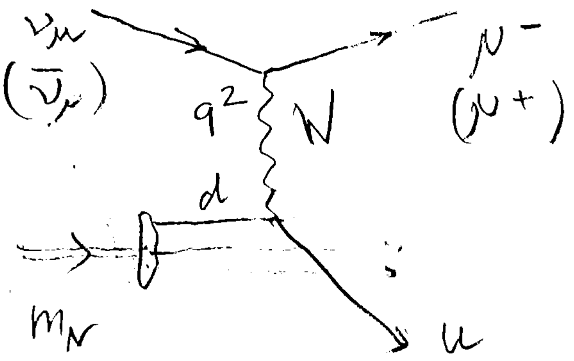
Neutrino scattering (Part 1)

Neutrino nucleon scattering played an important role to understand the weak interaction and also to resolve the proton and neutron structure in DIS:



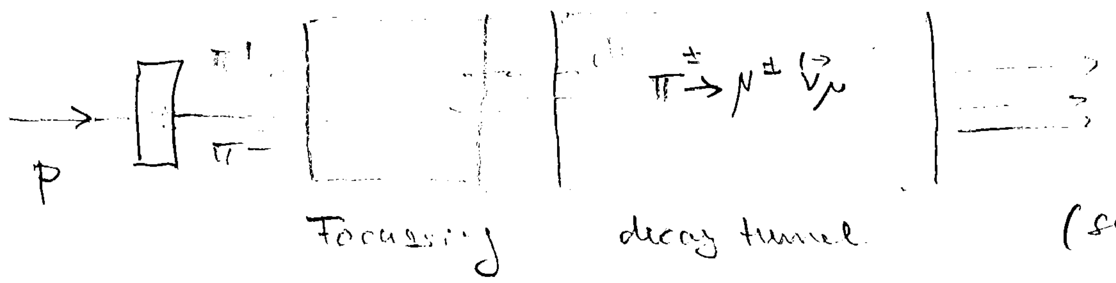
Fixed target configuration:

$$s = (E_\nu + m_N)^2 - E_\nu^2$$
$$= 2 m_N E_\nu - m_N^2$$
$$\simeq 2 m_N E_\nu \quad \text{for } E_\nu \gg m_N$$

Fundamental process: ν-Quark scattering, discussed in the following.

A few words on neutrino beams

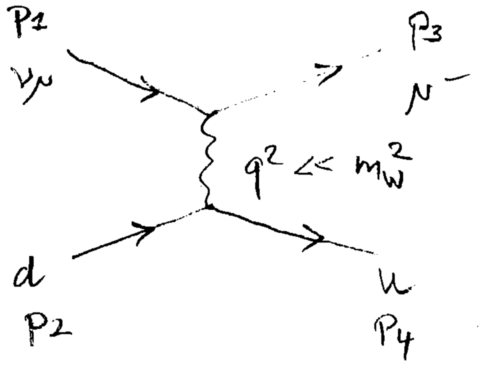
$\overset{(-)}{\nu}_\mu$ beams are produced from high-energy proton beams



(see slide)

Focussing is done with a so called „magnetic horn" (see slide)

ν – Quark scattering



Matrix element w/ $q^2 \ll m_W^2$: $\frac{1}{q^2 - m_W^2} \to \frac{1}{m_W^2}$

$$-iM_{fi} = \left[-i\frac{g}{\sqrt{2}}\, \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5) u(p_1) \right] \cdot \frac{i\, g_{\mu\nu}}{m_W^2} \cdot \left[-i\frac{g}{\sqrt{2}}\, \bar{u}(p_4)\gamma^\nu \frac{1}{2}(1-\gamma^5) u(p_2) \right]$$

$\frac{1}{2}(1-\gamma^5)$ projects out the LH spinor components

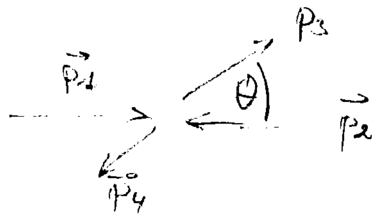
γ^μ coupling ensures that only $\bar{u}_L \gamma^\mu u_L$ term contribute.

