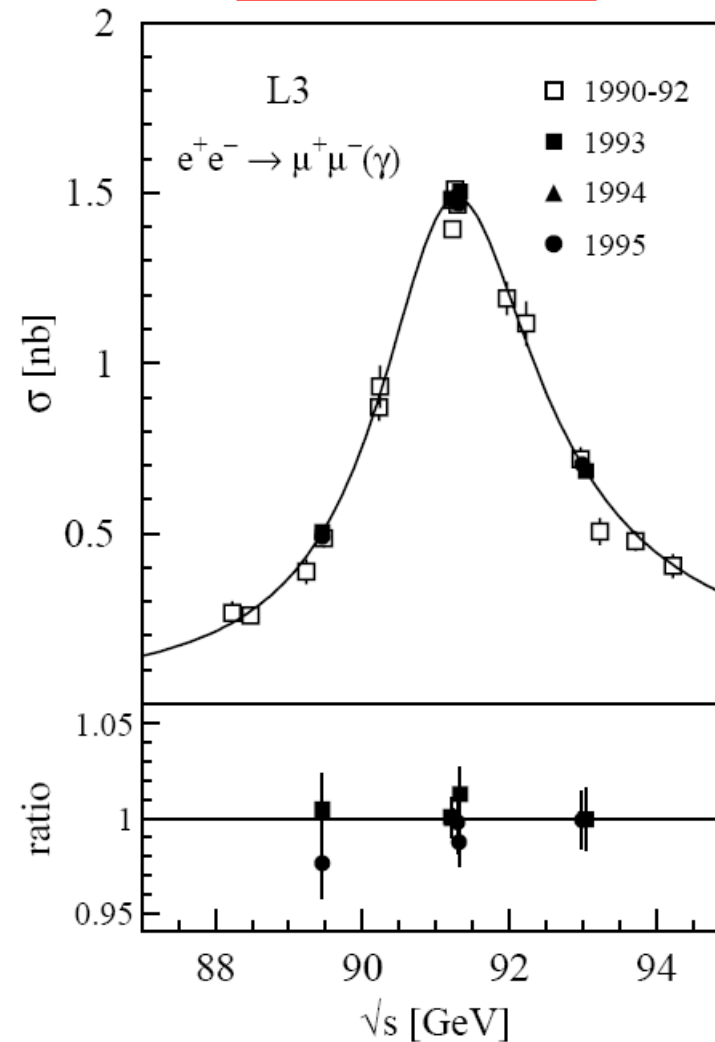
- Tidal effects and ground water level → changes the circumference of the LEP ring (1 – 2mm →  $\pm 5$ MeV )
- French TGV passing in the neighborhood → vagabonding electrical currents ( $\sim 1$ A) produce additional B field which modifies the energy (MeV)

