Introduction to the (Electro-) Weak Interaction

Reminder: helicity vs. chirality

- Chiral eigenstates are Lorentz-invariant but are <u>not</u> stationary states under time evolution

$$\left.\begin{aligned} P_R &= \tfrac{1}{2}(1+\gamma_5) \\ P_L &= \tfrac{1}{2}(1-\gamma_5) \end{aligned}\right\} P_R + P_L = 1$$

- Helicity eigenstates are <u>not</u> Lorentz-invariant in general, but are stationary states of the motion

$$\vec{\Sigma}\cdot\hat{p} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & \vec{0} \\ \vec{0} & \vec{\sigma}\cdot\hat{p} \end{pmatrix} = 2\vec{S}\cdot\hat{p}$$

(here $\vec{S} \sim \frac{1}{2}\mathbb{1}_4 \otimes \vec{\sigma}$)

also written sometimes as $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$$\Sigma^i = \tfrac{1}{2}\epsilon_{ijk}\sigma^{jk} = \gamma_5\gamma^0\gamma^i$$

do not confuse with relativistic 4-polarization:

$s^\mu = (0, \vec{S})$ in rest frame (zero helicity)

here, $\frac{1}{2}\vec{\sigma}$

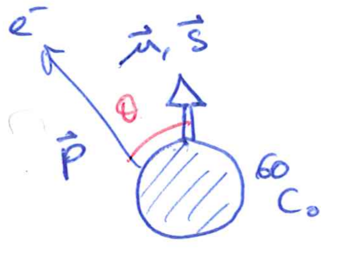
Lorentz boost:

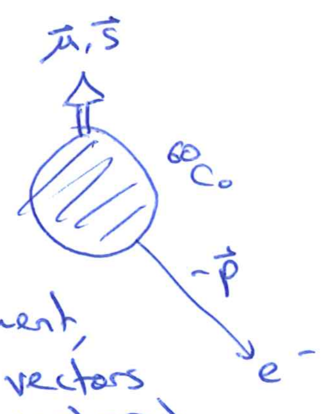
$s^0 \to \vec{\beta}\cdot\vec{S}$, i.e. helicity

$$\vec{S} \to \vec{S} + \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{S}) - \gamma\vec{\beta}s^0$$

Reminder: Wu experiment, 1957

$$^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$$
$$^{60}Ni^* \to {}^{60}Ni + 2\gamma$$





Positive magnetic moment,

$\vec{B}$ and $\vec{\mu}$ are axial vectors (pseudovectors)

Experimentally check the direction of e^- emission and look for asymmetry:

$$I(\theta) \sim 1 + \alpha\left(\frac{\vec{\sigma}\cdot\vec{p}}{E}\right) = 1 + \alpha\frac{v}{c}\cos\theta$$

contrast/visibility: $\dfrac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

^{60}Co has a spin ($S=5$) and can be polarized (at low T) in a $\vec{B}$ field. $^{60}Ni^*$ has $S=4$

$$\underbrace{^{60}Co \longrightarrow {}^{60}Ni^*}_{\Delta S=1} + \underset{S=\frac{1}{2}}{e^-} + \underset{S=\frac{1}{2}}{\bar{\nu}_e} \Rightarrow$$ e^- <u>and</u> $\bar{\nu}_e$ emitted with spin up to conserve angular momentum

Observation: electrons emitted preferentially with momentum <u>opposite</u> the ^{60}Co spin direction

$\Rightarrow$ negative helicity inferred from this momentum asymmetry

$\Rightarrow$ <u>positive</u> helicity for $\bar{\nu}_e$ (inferred)

Recall that for $m \to 0$ (or ultra-relativistic limit) the helicity states go over to the chiral ones:

$\vec{\sigma} \cdot \hat{p} = +1$ (positive) $\sim$ RH

$\vec{\sigma} \cdot \hat{p} = -1$ (negative) $\sim$ LH

$m_{\bar{\nu}} \approx 0 \Rightarrow$ "<u>anti</u>-neutrino is right-handed"

<u>Goldhaber</u>: "neutrinos are left-handed" (as we will see later on...)