If we neglect the particle masses : $u_L = u_\downarrow$ i.e. chiral states = helicity state

$$\Rightarrow M_{fi} = \frac{g^2}{2 m_W^2} \cdot \underbrace{\left[\bar{u}_\downarrow(p_3)\, \gamma_\mu\, u_\downarrow(p_1) \right]}_{j_{e\mu}} \underbrace{\left[\bar{u}_\downarrow(p_4)\, \gamma^\mu\, u_\downarrow(p_2) \right]}_{j_q^\mu}$$

Using the Dirac-Pauli representation of the γ-matrices and of the spinors (see exercise sheets 2 and 3) :

$$\boxed{\begin{aligned} u_\downarrow(p_1) &= \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \quad (\varphi_1 = 0,\ \theta_1 = 0) & u_\downarrow(p_2) &= \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \varphi_2 = \pi,\ \theta_2 = \pi \\ u_\downarrow(p_3) &= \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix} \quad \varphi_3 = 0,\ \theta_3 = \theta & u_\downarrow(p_4) &= \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix} \quad \varphi_4 = \pi,\ \theta_4 = \pi - \theta \end{aligned}}$$



in CMS, $m_i \approx 0$:
$|\vec{p}_i| = |E_i| = E$
$\hat{s} = 4E^2$
with $s = \sin\theta$
$c = \cos\theta$

see slide

one finds : $j_e^\mu = \bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) = 2E(c, s, -is, c)$

$$j_q^\mu = \bar{u}_\downarrow(p_4)\gamma^\mu u_\downarrow(p_2) = 2E(c, -s, -is, -c)$$

$$\Rightarrow M_{fi} = \frac{g^2}{2 m_W^2} \cdot 4E^2 (c^2 + s^2 + s^2 + c^2) = \frac{g^2}{m_W^2} \cdot \hat{s}$$

For the cross section we need to average M_{fi} over all initial spin configurations and sum of all possible final spin configs:

Initial: ν is always LH; quarks can be LH, RH however only LH contribute. Averaging $\rightarrow \frac{1}{2}$

final: only LH configuration contribute.

$$\Rightarrow \langle |M_{fi}|^2 \rangle = \frac{1}{2} \cdot |M_{fi}|^2 = \frac{1}{2} \left(\frac{g^2}{M_W^2} \hat{s} \right)^2$$

With formula for diff. cross section:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{s} \langle |M_{fi}|^2 \rangle \Rightarrow \frac{d\sigma_{\nu q}}{d\Omega_\mu} = \frac{1}{64\pi^2} \cdot \frac{1}{\hat{s}} \cdot \frac{1}{2} \left(\frac{g^2}{m_W^2} \right)^2 \hat{s}^2$$

$$\Rightarrow \frac{d\sigma}{d\Omega_\mu} = \frac{G_F^2}{4\pi^2} \cdot \hat{s} \qquad (\text{in CMS system}).$$

with $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$

Total cross section (integration over 4π):

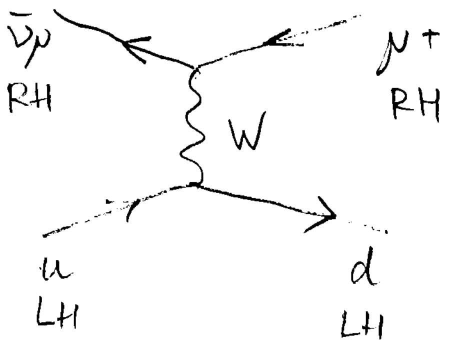
$$\sigma_{\nu q} = \frac{G_F^2}{\pi} \cdot \hat{s}$$

For very large $\hat{s}$, i.e. at large E_ν result violates unitarity: Used „effective Theorie" is not valid anymore. We should have used „full theory" with Propagator $\frac{1}{q^2 - M_W^2}$

→ additional factor $\frac{1}{\hat{s}^2}$ which cures the unitarity problem.