# Cross section measurements $\sigma(\sqrt{s})$

$$e^+ e^- \rightarrow \text{hadrons}$$



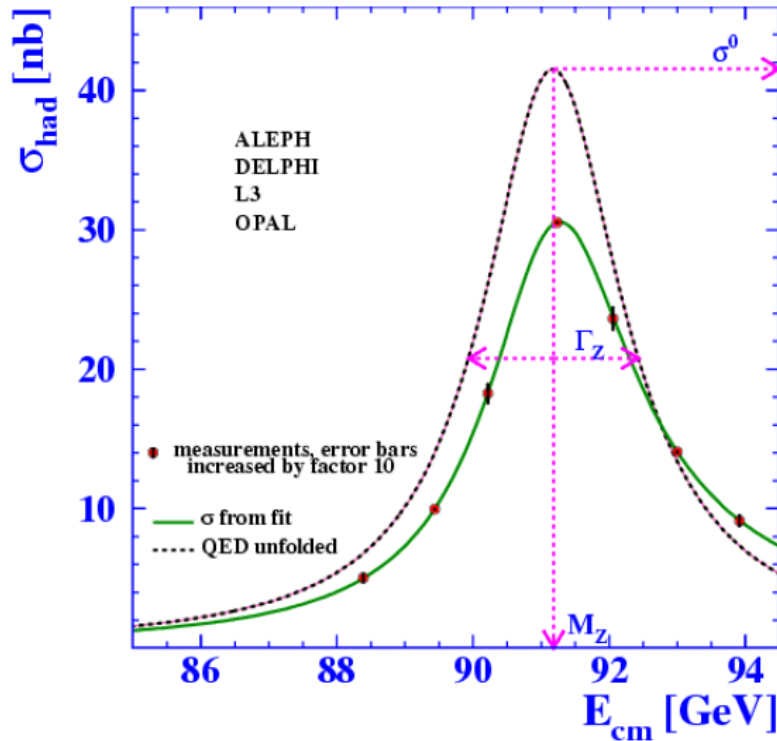
$$e^+ e^- \rightarrow \mu^+ \mu^-$$



Resonance shape is the same, independent of final state: same propagator



## Reminder:



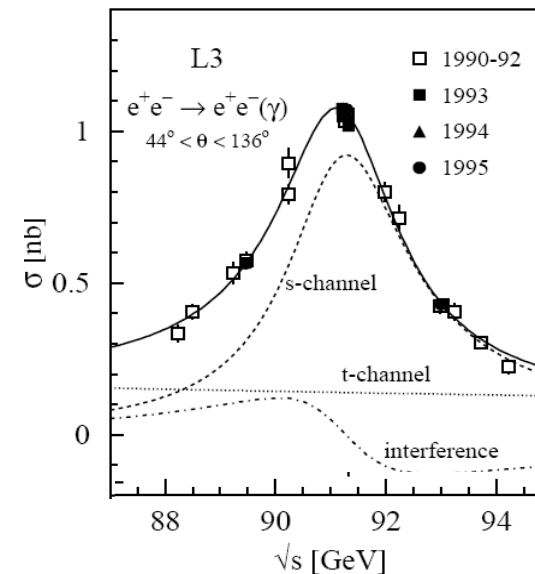
### Measurement of $e^+e^- \rightarrow e^+e^-$ :

t-channel contribution (mostly  $\gamma$ ) needs to be subtracted (see lecture on  $ee$ -annihilation). This is done using QED predictions.

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: 
$$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position  $\rightarrow M_Z$
- Height  $\rightarrow \Gamma_e \Gamma_\mu$
- Width  $\rightarrow \Gamma_Z$



# Z line shape parameters (LEP average)

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV} \pm 23 \text{ ppm} (*)$$

\*) error of the LEP energy determination:  $\pm 1.7 \text{ MeV}$  (19 ppm)

<http://lepewwg.web.cern.ch/>

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV} \pm 0.09 \%$$

$$\Gamma_{\text{had}} = 1.7458 \pm 0.0027 \text{ GeV}$$

$$\Gamma_e = 0.08392 \pm 0.00012 \text{ GeV}$$

$$\Gamma_\mu = 0.08399 \pm 0.00018 \text{ GeV}$$

$$\Gamma_\tau = 0.08408 \pm 0.00022 \text{ GeV}$$

3 leptons are treated independently



test of lepton universality

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\Gamma_{\text{had}} = 1.7444 \pm 0.0022 \text{ GeV}$$

$$\Gamma_e = 0.083985 \pm 0.000086 \text{ GeV}$$

Assuming lepton universality:  $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

(predicted by SM:  $g_A$  and  $g_V$  are the same:)

$$\Gamma_f = \frac{\alpha M_Z}{12 \sin^2 \theta_w \cos^2 \theta_w} \cdot [(g_V^f)^2 + (g_A^f)^2]$$

# Number of light neutrinos

In the Standard Model:

$$\Gamma_Z = \Gamma_{had} + 3 \cdot \Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible: } = \Gamma_{inv}} \rightarrow \left\{ \begin{array}{l} e^+ e^- \rightarrow Z \rightarrow \nu_e \bar{\nu}_e \\ e^+ e^- \rightarrow Z \rightarrow \nu_\mu \bar{\nu}_\mu \\ e^+ e^- \rightarrow Z \rightarrow \nu_\tau \bar{\nu}_\tau \end{array} \right.$$

$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had} - 3 \cdot \Gamma_\ell$$

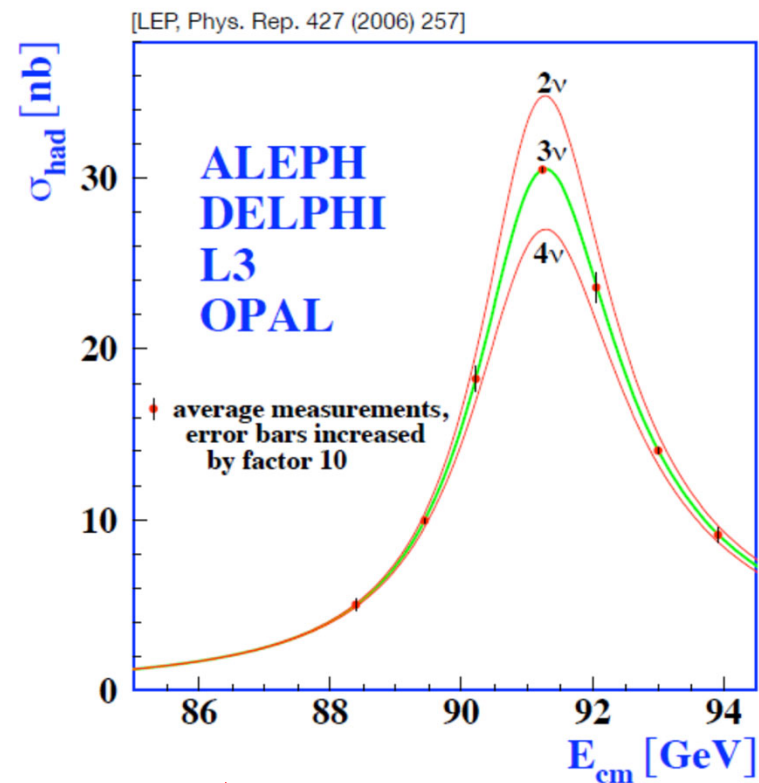
$$\Gamma_{inv} = 0.4990 \pm 0.0015 \text{ GeV}$$

To determine the number of light neutrinos:

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu, SM}} = \underbrace{\left( \frac{\Gamma_{inv}}{\Gamma_\ell} \right)_{\text{exp}}}_{5.9431 \pm 0.0163} \cdot \underbrace{\left( \frac{\Gamma_\ell}{\Gamma_\nu} \right)_{SM}}_{=1/1.991 \pm 0.001}$$

(small theo. uncertainties from  $m_{\text{top}} M_H$ )

$$N_\nu = 2.9840 \pm 0.0082$$



No room for new physics:  $Z \rightarrow$  ~~new invisible particles~~

# Lepton couplings to the Z-boson

*In the following we ignore the difference between chirality and helicity:  
good approximation as leptons are produced with energies  $\gg$  mass.*

Z boson couples differently to LH and RH leptons:

$$\left| g_L = \frac{1}{2}(g_V + g_A) \right| > \left| g_R = \frac{1}{2}(g_V - g_A) \right|$$

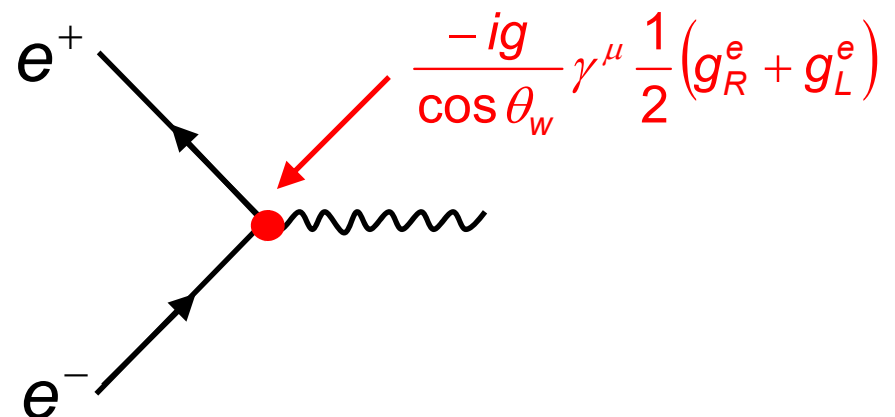
➡ Coupling to LH leptons stronger

Z produced in e+e- collisions is polarized.

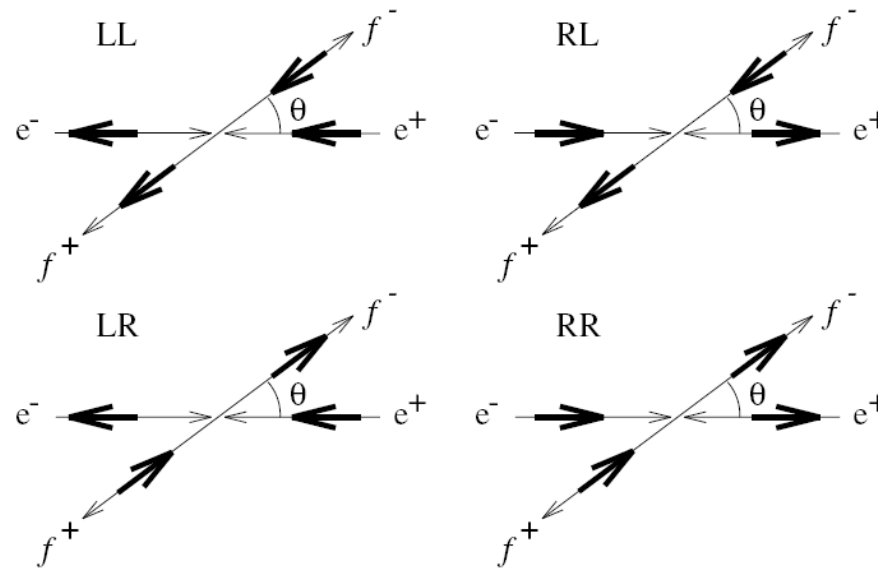
Experimental configuration:

$$e^- \text{ (left)} \quad e^+ \text{ (right)} \Rightarrow g_L$$

$$e^- \text{ (right)} \quad e^+ \text{ (left)} \Rightarrow g_R$$



Instead of measuring the spin averaged transition amplitudes try to decompose the different “helicity” components to the cross section, i.e.:



Vector coupling removes all other combinations.

### Observables:

Will be discussed in the following

$$\sigma_{LL} + \sigma_{RR}$$

$$\sigma_{RL} + \sigma_{LR}$$

Related to  $\sigma_F$  and  $\sigma_B$ .

