Discovery of the W- and Z-boson (recap)

https://cds.cern.ch/record/2103277/files/9789814644150_0006.pdf

Historical situation at the the end of the 1970:

- Standard Model unifying the electromagnetic and the weak interaction was developed in the course of the 1960s (S. Glashow (1959), A. Salam (1959), S. Weinberg (1967)): Theory predicts massive W and Z bosons.

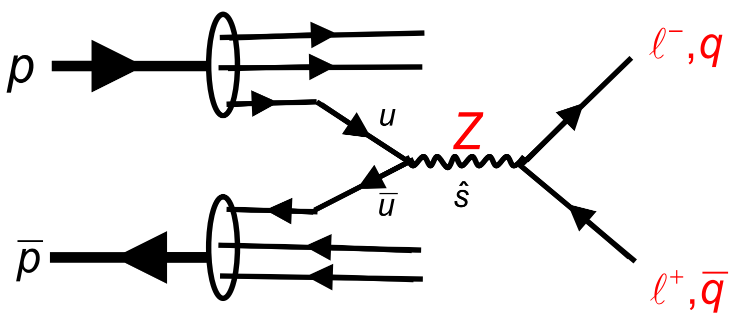
- Experimental evidence in favor of a unique description of the weak and electromagnetic interactions was obtained in 1973, with the observation of neutral current neutrino interaction which could only be explained by the exchange of a virtual heavy neutral particle. The measurements allowed a first estimation of $\sin^2\theta_w$ and together with the coupling G_F of the muon decays an estimation of the masses of the W and the Z-boson:

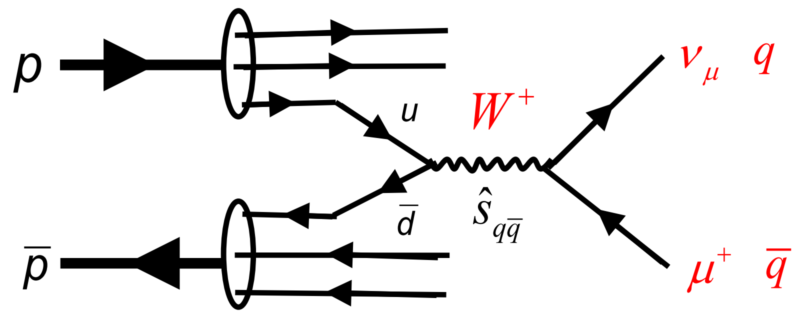
$$m_W \approx 60...80 \text{ GeV} \qquad m_Z \approx 75...92 \text{ GeV}$$

Too large to be accessible by any accelerator in operation at that time.

In 1976 Rubbia, Cline and McIntyre proposed the transformation of an existing high-energy proton accelerator (SPS) into a proton–antiproton collider (SppS) as a quick and cheap way to achieve collisions above thresholds for W and Z.

$$p\bar{p} \to Z \to f\bar{f} + X \qquad\qquad p\bar{p} \to W \to \ell\,\bar{\nu}_\ell + X$$





Colliding quarks (at the envisaged CMS energy dominantly valence quarks) carry momentum fraction x of the proton/antiproton momentum w/ <x> ≈ 0.17.

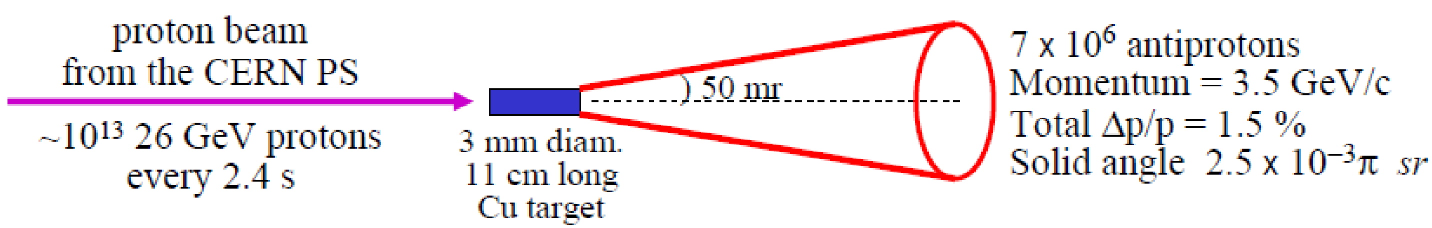
To achieve a quark-antiquark CMS energy of ~90 GeV proton-antiproton CMS energies of ~540 GeV are necessary: $\sqrt{\hat{s}_{q\bar{q}}} = x \cdot \sqrt{s_{p\bar{p}}} \to 90\,\text{GeV} \approx 0.17 \cdot 540\,\text{GeV}$

For a luminosity of L ≈ 2.5 × 10^{29} $cm^{-2}\,s^{-1}$ one expects typically only one Z→ee event per day: $\sigma(pp\to Z\to e^+e^-) \approx 50$ pb = $5\cdot10^{-35}cm^{-2}$ w/ BR(Z→ee)≈3%.
Beside the energy the luminosity thus was a challenge to measure sufficient Zs!

Antiproton beam

Antiproton source must be capable to deliver daily $\sim 3 \times 10^{10}$ anti-protons distributed in few (3–6) tightly collimated bunches within the angular and momentum acceptance of the CERN SPS.

CERN 26 GeV proton synchrotron (PS) is capable of producing antiprotons at the desired rates:

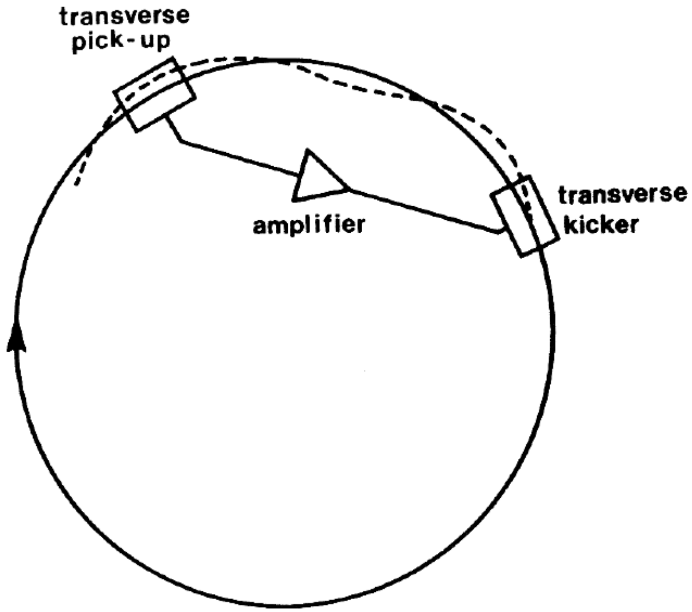


Sufficient antiprotons but they occupy a phase space volume which is too large by a factor $\geq 10^8$ to fit into the SPS acceptance, even after acceleration to the SPS injection energy of 26GeV (emittance of the beam is too large!).

→ Increase the antiproton phase space density at least 10^8 times before sending the p beam to the SPS. This process is called “cooling” (in analogy to a hot gas where the particle have very different momenta)

Stochastic cooling (S. van der Meer, 1972)

Reminder: Liouville theorem forbids any compression of phase volume by conservative forces such as the electromagnetic fields → emittance (defining the area of the phase space ellipse) cannot be reduced by fields acting on all particles of the beam. Need to act on individual (group of individual) particles.



Idea to cool betatron oscillation:

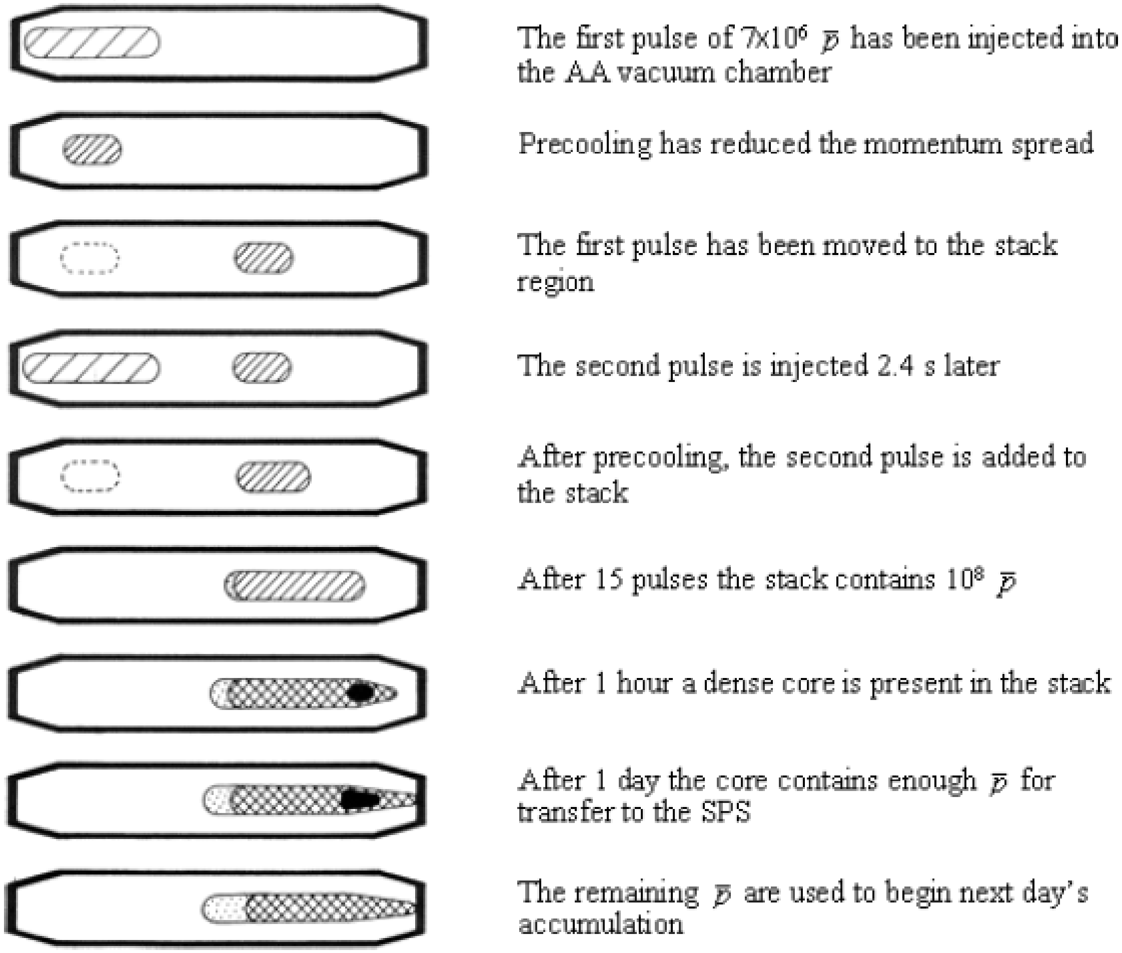
Measure deviation of group of particles at point where maximal. Pick-up signal proportional to deviation is amplified and provided to kicker at a place where particle crosses central orbit.

Pick-up is sensitive only to a group of particles (depends on geometry and on frequency response)

Cooling was performed in the Antiproton Accumulator AA: includes several independent cooling systems to cool horizontal and vertical oscillations, and also to decrease the beam momentum spread (cooling of longitudinal motion).

AA - a large aperture ring of different magnets - during construction.

Cooling and injection cycle:



When a sufficiently dense anti-proton stack has been accumulated in the AA, beam injection into the SPS is achieved using consecutive PS cycles.

Firstly, three proton bunches (six after 1986), each containing $\sim 10^{11}$ protons, are accelerated to 26 GeV in the PS and injected into the SPS. Then three p bunches (six after 1986), of typically $\sim 10^{10}$ antiprotons each, are extracted from the AA and injected into the PS accelerated to 26 GeV and injected into SPS.

