

Introduction to the (Electro-) Weak Interaction

Reminder: helicity vs. chirality

- Chiral eigenstates are Lorentz-invariant but are not stationary states under time evolution

$$\left. \begin{aligned} P_R &= \frac{1}{2}(1 + \gamma_5) \\ P_L &= \frac{1}{2}(1 - \gamma_5) \end{aligned} \right\} P_R + P_L = 1$$

- Helicity eigenstates are not Lorentz-invariant in general, but are stationary states of the motion

$$\vec{\Sigma} \cdot \hat{p} = \begin{pmatrix} \vec{\sigma} \cdot \hat{p} & 0 \\ 0 & \vec{\sigma} \cdot \hat{p} \end{pmatrix} = 2\vec{S} \cdot \hat{p}$$

↑ (here $\vec{S} \sim \frac{1}{2} \mathbb{1}_4 \otimes \vec{\sigma}$)

also written sometimes as $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$

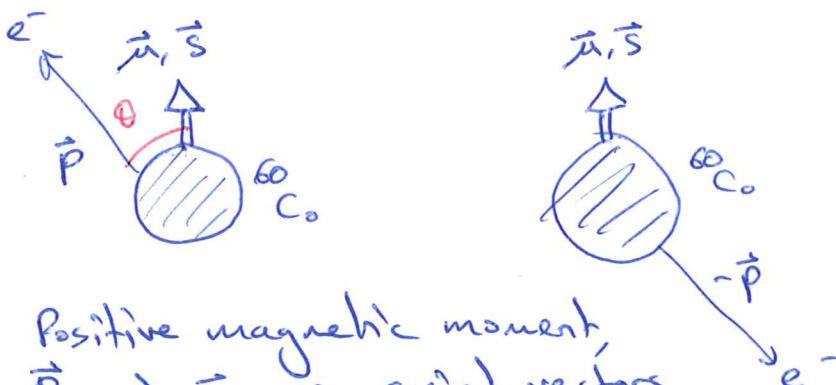
$$\begin{aligned} \Sigma^i &= \frac{1}{2} \epsilon_{ijk} \sigma^{jk} \\ &= \gamma_5 \gamma^0 \gamma^i \end{aligned}$$

do not confuse with relativistic 4-polarization: $s^\mu = (0, \vec{S})$ in rest frame here, $\frac{1}{2} \vec{\sigma} \uparrow$ (zero helicity)

Lorentz boost:

$$\begin{aligned} s^0 &\rightarrow \vec{\beta} \cdot \vec{S}, \text{ i.e. helicity} \\ \vec{S} &\rightarrow \vec{S} + \frac{\gamma^2}{\gamma+1} \vec{\beta} (\vec{\beta} \cdot \vec{S}) - \gamma \vec{\beta} s^0 \end{aligned}$$

Reminder: Wu experiment, 1957



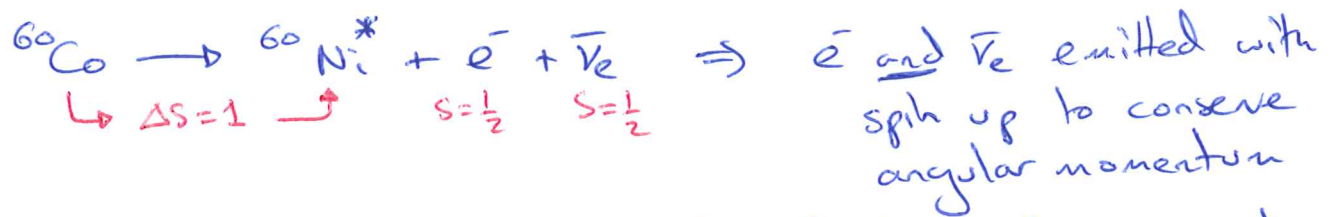
Positive magnetic moment, \vec{B} and $\vec{\mu}$ are axial vectors (pseudovectors)

Experimentally check the direction of e^- emission and look for asymmetry:

$$\begin{aligned} I(\theta) &\sim 1 + \alpha \left(\frac{\vec{\sigma} \cdot \vec{p}}{E} \right) \\ &= 1 + \alpha \frac{v}{c} \cos\theta \end{aligned}$$