→ Fermi's Theory w/ 4-fermion interaction is not valid anymore and we need a theory with „propagating W's" (i.e. SM)

$\bar{\nu}_\mu$ - quark scattering



$$\mathcal{M}_{fi} = \frac{g^2}{2 m_W^2} \cdot \underbrace{\left[\bar{v}_\uparrow(p_3)\gamma_\mu v_\uparrow(p_1)\right]}_{j_{e,\mu}} \underbrace{\left[\bar{u}_\downarrow(p_4)\gamma^\mu u_\downarrow(p_2)\right]}_{j_{q_1}^{\mu}}$$

using the anti-spinors:

$$v_\uparrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \qquad v_\uparrow(p_3) = \sqrt{E}\begin{pmatrix} s \\ -c \\ s \\ c \end{pmatrix} \qquad s = \sin\frac{\theta}{2}, \quad c = \cos\frac{\theta}{2}$$

$\varphi_1 = 0 \quad \theta_1 = 0 \qquad\qquad \varphi_3 = \pi \quad \theta_3 = \theta$

one finds in analogy: $\mathcal{M}_{fi} = \frac{g^2}{m_W^2} \cdot \frac{1}{2}(1+\cos\theta)\,\hat{s}$

or $\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{1}{2}\left(\frac{g^2}{m_W^2}\right)^2 \cdot \frac{1}{4}(1+\cos\theta)^2\,\hat{s}^2$

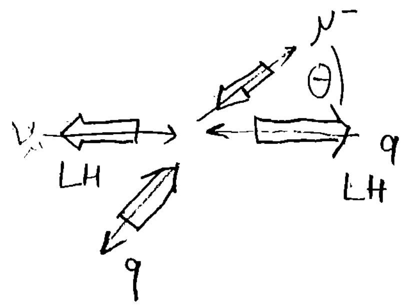
diff. cross section:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega_\mu} = \frac{G_F^2}{4\pi^2}\cdot\hat{s}\cdot\underbrace{\frac{1}{4}(1+\cos\theta)^2}_{\text{integration } \int_\Omega \cdots = \frac{4\pi}{3}} \qquad \text{(in CMS)}$$

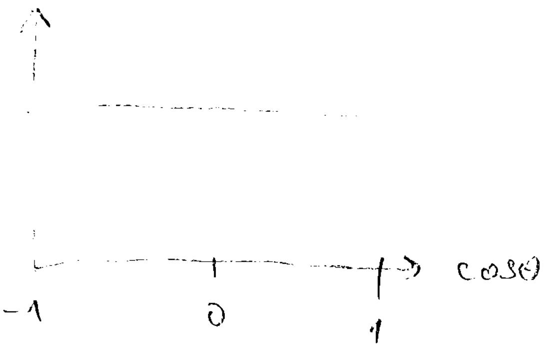
$$\Rightarrow \quad \sigma_{\bar{\nu}q} = \frac{G_F^2}{4\pi^2}\,\hat{s}\cdot\frac{4\pi}{3} = \frac{G_F^2\cdot\hat{s}}{3\pi} = \frac{1}{3}\,\sigma_{\nu q}$$

At the first view, this result is surprising.
Becomes clear if one analyses the spin configuration

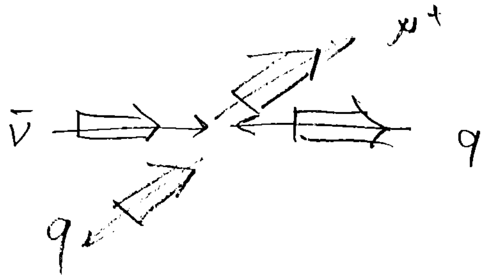
νq - scattering



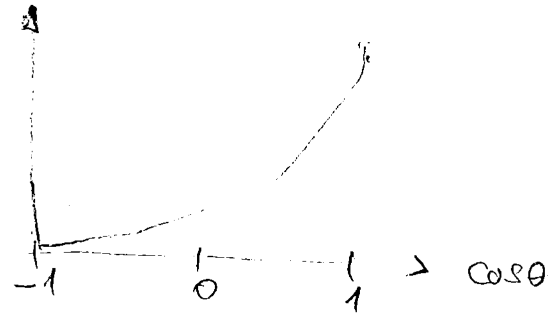
total spin = 0
flat $\cos\theta$ - distribution of outgoing particles