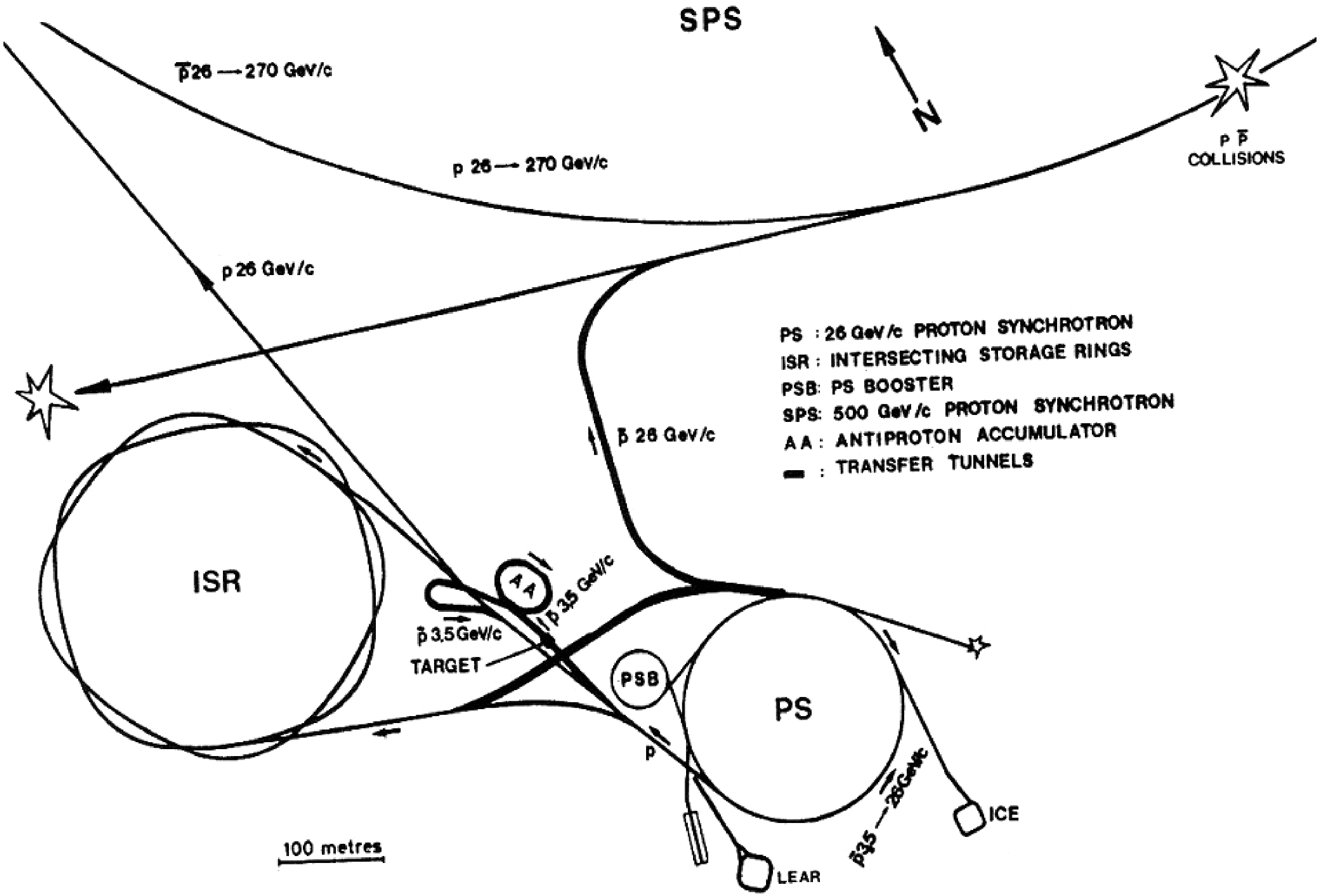
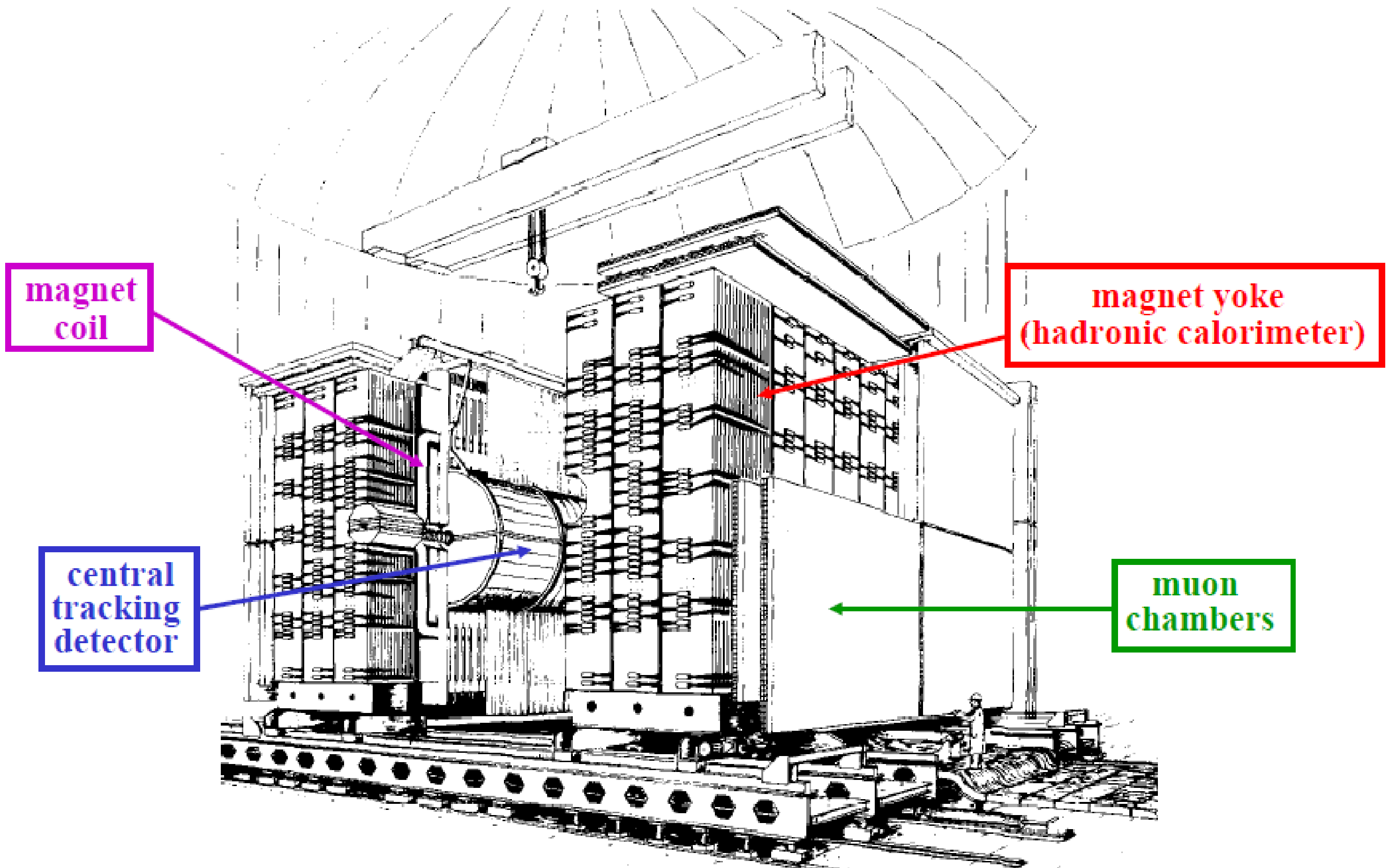


Table 1 CERN proton–antiproton collider operation, 1981–1990.

Year	Collision energy (GeV)	Peak luminosity ($cm^{-2}\ s^{-1}$)	Integrated luminosity (cm^{-2})
1981	546	$\sim 10^{27}$	2×10^{32}
1982	546	5×10^{28}	2.8×10^{34}
1983	546	1.7×10^{29}	1.5×10^{35}
1984–85	630	3.9×10^{29}	1.0×10^{36}
1987–90	630	3×10^{30}	1.6×10^{37}

Two Detectors: UA1 and UA2

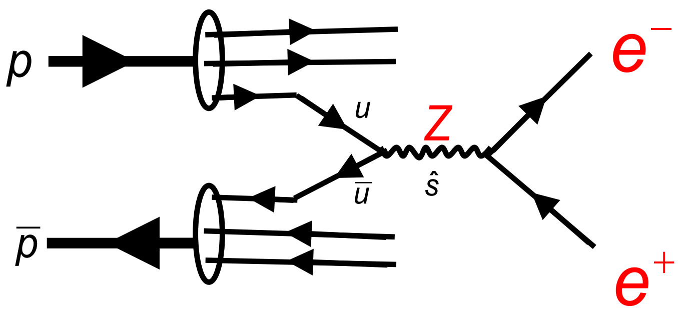
UA1 detector

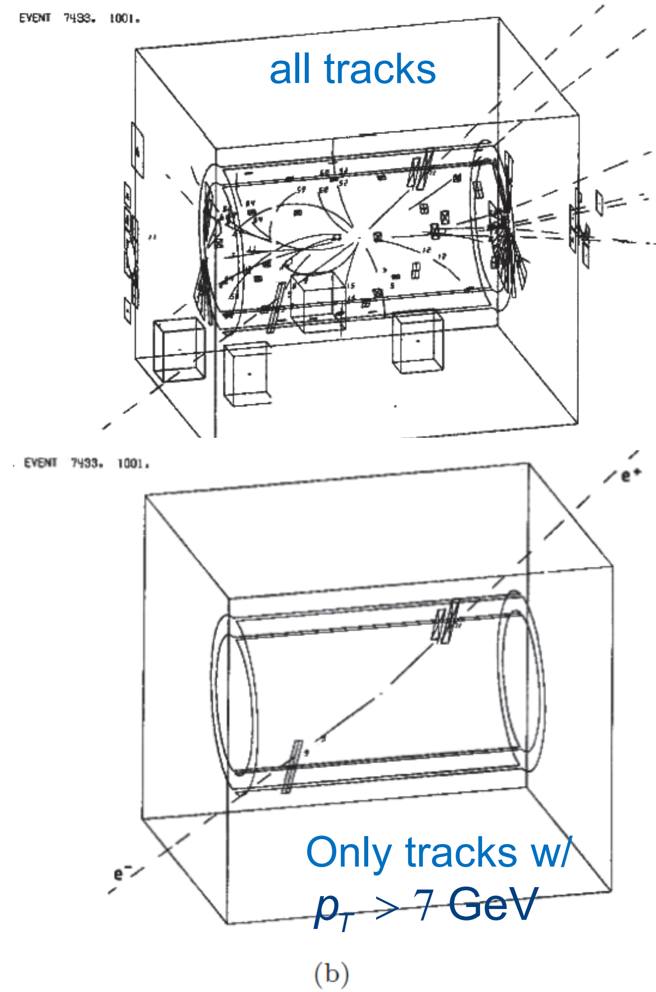


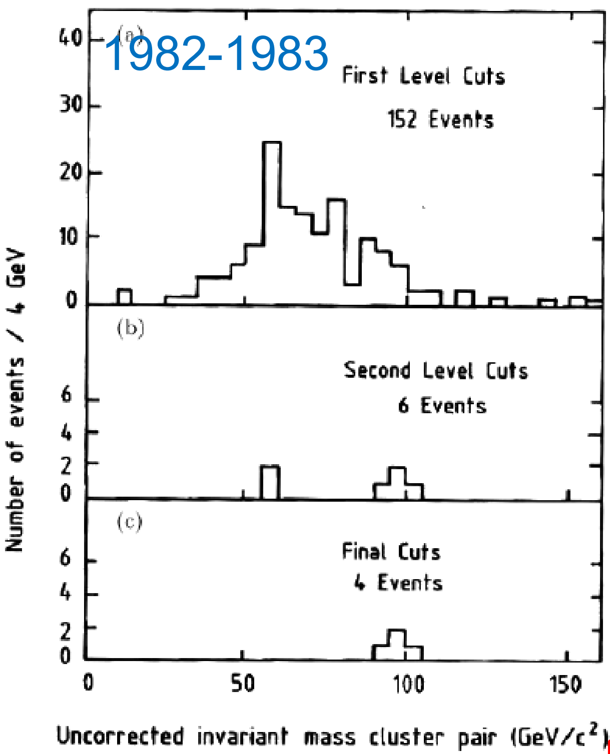
(shown with the two halves of the dipole magnet opened up)

Discovery of the Z-boson

(historically W discovery was a few weeks earlier)







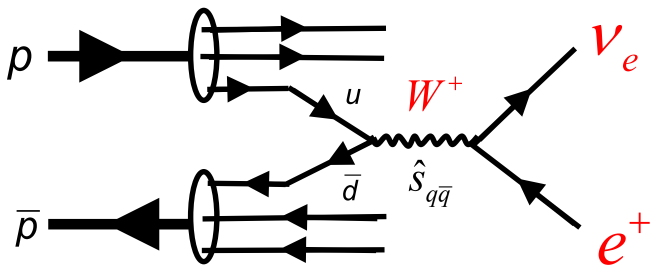
2 e.m. clusters w/ $E_T > 25$ GeV

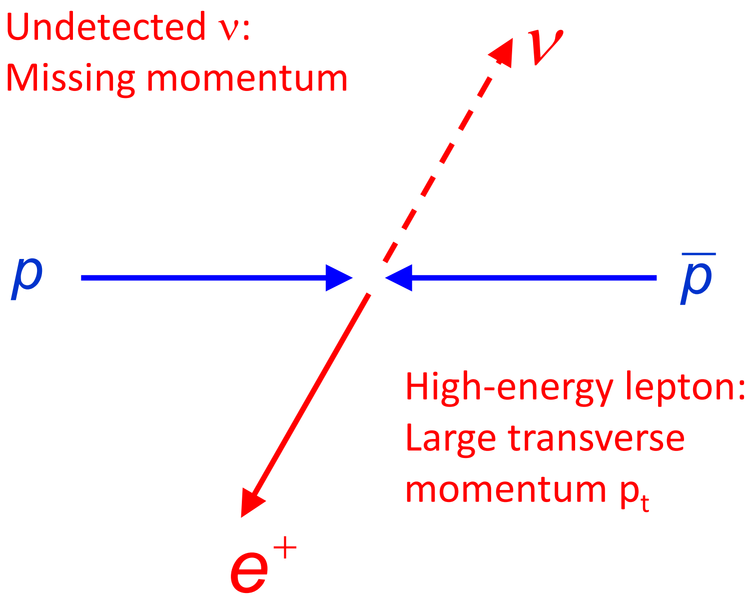
isolated track w/ $p_T > 7$ GeV

2 tracks w/ $p_T > 7$ GeV Pointing to clusters

Invariant mass: $m_Z = 95.2 \pm 2.5 \pm 3.0$ GeV

Discovery of the W-boson





How can the W mass be reconstructed ?

