CERN Summer School 2025

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e^+e^- - Annihilation:

1. e^+e^- - annihilation: a wrap-up
2. Cross section measurements of $e^+e^- \rightarrow ff$ and the measurement of R_{had}
3. Discovery of heavy quarks and τ-lepton
4. Test of QED and search for possible high-energy effects

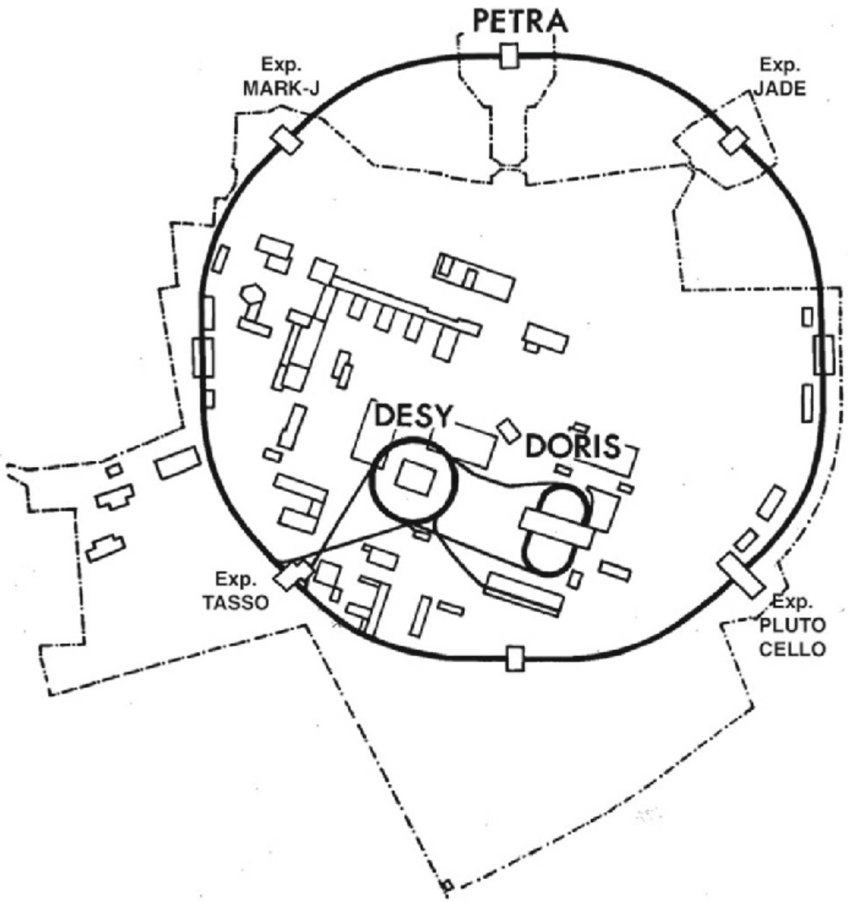
e^+e^- - machines (a selection)

Accelerator	Lab		L_{int} / Exper.
SPEAR	SLAC	2 – 8 GeV	
PEP	SLAC	→29 GeV	220 - 300 pb^{-1}
PETRA	DESY	12 - 47 GeV	~20 pb^{-1}
TRISTAN	KEK	50 – 60 GeV	~20 pb^{-1}
LEP	CERN	90 GeV	~200 pb^{-1}

In addition, there were/are the so called ee B-factories working at a centre-of-mass energy of 10.58 GeV ($e^+e^- \rightarrow \Upsilon(4S) \rightarrow BB$) and a tau-charm-factory working between 3 and 4 GeV.

DESY PETRA: Positron-Elektron-Tandem-Ring-Anlage

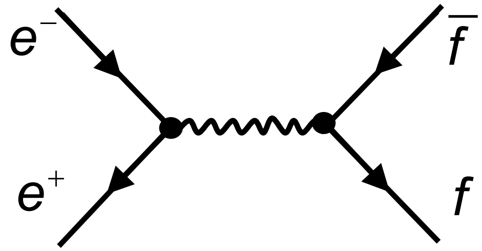
Operation: 1978 – today, circumference: 2.304 m., e^+e^- 1978 -1986, $\sqrt{s} \rightarrow 38$ GeV
Experiments **JADE**, MARK-J, PLUTO (CELLO) and TASSO).



Event display of the JADE detector

1. e^+e^- - annihilation: a wrap-up

Feynman rules:



$$-i\mathcal{M} = \underbrace{\bar{v}(p_2)(ieQ_e\gamma^{\mu})u(p_1)}_{J_e^{\mu}} \frac{g_{\mu\nu}}{q^2} \underbrace{\bar{u}(p_3)(ieQ_f\gamma^{\nu})v(p_4)}_{J_{\mu}^{\nu}}$$

If incoming electrons are not polarized and the spins of outgoing particle are not observed one needs to average over all incoming spin configurations and to sum over all possible outgoing configurations to obtain the average matrix element:

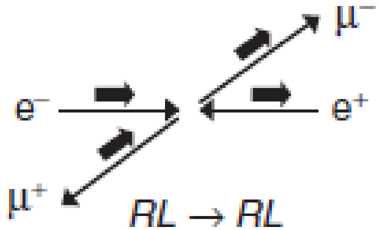
$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{1}{4}\sum_{s_i,s_f} \left|\mathcal{M}_{s_i,s_f}\right|^2$$

¼ arises from the average over 4 diff. initial spin states

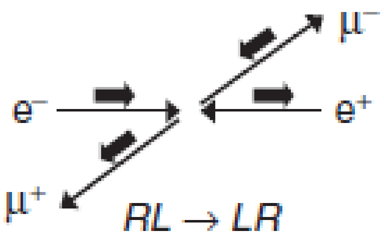
For massless fermions spin / helicity configurations ($u_{\uparrow}$ and $u_{\downarrow}$) are identical with the chirality configurations u_R and u_L and one can consider the 16 different contributions of type:

$$\mathcal{M}_{R(L)R(L)\to R(L)R(L)} \sim \bar{v}_{R(L)}\gamma^{\mu}u_{R(L)} \frac{g_{\mu\nu}}{q^2} \bar{u}_{R(L)}\gamma^{\nu}v_{R(L)}$$

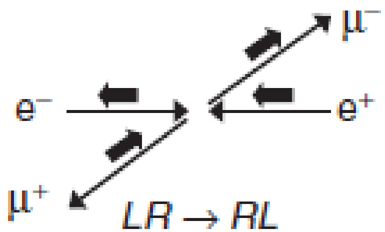
Due to the vector structure γ^μ of the coupling, only terms such as $\bar{v}_R \gamma^\mu u_L, \bar{u}_R \gamma^\mu v_L, \ldots$ don't vanish – which leaves only 4 non-zero $|M_{kl\to mn}|^2$ out of the possible 16:



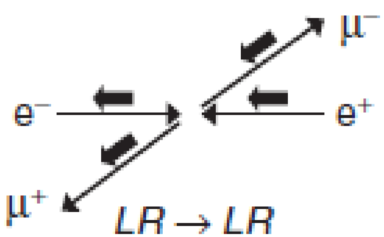
$$\left|\mathcal{M}_{RL\to RL}\right|^2 = \left(e^2 Q_f\, \bar{v}_R \gamma_\mu u_L \,\frac{1}{q^2}\, \bar{u}_R \gamma^\mu v_L \right)^2 = \left(4\pi\alpha Q_f\right)^2 \left(1+\cos\theta\right)^2$$



$$\left|\mathcal{M}_{RL\to LR}\right|^2 = \left(e^2 Q_f\, \bar{v}_R \gamma_\mu u_L \,\frac{1}{q^2}\, \bar{u}_L \gamma^\mu v_R \right)^2 = \left(4\pi\alpha Q_f\right)^2 \left(1-\cos\theta\right)^2$$



$$\left|\mathcal{M}_{LR\to RL}\right|^2 = \left(e^2 Q_f\, \bar{v}_L \gamma_\mu u_R \,\frac{1}{q^2}\, \bar{u}_R \gamma^\mu v_L \right)^2 = \left(4\pi\alpha Q_f\right)^2 \left(1-\cos\theta\right)^2$$



$$\left|\mathcal{M}_{LR\to LR}\right|^2 = \left(e^2 Q_f\, \bar{v}_L \gamma_\mu u_R \,\frac{1}{q^2}\, \bar{u}_L \gamma^\mu v_R \right)^2 = \left(4\pi\alpha Q_f\right)^2 \left(1+\cos\theta\right)^2$$