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Forward-backward asym.

$$\sigma_L = \sigma_{LL} + \sigma_{LR}$$

$$\sigma_R = \sigma_{RL} + \sigma_{RR}$$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Left right asym. (initial state)

$$\sigma_- = \sigma_{LL} + \sigma_{RL}$$

$$\sigma_+ = \sigma_{RR} + \sigma_{LR}$$

$$\mathcal{P}_f = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

fermion polarization (final state) 41

# Forward-backward asymmetry

Angular distribution: (see above)

$$F_{\gamma Z}(\cos \theta) = \frac{Q_e Q_\mu}{4 \sin^2 \theta_W \cos^2 \theta_W} \left[ 2g_V^e g_V^\mu (1 + \cos^2 \theta) + 4g_A^e g_A^\mu \cos \theta \right]$$

$$F_Z(\cos \theta) = \frac{1}{16 \sin^4 \theta_W \cos^4 \theta_W} \left[ (g_V^{e^2} + g_A^{e^2})(g_V^{\mu^2} + g_A^{\mu^2})(1 + \cos^2 \theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos \theta \right]$$

Very small:  $g_V \approx 0$

Forward-backward asymmetry  $A_{FB}$   $\frac{d\sigma}{d\cos \theta} \sim (1 + \cos^2 \theta) + \frac{8}{3} A_{FB} \cos \theta$

- Away from the resonance large  $\rightarrow$  interference term dominates

$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \rightarrow \text{Large, } g_A \text{ is large}$$

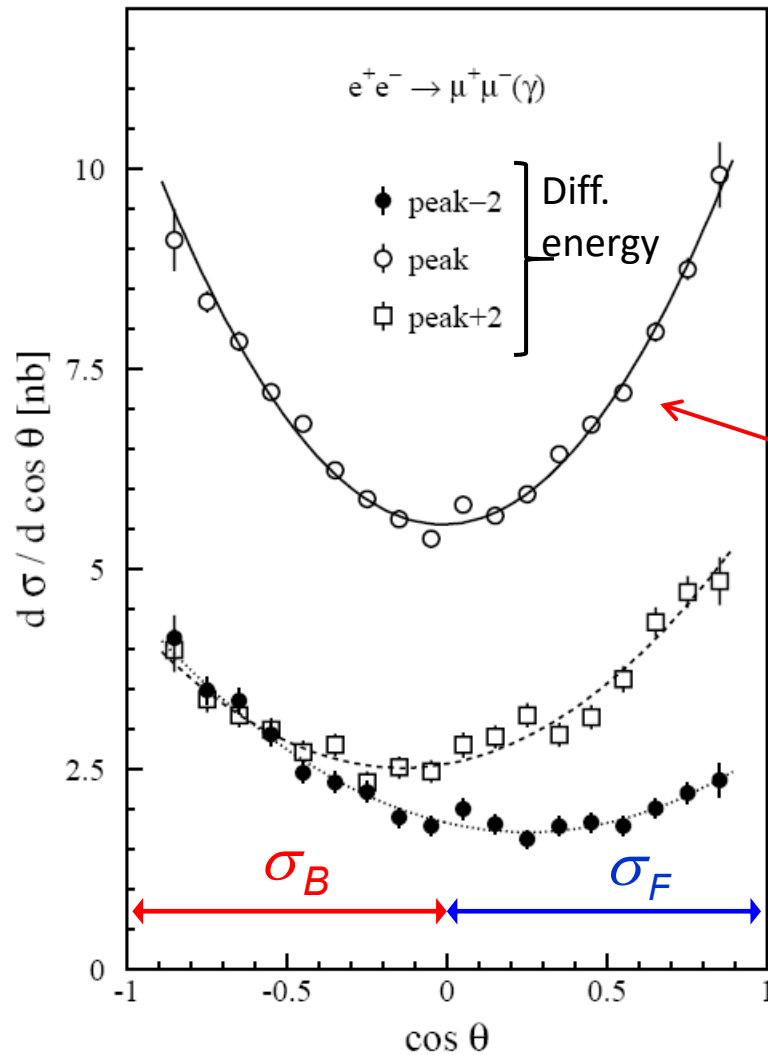
$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

- On the resonance (no interference) – only Z exchange (i.e.  $F_Z$ ) contributes:

$$A_{FB} = 3 \cdot \frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \cdot \frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2}$$

$\rightarrow$  very small because  $g_V^l$  small in SM

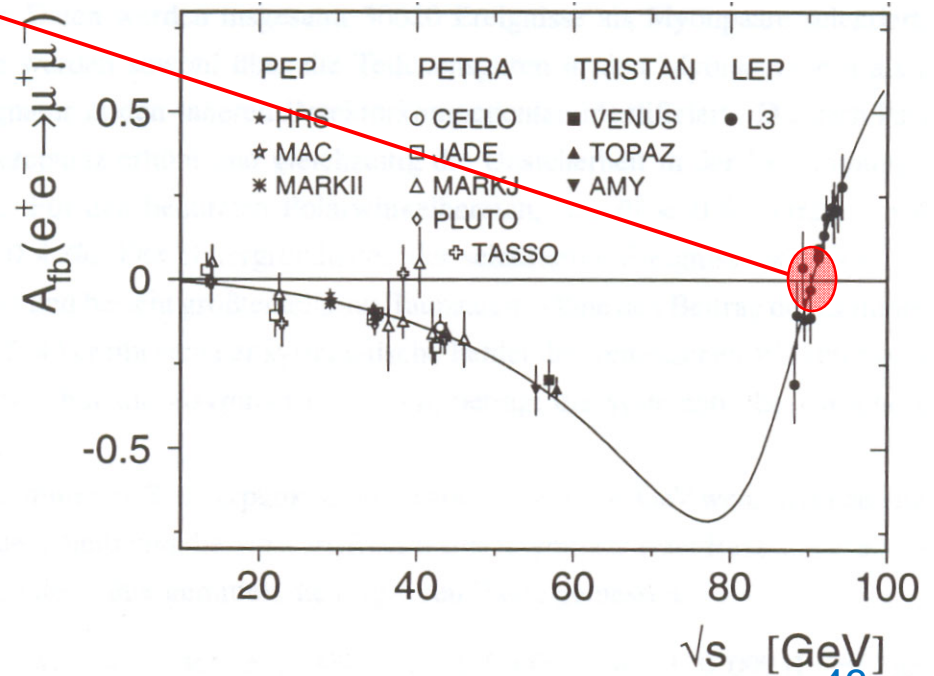
$$e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^-$$



