

Discovery of the W- and Z-boson (recap)

https://cds.cern.ch/record/2103277/files/9789814644150_0006.pdf

Historical situation at the end of the 1970:

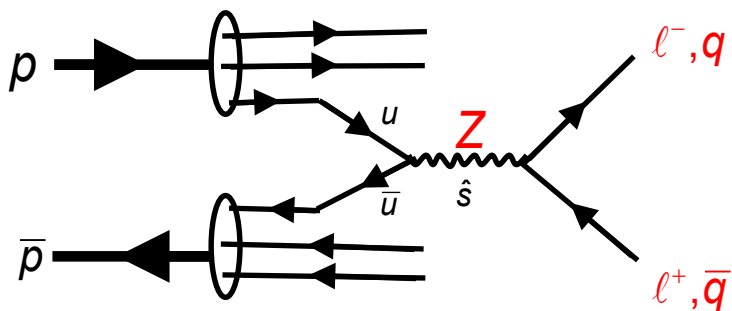
- Standard Model unifying the electromagnetic and the weak interaction was developed in the course of the 1960s (S. Glashow (1959), A. Salam (1959), S. Weinberg (1967)): Theory predicts massive W and Z bosons.
- Experimental evidence in favor of a unique description of the weak and electromagnetic interactions was obtained in 1973, with the observation of neutral current neutrino interaction which could only be explained by the exchange of a virtual heavy neutral particle. The measurements allowed a first estimation of $\sin^2\theta_w$ and together with the coupling G_F of the muon decay an estimation of the masses of the W and the Z-boson:

$$m_W \approx 60\dots 80 \text{ GeV} \quad m_Z \approx 75\dots 92 \text{ GeV}$$

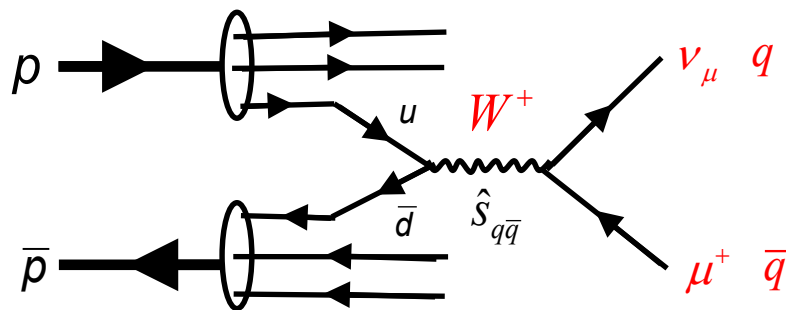
Masses too large to be accessible by any accelerator in operation at that time.

In 1976 Rubbia, Cline and McIntyre proposed the transformation of an existing high-energy proton accelerator (SPS) into a proton–antiproton collider (Sp \bar{p} S) as a quick and cheap way to achieve collisions above thresholds for W and Z.

$$p\bar{p} \rightarrow Z \rightarrow f\bar{f} + X$$



$$p\bar{p} \rightarrow W \rightarrow \ell \bar{\nu}_\ell + X$$



Colliding quarks (at the envisaged CMS energy dominantly valence quarks) carry momentum fraction x of the proton/antiproton momentum w/ $\langle x \rangle \approx 0.17$.

To achieve a quark-antiquark CMS energy of ~ 90 GeV proton-antiproton CMS energies of ~ 540 GeV are necessary: $\sqrt{\hat{s}_{q\bar{q}}} = x \cdot \sqrt{s_{p\bar{p}}} \rightarrow 90 \text{ GeV} \approx 0.17 \cdot 540 \text{ GeV}$

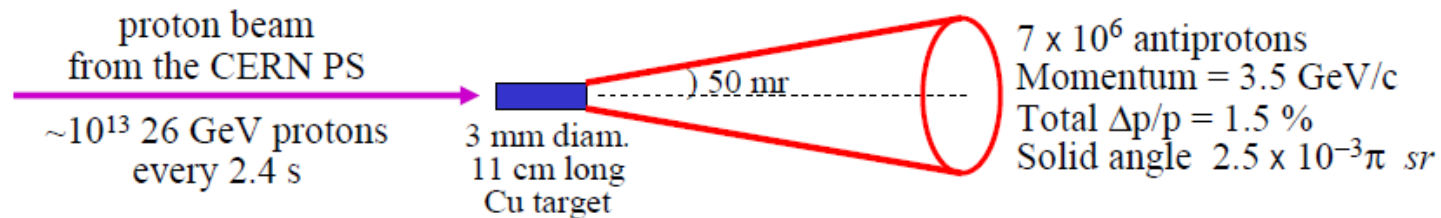
For a luminosity of $L \approx 2.5 \times 10^{29} \text{ cm}^{-2} \text{ s}^{-1}$ one expects typically only one $Z \rightarrow e\bar{e}$ event per day: $\sigma(pp \rightarrow Z \rightarrow e^+e^-) \approx 50 \text{ pb} = 5 \cdot 10^{-35} \text{ cm}^{-2}$ w/ $\text{BR}(Z \rightarrow e\bar{e}) \approx 3\%$.

Beside the energy the luminosity thus was a challenge to measure sufficient Z, Ws!

Antiproton beam

Antiproton source must be capable to deliver daily $\sim 3 \times 10^{10}$ anti-protons distributed in few (3–6) tightly collimated bunches within the angular and momentum acceptance of the CERN SPS.

CERN 26 GeV proton synchrotron (PS) is capable of producing antiprotons at the desired rates:

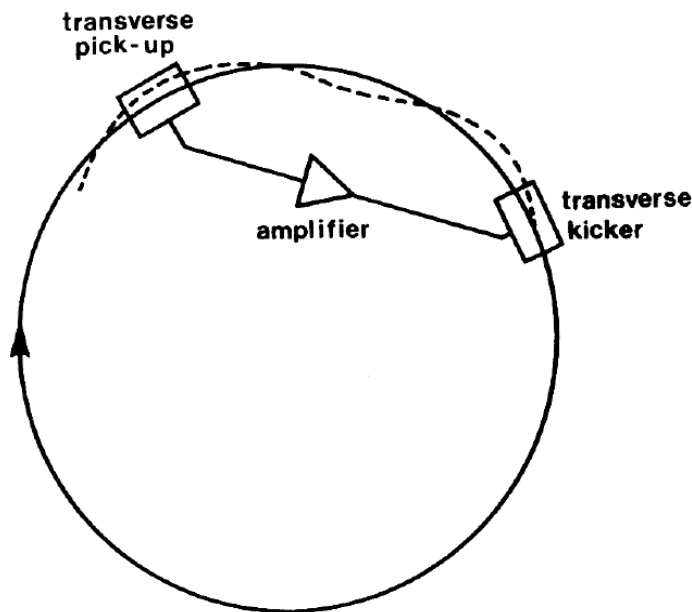


Sufficient antiprotons but they occupy a phase space volume which is too large by a factor $\geq 10^8$ to fit into the SPS acceptance, even after acceleration to the SPS injection energy of 26 GeV (emittance of the beam is too large!).

→ Increase the antiproton phase space density at least 10^8 times before sending the antiproton beam to the SPS. This process is called “cooling” (in analogy to a hot gas where the particles have very different momenta)

Stochastic cooling (S. van der Meer, 1972)

Reminder: Liouville theorem forbids any compression of phase volume by conservative forces such as electromagnetic fields → emittance (defining the area of the phase space ellipse) cannot be reduced by fields acting on all particles of the beam. Need to act on individual (or group of individual) particles.