$W^- \rightarrow e\,\bar{\nu}$

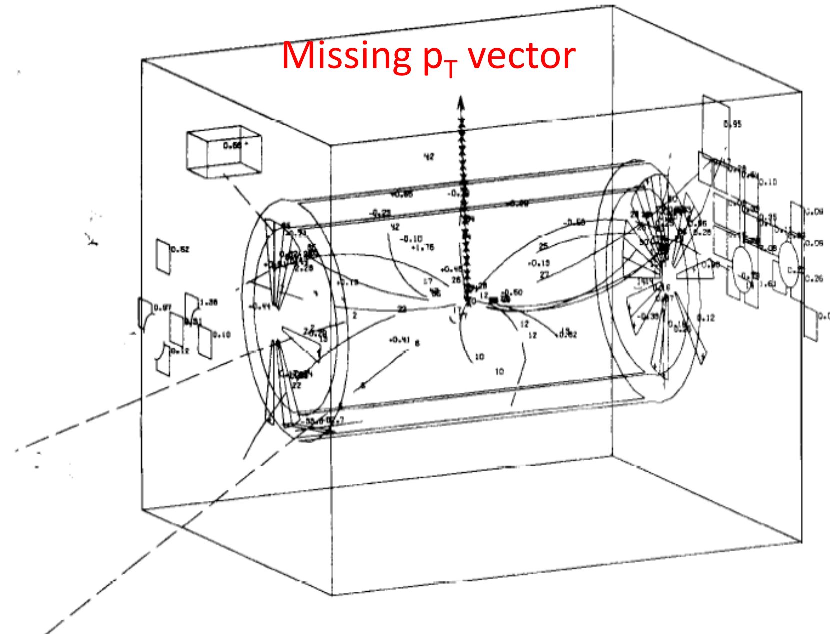


Fig. 16a. Event of the type $W^- \rightarrow e^- + \bar{\nu}_e$. All tracks and calorimeter cells are displayed.

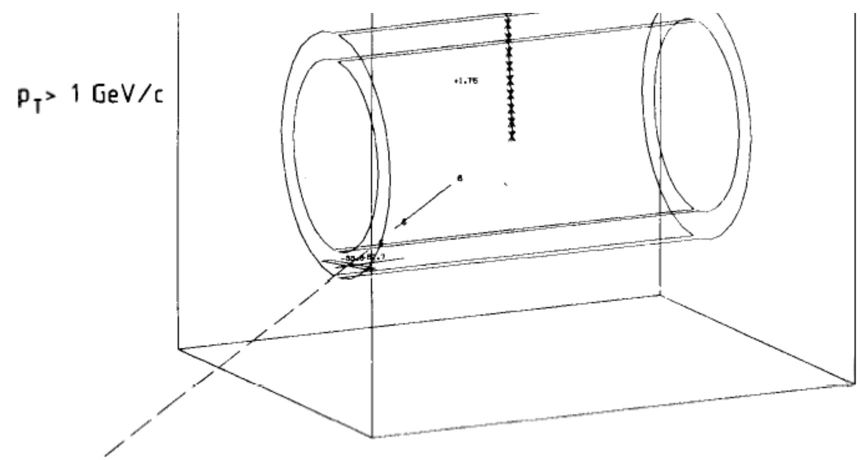


Fig. *16b.* The same as picture (a), except that now only particles with $p_T > 1$ GeV/c and calorimeters with $E_T > 1$ GeV are shown.

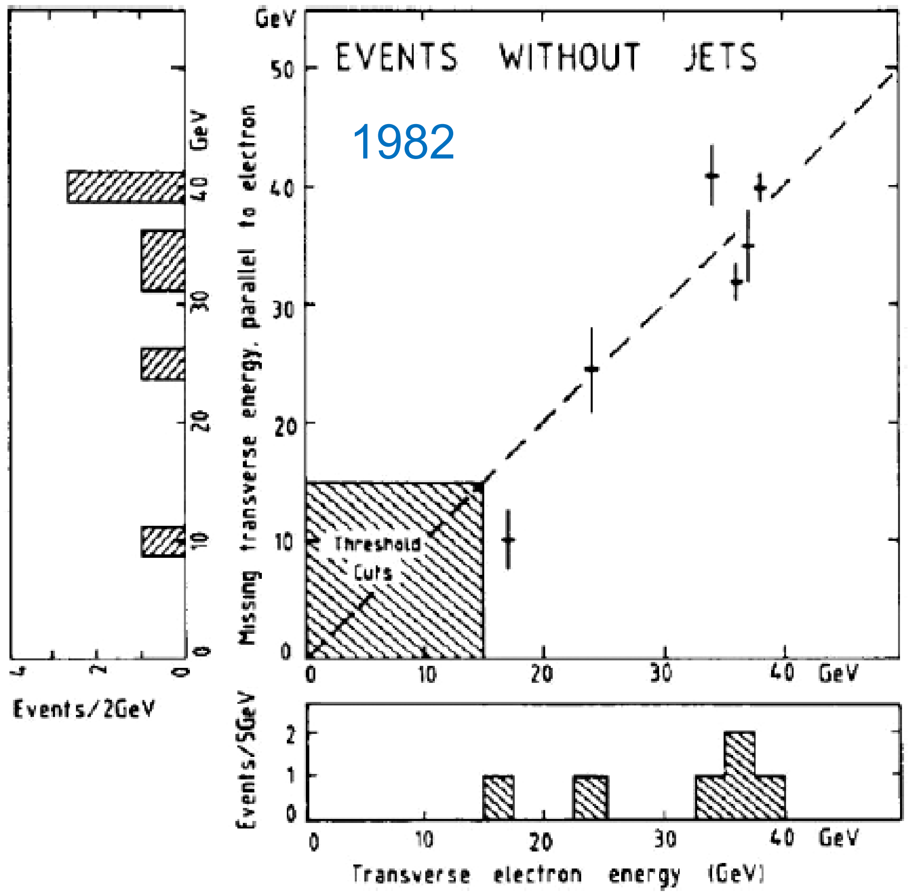
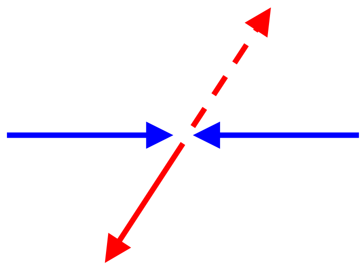


Fig. 11. UA1 scatter plot of all the events from the 1982 data which contain a high-pT electron and large $|\vec{p}_T^{miss}|$. The abscissa is the electron $|\overline{p}_T|$ and the ordinate is the $\vec{p}_T^{miss}$component antiparallel to the electron $\vec{p}_T$.

W mass measurement



In the W rest frame:

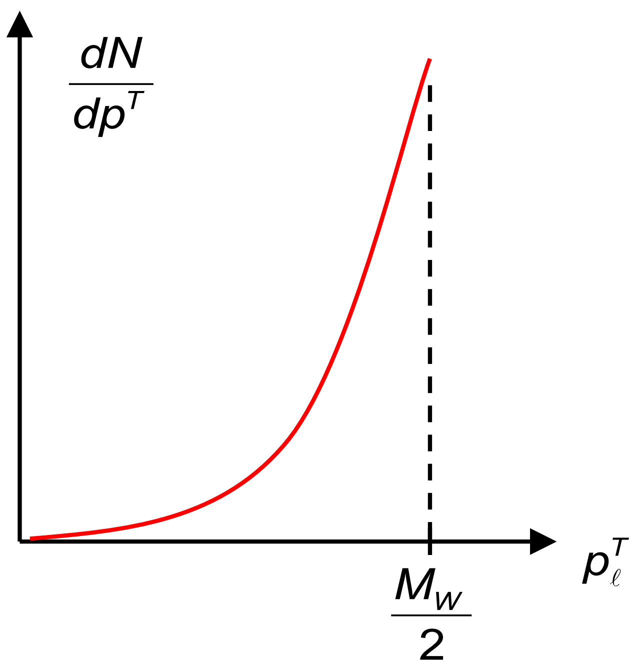
- $$|\vec{p}_\ell| = |\vec{p}_\nu| = \frac{M_W}{2}$$
- $$|p_\ell^T| \le \frac{M_W}{2}$$

In the lab system:

- W system boosted only along z axis
- p_T distribution is conserved

Jacobian Peak:

$$\frac{dN}{p_T} \sim \frac{2p_T}{M_W} \cdot \left(\frac{M_W^2}{4} - p_T^2 \right)^{-1/2}$$



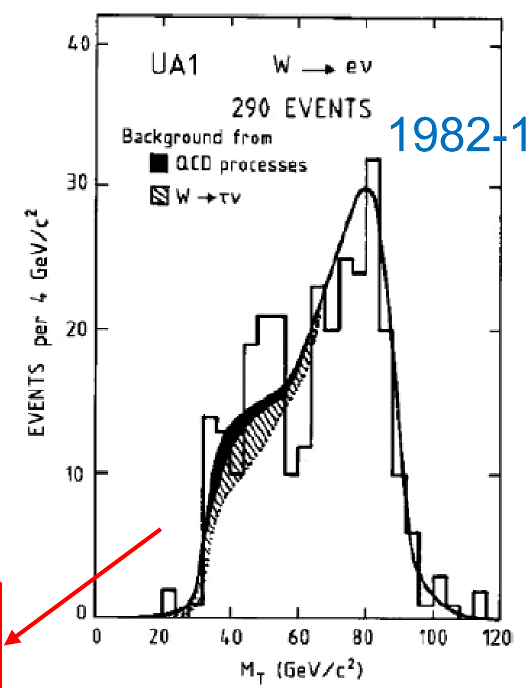
Transverse Mass M_T

$$M_T = 2p_T$$

Additional effects:

- trans. movement of the W
- Finite W decay width
- W decay not isotropics

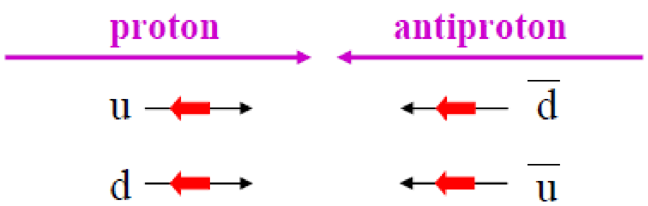
$$\boxed{m_W = 82.7 \pm 1.0 \pm 2.7 \text{ GeV},}$$



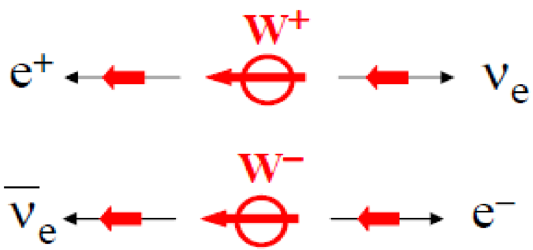
1982-1985

V-A coupling of the W-boson

Because of V-A coupling W bosons polarized in direction of antiproton beam:



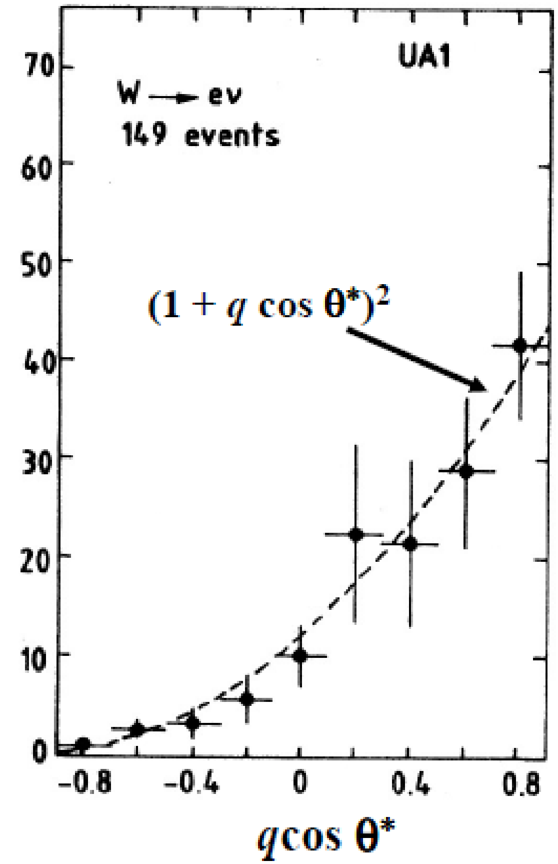
Decay to eν in W-rest frame:



Electron (positron) angular distribution in lab:

$$\frac{dN}{d\cos\theta^*} \propto \left(1 + q\cos\theta^*\right)^2$$

q = +1 for positrons, -1 for electrons
θ^* = angle w/r to antiproton directions



Confirms V-A coupling to quarks and leptons: charge asymmetry between proton / antiproton direction

Precision study of the Z boson at LEP

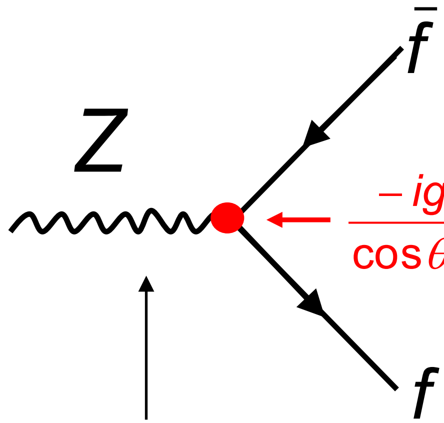
All measurements and plots (if not mentioned differently) from:

Precision Electroweak Measurements on the Z Resonance

ALEPH, DELPHI, OPAL, L3, SLD Collaborations,
Phys.Rept.427:257-454,2006. arXiv:hep-ex/0509008

Recap: Z couplings

$$g = \frac{e}{\sin\theta_w} \qquad g' = \frac{e}{\cos\theta_w}$$



$$\frac{-ig}{\cos\theta_w}\gamma^{\mu}\frac{1}{2}\left(g_V^f - g_A^f\gamma^5\right)$$

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M^2\right)}{q^2 - M^2}$$

$$g_Z = \frac{g}{\cos\theta_w}$$

Fermion current:

$$J_f = \frac{1}{2}\frac{e}{\sin\theta_w \cos\theta_w}\bar{\psi}_f\,\gamma^{\mu}\left(g_V^f - g_A^f\gamma^5\right)\psi_f$$

$$g_V^f = I_3^f - 2Q_f\sin^2\theta_W \quad \text{and} \quad g_A^f = I_3^f$$

$$\rho = \frac{g_Z^2}{M_Z^2}\bigg/\frac{g^2}{M_W^2} = \frac{g^2}{M_Z^2\cos^2\theta_w}\bigg/\frac{g^2}{M_W^2} = \frac{M_W^2}{M_Z^2\cos^2\theta_w} = 1$$

(at tree level)

Fermion couplings to the Z-boson: (Recap)

V, A couplings: $c_V^f = I_3^f - 2Q_f \sin^2\theta_W \qquad c_A^f = I_3^f$

L, R couplings: $c_L^f = \frac{1}{2}(c_V^f + c_A^f) \qquad c_R^f = \frac{1}{2}(c_V^f - c_A^f)$

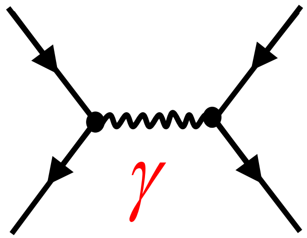
In the lecture we often use $g_{V,A}$ and $g_{L,R}$ instead of $c_{V,A}$ and $c_{L,R}$.
It is just the same – only different symbols!

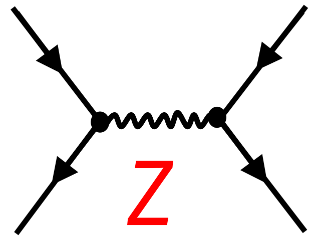
	g_V	g_A	g_V	g_A	g_L	g_R
ν	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	0
ℓ^-	$-1/2 + 2\sin^2\theta_W$	$-1/2$	-0.04	$-1/2$	-0.27	$+0.23$
$u-quark$	$+1/2 - 4/3 \sin^2\theta_W$	$1/2$	$+0.19$	$1/2$	$+0.35$	-0.15
$d-quark$	$-1/2 + 2/3 \sin^2\theta_W$	$-1/2$	-0.35	$-1/2$	-0.42	$+0.08$

$\sin^2\theta_w \approx 0.231$

1.1 Z-Boson parameters

Cross section for $e^+ e^- \to \gamma / Z \to f\bar{f}$

$|M|^2 = \Big|$ 

$+$ 

$\Big|^2$

for $e^+ e^- \to \mu^+ \mu^-$

Vanishes for massless positrons:

$$\frac{1}{2} k_\sigma \bar{v}_e \gamma^\sigma = \frac{1}{2} \bar{v}_e \not{k} = 0$$

= Dirac Eq for ingoing positron

$$M_\gamma = -ie^2 (\bar{u}_\mu \gamma^\nu v_\mu) \frac{g_{\nu\rho}}{q^2} (\bar{v}_e \gamma^\rho u_e)$$

$$M_Z = -i \frac{g^2}{\cos^2 \theta_W} \left[\bar{u}_\mu \gamma^\nu \frac{1}{2} (g_V^\mu - g_A^\mu \gamma^5) v_\mu \right] \underbrace{\frac{g_{\nu\rho} - q_\nu q_\rho / M_Z^2}{(q^2 - M_Z^2) + iM_Z \Gamma_Z}} \left[\bar{v}_e \gamma^\rho \frac{1}{2} (g_V^e - g_A^e \gamma^5) u_e \right]$$

Unphysical pole: Z propagator must be modified to account for finite Z width for $q^2 \approx M_Z^2$ (real particle w/ finite lifetime)

With a "little bit" of algebra similar as for M_γ in QED one obtains $<|M_Z|^2>$

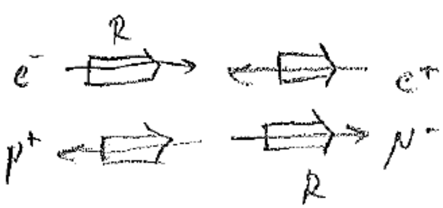
If you want to do the calculation yourself - here is the Z amplitude:

$$M_{fi} = -\underbrace{\frac{g_Z^2}{(s - M_Z^2) + i M_Z \Gamma_Z}}_{P_Z(s)} \cdot \left[\bar{v}_e(p_2)\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5)\, u_e(p_1)\right] g_{\mu\nu} \left[\bar{u}_\mu(p_3)\gamma^\nu \frac{1}{2}(c_V^f - c_A^f\gamma^5)\, v_\mu(p_4)\right]$$

with $c_V = c_L + c_R$ and $c_A = c_L - c_R$ and projectors $P_{L,R}\, u = \frac{1}{2}(1 \mp \gamma^5)\, u = u_{L,R}$

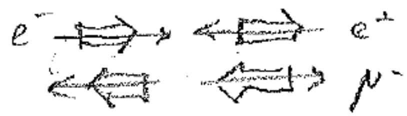
one can calculate the different chiral / helicity combinations

assuming massless fermions.



$$M_{RR} = P_Z(s)\, g_Z^2 \cdot c_R^e\, c_R^\mu\, \underbrace{g_{\mu\nu} \cdot \left[v_L(p_2)\gamma^\mu u_R(p_1)\right]\left[\bar{u}_R(p_3)\gamma^\nu v_L(p_4)\right]}_{s\cdot(1+\cos\theta)} \qquad (3)$$

$$|M_{RR}|^2 = |P_Z(s)|^2 \, g_Z^4 \, s^2 \cdot (c_R^e c_R^\mu)^2 \cdot (1+\cos\theta)^2$$



$$|M_{LL}|^2 = \cdots\cdots \, (c_L^e c_L^\mu)^2 \cdot (1+\cos\theta)^2$$

$$|M_{RL}|^2 = \cdots\cdots \, (c_R^e c_L^\mu)^2 \, (1-\cos\theta)^2$$

$$|M_{LR}|^2 = \cdots\cdots \, (c_L^e c_R^\mu)^2 \, (1-\cos\theta)^2$$

$$\Rightarrow \quad \langle |M|^2 \rangle = \frac{1}{4} \sum |M_{ij}|^2 \qquad (\text{see } e^+e^- \to \mu^+\mu^-)$$

with $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \underbrace{\left(\frac{p_f^*}{p_i^*}\right)}_{=1 \text{ for mass less fermions}} \cdot \langle |M|^2 \rangle$$ one finds:

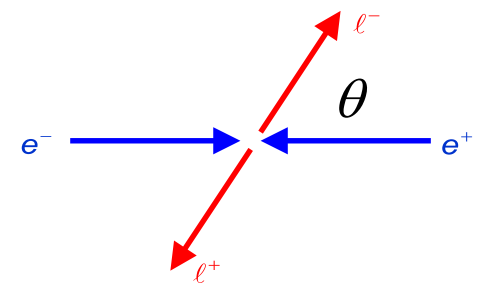
$$\frac{d\sigma}{d\Omega}\left(e^+e^- \to Z \to \mu^+\mu^-\right) = \frac{1}{256\pi^2 s} \cdot \frac{g_Z^4 \cdot s^2}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \cdot$$

$$\left\{ \frac{1}{4}\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right](1+\cos^2\theta) + 2 c_V^e c_A^e c_V^\mu c_A^\mu \cdot \cos\theta \right\}$$

For the total differential cross section of γ and Z contribution one finds:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[\underbrace{F_\gamma(\cos\theta)}_{\gamma} + \underbrace{F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}}_{\gamma/Z\ \text{interference}} + \underbrace{F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}}_{Z}\right]$$

Vanishes at $\sqrt{s} \approx M_Z$: finite lifetime decouples initial/final state.



$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

symmetric in $\cos\theta$

(QED part)

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W\cos^2\theta_W}\left[2g_V^e g_V^\mu (1+\cos^2\theta) + 4g_A^e g_A^\mu \cos\theta\right]$$

asymmetric in $\cos\theta$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W\cos^4\theta_W}\left[(g_V^{e^2}+g_A^{e^2})(g_V^{\mu^2}+g_A^{\mu^2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta\right]$$

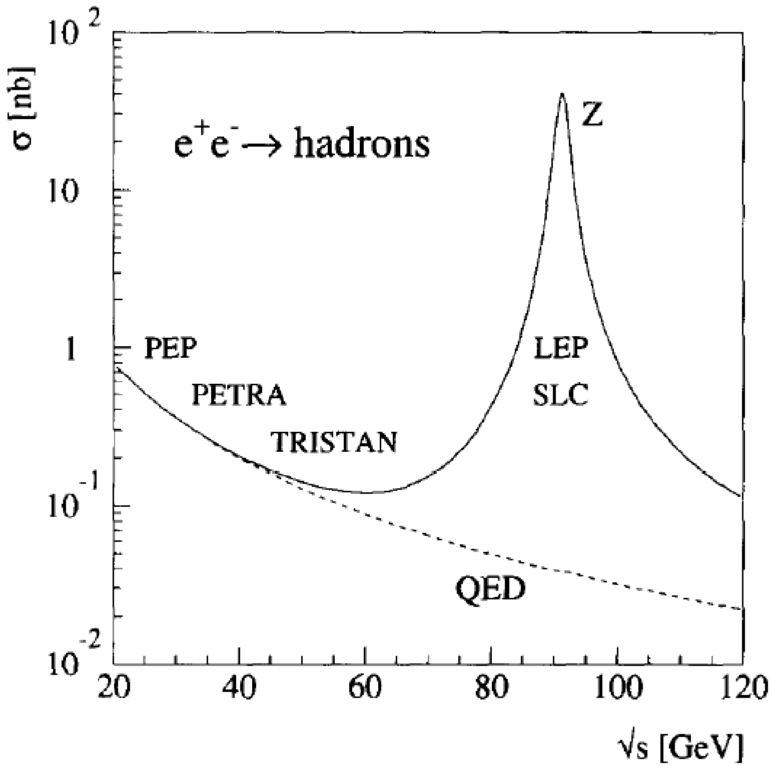
Asymmetric angular distribution → forward-backward asymmetry