Contrast/visibility: $\frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

^{60}Co has a spin ($S=5$) and can be polarized (at low T) in a \vec{B} field. $^{60}\text{Ni}^*$ has $S=4$



Observation: electrons emitted preferentially with momentum opposite the ^{60}Co spin direction

\Rightarrow negative helicity inferred from this momentum asymmetry

\Rightarrow positive helicity for $\bar{\nu}_e$ (inferred)

Recall that for $m \rightarrow 0$ (or ultra-relativistic limit) the helicity states go over to the chiral ones:

$$\vec{\sigma} \cdot \hat{p} = +1 \text{ (positive)} \sim \text{RH}$$

$$\vec{\sigma} \cdot \hat{p} = -1 \text{ (negative)} \sim \text{LH}$$

$m_{\bar{\nu}_e} = 0 \Rightarrow$ "anti-neutrino is right-handed"

Goldhaber: "neutrinos are left-handed" (as we will see later on...)

The two experimental scenarios from the Wu measurement are also related by a parity symmetry \Rightarrow the interaction which dominates this decay is not parity-conserving!

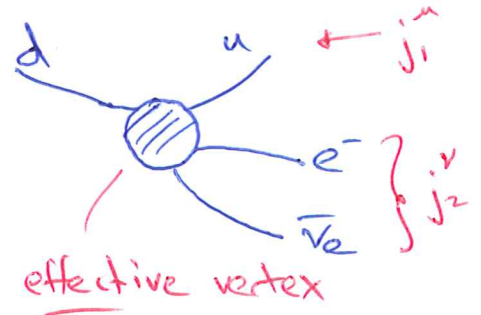
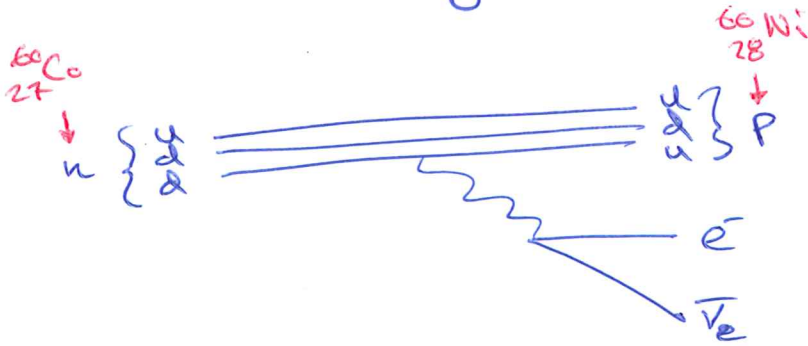
Recall from the theory part, that the vector interaction vertex from QED is parity-conserving ($j^\mu \sim \bar{u} \gamma^\mu u$)

\Rightarrow basic difference of weak interaction to QED (and QCD)

Question: what if we took $j_A^\mu \sim \bar{u} \gamma_5 \gamma^\mu u$

... would this interaction violate parity?

- Measured quantities involve squared amplitudes
- Already at the level of the matrix element, we are considering an interaction of 2 currents:



$$M \sim j_1 \cdot j_2 \sim \underbrace{g_V^2 + g_A^2}_{\text{conserves parity}} + \underbrace{g_V g_A + g_A g_V}_{\text{violates parity}}$$

$$\Rightarrow \frac{2g_A g_V}{g_A^2 + g_V^2}$$

gives the relative strength of parity-violation in relation to the parity-conserving part

- "maximal" for $|g_A| = |g_V|$
- vanishing for $g_A = 0$ or $g_V = 0$

The SM charged-current interaction (mediated by W) turns out to have equal weighting of the V and A contributions.

The neutral-current (mediated by Z) has weighting that depends on electric charge and weak isospin: a new quantum number.

Standard Model symmetries and particle content:

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

\uparrow
like spin

discrete symmetries C, P, T

- Spin:
- spin-0 \rightarrow singlet (π, η)
 - spin- $\frac{1}{2}$ \rightarrow doublet (e, μ, ν, \dots)
 - spin-1 \rightarrow triplet ($d, J/\psi, \rho, \dots$)

$\left(\begin{array}{c} \uparrow \\ \downarrow \end{array} \right) \leftarrow$ by convention

Electroweak: experimentally there is a triplet: $\underbrace{W^-, W^0, W^+}_{\vec{W}}$

... and all quarks/leptons are observed to be either a singlet or member of a doublet

Take e^- as example, spinor $\psi_{e^-} = (P_R + P_L)\psi_{e^-} = \bar{e}_R + \bar{e}_L$

key point: L and R transform differently under $SU(2)$ electroweak

singlet
part of doublet with LH neutrino

$e_R^- \sim SU(2)$ singlet

$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \sim SU(2)$ doublet

Note ν_e on top (cf. ↑) unlike on the current Wikipedia particle chart!