Idea to cool betatron oscillation:

Measure deviation of group of particles at point where maximal. Pick-up signal proportional to deviation is amplified and provided to kicker at a place where particle crosses central orbit.

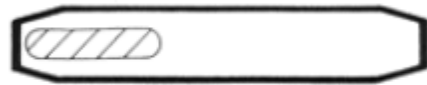
Pick-up is sensitive only to a group of particles (depends on geometry and on frequency response)

Cooling was performed in the Antiproton Accumulator AA: includes several independent cooling systems to cool horizontal and vertical oscillations, and also to decrease the beam momentum spread (cooling of longitudinal motion).

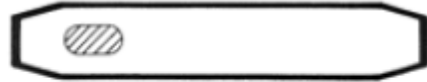


AA - a large aperture ring of different magnets - during construction.

Cooling and injection cycle:



The first pulse of $7 \times 10^6 \bar{p}$ has been injected into the AA vacuum chamber



Precooling has reduced the momentum spread



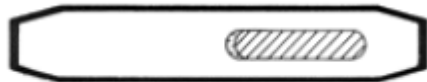
The first pulse has been moved to the stack region



The second pulse is injected 2.4 s later



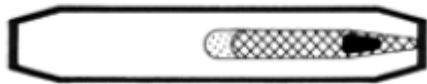
After precooling, the second pulse is added to the stack



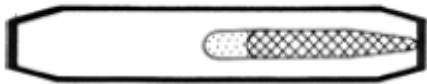
After 15 pulses the stack contains $10^8 \bar{p}$



After 1 hour a dense core is present in the stack



After 1 day the core contains enough \bar{p} for transfer to the SPS



The remaining \bar{p} are used to begin next day's accumulation

When a sufficiently dense anti-proton stack has been accumulated in the AA, beam injection into the SPS is achieved using consecutive PS cycles.

Firstly, three proton bunches (six after 1986), each containing $\sim 10^{11}$ protons, are accelerated to 26 GeV in the PS and injected into the SPS. Then three p bunches (six after 1986), of typically $\sim 10^{10}$ antiprotons each, are extracted from the AA and injected into the PS accelerated to 26 GeV and injected into SPS.

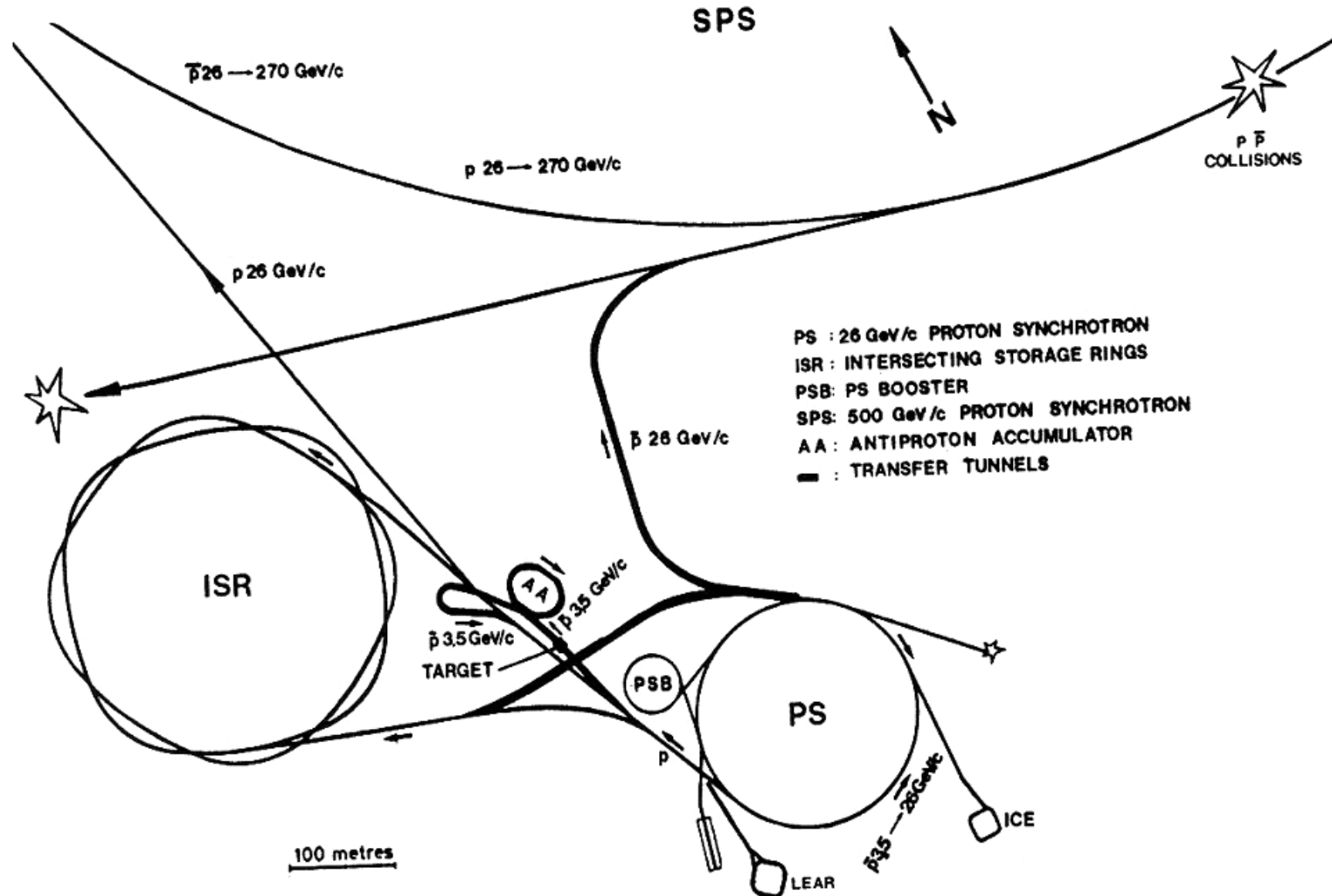
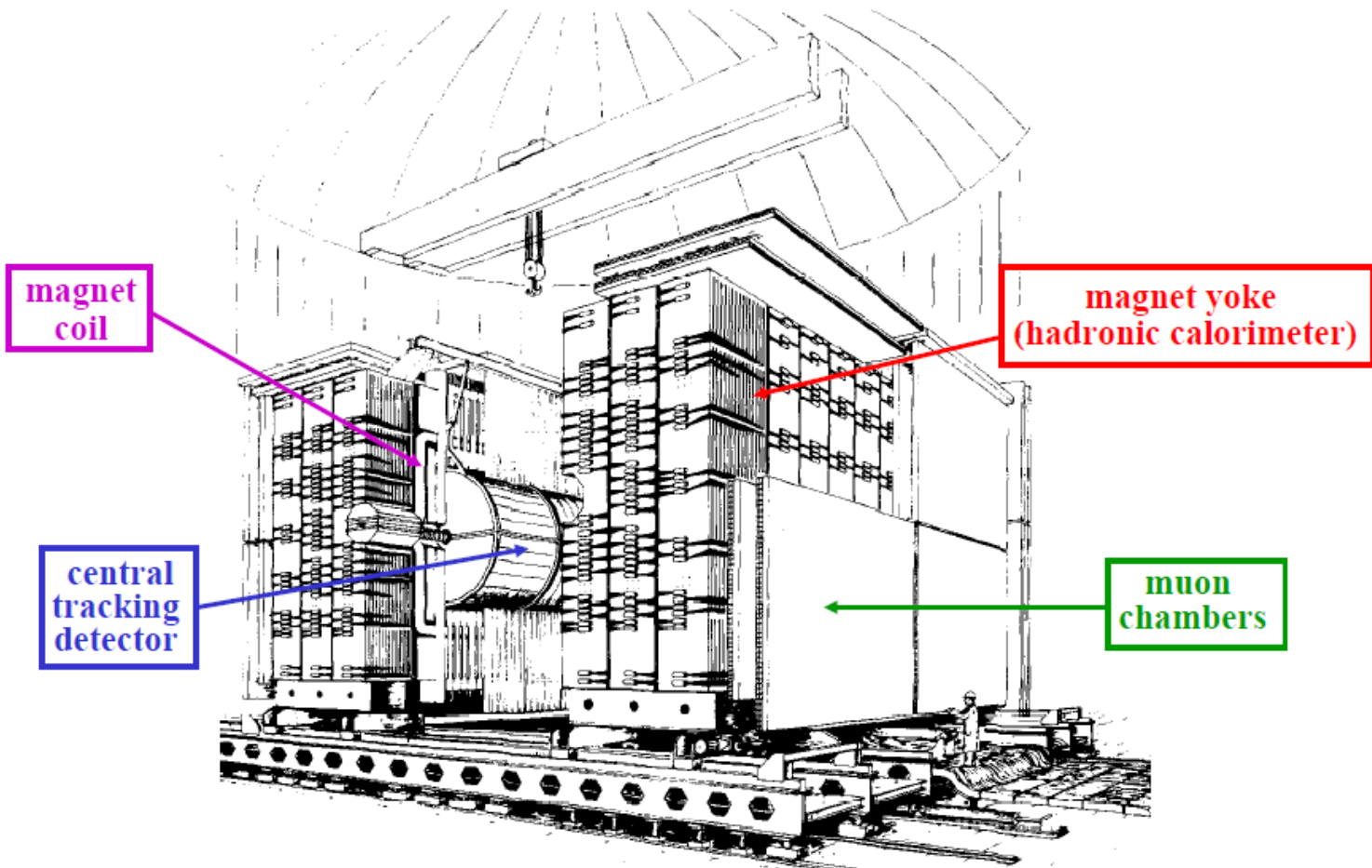