Summing and averaging the contributions:

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}(4\pi\alpha Q_f)^2\left[2(1+\cos\theta)^2+2(1-\cos\theta)^2\right]$$

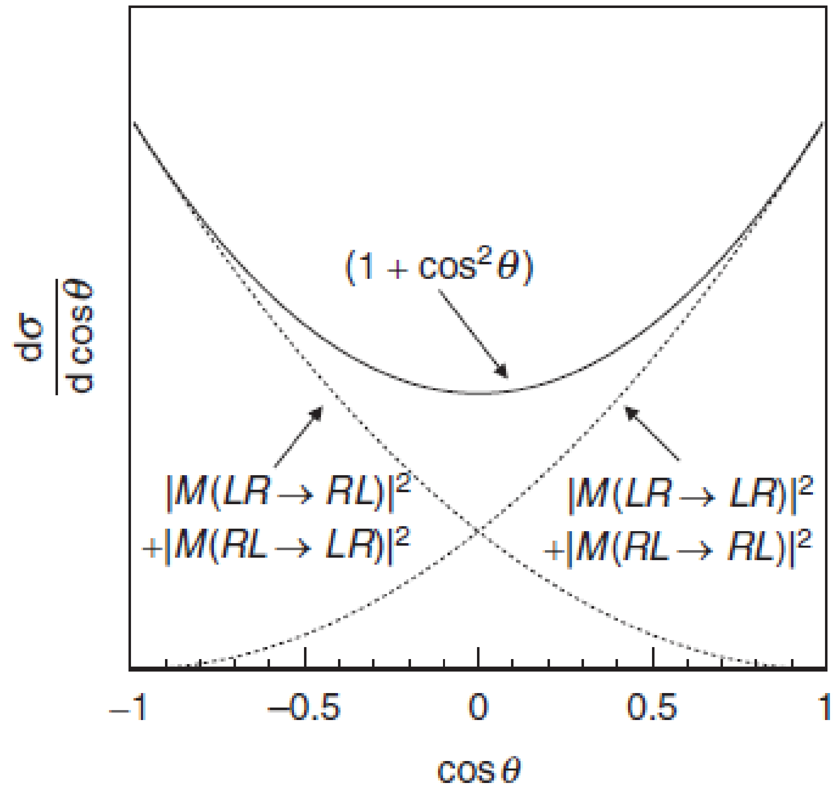
$$\langle|\mathcal{M}|^2\rangle = (4\pi\alpha Q_f)^2(1+\cos^2\theta)$$

And with the cross section formula:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\langle|\mathcal{M}|^2\rangle$$

$$= \frac{\alpha^2}{3s}Q_f^2\cdot(1+\cos^2\theta)$$

$$\sigma = \frac{4\pi\alpha^2}{3s}Q_f^2$$



2. Cross section measurements of $e^+e^- \rightarrow ff$ and measurement of R_{had}

Experimentally the cross section is given by the number of observed signal events - corrected for background, efficiency and acceptance - normalized to the integrated luminosity of the recorded data:

$$\sigma = \frac{N_{events}(1-b)}{\varepsilon A \cdot \mathcal{L}_{\text{int}}} \quad \text{with} \quad \begin{cases} N_{events} = \text{number of selected events} \\ b = \text{background fraction in sample} \\ \varepsilon A = \text{efficiency} \cdot \text{acceptance} \\ \mathcal{L}_{\text{int}} = \text{Integrated luminosity} \end{cases}$$

Remark: acceptance is defined by the detector coverage, ε is the "efficiency" within the acceptance.

$$\frac{d\sigma}{d\cos\theta} = \frac{\Delta N_{events}}{\Delta\cos\theta} \frac{(1-b)}{\varepsilon A \cdot \mathcal{L}_{\text{int}}}$$

Measured in bins of $\cos\theta$, assuming rotational symmetry in azimuthal angle φ. Correction b, εA might be θ-dependent.

εA are generally determined from MC-simulation: $N_{seleted}/N_{generated}$
Background fraction b can be determined from simulation or from control samples.

Determination of integrated luminosity:

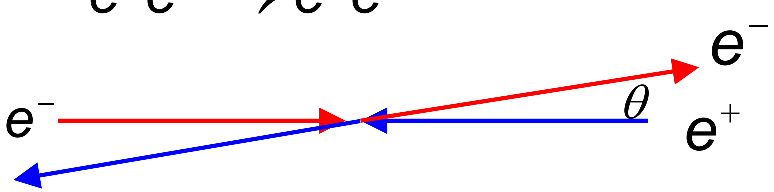
The determination of the (integrated) luminosity from machine parameters is often not accurate enough – the exact focussing of the beams (β*) and the exact positioning (head-on collision) is difficult to maintain constant and to reproduce, Also, the “availability” of the detectors for data-taking might vary – such that they cannot profit from the delivered luminosity.

Therefore the (integrated) luminosity is determined by the individual detectors using a reference process: $\mathcal{L}_{\text{int}} = N_{ref} / \sigma_{ref}$

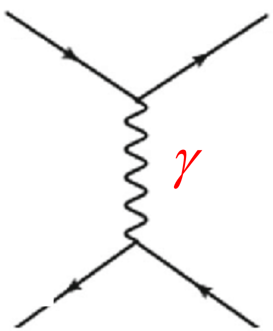
- Reference process should be independent from the processes to be measured
- Reference process should have a large cross section

For e^+e^- machines usually the small angle (t-channel) Bhabha scattering is used:

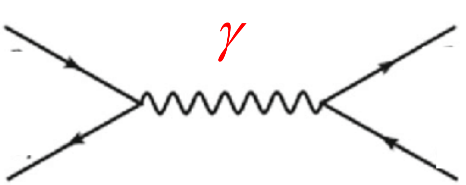
$$e^+e^- \rightarrow e^+e^-$$



For small θ, t-channel largely dominates cross section.

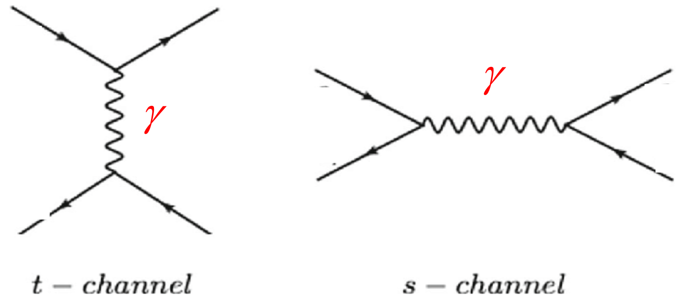


t − channel



s − channel

$$|\mathcal{M}|^2 \sim \Bigg|$$



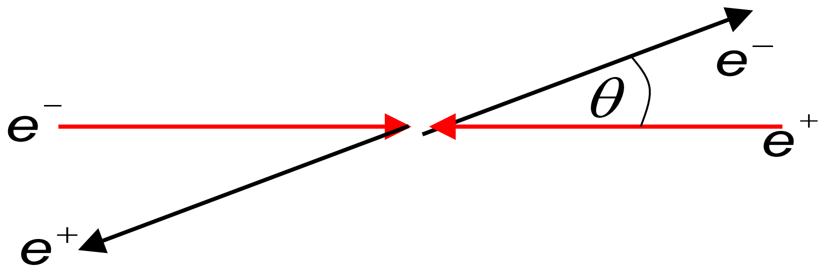
$$\Bigg|^2$$

interference

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}\right) = \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2 \sim \frac{1}{\theta^4}$$

With $t = -\frac{s}{2}(1-\cos\theta) \approx -\frac{s}{4}\theta^2$ for small θ: t-channel dominates

CM system:

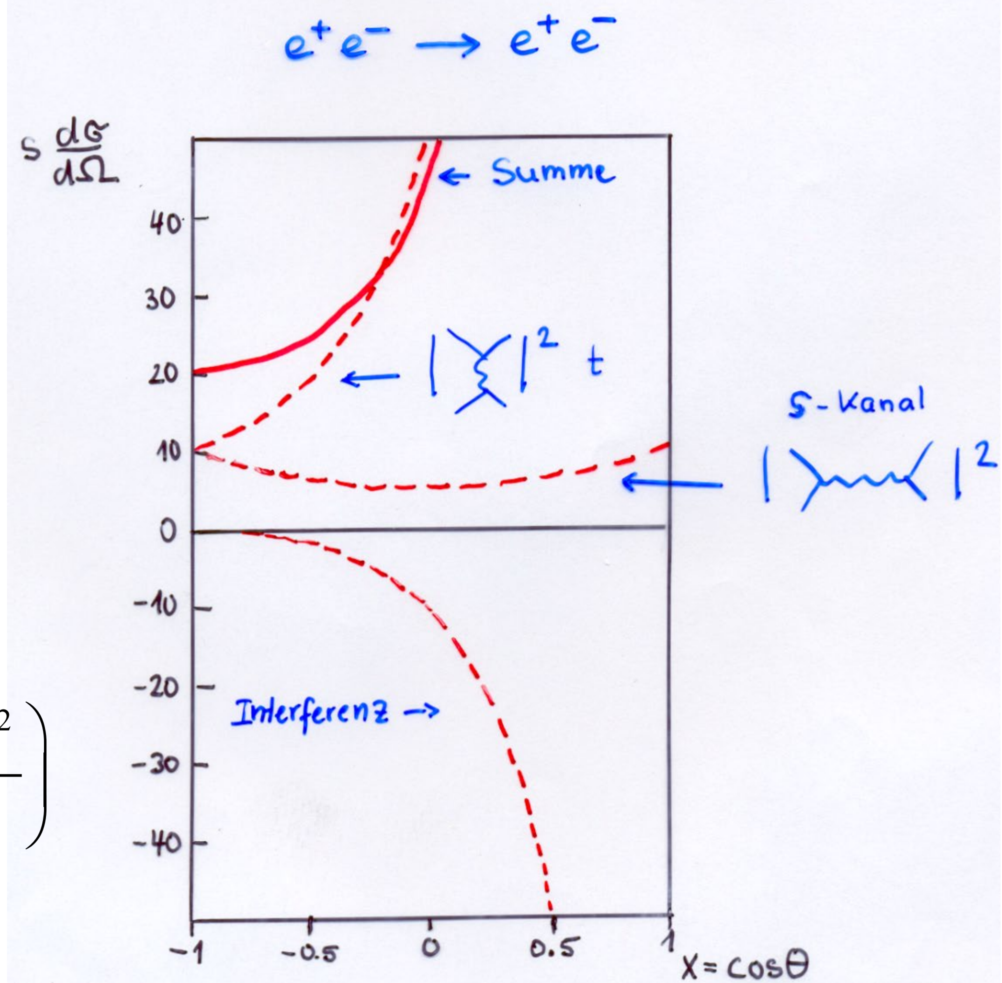


with x=cosθ

t-channel s-channel

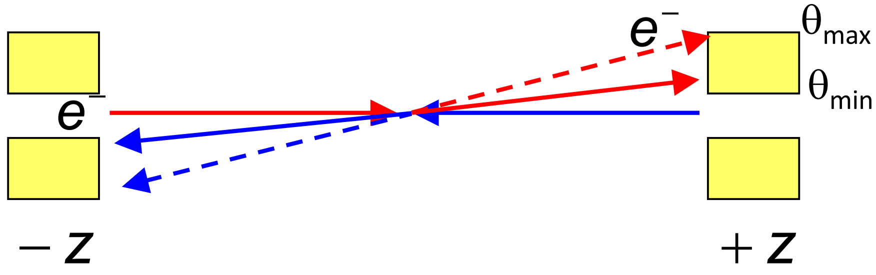
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{4+(1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2} \right)$$

$$= \frac{\alpha^2}{2s}\left(\frac{3+x^2}{1-x} \right)^2$$



Luminosity measurement:

Special "luminosity monitors" = calorimeters at very small angles.



$$\frac{d\sigma}{d\Omega} \approx \frac{4\alpha^2(\hbar c)^2}{E^2\theta^4} \quad \longrightarrow \quad \frac{d\sigma}{d\theta} \sim \frac{1}{\theta^3} \rightarrow \sigma \sim \left(\frac{1}{\theta_{min}{}^2} - \frac{1}{\theta_{max}{}^2}\right)$$

$$\frac{d\sigma}{d\Omega} \sim \frac{d\sigma}{d\cos\theta} \sim \frac{d\sigma}{\sin\theta d\theta} \sim \frac{1}{\theta}\frac{d\sigma}{d\theta}$$ (for small θ)

Inner acceptance determination very critical

Examples from LEP:

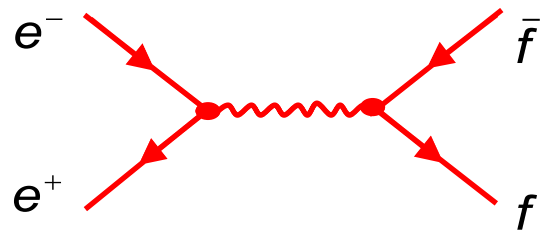
	distance (m)	R_{min} (cm)	R_{max} (cm)	Θ_{min} (mrad)	Θ_{max} (mrad)	technology
ALEPH LCAL	2.7	10	52	45	190	lead+prop. wire ch.
DELPHI SAT	2.5	10	40	43	135	lead+sc. fibers
L3 BGO	2.8	6.8	19	25	70	BGO
OPAL FD	2.4	11.5	29	48	120	lead+scintillator

Table 1: Basic parameters of the first generation detectors at LEP.

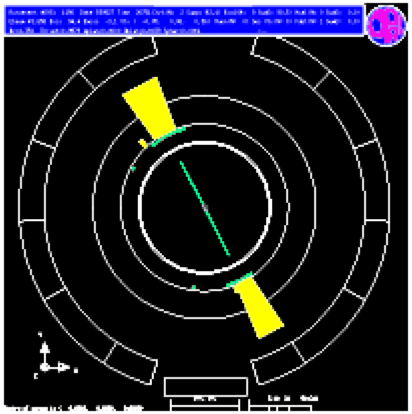
Typical luminosity error achieved: 0.3 - 0.5 % (1st generation lumi detector)
(dominated by acceptance knowledge) 0.07 – 0.15 % (2nd generation: Si strips)

Determination of N_{event}: select and count.