$$\frac{d\sigma}{d \cos \theta} \sim (1 + \cos^2 \theta) + \frac{8}{3} A_{FB} \cos \theta$$

with  $A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$

$$\sigma_{F(B)} = \int_{0(-1)}^{1(0)} \frac{d\sigma}{d \cos \theta} d \cos \theta$$





# Determination of the couplings $g_A$ and $g_V$

## Asymmetrie at the Z pole

$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

## Cross section at the Z pole

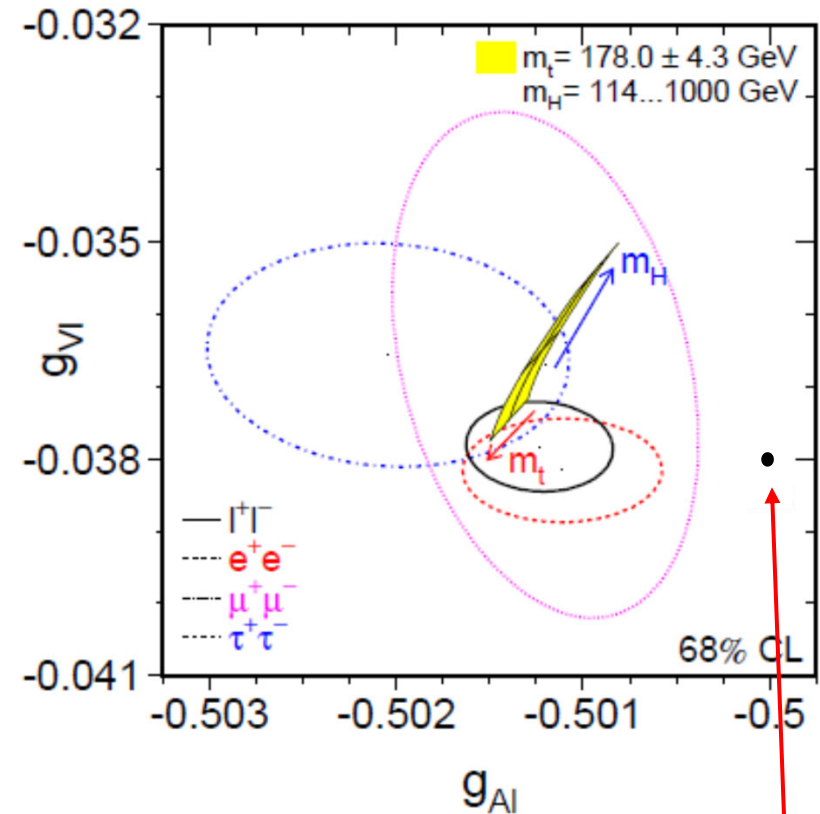
$$\sigma_Z \sim [(g_V^e)^2 + (g_A^e)^2] [(g_V^f)^2 + (g_A^f)^2]$$



Lepton asymmetries together with lepton pair cross sections allow the determination of the lepton couplings  $g_A$  and  $g_V$ . → **elliptical confidence regions**

Good agreement between the 3 lepton species confirms “lepton universality”

Different contour size: electrons are measured in all measurements; tau contour uses additional measurement (polarization)



Tree level SM prediction

Deviation from tree level SM prediction

$$g_V = T_3 - 2q \sin^2 \theta_w \quad g_A = T_3 \quad \sin^2 \theta_w = 1 - \frac{m_W^2}{m_Z^2}$$

is an effect of higher-order loop-corrections.

Assuming lepton universality the measurements result in:

$$g_V^\ell = -0.03783 \pm 0.00041$$

$$g_A^\ell = -0.50123 \pm 0.00026$$

$$g_R^\ell = +0.23170 \pm 0.00025$$

$$g_L^\ell = -0.26959 \pm 0.00024$$



From  $g_V$  one can calculate  $\sin^2 \theta_w$

$$\sin^2 \theta_w^{\text{eff}} = 0.23113 \pm 0.00021$$

(“effective mixing angle” – measurement includes higher order effects in couplings)

As predicted by the SM: Z boson couples stronger to LH leptons than to RH leptons.