The two experimental scenarios from the Wu measurement are also related by a parity symmetry $\Rightarrow$ the interaction which dominates this decay is not parity-conserving!

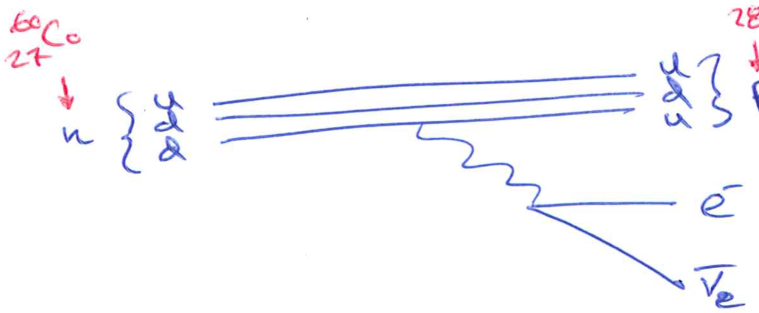
Recall from the theory part, that the vector interaction vertex from QED <u>is</u> parity-conserving ($j^\mu \sim \bar{u}\gamma^\mu u$)

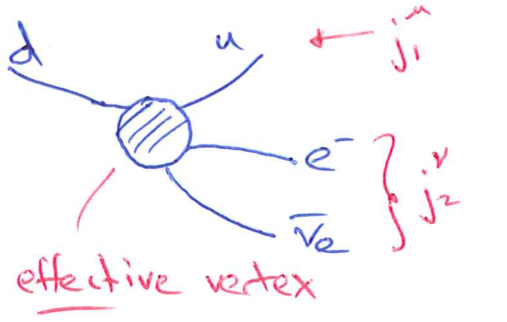
$\Rightarrow$ basic difference of weak interaction to QED (and QCD)

Question: what if we took $j_A^\mu \sim \bar{u}\gamma_5\gamma^\mu u$

... would this interaction violate parity?

- Measured quantities involve <u>squared</u> amplitudes
- Already at the level of the matrix element, we are considering an interaction of 2 <u>currents</u>:





suppose for both, we have

$$j^\mu_{V-A} \sim \bar{u}(\gamma^\mu - \gamma^\mu\gamma^5)u$$

or $\nu, \bar{\nu}$

insert g_V g_A to scale the relative strength (just numbers)

$$M \sim j^1 \cdot j^2 \sim \underbrace{g_V^2 + g_A^2}_{\text{conserves parity}} + \underbrace{g_V g_A + g_A g_V}_{\text{violates parity}}$$

$\Rightarrow \dfrac{2g_A g_V}{g_A^2 + g_V^2}$ gives the relative strength of parity-violation in relation to the parity-conserving part

- "maximal" for $|g_A| = |g_V|$
- vanishing for $g_A = 0$ <u>or</u> $g_V = 0$

The SM charged-current interaction (mediated by W) turns out to have equal weighting of the V and A contributions.

The neutral-current (mediated by Z) has weighting that depends on electric charge and <u>weak</u> <u>isospin</u>: ~~a new~~ quantum number.

Standard Model symmetries and particle content:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \qquad \text{discrete symmetries } C, P, T$$

like spin

<u>Spin</u>:
spin-0 $\rightarrow$ singlet (pion)
spin-$\frac{1}{2}$ $\rightarrow$ doublet $(e, p, n, \ldots)$ $\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$ ← by convention
spin-1 $\rightarrow$ triplet $(d, J/\psi, \rho, \ldots)$

Electroweak: experimentally there is a triplet: $\underbrace{W^-, W^0, W^+}_{\vec{W}}$

...and all quarks/leptons are observed to be either a singlet or member of a doublet

singlet

Take e^- as example, spinor $\psi_{e^-} = (P_R + P_L)\psi_{e^-} = e^-_R + e^-_L$

~~ ~~ part of doublet with LH neutrino

key point: L and R transform differently under SU(2) electroweak

$e^-_R \sim$ SU(2) singlet

$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \sim$ SU(2) doublet

Note ν_e on top (cf. ↑) unlike on the current wikipedia particle chart!