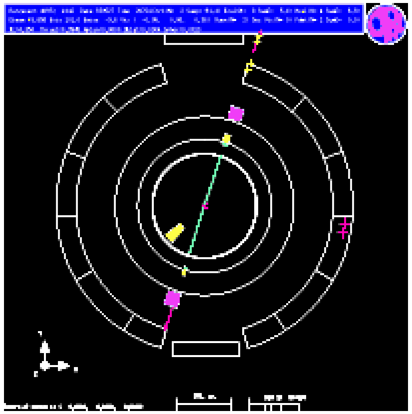
Event display from OPAL at LEP



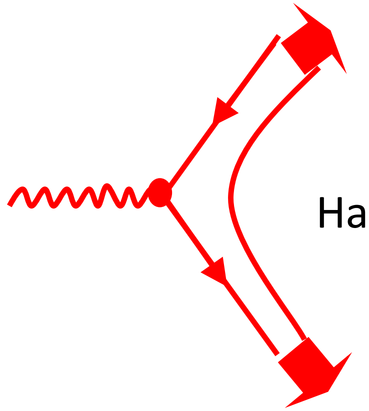
$e^- e^+$



$\mu^- \mu^+$

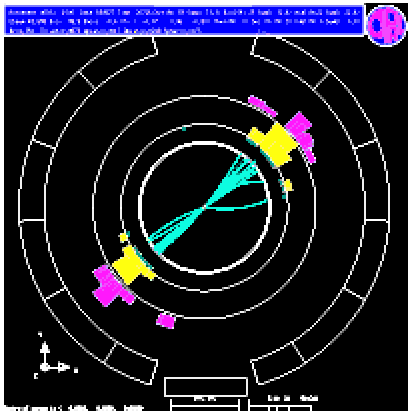


$q\,\bar{q}$ mit
q = u,d,s,c,b



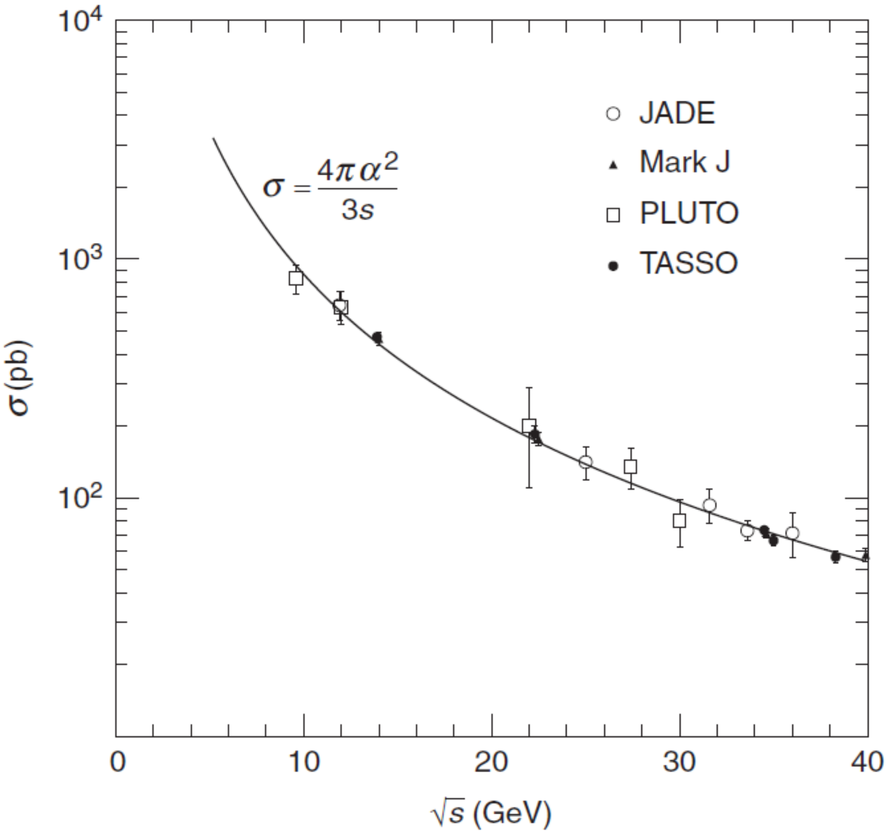
Hadron jets

$q\,\bar{q}$

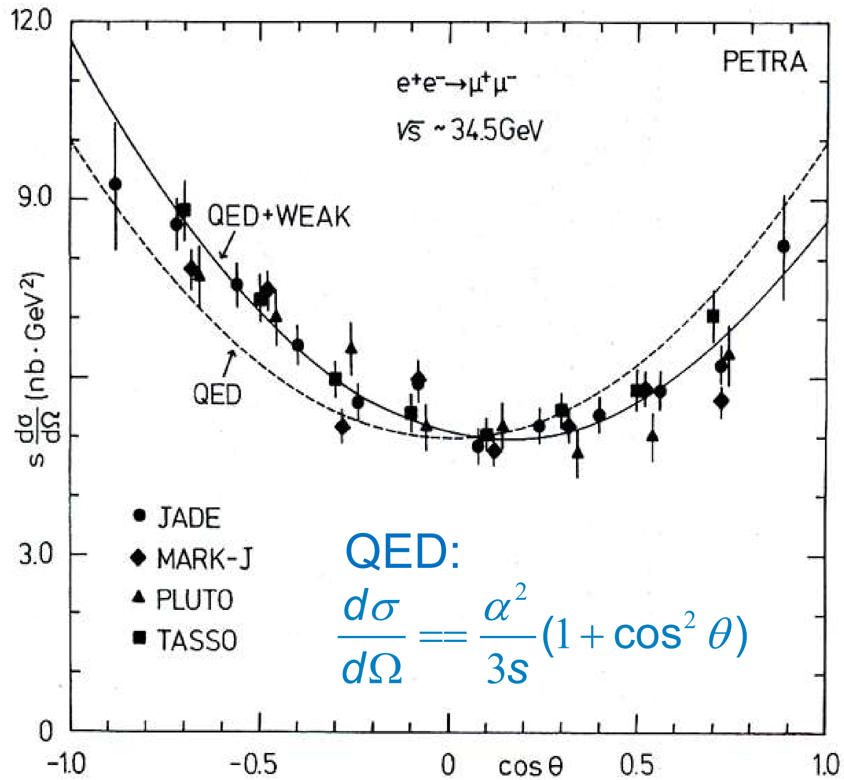


Cross section for $e^+e^- \to \mu^+\mu^-$

Total cross section



differential cross section



QED:

$$\frac{d\sigma}{d\Omega} == \frac{\alpha^2}{3s}(1+\cos^2\theta)$$

Total cross section follows the QED prediction very well.

Differential cross section deviates from QED because of γZ-interference. (will be discussed below)

Measurement of $e^+e^-\to$hadrons and R_{had}

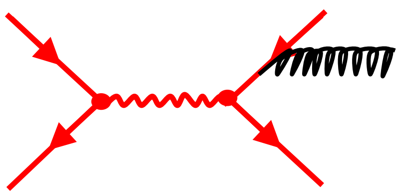
$$R_{had} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sigma_{had}}{\sigma_{\mu\mu}}$$

(see theory lecture)

$$R_{had} = N_C \cdot \sum_{quarks\ i} Q_i^2 =$$

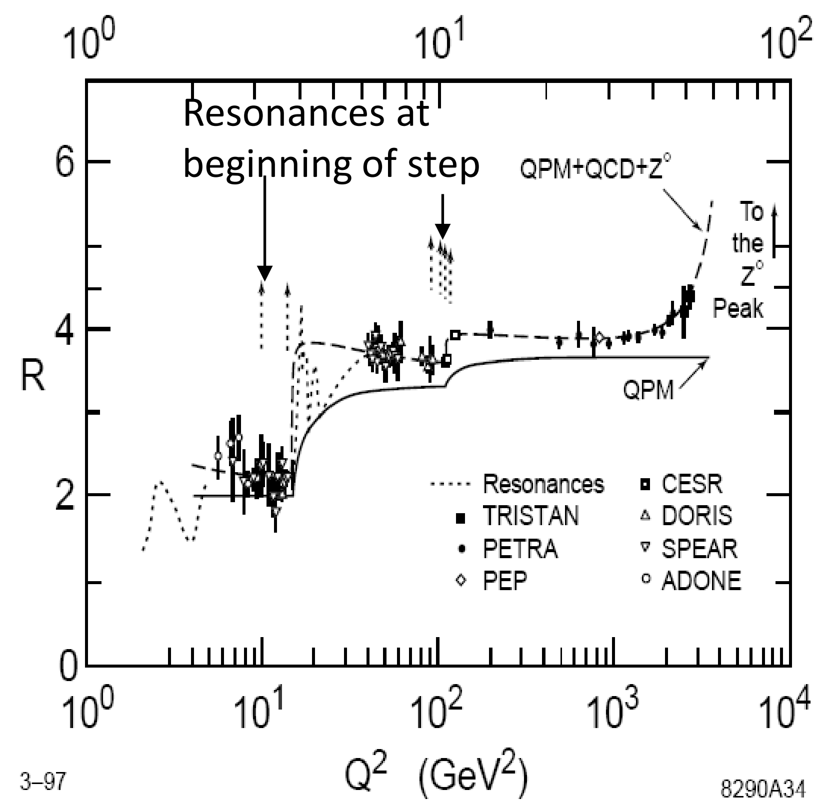
$\sqrt{s}$	Quarks	$R_{had} = 3\cdot\sum_i Q_i^2$
< ~3 GeV	uds	3·6/9=2.00
< ~10 GeV	udsc	3·10/9=3.33
< ~350 GeV	udscb	3·11/9=3.67

Data lies systematically higher than the prediction from Quark Parton Model (QPM)
$\to$ QCD corrections: gluon bremsstrahlung



$$\sigma(s) = \sigma_{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411\cdot\frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

~ 7%



3. Discovery of heavy particles

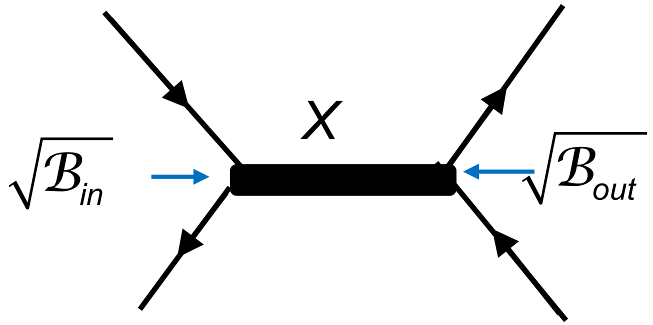
Hadronic resonances of heavy quarks:

Resonances and Breit-Wigner cross section:

Assume that there is a particle X (resonance) with mass m_X and the same quantum numbers than the photon. If particle X couples to e^+e^-, $\mu\mu$ and qq one would have an additional contribution:

$$e^+e^- \rightarrow X \rightarrow f\bar{f}$$

$\mathcal{B}_{in,out}$ branching ratios



This leads to a "resonance contribution" to the cross section. The resonance cross section can be calculated on very general grounds using partial wave analysis of the scattering amplitude. One finds the so called Breit-Wigner Resonance cross section:

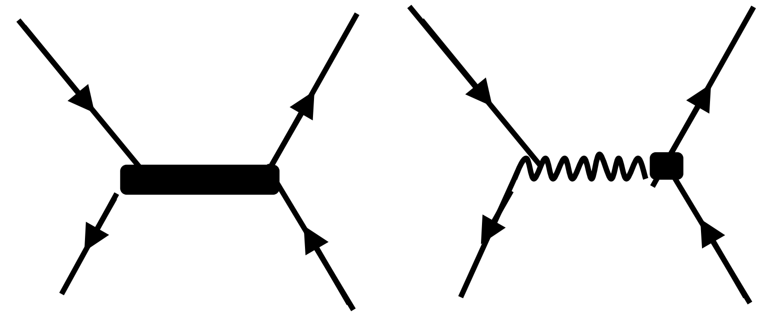
$$\sigma_{BW}(E) = \frac{2J+1}{(2s_1+1)(2s_2+1)} \frac{4\pi}{k^2} \left[\frac{\Gamma^2/4}{(E-m_X)^2 + \Gamma^2/4} \right] \mathcal{B}_{in}\mathcal{B}_{out}$$

For narrow resonances:

$$[\quad] = \pi\Gamma \cdot \delta(E - m_X)/2$$

Where k=CMS momentum "in" particles, J=spin of resonance, $s_{1,2}$=spin of in particles, Γ=total widths (sum of partial widths)

In case of a resonance there are thus two contributions to the same final state:

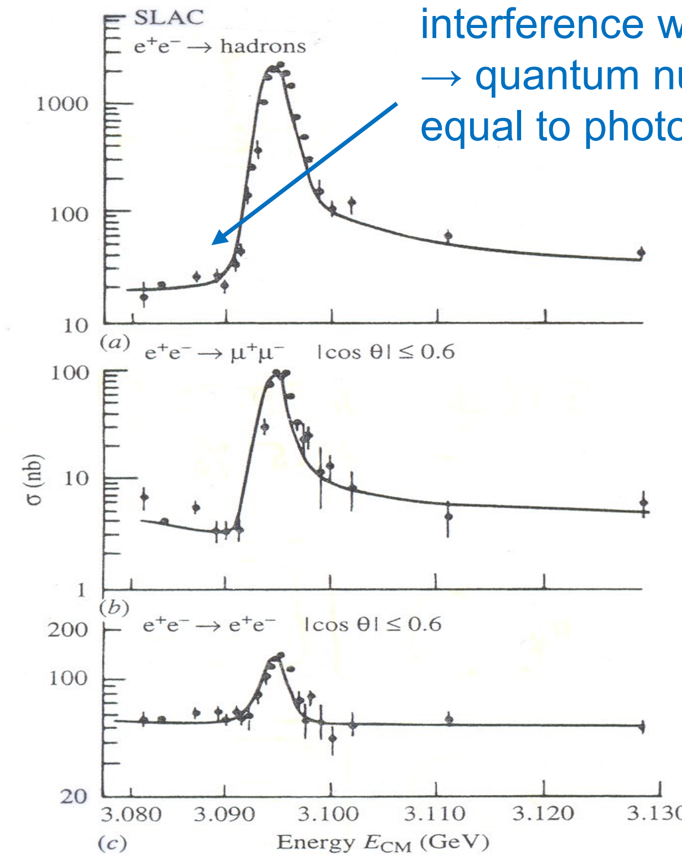


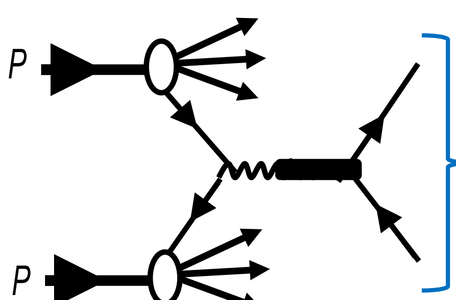
Discovery of the J/ψ ($c\bar{c}$):

In 1974, at SPEAR in e^+e^- @ ~3.1 GeV) a resonance has been observed which decays into e^+e^-, $\mu\mu$ and hadrons. The resonance has a very tiny widths $\Gamma\approx$90 keV much smaller than the energy resolution of the beams (B. Richter et al.).

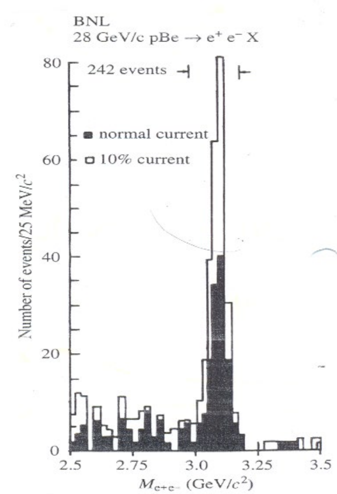
At the same time the resonance has been found in pBe fixed-target collisions (S.C:C Ting et al.)

Dispersion like behaviour: interference w/ γ → quantum numbers equal to photon





Invariant mass



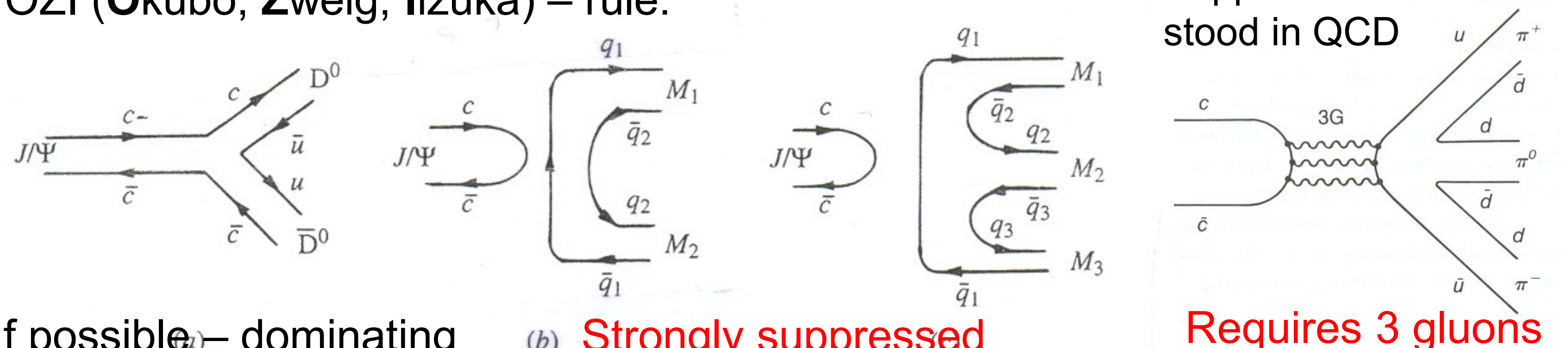
New “heavy” narrow resonance - discovery of c-quark

- New heavy meson
- Quantum numbers of the photon.
- High mass and extreme narrowness of J/ψ indicates that I cannot be understood in terms of u,d and s-quarks (the known quark at the time): not heavy enough, hadronic decays → large Γ
- However; Glashow, Iliopoulos & Maiani (1970) postulated the existence of a forth “heavy” quark: c-quark with charm quantum number. Thus, J/ψ could by a bound $c\bar{c}$ – state.