Using Z and W boson masses:

$$m_W = 80.3692 \pm 0.0133 \text{ GeV}$$

$$m_Z = 91.1880 \pm 0.0020 \text{ GeV}$$

} PDG

$$\sin^2 \theta_w = 1 - \frac{m_W^2}{m_Z^2} = 0.2232 \pm 0.0003$$

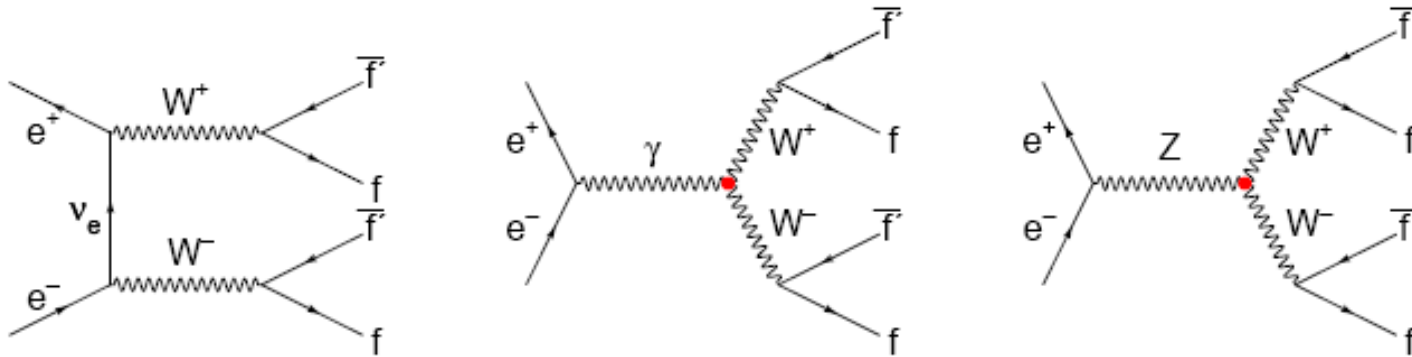
The difference between the two determination of the weak mixing angle is related to higher-order loop corrections which modify the  $\rho$ -parameter and leads to effective couplings and an effective mixing angle (next semester). These loop correction are functions of the top and the [Higgs mass](#).

# W – mass measurement at LEP

# W-pair production at LEP2 ( $\sqrt{s} > 161$ GeV)

At LEP:  $e^+e^- \rightarrow WW \rightarrow f\bar{f}f\bar{f}$

$\sim 10K$  WW events / LEP-experiment



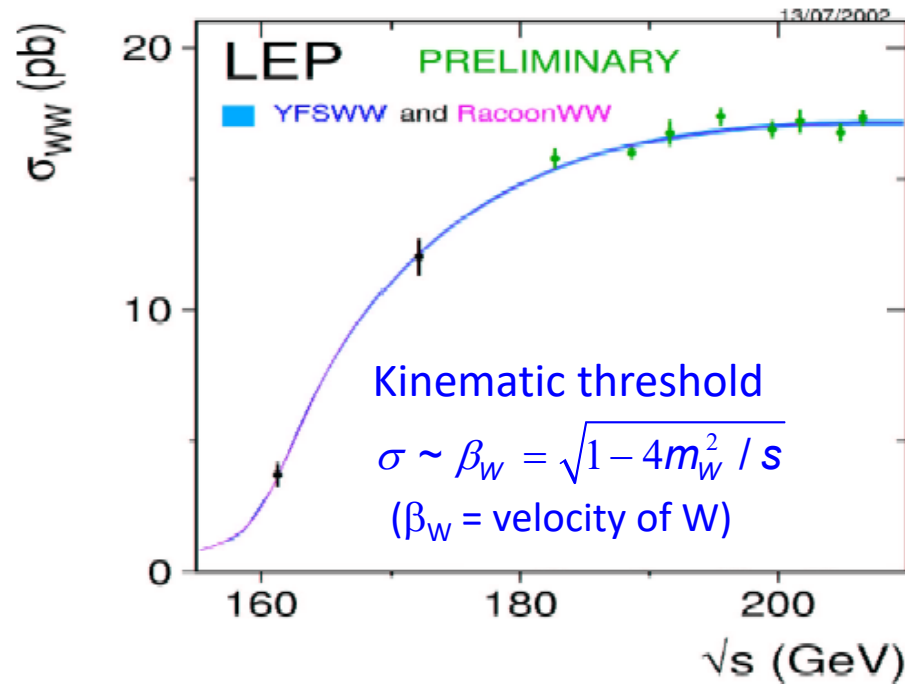
Threshold behavior of the cross section (kinematics, phase space) for  $ee \rightarrow WW$  production:



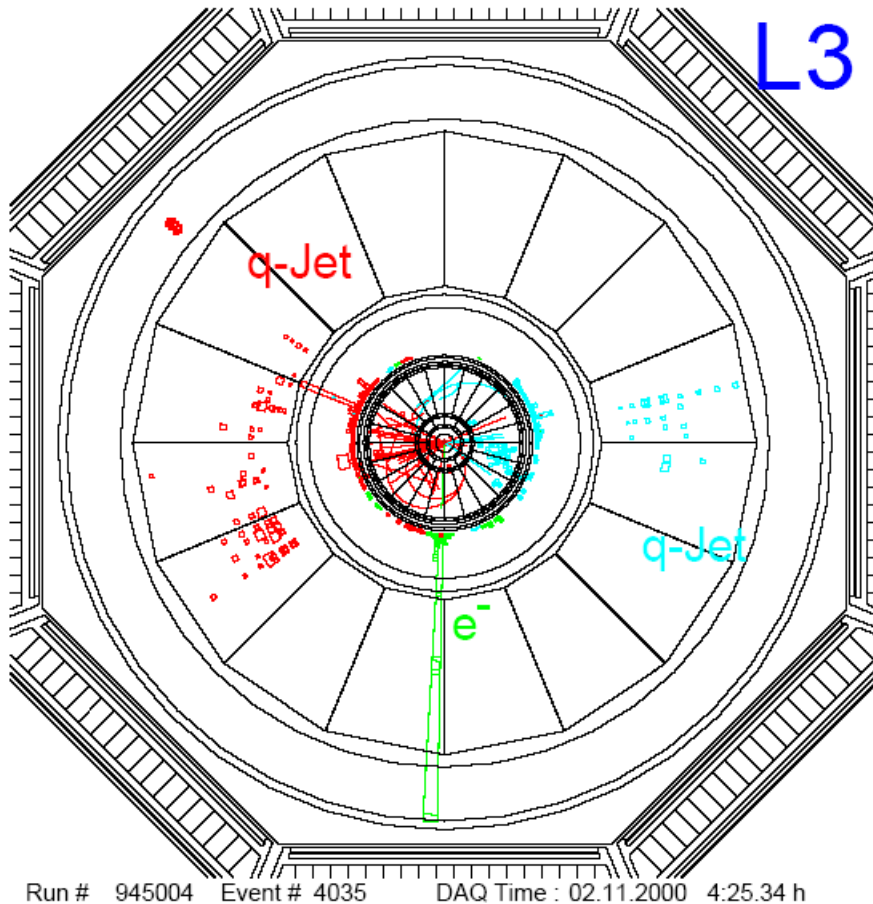
Phase space factor =  $f(m_W, \sqrt{s})$ :

→ Allows determination of  $m_W$

$$m_W = 80.42 \pm 0.20 \pm 0.03(E_{\text{LEP}}) \text{ GeV}$$

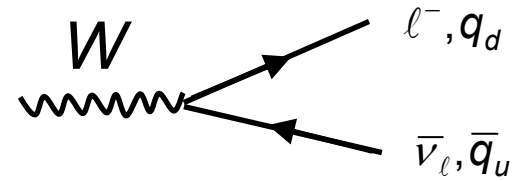


# Topology of WW decays

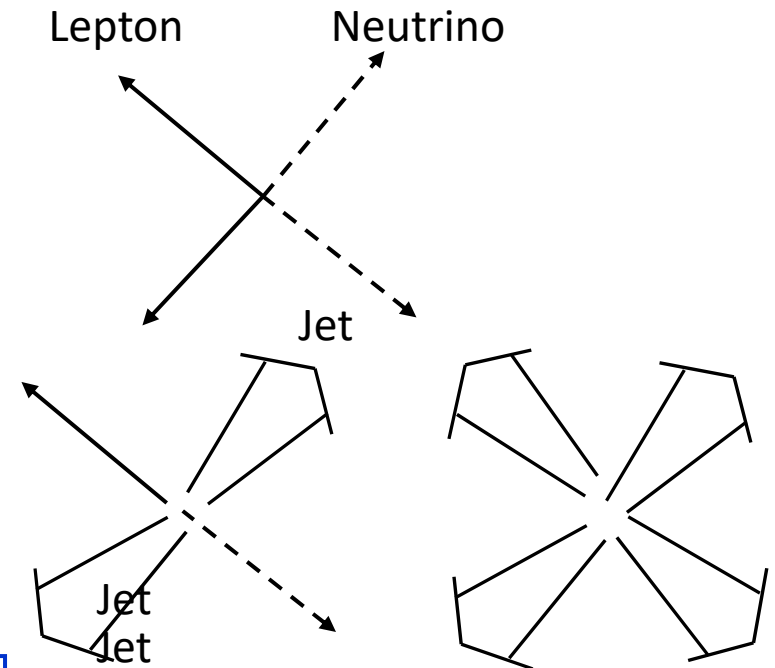


Signatures for a mass measurement:

$W_1 \rightarrow l\nu$   $W_2 \rightarrow \text{JetJet}$  or  $W_1 \rightarrow \text{JetJet}$   $W_2 \rightarrow \text{JetJet}$