I.e., when L points "up" in $SU(2)$ electroweak space, it represents ν_{eL} while "down" $\sim e^-$. Rotations in this space can produce transitions: $\nu_{eL} \leftrightarrow e^-$
(just like strong isospin: $p \leftrightarrow n$)

... except better!

Raising/lowering operators can be defined, like for angular momentum: but for e_R^- , electroweak transitions don't connect it to any other state.

Weak isospin: fermions with negative (LH) chirality have $T = \frac{1}{2}$ and can be grouped in doublets with $T_3 = \pm \frac{1}{2}$

e.g. $\begin{pmatrix} u \\ d \end{pmatrix}$ or $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{matrix} \leftarrow +\frac{1}{2} \\ \leftarrow -\frac{1}{2} \end{matrix}$

The right-handed ones are singlets: u_R, e_R , etc.

Anti-particles get the opposite sign of T_3 (as for other quantum numbers) \Rightarrow the antiparticle doublet is for RH and gets "flipped over".

RH particles and LH anti-particles have $T=0$ (singlet)

To be properly discussed in the EW theory block:

- charged current (W^\pm) only interacts with doublet states
- neutral current (Z^0) interacts with everything

Be careful: SM unifies $SU(2) \times U(1)$
 weak isospin \leftarrow $SU(2)$
 weak hypercharge \leftarrow $U(1)$

and $\vec{W} = (W^-, W^0, W^+)$ mixes w/ B^0
 $T=1, T_3=0$ $T=0, T_3=0$

\Rightarrow the Z^0 and γ (of QED) are observed below the electroweak scale of $v \approx 246 \text{ GeV}$

Electric charge: $Q = T_3 + \frac{1}{2}Y$ i.e., a mix of weak isospin and weak hypercharge

The correct vertex factors turn out to be:

charged current

$$-i \frac{g_W}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1 - \gamma_5)$$

$\underbrace{\hspace{10em}}_{\gamma^\mu P_L}$

neutral current

$$-i \frac{g_Z}{2} \gamma^\mu (C_V - C_A \gamma_5)$$

$C_L + C_R = T_3 - Q \sin^2 \theta_W$ $C_L - C_R = T_3$

Compare to $-i Q \gamma^\mu$ for QED...

Fortunately the parameter θ_W simplifies many things:

$$g_Z = \frac{g_W}{\cos \theta_W} = \frac{e}{\sin \theta_W \cos \theta_W} \quad \text{and} \quad m_W = m_Z \cos \theta_W$$

\uparrow \uparrow \uparrow \uparrow
 80.4 GeV 91.2 GeV $E=1, 13$

In general, we can construct fermion currents via any of the Lorentz tensor structures that were introduced in the first theory block - limited options to maintain a Lorentz-invariant result. Propagator will connect currents, e.g.

QED, $M \sim -Q_a Q_b (j_a \cdot j_b) / q^2 \rightarrow$ built out of scalar products

Classification by properties under Lorentz transformations and discrete symmetries:

<u>Lorentz</u>	<u>name</u>	<u>parity</u>	<u>example</u>
$\mathbb{1}$	scalar	+	mass, temperature
γ_5	pseudoscalar	-	helicity
γ^μ	vector	+	momentum, \vec{E} field
$\gamma^\mu \gamma_5$	pseudovector	-	angular momentum, \vec{B} field

General bilinear combination of 2 spinors:
 vertex factor drops ext. wavefn's, $\left. \begin{array}{l} v+A \\ v-A \end{array} \right\}$
 $\bar{u}(p) \Gamma u(p) \rightarrow$ e.g. $(\text{const.}) \times (\gamma^\mu \pm \gamma^\mu \gamma_5)$
 \leftarrow 4×4 matrix built out of γ^μ , products, etc. above

Propagator: recall QED, $-i \frac{g_{\mu\nu}}{q^2}$ where the photon (γ) has spin-1 and $g_{\mu\nu}$ expresses a sum over $(2S+1)+1$ polarization states for virtual photons (since these are not fixed by observation).