But: Why is it so narrow? Expect decays to D^+D^- or $D^0\bar{D}^0$ → large Γ

OZI (**O**kubo, **Z**weig, **I**izuka) – rule:

Suppression under-stood in QCD



If possible – dominating | Strongly suppressed | Requires 3 gluons

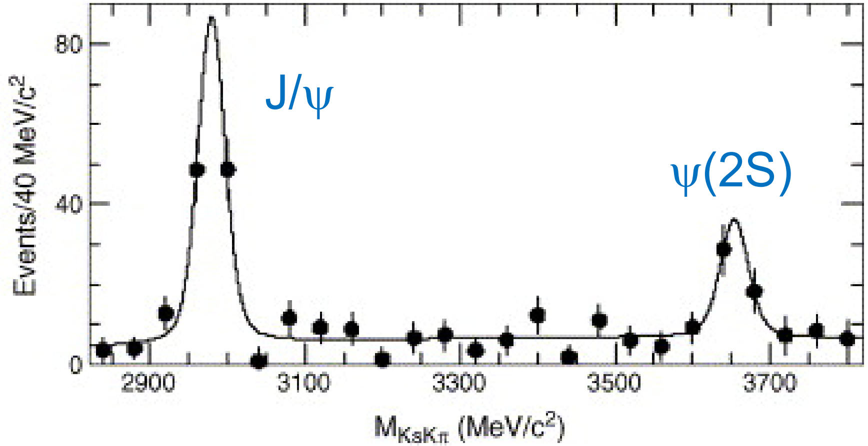
Not possible: $2m_D > m_{J\psi}$ ⟹ J/ψ decays to hadrons strongly suppressed.

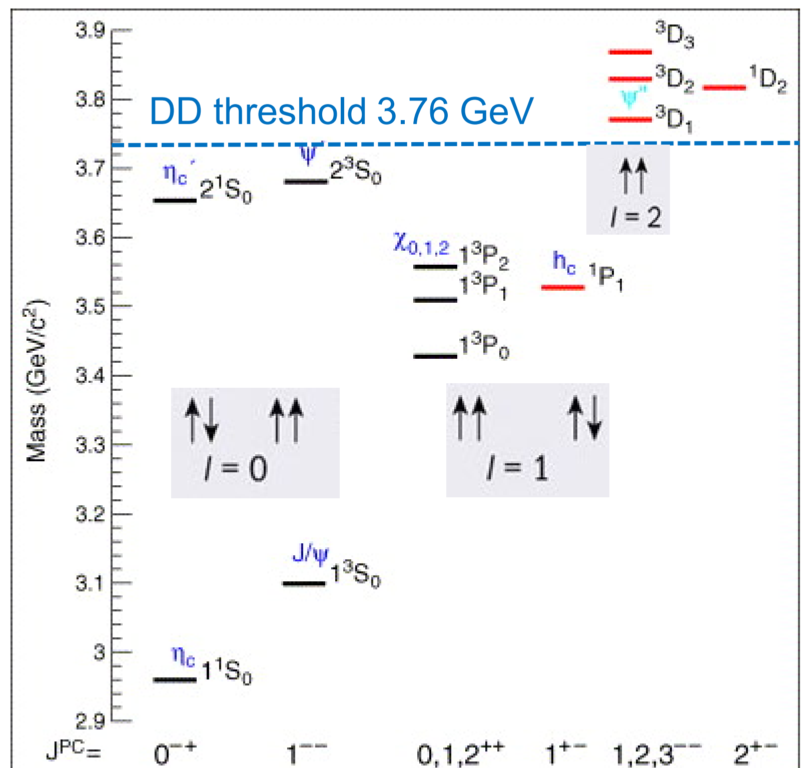
Today we know that the J/ψ or the shortly afterwards observed $\psi(2S)$ are members of the family of bound $c\bar{c}$ states (**charmonia**).

Spectroscopy (exact measurement of particle masses and their decays) reveals information on the QCD potential between the two quarks.

See spectroscopy of positronium.

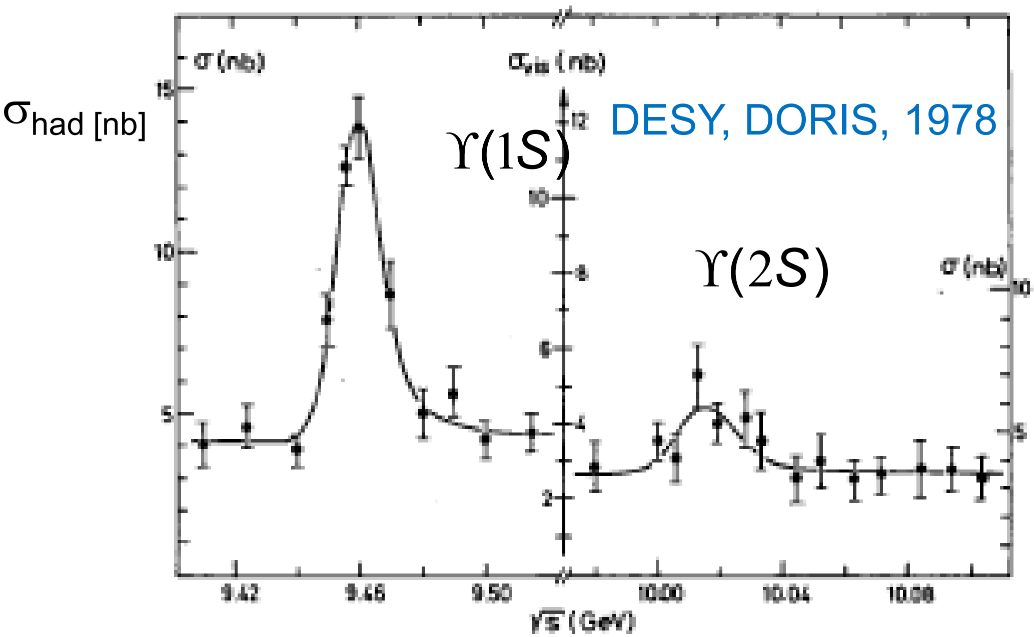
https://doi.org/10.1016/j.ppnp.2004.12.001

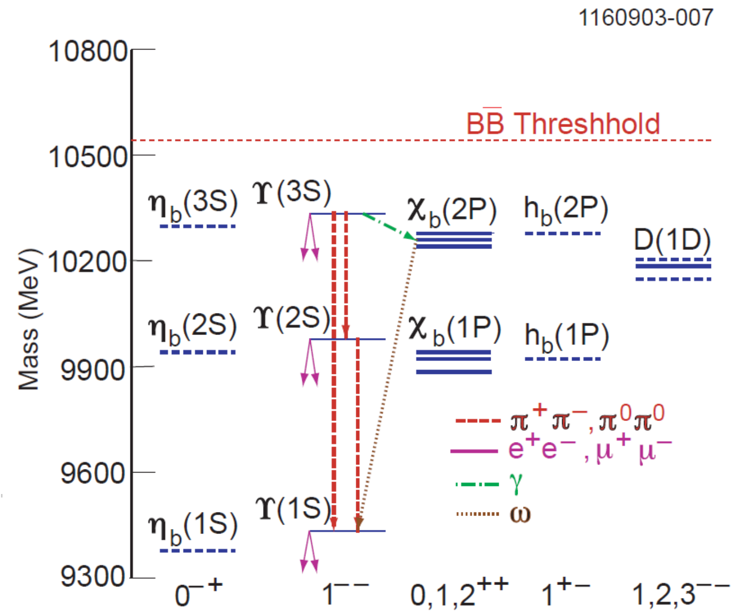




Discovery of the Υ ($b\bar{b}$):

In 400 GeV proton fixed-target collsion other even higher-mass resonance (~9.5 GeV) was observed (S. Herb et al., 1977). Quickly afterwards its existence was confirmed in e^+e^- collisions (DESY, DORIS) – in addition a 2nd excited state was observed.



