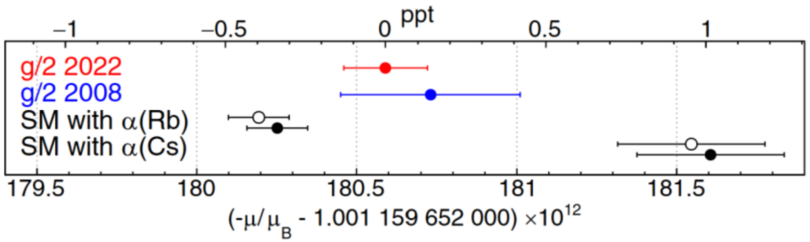
Anomalous Magnetic Moments of Leptons

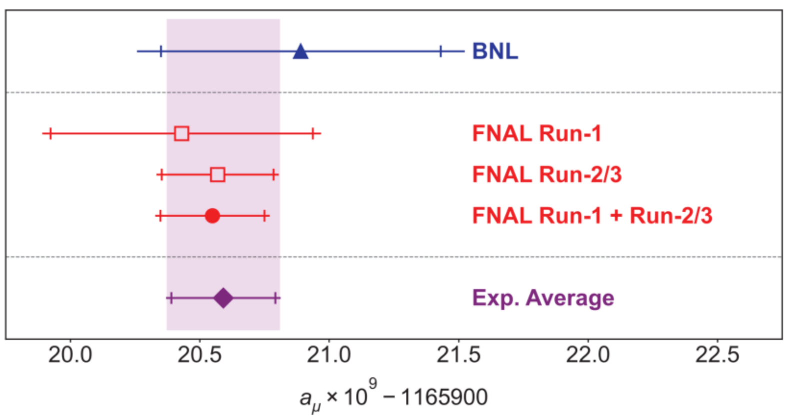
Electron:

Fan *et al.*, Phys. Rev. Lett. **130**, 071801 (2023)



Muon:

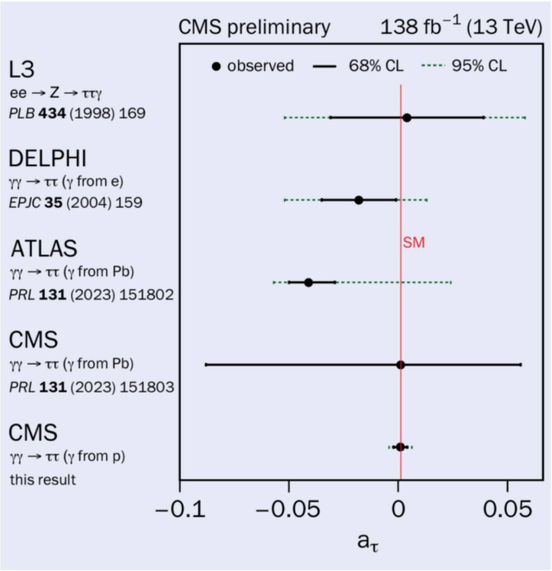
D. P. Aguillard *et al.* Phys. Rev. Lett. **131**, 161802 (2023)



Tau:

ATLAS, Phys. Rev. Lett. **131**, 151802 (2023)

CMS, Phys. Rev. Lett. **131**, 151803 (2023)