Table 1 CERN proton–antiproton collider operation, 1981–1990.

Year	Collision energy (GeV)	Peak luminosity ($\text{cm}^{-2} \text{ s}^{-1}$)	Integrated luminosity (cm^{-2})
1981	546	$\sim 10^{27}$	2×10^{32}
1982	546	5×10^{28}	2.8×10^{34}
1983	546	1.7×10^{29}	1.5×10^{35}
1984–85	630	3.9×10^{29}	1.0×10^{36}
1987–90	630	3×10^{30}	1.6×10^{37}

Two Detectors: UA1 and UA2

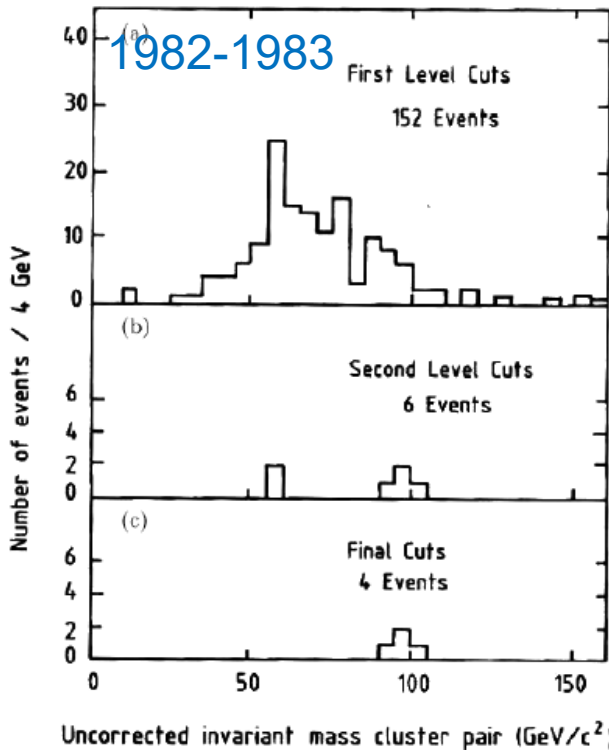
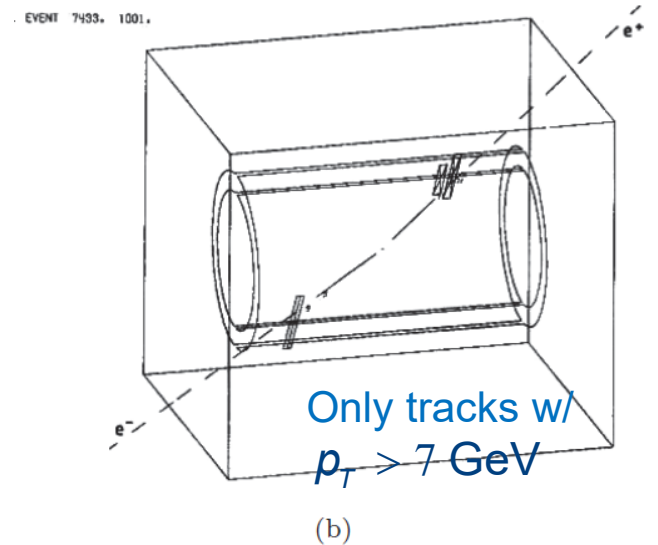
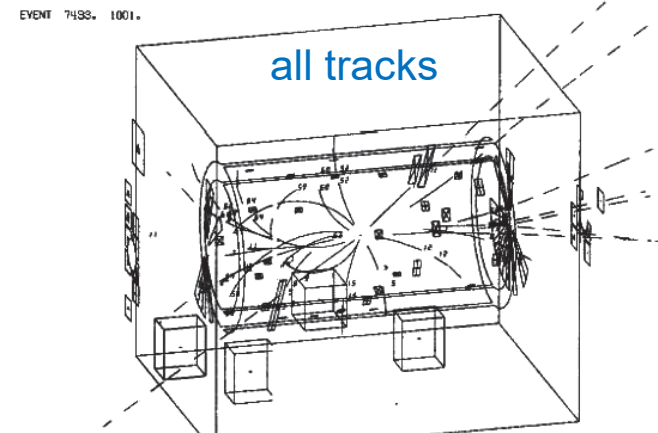
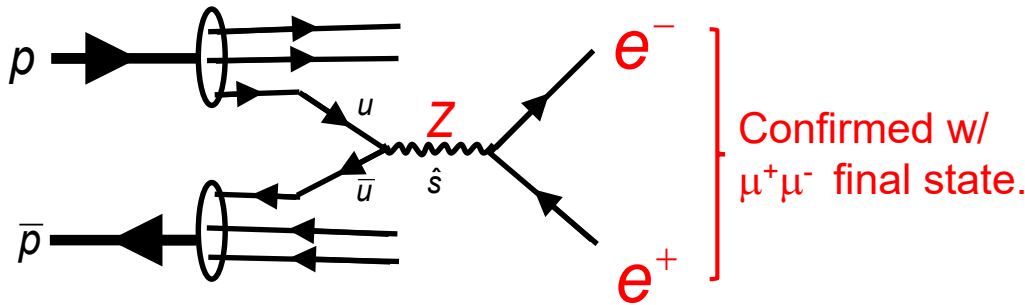
UA1 detector