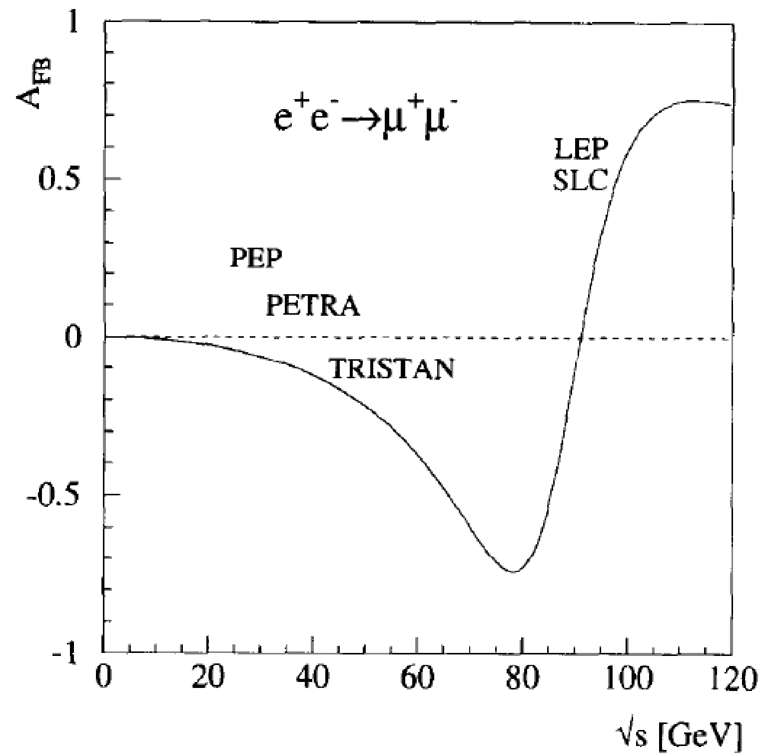
$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta \qquad \text{with} \quad \begin{cases} A_{FB} = \dfrac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ \sigma_{F(B)} = \displaystyle\int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta \end{cases}$$

At this point A_{FB} is an observable → linear in couplings.

J.Mnich, Experimental Tests of Standard Model in ee→ff at the Z resonance

Expectations:





Large forward-backward asymmetries away from the Z pole caused by γ/Z interference.

Cross section at the Z-pole $\sqrt{s} \approx M_Z$: Breit-Wigner Resonance

(ignore QED contribution, interference vanishes because of finite Z lifetime)

$$\sigma_{tot} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16\sin^4\theta_w \cos^4\theta_w} \cdot \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$$

With partial and total widths: $\Gamma_f = \frac{\alpha M_Z}{12\sin^2\theta_w \cos^2\theta_w} \cdot \left[(g_V^f)^2 + (g_A^f)^2\right]$ $\qquad \Gamma_Z = \sum_i \Gamma_i$

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

(one could have immediately given this formular for the resonsnce)

Breit-Wigner Resonance: $\sigma_Z\left(\sqrt{s} \approx M_Z\right) \approx \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2} = \frac{12\pi}{M_Z^2} BR(Z \to ee) BR(Z \to ff)$

Cross sections and widths can be calculated within the Standard Model if $\sin^2\theta_w$ and M_Z are known.

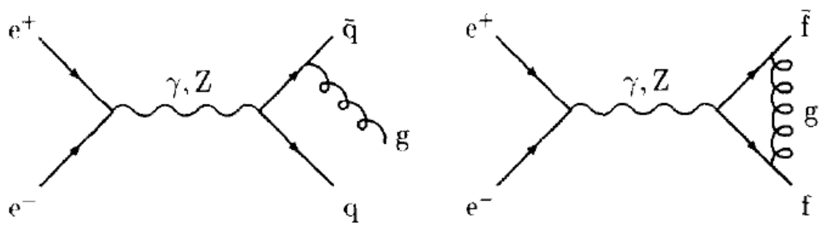
From the couplings one expects the following BR (independent of M_Z)

	$BR = \Gamma_f / \Gamma_Z$
e, μ, τ	3.5%
ν_e, ν_μ, ν_τ	7%
hadrons $(= \sum_q q\bar{q})$	69%

Remind color factor: $N_C=3$

No final state photon bremsstrahlung and no gluon bremsstrahlung considered.

Large corrections for hadronic final states from gluon final state bremsstrahlung:



$$R_{\mathrm{QCD}} = 1 + \frac{\alpha_S(m_Z^2)}{\pi} + \cdots .$$

Opens a way to measure α_s (M_Z).

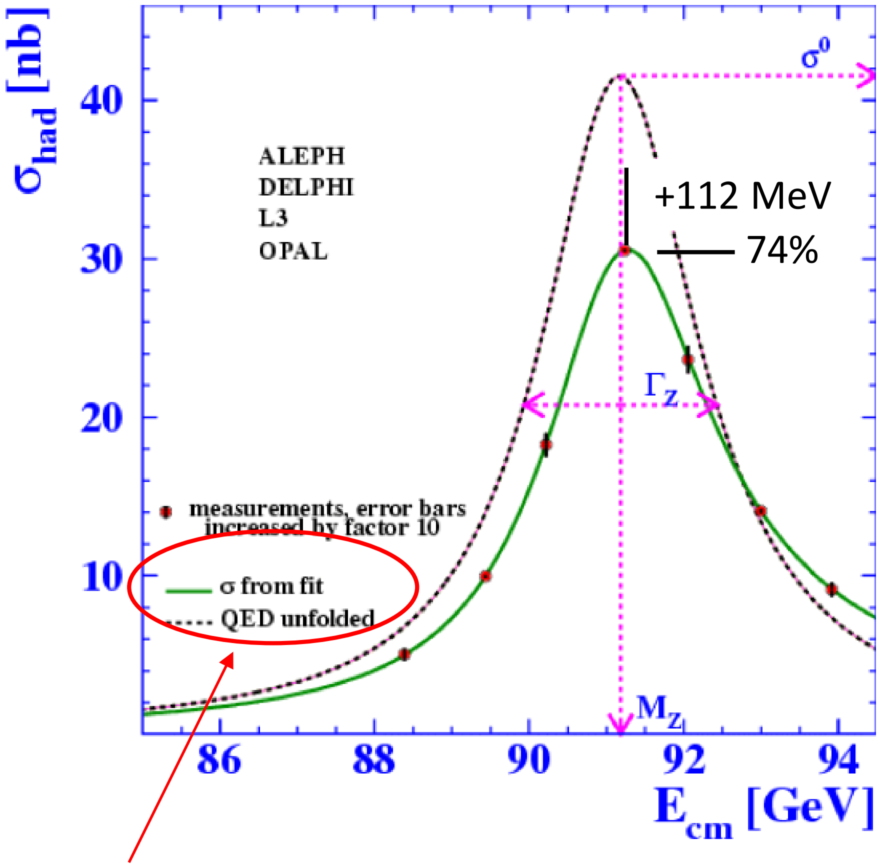
Similarly there are final state QED corrections to take into account (formally similar but much smaller):

$$R_{QED} = 1 + \frac{\alpha(m_Z^2)}{\pi} + \ldots$$

Important: $\alpha(m_Z^2) = \frac{1}{129}$

Measurement of Z-lineshape

Strongly influenced by Bremsstrahlung



$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

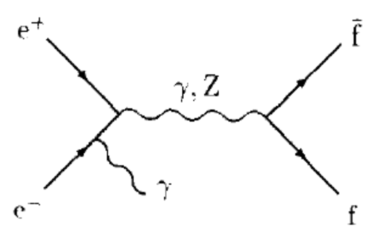
Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

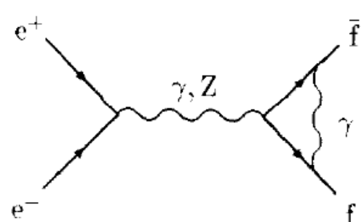
- Resonance position $\rightarrow$ M_Z
- Height $\rightarrow$ $\Gamma_e \Gamma_\mu$
- Width $\rightarrow$ Γ_Z

Bremsstrahlung corrections

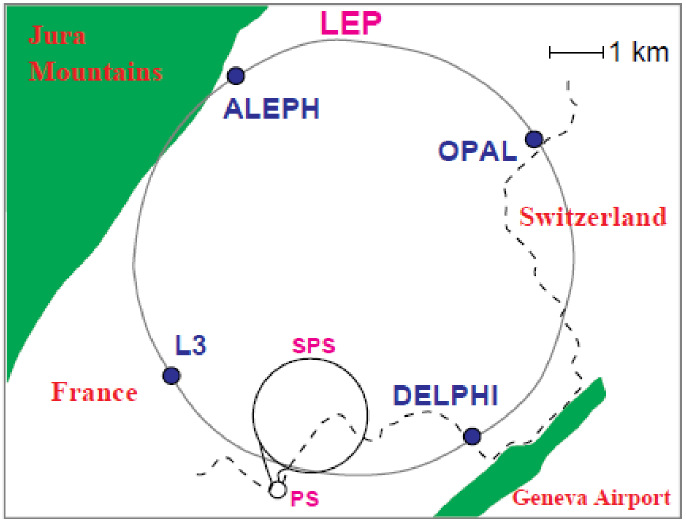
$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z,s)\sigma_{ff}^0(zs)dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

Radiator function G(z,s)





Large Electron Positron Collider (LEP)

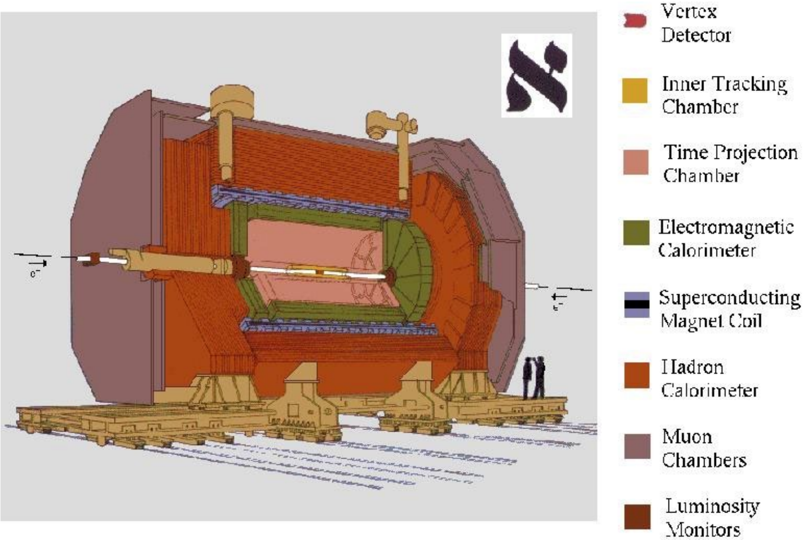


Circumference	~27 km
Centre-of-mass energy	92.1 GeV(LEP1) to 209 GeV(LEP 2)
Accelerating gradient	Up to 7 MV/m (SC cavities)
Number of bunches	4 x 4
Current per bunch	~ 750 μA
Luminosity (at Z0)	$\sim 24 \times 10^{30}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ (~1 Z0/sec)
Luminosity (at LEP2)	$\sim 50 \times 10^{30}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ (3 WW/hour)
Interaction regions	4 (ALEPH,DELPHI,L3,OPAL)
Energy calibration	< 1 MeV (at Z0)