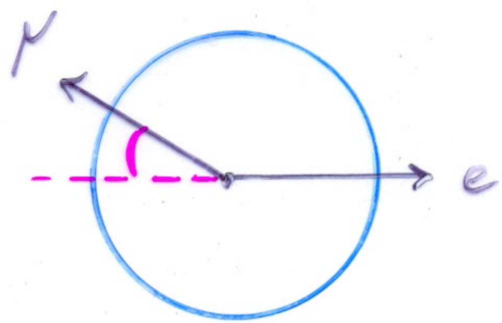


The resonances have been identified as $b\bar{b}$ bound states: bottonium states

Discovery of the Tau-Lepton

Evidence of anomalous lepton production in e^+e^- annihilation (M. L. Perl et al.1975):

Observation of eμ final states at $\sqrt{s} \approx 4.8$ GeV



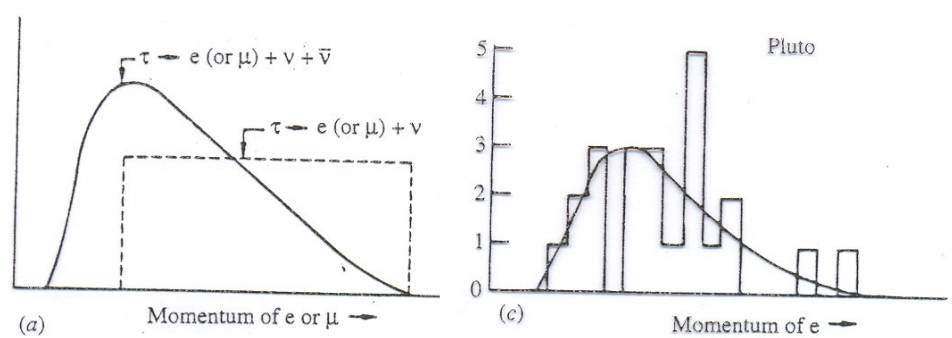
Particles \ N_γ	0	1	>1	0	1	>1
	Total charge = 0			Total charge = ±2		
$e-e$	40	111	55	0	1	0
$e-\mu$	24	8	8	0	0	3
$\mu-\mu$	16	15	6	0	0	0
$e-h$	20	21	32	2	3	3
$\mu-h$	17	14	31	4	0	5
$h-h$	14	10	30	10	4	6

Interpretation: Pair production of a new sequential heavy lepton (τ-lepton)

$$e^+e^- \to \tau^+\tau^-$$

$$\tau^- \to e^-\,\bar{\nu}_e\nu_\tau$$

$$\tau^+ \to \mu^+\,\nu_\mu\bar{\nu}_\tau$$



PLUTO (DESY) confirms the production of a new heavy lepton.

Tau-mass determination from threshold behaviour

In our derivation of the matrix element / cross section we have neglected possible masses of the out-going fermions. In case of CMS energies which are only marginally larger than $2m_f$ the masses of the out-going fermions needs to considered.

This leads to a slightly modified average matrix element square:

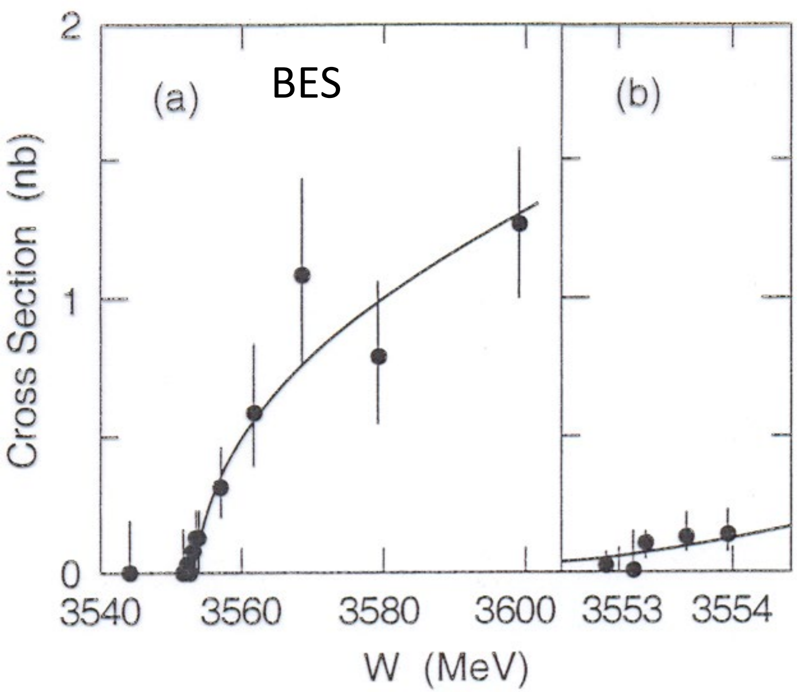
$$\left\langle |\mathcal{M}|^2 \right\rangle = \left(4\pi\alpha Q_f\right)^2 \left(2 - \beta^2 + \beta^2 \cos^2\theta\right)$$

with $\beta = p/E$ velocity of the out-going fermions.

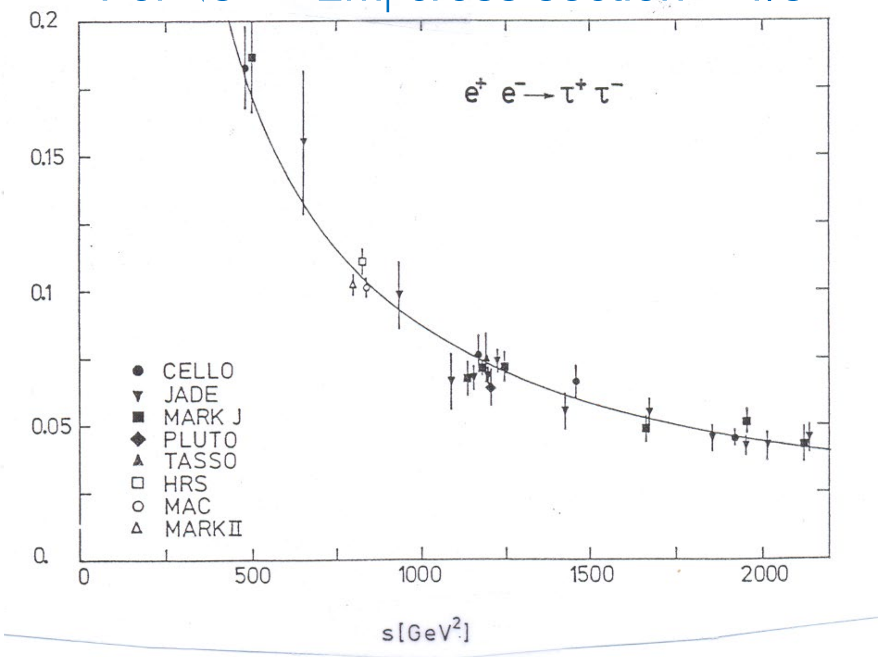
The total cross section is thus given by:

$$\sigma = \frac{4\pi\alpha^2}{3s} Q_f^2 \cdot \beta \left(\frac{3-\beta^2}{2}\right) \quad \text{with} \quad \beta^2 = p^2/s^2 = \left(1 - \frac{4m_f^2}{s}\right)$$

Tau lepton: a sequential heavy lepton



For $\sqrt{s} >> 2m_f$ cross section ~ 1/s



$$\sigma = \frac{4\pi\alpha^2}{3s} Q_f^2 \cdot \beta\left(\frac{3-\beta^2}{2}\right) \qquad \beta^2 = \left(1 - \frac{4m_f^2}{s}\right)$$

For large energies cross section behaves like QED prediction for

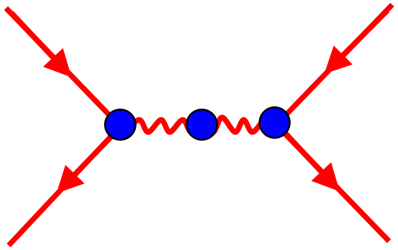
$$e^+e^- \to \mu^+\mu^-$$

$$m_\tau = 1776.96^{+0.18+0.20}_{-0.19-0.16}\ MeV$$

BES, 1994

4. Test of QED and search for possible high-energy effects

Possible break-down of QED:
- Are fundamental fermions really point-like?
- Is there a heavy photon w/ modified propagator?



Modified photon propagator assuming heavy photon w/ mass Λ and standard coupling α:

$$\frac{1}{q^2} \to \frac{1}{q^2} - \frac{1}{q^2 - \Lambda^2} = \frac{1}{q^2}\left(1 - \frac{q^2}{q^2 - \Lambda^2}\right)$$

Form factor $F(q^2)\left(1 - \frac{q^2}{q^2 - \Lambda^2}\right)$

Modified propagator or form factor corresponds to a modified electromagnetic potential: Additional Yukawa component to account for a non-point like structure of fermion / interaction.