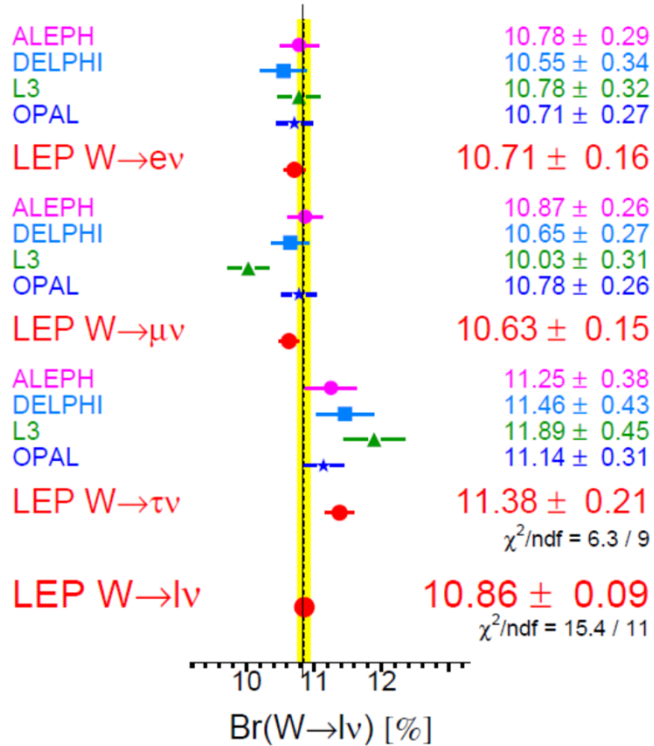


$WW \rightarrow$   $\left\{ \begin{array}{ll} qq\ell\nu & 44\% \\ qq\bar{q}q & 45\% \\ \ell\nu\ell\nu & 11\% \end{array} \right.$



Use dijet invariant mass.

## W Leptonic Branching Ratios

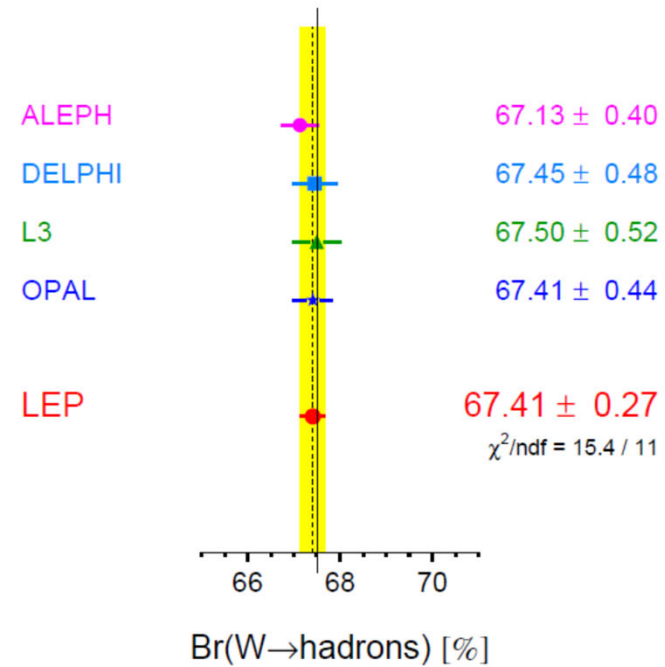


Agreement between leptons =  
test of lepton universality

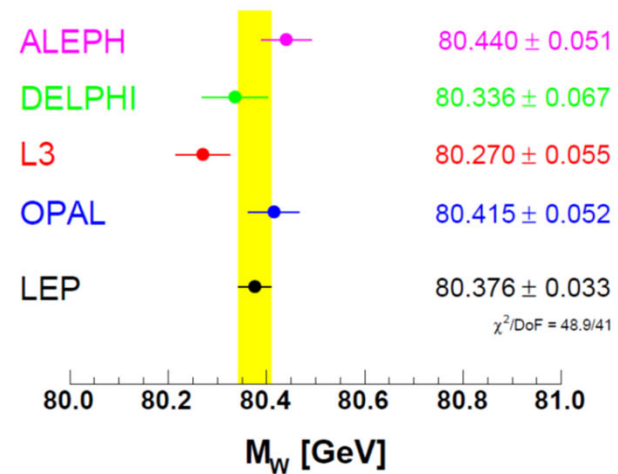
## W-mass:

Use  $WW \rightarrow qq \nu l$  or  $WW \rightarrow qq qq$   
events and calculate the qq  
invariant mass.

## W Hadronic Branching Ratio



## LEP W-Boson Mass

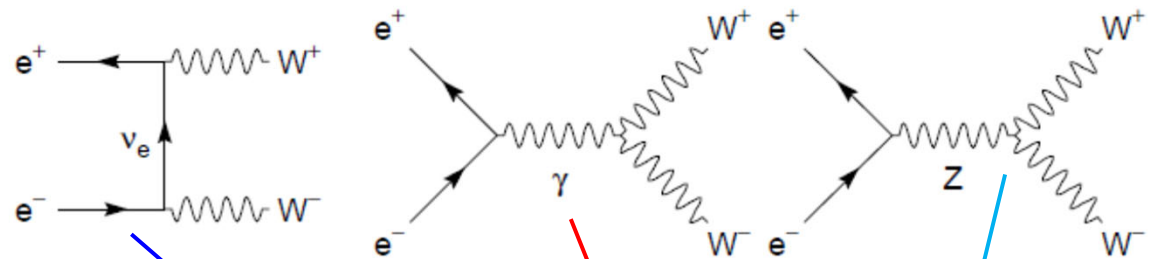


# Triple Gauge Boson Coupling

CERN-PH-EP/2013-022

arXiv:1302.3415

$$e^+ e^- \rightarrow WW$$



Cross section data confirms the existence of the  $\gamma/ZWW$  triple gauge boson vertex.