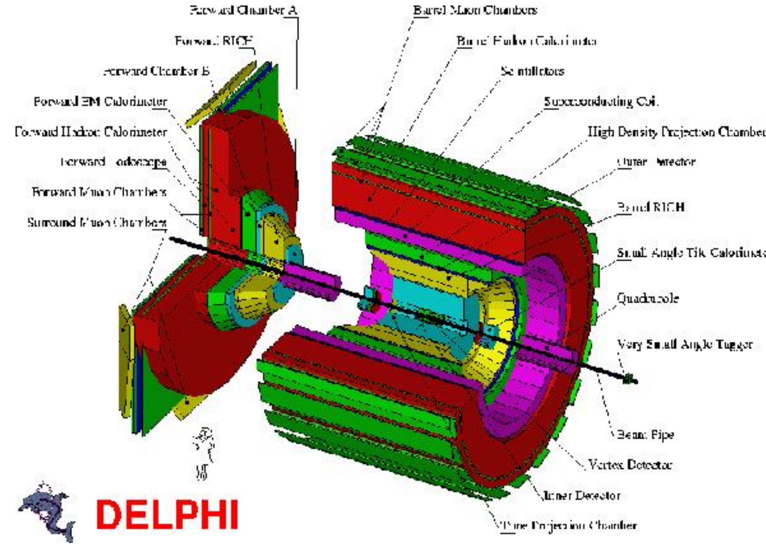
Number of Events										
	$Z \to q\overline{q}$					$Z \to \ell^+\ell^-$				
Year	A	D	L	O	LEP	A	D	L	O	LEP
1990/91	433	357	416	454	1660	53	36	39	58	186
1992	633	697	678	733	2741	77	70	59	88	294
1993	630	682	646	649	2607	78	75	64	79	296
1994	1640	1310	1359	1601	5910	202	137	127	191	657
1995	735	659	526	659	2579	90	66	54	81	291
Total	4071	3705	3625	4096	15497	500	384	343	497	1724

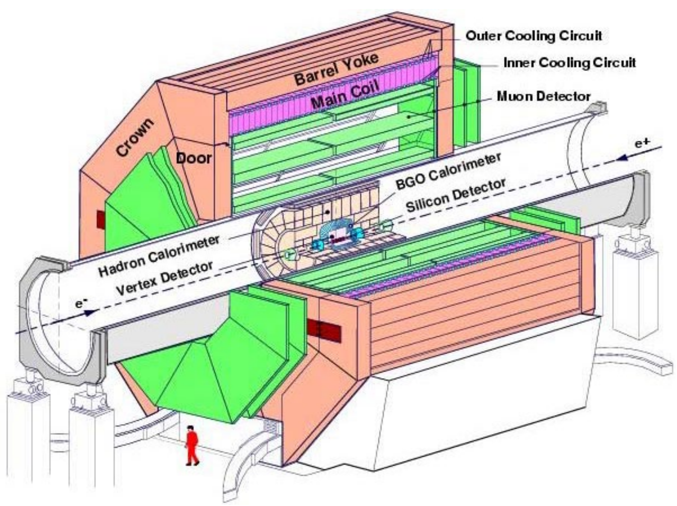
Events per experiment

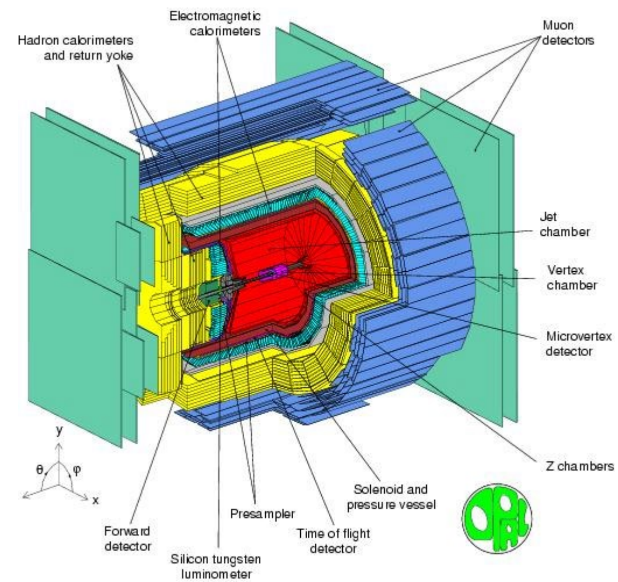
LEP Detectors



The ALEPH Detector

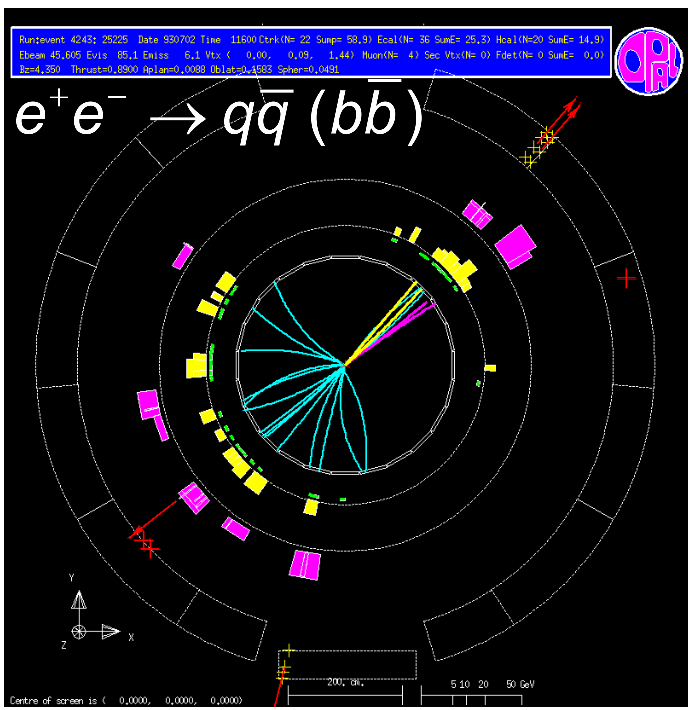


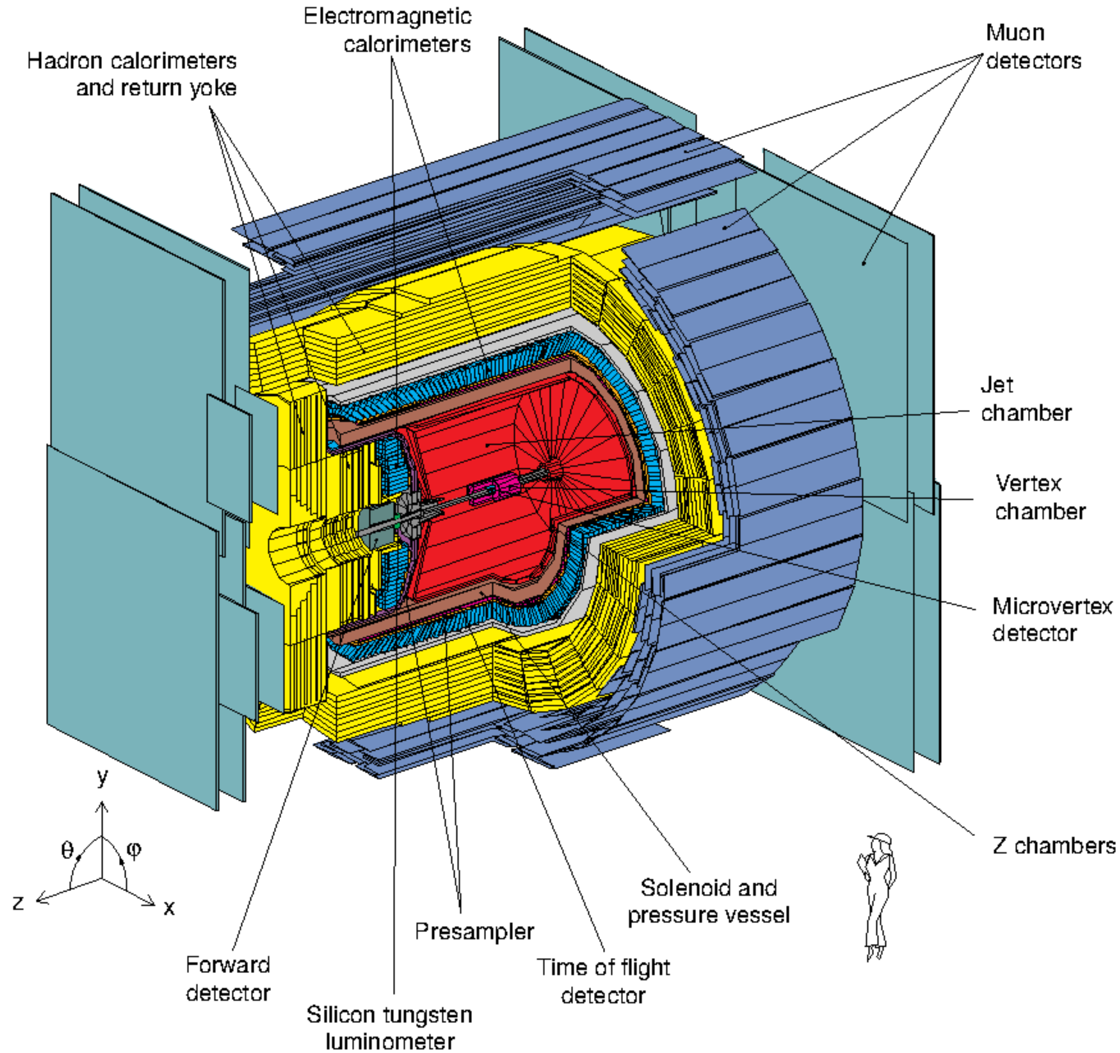




Example: OPAL Detector

Heidelberg:
Jet-Chamber





Measurement of Cross Section $\sigma(\sqrt{s})$:

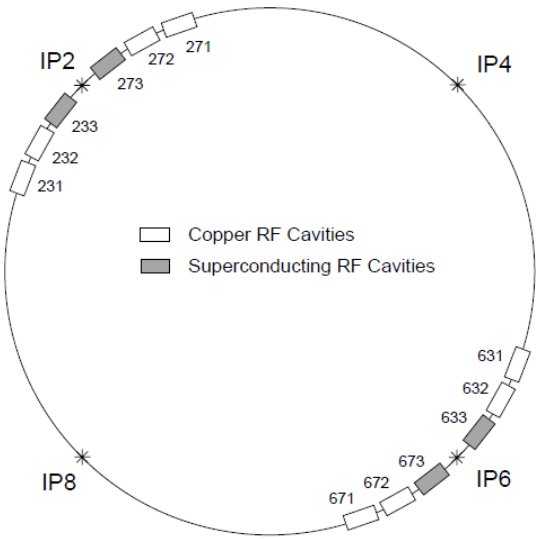
$$\sigma(\sqrt{s}) = \frac{N_{signal} - N_{back}}{\varepsilon \cdot L_{\text{int}}}$$

$= 2E_B$

Requires calibration of beam energy and experiment dependent correction (synchrotron loss). Uncertainties in the energy scale translates into an absolute error of M_Z.

LEP Beam Energy Calibration

$$E_B = \frac{e\,c}{2\pi} \oint_s B(s)ds$$



Beam energy calibration requires precise measurements of the average B-field along the LEP ring. In addition interaction point dependent corrections are necessary to account for the energy loss by synchrotron radiation (260 MeV / turn) and the asymmetric position of the RF cavities during LEP-1.

Different measurement to determine B-field of dipole magnets have been used:

- Field display: NMR probes / rotating coil inside a reference magnet powered in series with the LEP dipole magnets. Problem: different position and different environment. Used to extrapolate from periods w/o other measurements
- Flux loop measurements: induction loops in all 8 octants → measure induction voltage when the B field is ramped.
- NMR probes inside the ring dipole magnets (installed only in 1995)

Good reproducibility but no absolute calibration.

Measurements to calibrate flux-loops / NMR probes:

- Proton calibration: LEP ring was filled with 20 GeV protons.
  → precise determination of proton velocity → proton momentum → B field
  Method reached absolute accuracy of 10^{-4} at 20 GeV.
- Resonant depolarization (ultimate method, precision better than 1 MeV (10^{-5}).
  Method is a "g-2 experiment" where the electron g-2 is known and the average B-field / average electron energy E_B is determined instead.