I.e., when L points "up" in SU(2) electroweak space, it represents ν_{eL} while "down" $\sim e^-_L$. Rotations in this space can produce transitions: $\nu_{eL} \leftrightarrow e^-_L$

(just like strong isospin: $p \leftrightarrow n$)

... except better!

Raising/lowering operators can be defined, like for angular momentum: but for e^-_R, electroweak transitions don't connect it to any other state.

Weak isospin: fermions with negative (LH) chirality have $T = \frac{1}{2}$ and can be grouped in doublets with $T_3 = \pm\frac{1}{2}$

e.g. $\begin{pmatrix} u \\ d \end{pmatrix}$ or $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{matrix} \leftarrow +\frac{1}{2} \\ \leftarrow -\frac{1}{2} \end{matrix}$

The right-handed ones are singlets: u_R, e^-_R, etc.

Anti-particles get the opposite sign of T_3 (as for other quantum numbers) ⇒ the antiparticle doublet is for RH and gets "flipped over".

RH particles and LH anti-particles have $T = 0$ (singlet)

To be properly discussed in the EW theory block:

- charged current ($W^\pm$) only interacts with doublet states
- neutral current (Z^0) interacts with everything

Be careful: SM unifies $\underset{\text{weak isospin}}{SU(2)_L} \times \underset{\text{weak hypercharge}}{U(1)_Y}$

and $\vec{W} = (W^-, \underset{T=1,\, T_3=0}{W^0}, W^+)$ mixes w/ B^0 $\quad T=0, T_3=0$

$\Rightarrow$ the Z^0 and γ (of QED) are observed below the electroweak scale of $v \approx 246\,\text{GeV}$

Electric charge: $Q = T_3 + \frac{1}{2}Y$ i.e., a mix of weak isospin and weak hypercharge

The correct vertex factors turn out to be:

charged current

$$-i\frac{g_W}{2}\underbrace{\frac{1}{2}\gamma^\mu(1-\gamma_5)}_{\gamma^\mu P_L}$$

neutral current

$$-i\frac{g_Z}{2}\cdot\gamma^\mu(c_V - c_A\gamma_5)$$

$c_L - c_R = T_3$

$c_L + c_R = T_3 - \cancel{2}Q\sin\theta_W$

Compare to $-iQ\gamma^\mu$ for QED...

Fortunately the parameter θ_W simplifies many things:

$$g_Z = \frac{g_W}{\cos\theta_W} = \frac{e}{\sin\theta_W\cos\theta_W} \quad \text{and} \quad m_W = m_Z \underbrace{\cos\theta_W}_{[-1,1]}$$

m_W: 80.4 GeV $\quad m_Z$: 91.2 GeV

In general, we can construct fermion currents via any of the Lorentz tensor structures that were introduced in the first theory block – limited options to maintain a Lorentz-invariant result. Propagator will connect currents, e.g.

QED, $\mathcal{M} \sim -Q_a Q_b (j_a \cdot j_b)/q^2 \rightarrow$ built out of scalar products

Classification by properties under Lorentz transformations and discrete symmetries:

Lorentz	name	parity	example
$\mathbb{1}$	scalar	$+$	mass, temperature
γ_5	pseudoscalar	$-$	helicity
γ^μ	vector	$+$	momentum, $\vec{E}$ field
$\gamma^\mu\gamma_5$	pseudovector	$-$	angular momentum, $\vec{B}$ field

General bilinear combination of 2 spinors:

or $v, \bar{v}$

$\bar{u}(p)\,\Gamma\,u(p) \longrightarrow$ e.g. $(\text{const.}) \times (\gamma^\mu \pm \gamma^\mu\gamma_5)$ $\begin{cases} V+A \\ V-A \end{cases}$

vertex factor drops ext. wavefns,

Γ ← 4x4 matrix built out of γ^μ, products, etc. above

Propagator: recall QED, $-i\dfrac{g_{\mu\nu}}{q^2}$ where the photon (γ) has spin-1 and $g_{\mu\nu}$ expresses a sum over $(2J+1)+1$ polarization states for virtual photons (since these are not fixed by observation).