(shown with the two halves of the dipole magnet opened up)

Discovery of the Z-boson

(historically W discovery was a few weeks earlier)



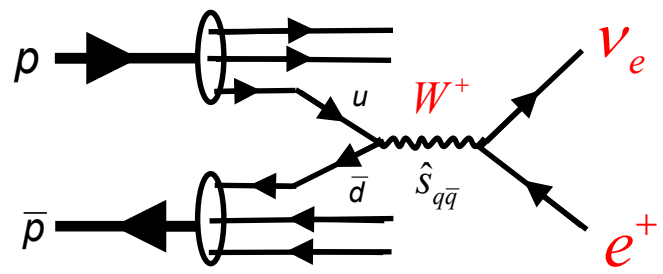
2 e.m. clusters w/
 $E_T > 25 \text{ GeV}$

isolated track w/
 $p_T > 7 \text{ GeV}$

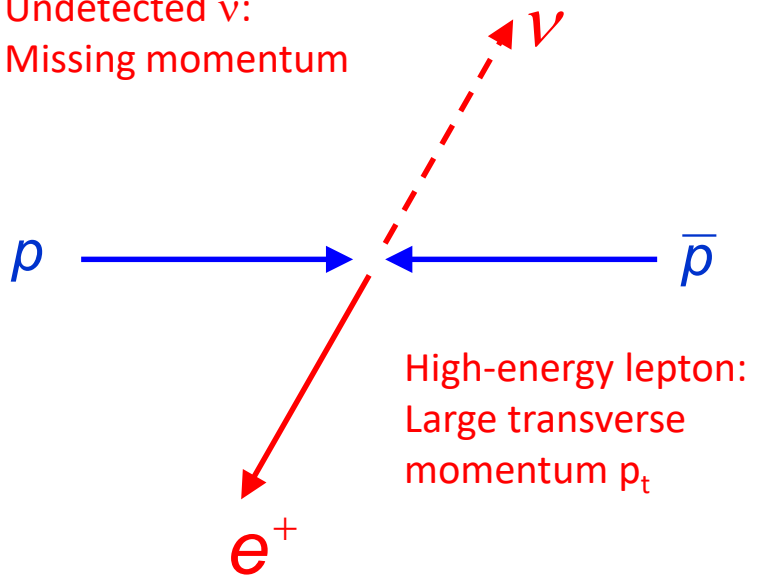
2 tracks w/
 $p_T > 7 \text{ GeV}$
Pointing to clusters

Invariant mass: $m_Z = 95.2 \pm 2.5 \pm 3.0 \text{ GeV}$

Discovery of the W-boson



Undetected ν :
Missing momentum



How can the W mass be reconstructed ?

$$W^- \rightarrow e^- \bar{\nu}$$

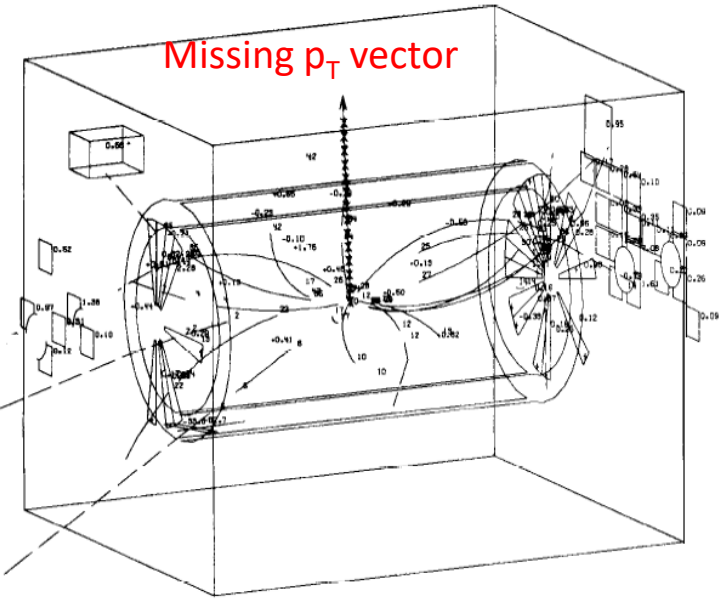


Fig. 16a. Event of the type $W^- \rightarrow e^- + \bar{\nu}_e$. All tracks and calorimeter cells are displayed.

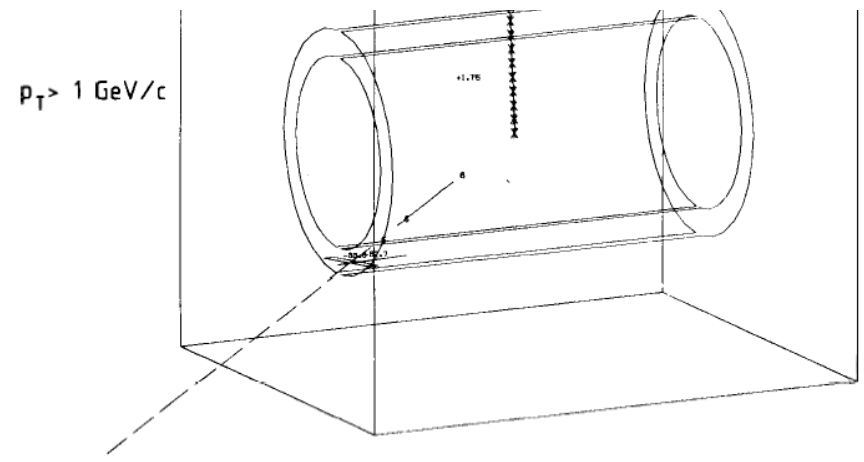


Fig. 16b. The same as picture (a), except that now only particles with $p_T > 1$ GeV/c and calorimeters with $E_T > 1$ GeV are shown.