Spin-tune: $$\Delta\nu = \frac{g-2}{2}\frac{E_B}{m_e c^2}$$

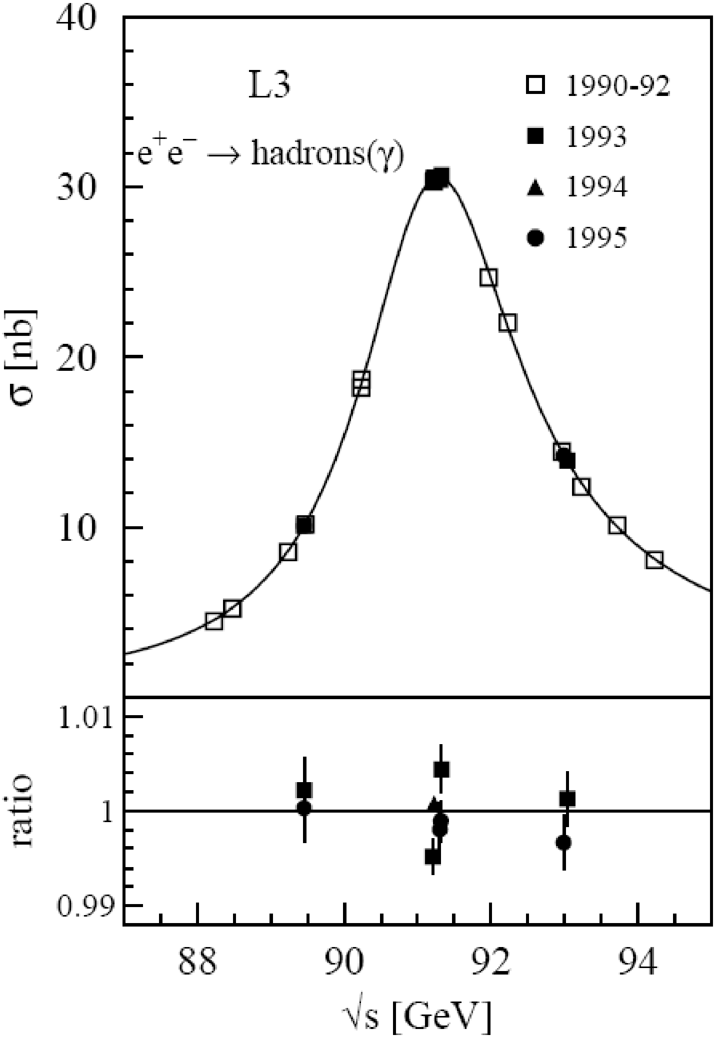
Absolute energy calibration at 17 ppm level has been achieved.

Requires the correction of many unexpected effects:

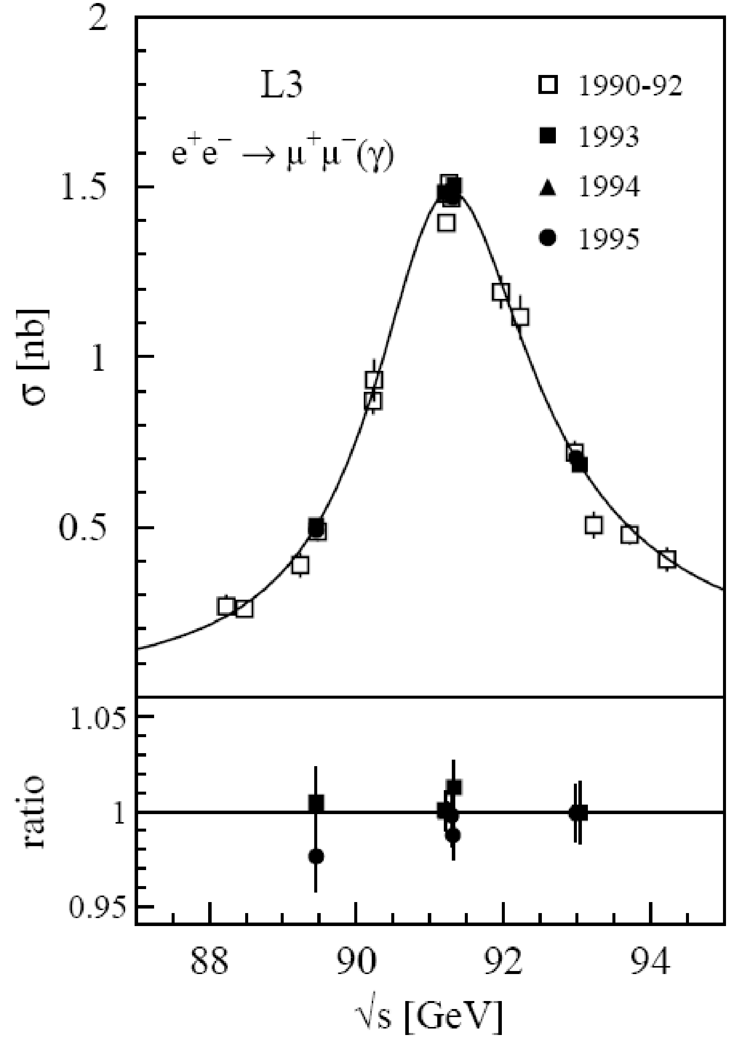
- Tidal effects and ground water level → changes the circumference of the LEP ring (1 – 2mm → ± 5MeV)
- French TGV passing in the neighborhood → vagabonding electrical currents (~1A) produce additional B field which modifies the energy (MeV)

Cross section measurements $\sigma(\sqrt{s})$

$e^+ e^- \rightarrow hadrons$

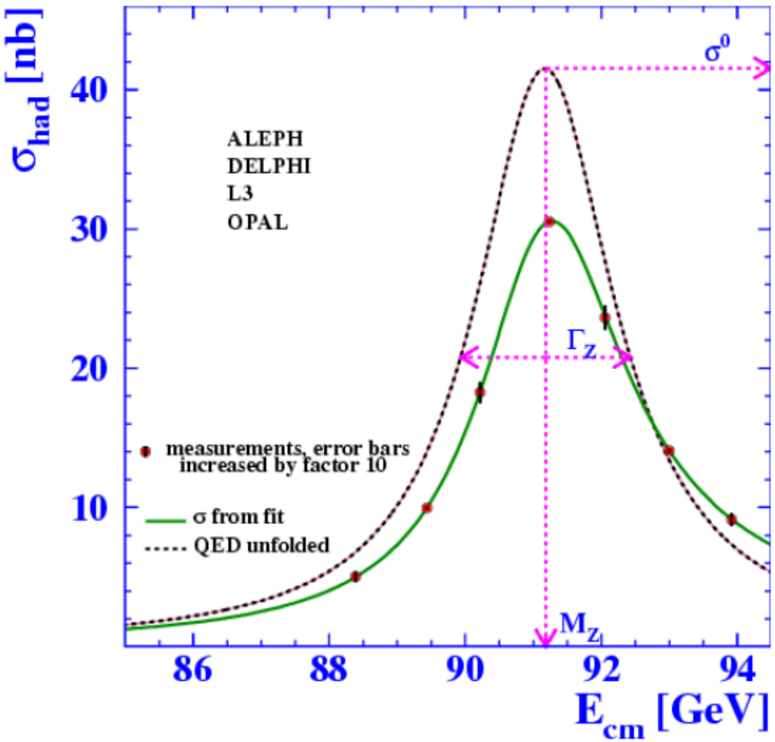


$e^+ e^- \rightarrow \mu^+ \mu^-$



Resonance shape is the same, independent of final state: Propagator the same!

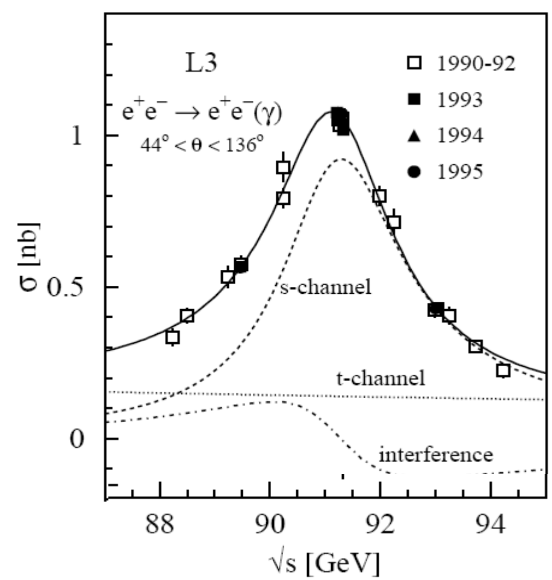
Reminder:



$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_\mu$
- Width $\rightarrow \Gamma_Z$



Measurement of $e^+e^- \rightarrow e^+e^-$:
t-channel contribution (mostly γ) needs to be subtracted (see lecture on ee-annihilation). This is done using QED prediction.

Z line shape parameters (LEP average)

M_Z = 91.1876 ± 0.0021 GeV ±23 ppm (*)

*) error of the LEP energy determination: ±1.7 MeV (19 ppm)
http://lepewwg.web.cern.ch/

Γ_Z = 2.4952 ± 0.0023 GeV ±0.09 %
Γ_{had} = 1.7458 ± 0.0027 GeV
Γ_e = 0.08392 ± 0.00012 GeV
Γ_μ = 0.08399 ± 0.00018 GeV
Γ_τ = 0.08408 ± 0.00022 GeV

3 leptons are treated independently

test of lepton universality

Γ_Z = 2.4952 ± 0.0023 GeV
Γ_{had} = 1.7444 ± 0.0022 GeV
Γ_e = 0.083985 ± 0.000086 GeV

Assuming lepton universality: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

(predicted by SM: g_A and g_V are the same:)

$$\Gamma_f = \frac{\alpha M_Z}{12 \sin^2\theta_w \cos^2\theta_w} \cdot \left[(g_V^f)^2 + (g_A^f)^2 \right]$$

Number of light neutrinos

In the Standard Model:

$$\Gamma_Z = \Gamma_{had} + 3\cdot\Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible}:\ \Gamma_{inv}} \rightarrow \begin{cases} e^+e^- \rightarrow Z \rightarrow \nu_e\bar{\nu}_e \\ e^+e^- \rightarrow Z \rightarrow \nu_\mu\bar{\nu}_\mu \\ e^+e^- \rightarrow Z \rightarrow \nu_\tau\bar{\nu}_\tau \end{cases}$$

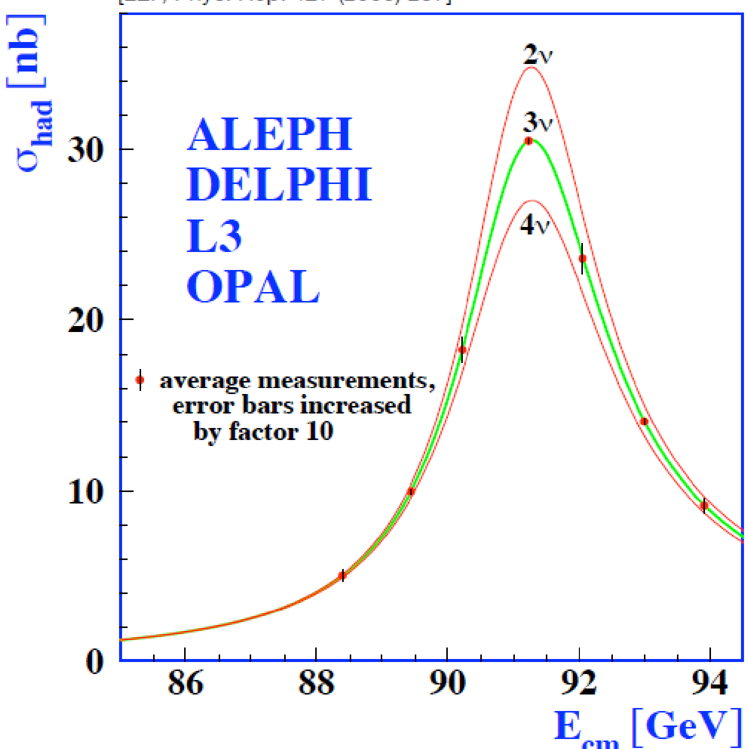
$$\boxed{\Gamma_{inv} = 0.4990 \pm 0.0015\ \text{GeV}}$$

To determine the number of light neutrinos:

$$\boxed{N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu,SM}}} = \underbrace{\left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp}}_{5.9431\pm0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}}_{=1/1.991\pm0.001}$$

(small theo. uncertainties from m_{top} M_{H})

[LEP, Phys. Rep. 427 (2006) 257]



$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

No room for new physics: $Z \rightarrow$ ~~new~~ invisible particles

Lepton couplings to the Z-boson

In the following we ignore the difference between chirality and helicity: good approximation as leptons are produced with energies >> mass.

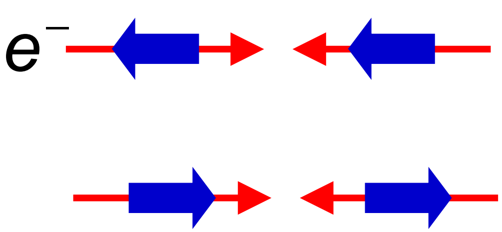
Z boson couples differently to LH and RH leptons:

$$\left| g_L = \frac{1}{2}(g_V + g_A) \right| \quad > \quad \left| g_R = \frac{1}{2}(g_V - g_A) \right|$$

Coupling to LH leptons stronger

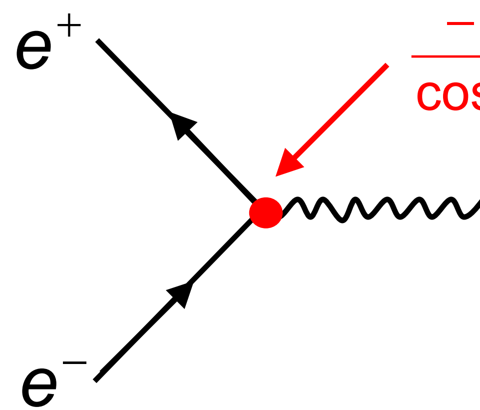
Z produced in e+e- collisions is polarized.