$\bar{\nu} q$ - scattering



total spin = 1
angular distribution follows
$\sim (1+\cos\theta)^2$



ν - antiquark scattering

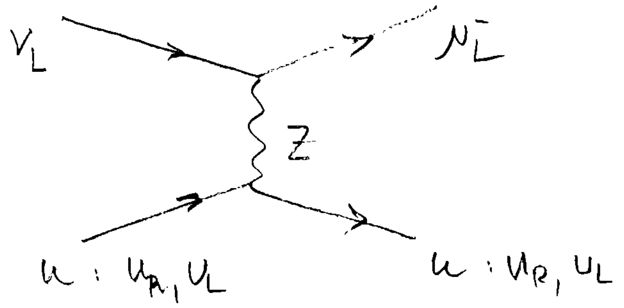
In case of $\nu\bar{q}$ and $\bar{\nu}\bar{q}$ one can easily convince oneself that the roles are exchanged

$$\Rightarrow \begin{cases} \dfrac{d\sigma_{\nu\bar{q}}}{d\Omega_\mu} = \dfrac{G_F^2}{4\pi^2} \cdot \hat{s} \cdot \dfrac{1}{4}\left(1+\cos\theta\right)^2 \\[2ex] \dfrac{d\sigma_{\bar{\nu}\bar{q}}}{d\Omega_\mu} = \dfrac{G_F^2}{4\pi^2} \cdot \hat{s}. \end{cases}$$

Measured $\nu N / \bar{\nu} N$ cross section ratio (s. slide) shows ~ 0.5 instead of $1/3$ $\Rightarrow$ evidence that nucleons also contain anti-quarks!

„Neutral current" ν-scattering

One of the predictions of the SM is the NC (i.e. Z exchange) ν-scattering process:



Z couples to LH and RH quarks with different strength:

$$C_L^q = \frac{1}{2}\left(C_V^q + C_A^q\right)$$
$$C_R^q = \frac{1}{2}\left(C_V^q - C_A^q\right)$$

Reminder: $C_V = T_3 - 2Q\sin^2\theta_W \qquad C_A = T_3 \qquad \sin^2\theta_W \approx 0.231$

	T_3	Q	C_V	C_A	C_L	C_R
u-type	$+1/2$	$+2/3$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W \approx +0.19$	$+1/2$	≈ 0.35	≈ -0.15
d-type Quarks	$-1/2$	$-1/3$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \approx -0.35$	$-1/2$	≈ -0.42	$\approx +$

We should thus discuss to different spin configurations:

ν (LH) — q (LH) $\sim g_L^2$ gives flat angular distribution.

ν (LH) — q (RH) $\sim g_R^2$ gives angular distr. $\sim \frac{1}{4}(1+\cos\theta)^2$

Instead of redoing the calculation one can „adapt" the CC-result.

$$\frac{d\sigma_{\nu q}^{NC}}{d\Omega_\mu} = \frac{G_F^2}{4\pi^2}\cdot\hat{s}\cdot\left[\left(g_L^q\right)^2 + \frac{1}{4}\left(g_R^q\right)^2\left(1+\cos\theta\right)^2\right]$$

$$\Rightarrow \quad \sigma_{\nu q}^{NC} = \frac{G_F^2}{\pi}\hat{s}\left[\left(g_L^q\right)^2 + \frac{1}{3}\left(g_R^q\right)^2\right]$$

Remarks: - Observation of ν-scattering w/o $\mu^\pm$ confirms Z exchange predicted by SM

$$- \; R = \frac{\sigma_{\nu q}^{NC}}{\sigma_{\nu q}^{CC}} \sim \left(g_L^q\right)^2 + \frac{1}{3}\left(g_R^q\right)^2 \sim f\left(\sin^2\theta_W\right)$$

→ see slide: Discovery of NC