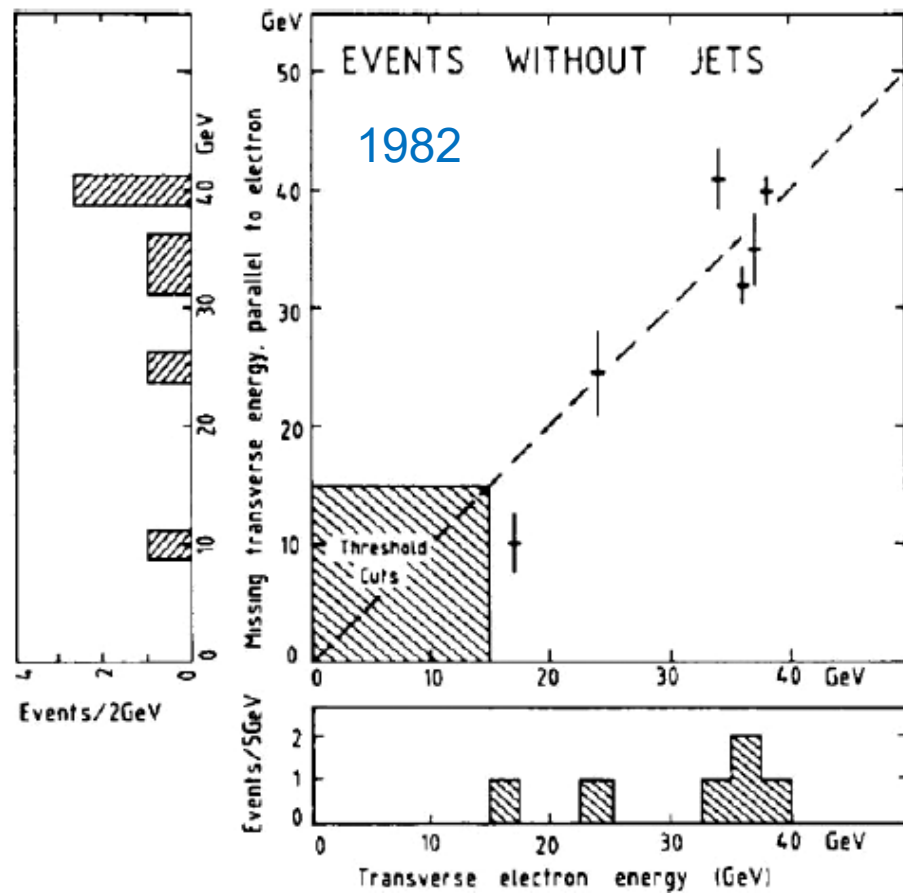
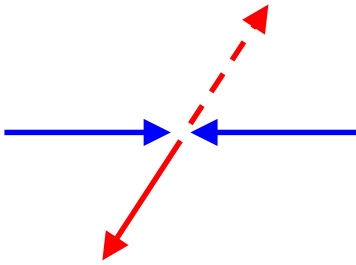


Fig. 11. UA1 scatter plot of all the events from the 1982 data which contain a high- p_T electron and large $|\vec{p}_T^{\text{miss}}|$. The abscissa is the electron $|\vec{p}_T|$ and the ordinate is the \vec{p}_T^{miss} component antiparallel to the electron \vec{p}_T .

W mass measurement



In the W rest frame:

- $|\vec{p}_\ell| = |\vec{p}_\nu| = \frac{M_W}{2}$
- $|p_\ell^T| \leq \frac{M_W}{2}$

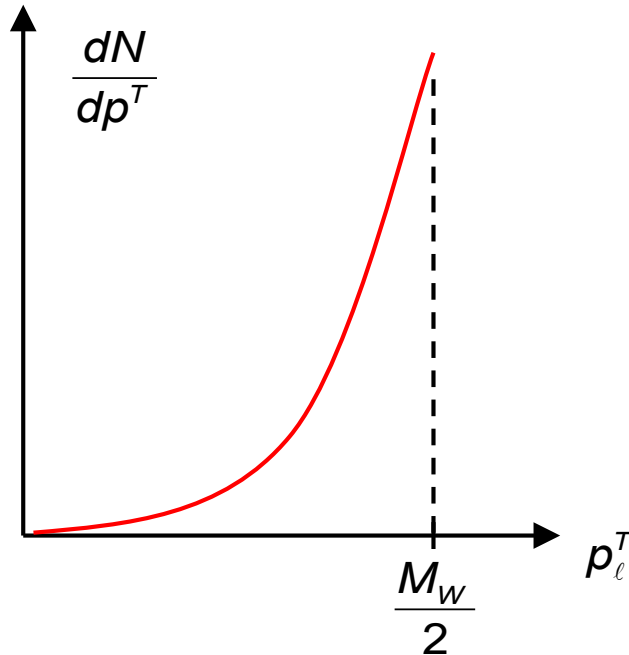
In the lab system:

- W system boosted only along z axis
- p_T distribution is conserved

Jacobian Peak:

$$\frac{dN}{p_T} \sim \frac{2p_T}{M_W} \cdot \left(\frac{M_W^2}{4} - p_T^2 \right)^{-1/2}$$

(assumes flat distribution of the electron – not correct, see next page)

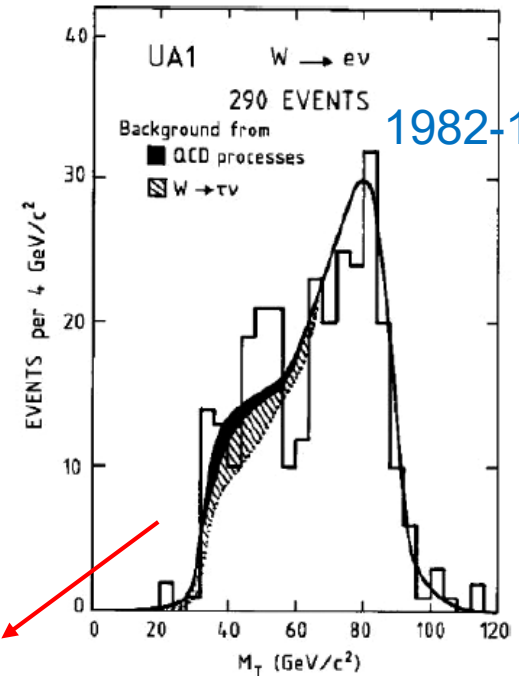


Transverse Mass M_T
 $M_T = 2p_T$

Additional effects:

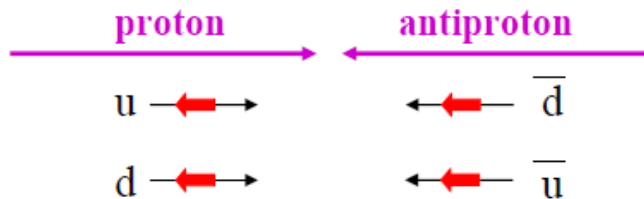
- trans. movement of the W
- Finite W decay width
- W decay not flat

$$m_W = 82.7 \pm 1.0 \pm 2.7 \text{ GeV,}$$

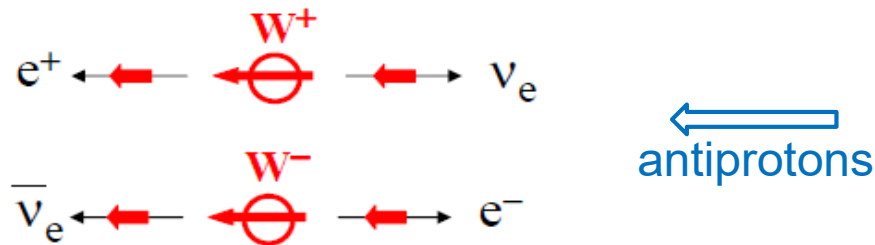


V-A coupling of the W-boson

Because of V-A coupling W bosons polarized in direction of antiproton beam:



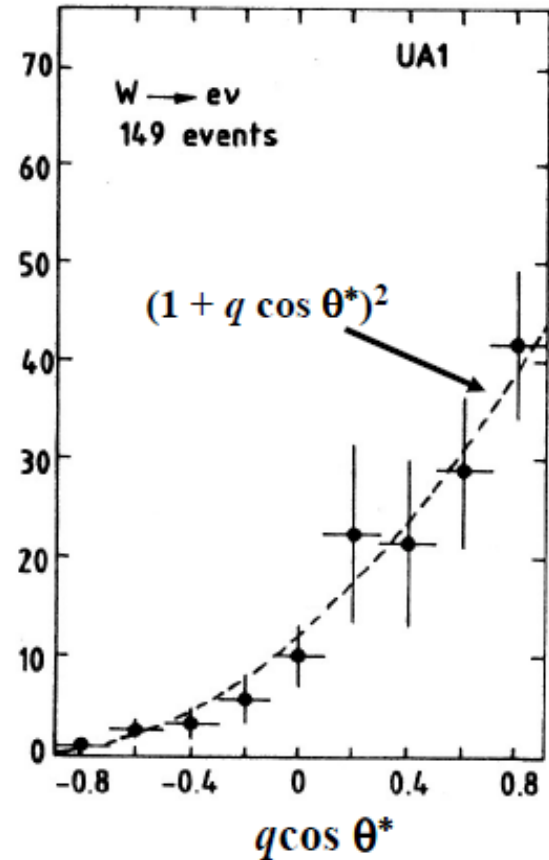
Decay to $e\nu$ in W-rest frame:



Electron (positron) angular distribution in lab:

$$\frac{dN}{d\cos\theta^*} \propto (1 + q \cos\theta^*)^2$$

$q = +1$ for positrons, -1 for electrons
 θ^* = angle w/r to antiproton directions



Confirms V-A coupling to quarks and leptons: charge asymmetry between proton / antiproton direction

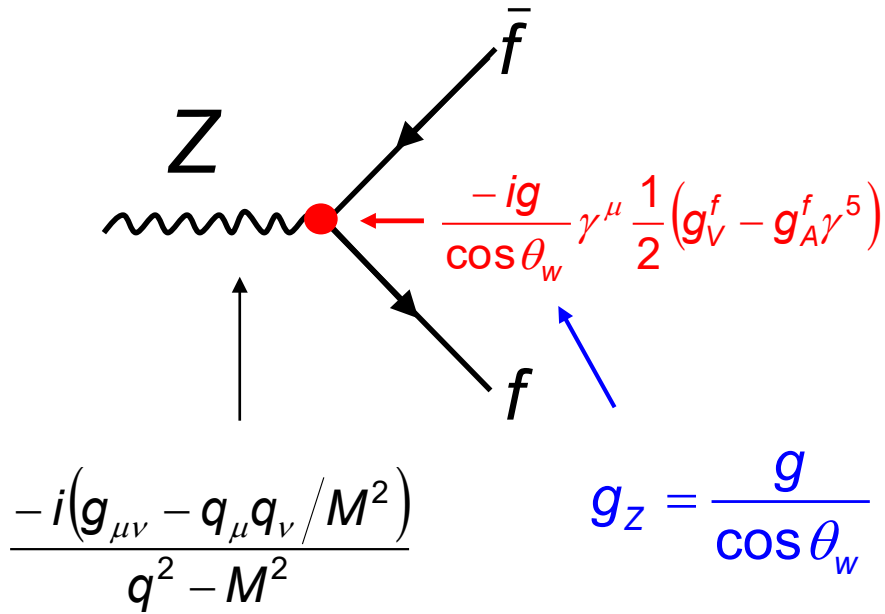
Precision study of the Z boson at LEP

All measurements and plots (if not mentioned differently) from:

Precision Electroweak Measurements on the Z Resonance

ALEPH, DELPHI, OPAL, L3, SLD Collaborations,
Phys.Rept.427:257-454,2006. arXiv:hep-ex/0509008

Recap: Z couplings



$$g = \frac{e}{\sin \theta_w} \quad g' = \frac{e}{\cos \theta_w}$$

Fermion current:

$$J_f = \frac{e}{\sin \theta_w \cos \theta_w} \bar{\psi}_f \gamma^\mu \frac{1}{2} (g_V^f - g_A^f \gamma^5) \psi_f$$

$$g_V^f = I_3^f - 2Q_f \sin^2 \theta_w \quad \text{and} \quad g_A^f = I_3^f$$

$$\rho = \frac{g_Z^2}{M_Z^2} \Big/ \frac{g^2}{M_W^2} = \frac{g^2}{M_Z^2 \cos^2 \theta_w} \Big/ \frac{g^2}{M_W^2} = \frac{M_W^2}{M_Z^2 \cos^2 \theta_w} = 1 \quad (\text{at tree level})$$


Fermion couplings to the Z-boson: (Recap)

V, A couplings: $c_V^f = I_3^f - 2Q_f \sin^2 \theta_W$ $c_A^f = I_3^f$

L, R couplings: $c_L^f = \frac{1}{2}(c_V^f + c_A^f)$ $c_R^f = \frac{1}{2}(c_V^f - c_A^f)$

In the lecture we often use $g_{V,A}$ and $g_{L,R}$ instead of $c_{V,A}$ and $c_{L,R}$.
It is just the same – only different symbols!

	g_V	g_A	g_V	g_A	g_L	g_R
ν	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
l^-	$-\frac{1}{2} + 2\sin^2 \theta_W$	$-\frac{1}{2}$	-0.04	$-\frac{1}{2}$	-0.27	+0.23
<i>u</i> – quark	$+\frac{1}{2} - \frac{4}{3}\sin^2 \theta_W$	$\frac{1}{2}$	+0.19	$\frac{1}{2}$	+0.35	-0.15
<i>d</i> – quark	$-\frac{1}{2} + \frac{2}{3}\sin^2 \theta_W$	$-\frac{1}{2}$	-0.35	$-\frac{1}{2}$	-0.42	+0.08


 $\sin^2 \theta_W \approx 0.231$

1.1 Z-Boson parameters

Cross section for $e^+ e^- \rightarrow \gamma / Z \rightarrow f\bar{f}$

$$|M|^2 = \left| \begin{array}{c} \text{Diagram 1: } e^+ e^- \rightarrow \gamma \rightarrow f\bar{f} \\ \text{Diagram 2: } e^+ e^- \rightarrow Z \rightarrow f\bar{f} \end{array} \right|^2$$

for $e^+ e^- \rightarrow \mu^+ \mu^-$

$$M_\gamma = -ie^2 (\bar{u}_\mu \gamma^\nu v_\mu) \frac{g_{\nu\rho}}{q^2} (\bar{v}_e \gamma^\rho u_e)$$

$$M_Z = -i \frac{g^2}{\cos^2 \theta_W} \left[\bar{u}_\mu \gamma^\nu \frac{1}{2} (g_V^\mu - g_A^\mu \gamma^5) v_\mu \right] \underbrace{\frac{g_{\nu\rho} - q_\nu q_\rho / M_Z^2}{(q^2 - M_Z^2) + iM_Z \Gamma_Z}}_{\text{Unphysical pole}} \left[\bar{v}_e \gamma^\rho \frac{1}{2} (g_V^e - g_A^e \gamma^5) u_e \right]$$

Vanishes for massless positrons:
 $\frac{1}{2} k_\sigma \bar{v}_e \gamma^\sigma = \frac{1}{2} \bar{v}_e \not{K} = 0$
 = Dirac Eq for ingoing positron