4 degrees of freedom \leftrightarrow 4 components of γ^μ
 s. scalar/pseudoscalar \sim spin-0
 tensor \sim spin 2 \Rightarrow 6 components

i.e., $\sum_{\lambda} \epsilon_{\mu}^{\lambda*} \epsilon_{\nu}^{\lambda} = -g_{\mu\nu}$ for virtual photons
 (cf. spinor completeness relations from sheet 3, $\sum_s u_s \bar{u}_s$)

For weak interactions, it turns out the mediating boson is spin-1 with $m_W \neq 0 \Rightarrow$ additional degree of freedom for a longitudinal polarization state

EW: $\sum_{\lambda} \epsilon_{\mu}^{\lambda*} \epsilon_{\nu}^{\lambda} = -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{m_W^2}$ can be shown that the corresponding term in QED does not contribute to matrix element

The Lorentz-invariant form of the matrix element for 2 vertices can also be derived via perturbation theory:

$$M \sim \frac{g_a g_b}{q^2 - m^2} \leftarrow \begin{array}{l} \text{2 vertices} \\ \text{propagator for virtual particle} \\ \text{with mass } m \text{ coupling them} \end{array}$$

Putting it together,

$$P_W = \frac{-i}{q^2 - m_W^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2} \right) \xrightarrow{q^2 \ll m_W^2} \frac{-i g_{\mu\nu}}{q^2 - m_W^2} \xrightarrow{q^2 \ll m_W^2} \frac{i g_{\mu\nu}}{m_W^2} \sim G_F$$

\uparrow for P_Z just replace $m_W \rightarrow m_Z$

Now $m_W \approx 80 \text{ GeV}$ and $m_\mu \approx 100 \text{ MeV} \Rightarrow \frac{m_\mu^2}{m_W^2} \approx 10^{-6}$

$$P_{\text{QED}} \sim \frac{1}{q^2} \quad P_W \sim \frac{1}{q^2 - m_W^2} \quad \left. \begin{array}{l} \text{high-E} \\ \text{low-E} \end{array} \right\}$$

In the "high-energy" limit, m_W is not important in comparison to $q^2 \Rightarrow$ similar strength of QED vs EW

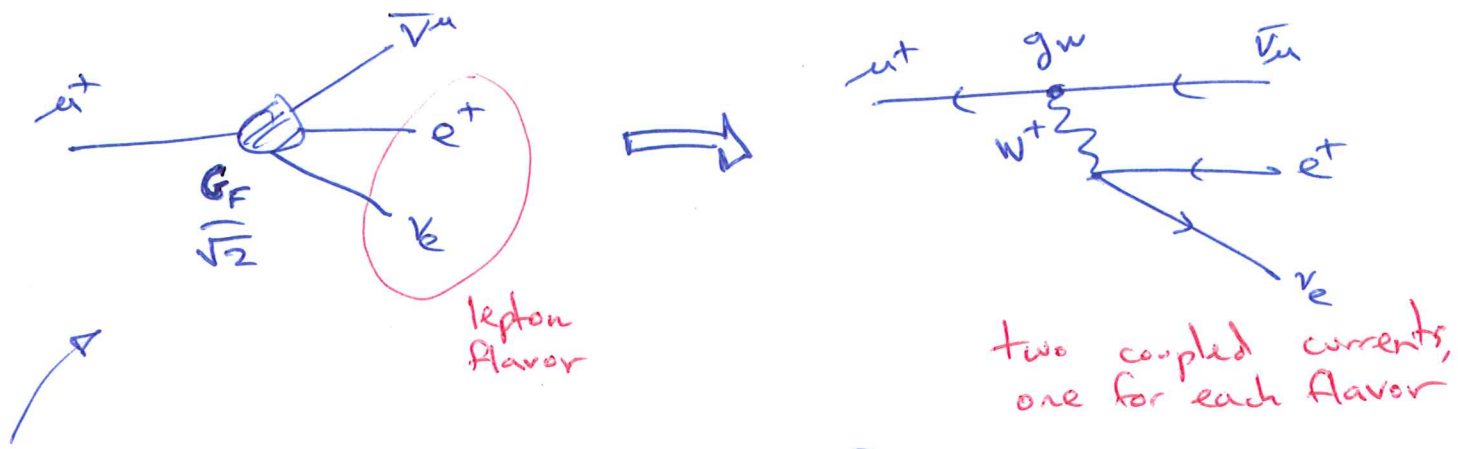
In "low-energy" limit, W -mediated interactions are suppressed by $\frac{q^4}{m_W^4} \Rightarrow$ this is the sense in which it is "weak"

We now have the Feynman rules (vertex and propagator), noting that only LH particles (RH anti-particles) participate in charged-current interactions.