4 degrees of freedom $\longleftrightarrow$ 4 components of j^μ

cf. scalar/pseudoscalar ~ spin-0

tensor ~ spin 2 $\Rightarrow$ 6 components

i.e., $\sum_\lambda \epsilon_\mu^{\lambda *}\epsilon_\nu^\lambda = -g_{\mu\nu}$ for virtual photons

(cf. spinor completeness relations from sheet 3, $\sum_s u_s\bar{u}$)

For weak interactions, it turns out the mediating boson is spin-1 with $m_W \neq 0$ $\Rightarrow$ additional degree of freedom for a longitudinal polarization state

EW: $\sum_\lambda \epsilon_\mu^{\lambda *}\epsilon_\nu^\lambda = -g_{\mu\nu} + \boxed{\dfrac{q_\mu q_\nu}{m_W^2}}$

can be shown that the corresponding term in QED does not contribute to matrix element

The Lorentz-invariant form of the matrix element for 2 vertices can also be derived via perturbation theory:

$$M \sim \frac{g_a g_b}{q^2 - m^2}$$

$\leftarrow$ 2 vertices

$\leftarrow$ propagator for virtual particle with mass m coupling them

Putting it together,

$$P_W = \frac{-i}{q^2 - m_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right) \xrightarrow{q^2 \ll m_W^2} \frac{-i g_{\mu\nu}}{q^2 - m_W^2} \xrightarrow{q^2 \ll m_W^2} \frac{i g_{\mu\nu}}{m_W^2} \sim G_F$$

for P_Z just replace $m_W \to m_Z$

<u>Now</u> $m_W \approx 80$ GeV and $m_\mu \approx 100$ MeV $\Rightarrow \frac{m_\mu^2}{m_W^2} \approx 10^{-6}$

$$P_{QED} \sim \frac{1}{q^2} \qquad P_W \sim \frac{1}{q^2 - m_W^2} \begin{cases} \text{high-E} \\ \text{low-E} \end{cases}$$

In the "high-energy" limit, m_W is not important in comparison to $q^2 \Rightarrow$ similar strength of QED vs EW

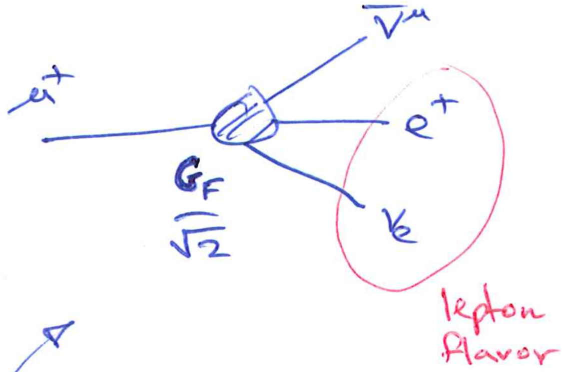
In "low-energy" limit, W-mediated interactions are suppressed by $q^4/m_W^4 \Rightarrow$ this is the sense in which it is "weak"

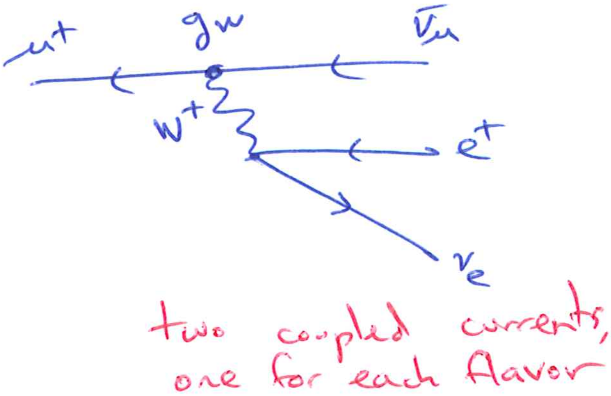
We now have the Feynman rules (vertex and propagator), noting that only LH particles (RH anti-particles) participate in charged-current interactions.