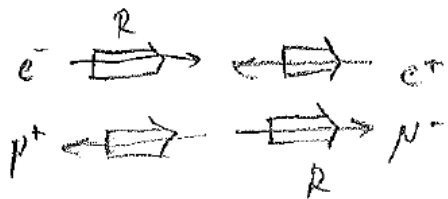
Unphysical pole: Z propagator must be modified to account for finite Z width for $q^2 \approx M_Z^2$ (real particle w/ finite lifetime)

With a “little bit” of algebra similar as for M_γ in QED one obtains $\langle |M_Z|^2 \rangle$

If you want to do the calculation yourself - here is the Z amplitude:

$$M_{fi} = - \frac{g_Z^2}{\underbrace{(s - m_Z^2) + im_Z \Gamma_Z}_{P_Z(s)}} \cdot \left[\bar{v}_e(p_2) \gamma^\mu \frac{1}{2} (c_V^e - c_A^e \gamma^5) u_e(p_1) \right] g_{\mu\nu} \left[\bar{u}_\nu(p_3) \gamma^\nu \frac{1}{2} (c_V^\nu - c_A^\nu \gamma^5) v_\nu(p_4) \right]$$

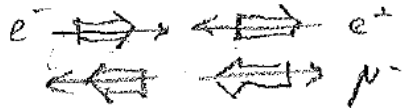
With $c_V = c_L + c_R$ and $c_A = c_L - c_R$ and projectors $P_{L,R} u = \frac{1}{2}(1 \mp \gamma^5)u = u_{L,R}$ one can calculate the different chiral / helicity combinations assuming massless fermions.



$$M_{RR} = P_Z(s) g_Z^2 \cdot c_R^e c_R^\nu \underbrace{g_{\mu\nu} \cdot [v_L(p_2) \gamma^\mu u_R(p_1)] [\bar{u}_R(p_3) \gamma^\nu v_L(p_4)]}_{s \cdot (1 + \cos \theta)} \quad (3)$$

$$|M_{RR}|^2 = |P_2(s)|^2 g_Z^4 s^2 (c_R^e c_R^\nu)^2 \cdot (1 + \cos\theta)^2$$

$$|M_{LL}|^2 = \dots \dots \dots (c_L^e c_L^\nu)^2 \cdot (1 + \cos\theta)^2$$



$$|M_{RL}|^2 = \dots \dots \dots (c_R^e c_L^\nu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = \dots \dots \dots (c_L^e c_R^\nu)^2 (1 - \cos\theta)^2$$

$$\Rightarrow \langle |M|^2 \rangle = \frac{1}{4} \sum |M_{ij}|^2 \quad (\text{see } e^+e^- \rightarrow \nu^+\nu^-)$$

with $\frac{d\sigma}{ds} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \underbrace{\left(\frac{P_3^\mu}{P_1^\mu} \right)}_{=1 \text{ for massless fermions}} \cdot \langle |M|^2 \rangle$ one finds:

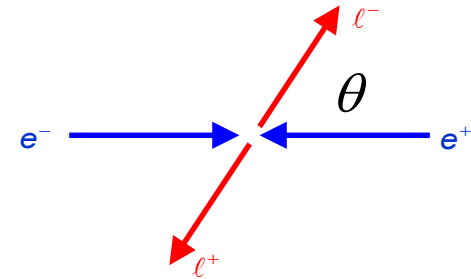
$$\frac{d\sigma}{d\Omega} (e^+e^- \rightarrow Z \rightarrow \nu^i \bar{\nu}^i) = \frac{1}{256\pi^2 s} \cdot \frac{g_2^4 \cdot s^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

$$\left\{ \frac{1}{4} \left[(C_V^e)^2 + (C_A^e)^2 \right] \left[(C_V^\nu)^2 + (C_A^\nu)^2 \right] (1 + \cos^2\theta) + 2 C_V^e C_A^e C_V^\nu C_A^\nu \cos\theta \right\}$$

For the total differential cross section of γ and Z contribution one finds:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \left[\underbrace{F_\gamma(\cos\theta)}_{\gamma} + \underbrace{F_{\gamma Z}(\cos\theta)}_{\gamma/Z \text{ interference}} + F_Z(\cos\theta) \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right]$$

Vanishes at $\sqrt{s} \approx M_Z$: finite lifetime decouples initial/final state.



$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1 + \cos^2\theta) = (1 + \cos^2\theta) \quad \begin{array}{l} \text{symmetric in } \cos\theta \\ \text{(QED part)} \end{array}$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4 \sin^2 \theta_W \cos^2 \theta_W} \left[2g_V^e g_V^\mu (1 + \cos^2\theta) + 4g_A^e g_A^\mu \cos\theta \right]$$

asymmetric in $\cos\theta$

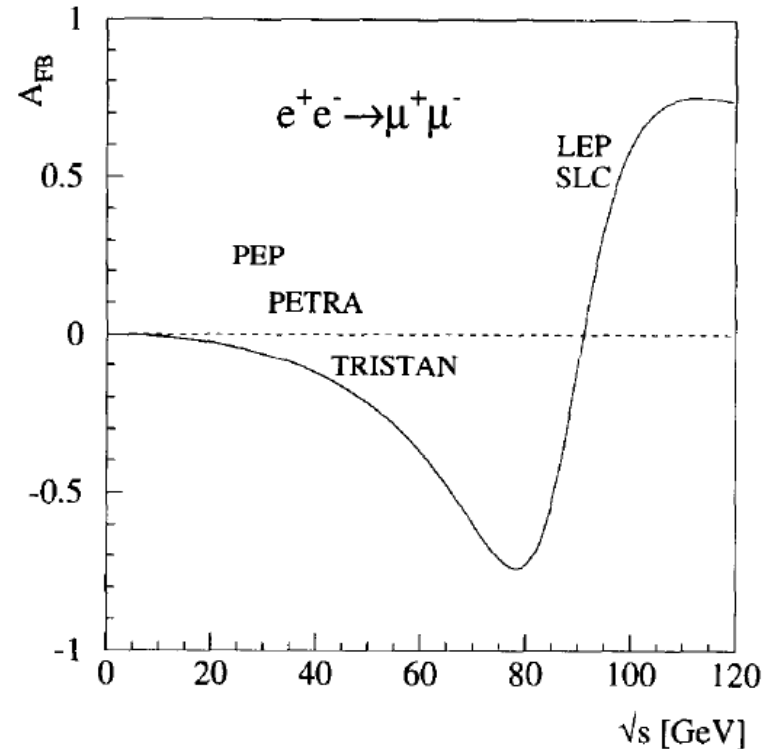
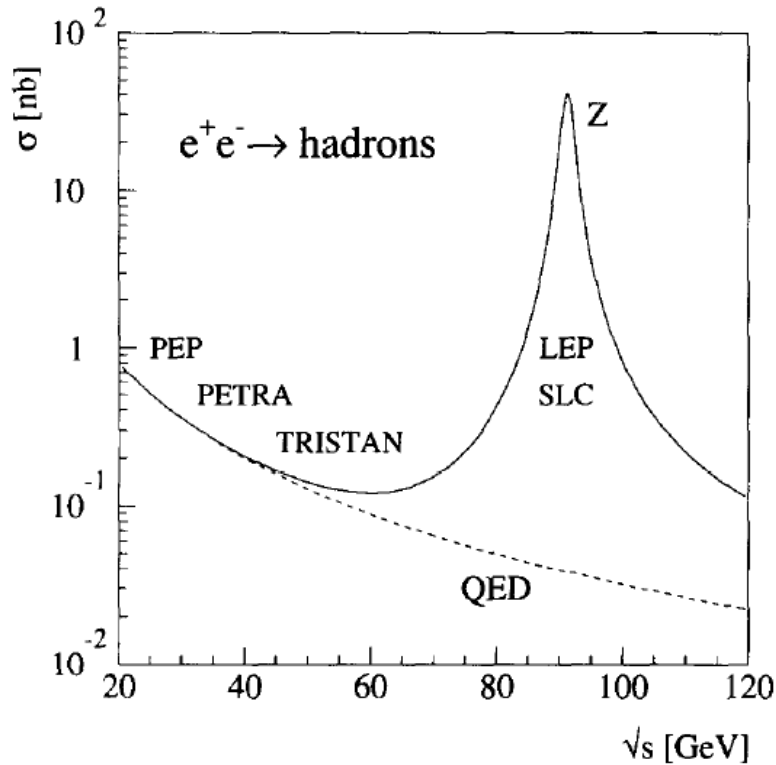
$$F_Z(\cos\theta) = \frac{1}{16 \sin^4 \theta_W \cos^4 \theta_W} \left[(g_V^{e^2} + g_A^{e^2})(g_V^{\mu^2} + g_A^{\mu^2})(1 + \cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta \right]$$