<u>Experimental configuration:</u>



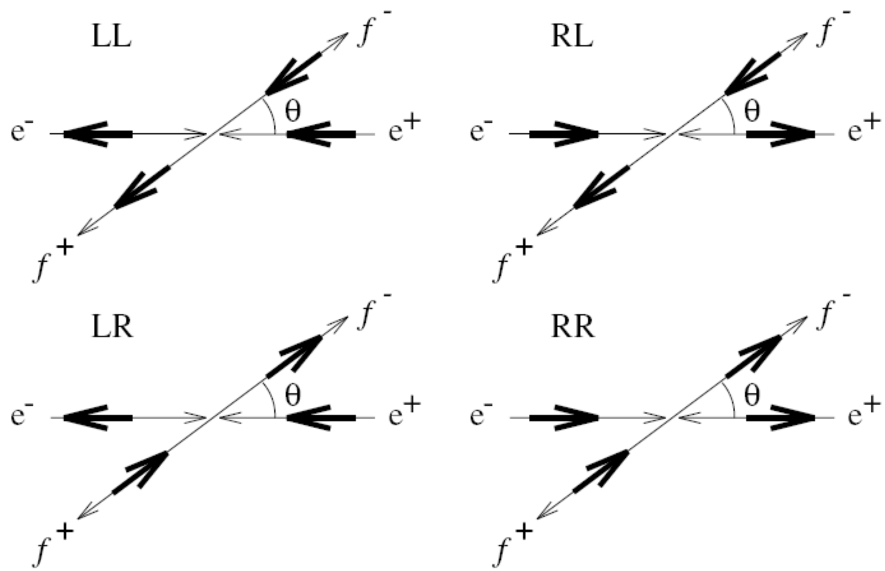
$\Rightarrow \quad g_L$

$\Rightarrow \quad g_R$



$$\frac{-ig}{\cos\theta_w}\gamma^\mu \frac{1}{2}\left(g_R^e + g_L^e\right)$$

Instead of measuring the spin averaged transition amplitudes try to decompose the different "helicity" components to the cross section:



Vector coupling removes all other combinations.

Observables:

$\sigma_{LL} + \sigma_{RR}$ $\sigma_{RL} + \sigma_{LR}$

Related to σ_F and σ_B.

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Forward-backward asym.

$\sigma_L = \sigma_{LL} + \sigma_{LR}$ $\sigma_R = \sigma_{RL} + \sigma_{RR}$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Left right asym. (initial)

$\sigma_- = \sigma_{LL} + \sigma_{RL}$ $\sigma_+ = \sigma_{RR} + \sigma_{LR}$

$$\mathcal{P}_f = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

fermion polarization (final)

Forward-backward asymmetry

Angular distribution: (see above)

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W}\left[2g_V^e g_V^\mu (1+\cos^2\theta) + 4 g_A^e g_A^\mu \cos\theta\right]$$

Very small: $g_V \approx 0$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W}\left[({g_V^e}^2 + {g_A^e}^2)({g_V^\mu}^2 + {g_A^\mu}^2)(1+\cos^2\theta) + 8 g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta\right]$$

Forward-backward asymmetry A_{FB}

$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

- Away from the resonance large $\rightarrow$ interference term dominates

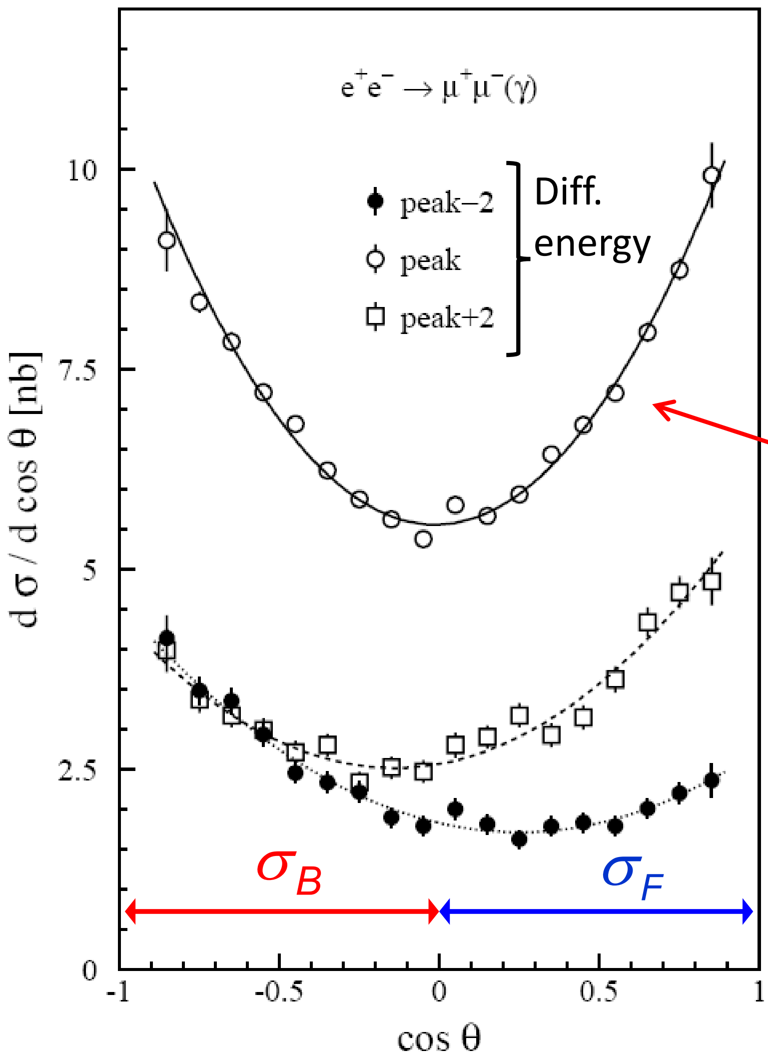
$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \qquad \rightarrow \text{large}$$

- At the Z pole: Interference = 0 (see energy dependence of interference term)

$$A_{FB} = 3 \cdot \frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \cdot \frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2}$$

$\rightarrow$ very small because g_V^l small in SM

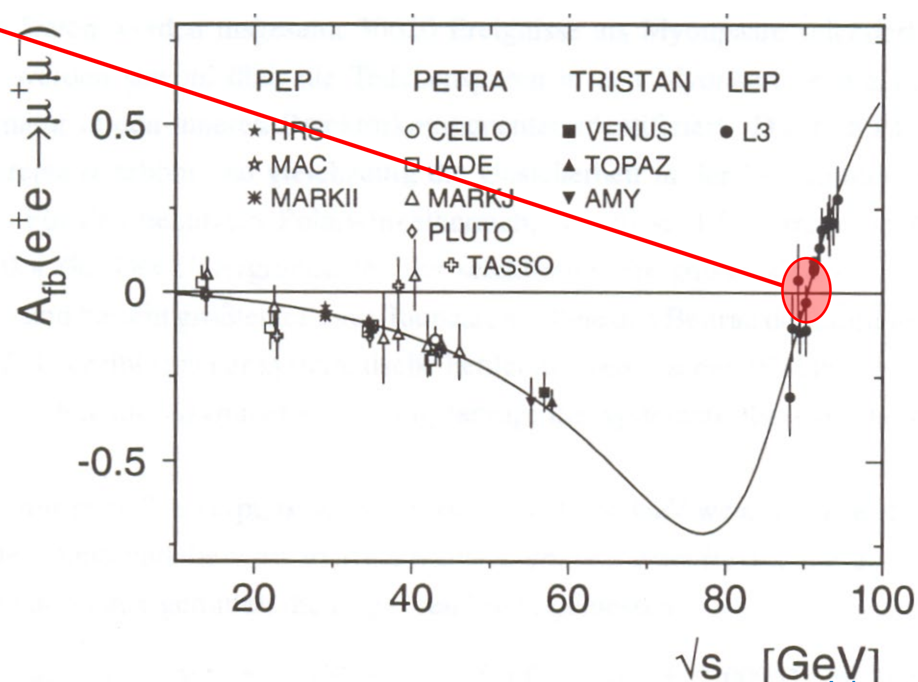
$$e^+ e^- \to Z \to \mu^+ \mu^-$$



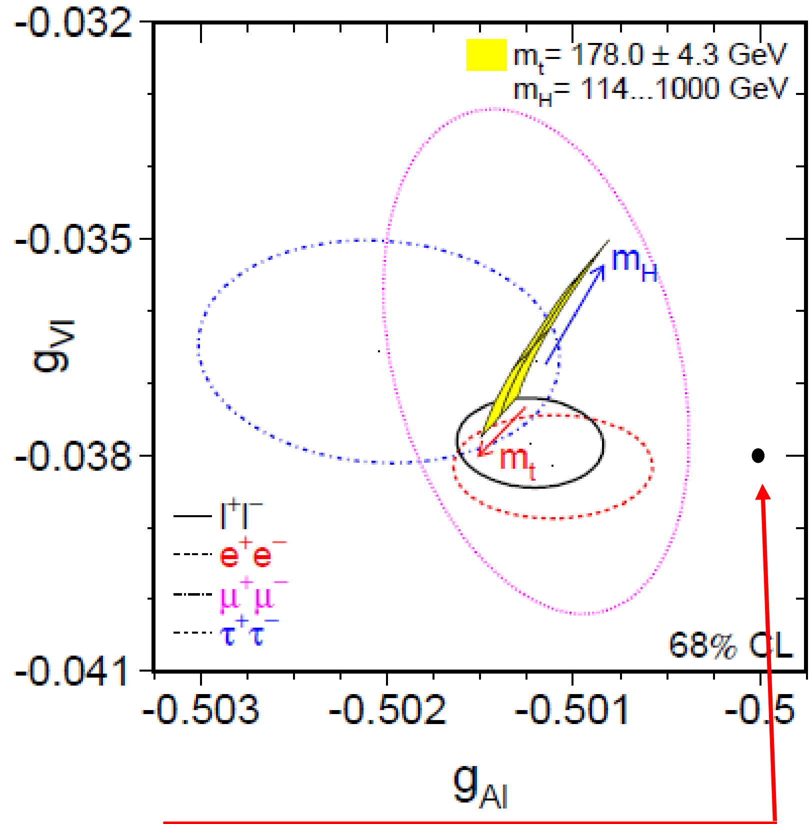
$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with $$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$



Determination of the couplings g_A and g_V



Asymmetrie at the Z pole

$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

Cross section at the Z pole

$$\sigma_Z \sim \left[(g_V^e)^2 + (g_A^e)^2 \right] \left[(g_V^f)^2 + (g_A^f)^2 \right]$$

Lepton asymmetries together with lepton pair cross sections allow the determination of the lepton couplings g_A and $g_{V.}$ → elliptical confidence regions

Good agreement between the 3 lepton species confirms “lepton universality”

Different contour size: electrons are measured in all measurements; tau contour uses additional measurement (polarization)

Tree level SM prediction

Deviation from tree level SM prediction

$$g_V = T_3 - 2q\sin^2\theta_W \quad g_A = T_3 \quad \sin^2\theta_w = 1 - \frac{m_W^2}{m_Z^2}$$

is an effect of higher-order loop-corrections.

Assuming lepton universality the measurements result in:

$$g_V^\ell = -0.03783 \pm 0.00041$$

$$g_A^\ell = -0.50123 \pm 0.00026$$

$$g_R^\ell = +0.23170 \pm 0.00025$$

$$g_L^\ell = -0.26959 \pm 0.00024$$

From g_v one can calculate $\sin^2\theta_w$

$$\sin^2\theta_w = 0.23113 \pm 0.00021$$

("effective mixing angle" – measurement includes higher order effects in couplings)

As predicted by the SM: Z boson couples stronger to LH leptons than to RH leptons.

Using Z and W boson masses:

$$\left.\begin{aligned} m_W &= 80.3692 \pm 0.0133 \text{ GeV} \\ m_z &= 91.1880 \pm 0.0020 \text{ GeV} \end{aligned}\right\} \text{PDG}$$

$$\sin^2\theta_w = 1 - \frac{m_w^2}{m_Z^2} = 0.2232 \pm 0.0003$$

The difference between the two determination of the weak mixing angle is related to higher-order loop corrections which modify the ρ-parameter and leads to effective couplings and an effective mixing angle (next semester).