- unlike QED and QCD, all SM particles participate in weak interactions (ok, all the fermions...)
- every process appears consistent with a universal, dimensionless coupling: compare

$$\alpha_W = \frac{g_W^2}{4\pi c} \sim \frac{1}{30} \quad \text{vs} \quad \alpha_{\text{QED}} = \frac{e^2}{4\pi c} \sim \frac{1}{137}$$

Historically the weak interaction was understood bottom-up, i.e., decay via weak charged currents involved a nonlocal current-current coupling (4-fermion)



This is a convenient effective field theory (EFT) for charged-current weak interactions, and for defining G_F (Note G_F was originally written for a pure vector current, so there is a factor of $\sqrt{2}$ to properly normalize the V-A theory.)

- V-A was an experimental conclusion

Matrix elements for leptons ($l \sim \mu, \tau$, etc. for $l \sim e, \mu, \tau$)

Effective (Fermi) theory:
$$M_F = \frac{G_F}{\sqrt{2}} [\bar{l} \gamma_\mu (1-\gamma_5) \nu_l] [\bar{\nu}_l \gamma^\mu (1-\gamma_5) l']$$

Standard Model:
$$M_{SM} = \frac{g_w^2}{8} [\bar{l} \gamma_\mu (1-\gamma_5) \nu_l] \frac{-i(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2})}{q^2 - m_W^2} [\bar{\nu}_l \gamma_\nu (1-\gamma_5) l']$$

(note the fermion currents are the same)

We might have expected $G_F \sim \frac{g_w^2}{m_W^2}$ from the propagator but including the $\sqrt{2}$ and chiral projectors we get

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8m_W^2} = \frac{1}{2v^2}$$

where again $v \approx 246$ GeV is the electroweak scale or "Higgs vev"

Note that in the $q^2 \rightarrow 0$ limit, $M_{SM} \rightarrow M_F$ and we recover the direct current-current coupling of the Fermi theory as a low-energy effective coupling (interaction).

$\Lambda \sim 246 \text{ GeV}$ is a typical energy where the full Electroweak theory would be required...

Recall in natural units, $[L] = [E]^{-1}$ i.e.

high energy
 \updownarrow
short distance

... so we can also think of this as defining (vaguely) a resolution scale for the size of structures or range of interactions that can be resolved

The Fermi theory is not renormalizable (\rightarrow later theory block) but:

- QED corrections to the leading Fermi interaction are finite to all orders (\Rightarrow we can separately parameterize the weak interaction and electromagnetic corrections)
- G_F includes (in one number!) all weak interaction effects in the low-energy EFT (full dynamics of W etc., insofar as they impact low-energy observables)

The muon lifetime, experimentally, is a powerful way to determine G_F

- no hadronic corrections until sub-ppm precision (via 2-loop QED) due to mass
- τ_μ well suited to precise experimental measurement
- clear theory interpretation



The decay rate is

$$\Gamma_{\mu}^{-1} = \frac{G_F^2 m_{\mu}^5}{192 \pi^3} \left(1 + \sum_i \Delta q_i \right)$$

corrections

$\Delta q_0 \sim$ phase space
 $\Delta q_1 \sim$ 1st order QED
 $\Delta q_2 \sim$ 2nd order QED
etc.

Sargent's rule: weak interactions have rate $\propto Q^5$

we know $[\Gamma] \sim [E] \sim [M]^{+1}$

$$[\Gamma] \sim [M]^{-1}$$

$$\Gamma \propto G_F^2 \text{ from } |M_F|^2$$

$$[G_F] \sim [M]^{-2}$$

$$\Rightarrow G_F^2 Q^5 \sim [M]^{+1}$$

rest frame, $Q \sim m_{\mu}$

↑
available energy

matrix element

mass