Asymmetric angular distribution → forward-backward asymmetry

$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{8}{3}A_{FB}\cos\theta \quad \text{with} \quad \left\{ \begin{array}{l} A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ \sigma_{F(B)} = \int_{0(-1)}^{1(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta \end{array} \right.$$

At this point A_{FB} is an observable → linear in couplings.

Expectations:



Large forward-backward asymmetries away from the Z pole caused by γ/Z interference.

Cross section at the Z-pole $\sqrt{s} \approx M_Z$: Breit-Wigner Resonance

(ignore QED contribution, **interference vanishes because of finite Z lifetime**)

$$\sigma_{tot} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16 \sin^4 \theta_w \cos^4 \theta_w} \cdot [(g_V^e)^2 + (g_A^e)^2] [(g_V^\mu)^2 + (g_A^\mu)^2] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$$

With partial and total widths: $\Gamma_f = \frac{\alpha M_Z}{12 \sin^2 \theta_w \cos^2 \theta_w} \cdot [(g_V^f)^2 + (g_A^f)^2]$ $\Gamma_Z = \sum_i \Gamma_i$

(one could have immediately given this formular for the resonance)

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Breit-Wigner Resonance: $\sigma_Z(\sqrt{s} \approx M_Z) \approx \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2} = \frac{12\pi}{M_Z^2} BR(Z \rightarrow ee) BR(Z \rightarrow ff)$

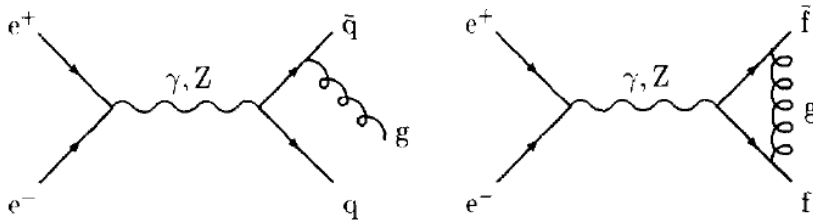
Cross sections and widths can be calculated within the Standard Model if $\sin^2 \theta_w$ and M_Z are known.

From the couplings one expects the following BR (independent of M_Z)

	$BR = \Gamma_i / \Gamma_Z$	
e, μ, τ	3.5%	
ν_e, ν_μ, ν_τ	7%	
hadrons ($= \sum_q q\bar{q}$)	69%	← Remind color factor: $N_C=3$

No final state photon bremsstrahlung and no gluon bremsstrahlung considered.

Large corrections for hadronic final states from gluon final state bremsstrahlung:



$$R_{\text{QCD}} = 1 + \frac{\alpha_S(m_Z^2)}{\pi} + \dots$$

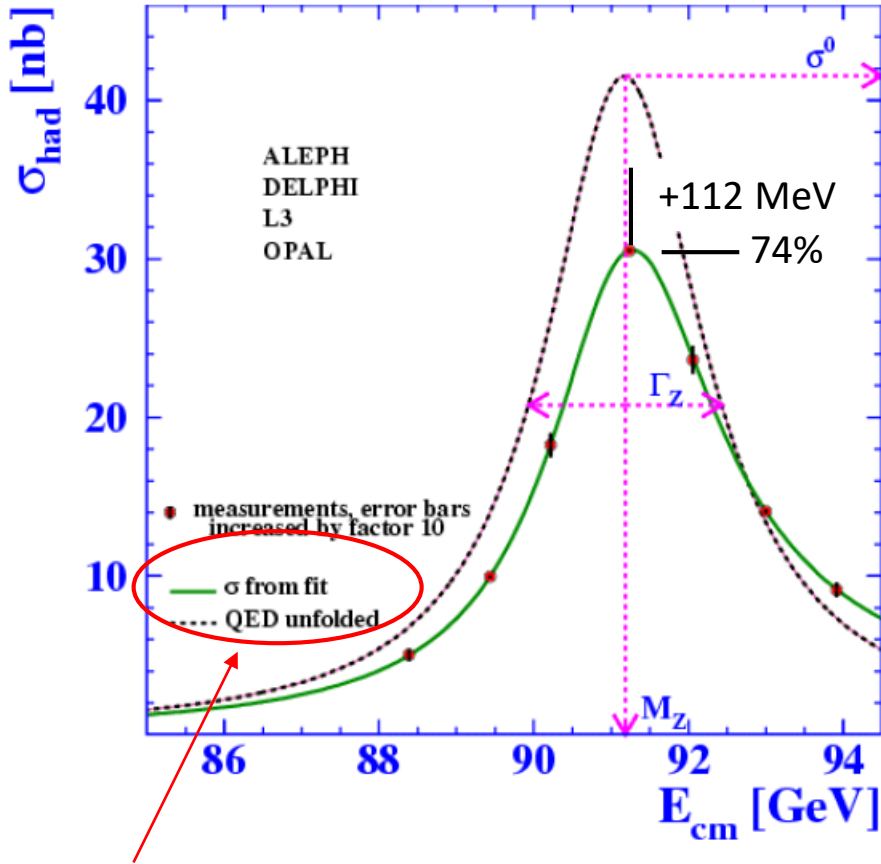
Opens a way to measure $\alpha_s (M_Z)$.

Similarly there are final state QED corrections to take into account (formally similar but much smaller):

$$R_{\text{QED}} = 1 + \frac{\alpha(m_Z^2)}{\pi} + \dots \quad \text{Important: } \alpha(m_Z^2) = \frac{1}{129}$$

Measurement of Z-lineshape

Strongly influenced by Bremsstrahlung



$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak:
$$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_\mu$
- Width $\rightarrow \Gamma_Z$

Bremsstrahlung corrections

$$\sigma_{ff(\gamma)} = \int_0^1 G(z,s) \sigma_{ff}^0(zs) dz \quad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

Radiator function $G(z,s)$