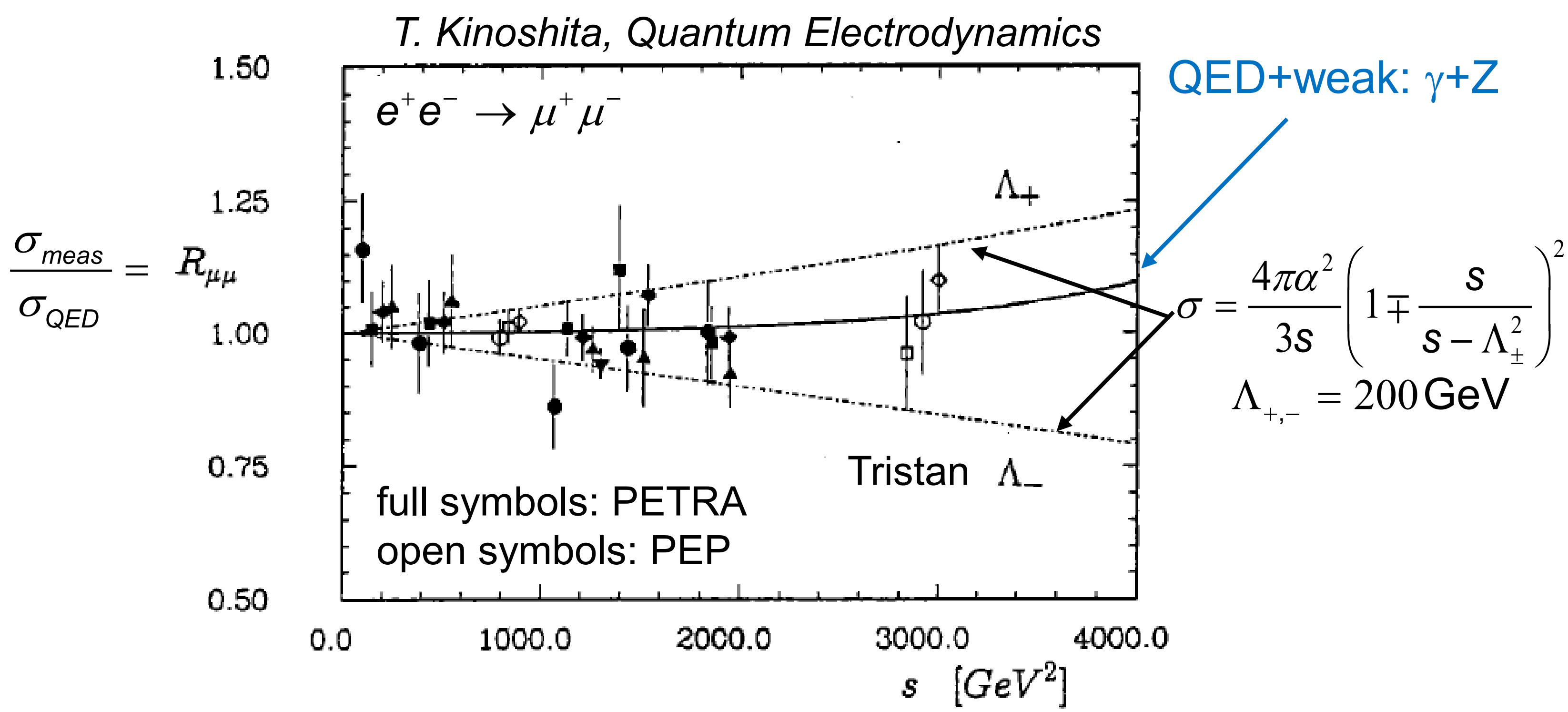
Potential $\frac{1}{r} \to \frac{1}{r}\left(1 - e^{-\Lambda/r}\right)$

<u>Modified cross section:</u> $e^+e^- \to \mu^+\mu^-$

$$\sigma = \frac{4\pi\alpha^2}{3s}\left(1 \mp \frac{s}{s - \Lambda_\pm^2}\right)^2$$

Term Λ_- has no simple physical interpretation but is added to also account for a higher cross section

For Bhabha cross section more involved: s, t, s·t terms at different scales.



Interpretation:

Λ_+ contribution reduces cross section – curve reflects the smallest cross section prediction consistent w/ data → lower bound on Λ_+.

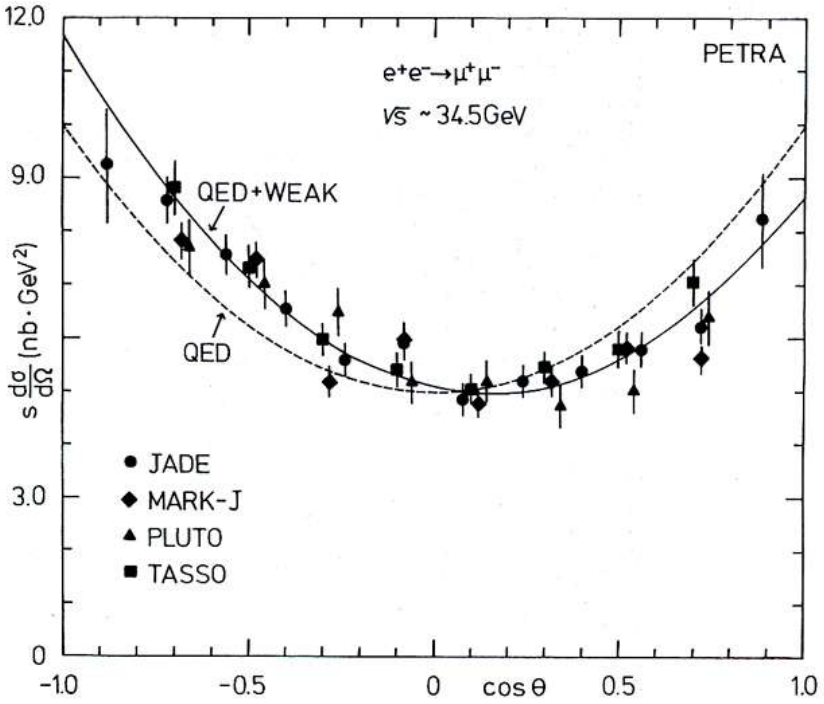
Λ_- contribution increases cross section – curve reflects the largest cross section prediction consistent w/ data → lower bound on Λ_-.

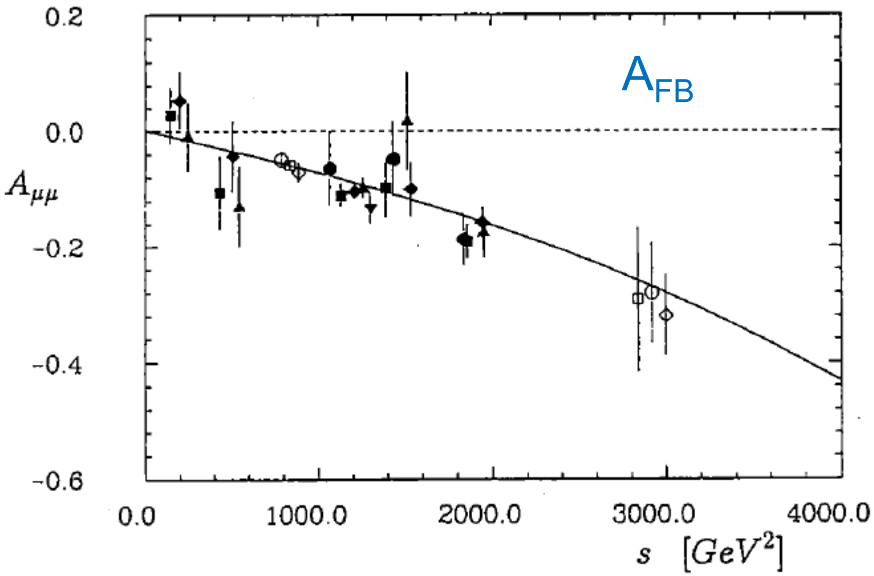
Experimental limits obtained from corrected cross sections (Z contribution) vary between 250 – 350 GeV (no common analyses): μ is point-like down to 10^{-18}m. From ee → ee, ττ similar limits can be obtained for electron and tau.

Effect of Z-exchange: "heavy photon" but with slightly different couplings

Form the absolute cross section it is hard to see the effect of Z exchange:

But: Z couplings violate parity → large γZ interference leads to a large asymmetric angular distribution even at small energies (discusses later)





$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{3s} Q_f^2 \cdot (1 + \cos^2\theta + A_{FB}\cos\theta)$$

A_{FB} parmetrizes effect of γZ interference