- unlike QED and QCD, <u>all</u> SM particles participate in weak interactions (ok, all the fermions...)
- every process appears consistent with a universal, dimensionless coupling: compare

$$\alpha_W = \frac{g_W^2}{\hbar c} \sim \frac{1}{30} \quad \text{vs} \quad \alpha_{QED} = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

Historically the weak interaction was understood bottom-up, i.e., decay via weak charged currents involved a nonlocal current-current coupling (4-fermion)





This is a convenient effective field theory (EFT) for charged-current weak interactions, and for defining G_F

(Note G_F was originally written for a pure vector current, so there is a factor of $\sqrt{2}$ to properly normalize the V-A theory.)

- V-A was an experimental conclusion

Matrix elements for leptons ($\ell \sim u_\ell$, etc. for $\ell \sim e, \mu, \tau$)

Effective (Fermi) Theory: $M_F = \frac{G_F}{\sqrt{2}} \left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right]\left[\bar{\nu}_{\ell'}\gamma^\mu(1-\gamma_5)\ell'\right]$

Standard Model: $M_{SM} = \frac{g_w^2}{8}\left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right] \cdot \frac{-i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_w^2}\right)}{q^2 - m_w^2} \cdot \left[\bar{\nu}_{\ell'}\gamma_\nu(1-\gamma_5)\ell'\right]$

(note the fermion currents are the same)

We might have expected $G_F \sim \frac{g_w^2}{m_w^2}$ from the propagator, but including the $\sqrt{2}$ and chiral projectors we get

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8 m_w^2} = \frac{1}{2v^2}$$

where again $v \simeq 246$ GeV is the electroweak scale or "Higgs vev"

Note that in the $q^2 \to 0$ limit, $M_{SM} \to M_F$ and we recover the direct current-current coupling of the Fermi theory as a low-energy effective coupling (interaction).

$\Lambda \sim 246$ GeV is a typical energy where the full Electroweak theory would be required...

Recall in natural units, $[L] = [E]^{-1}$ i.e.

high energy
&
short distance

... so we can also think of this as defining (vaguely) a resolution scale for the size of structures or range of interactions that can be resolved

The Fermi theory is not renormalizable ($\to$ later theory block)
but:

- QED corrections to the leading Fermi interaction are finite to all orders ($\Rightarrow$ we can separately parameterize the weak interaction and electromagnetic corrections)

- G_F includes (in one number!) all weak interaction effects in the low-energy EFT (full dynamics of W etc., insofar as they impact low-energy observables)

The muon lifetime, experimentally, is a powerful way to determine G_F

- no hadronic corrections until sub-ppm precision (via 2-loop QED) due to mass
- τ_μ well suited to precise experimental measurement
- clear theory interpretation

$\mu^\pm \to e^\pm \overset{(-)}{\nu_e} \overset{(-)}{\nu_\mu}$

The decay rate is

corrections

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 + \sum_i \Delta q_i\right)$$

$\Delta q_0 \sim$ phase space
$\Delta q_1 \sim 1^{st}$ order QED
$\Delta q_2 \sim 2^{nd}$ order QED
etc.

Sargent's rule: weak interactions have rate $\propto Q^5$

available energy

mass

we know $[\Gamma] \sim [E] \sim [M]^{+1}$
$[\tau] \sim [M]^{-1}$

matrix element

$$\left.\begin{array}{l}\Gamma \propto G_F^2 \text{ from } |M_F|^2 \\ [G_F] \sim [M]^{-2}\end{array}\right\} \Rightarrow G_F^2 Q^5 \sim [M]^{+1}$$

rest frame, $Q \sim m_\mu$