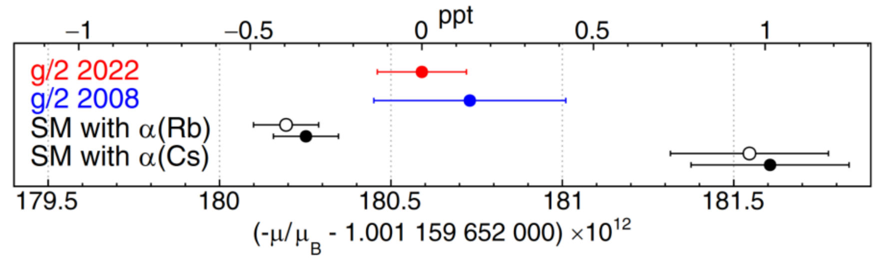


Electron: single-electron quantum cyclotron

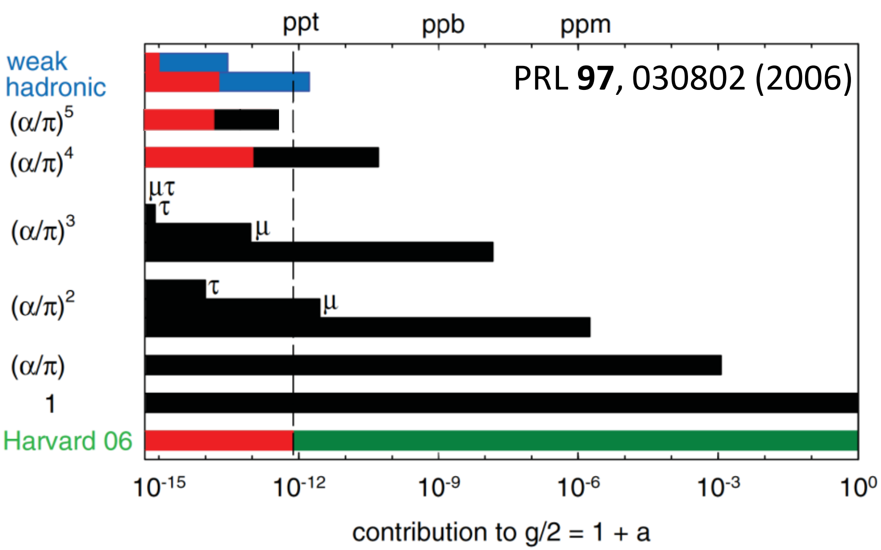
$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\mu,\tau} + a_{\text{hadron}} + a_{\text{weak}}$$

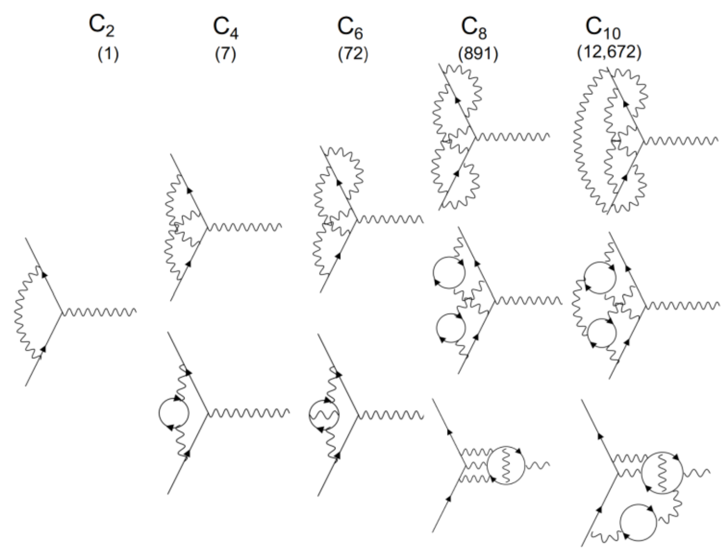
Xing Fan, 2022. Doctoral dissertation (Harvard)

term	contribution
tree level	1.000 000 000 000 000
$C_2\left(\frac{\alpha}{\pi}\right)$	0.001 161 409 731 851 (000)(093)
$C_4\left(\frac{\alpha}{\pi}\right)^2$	−0.000 001 772 305 060 (000)(000)
$C_6\left(\frac{\alpha}{\pi}\right)^3$	0.000 000 014 804 204 (000)(000)
$C_8\left(\frac{\alpha}{\pi}\right)^4$	−0.000 000 000 055 668 (000)(000)
$C_{10}\left(\frac{\alpha}{\pi}\right)^5$	0.000 000 000 000 456 (011)(000)
$a_{\mu,\tau}$	0.000 000 000 002 748 (000)
a_{hadron}	0.000 000 000 001 693 (012)
a_{weak}	0.000 000 000 000 031 (000)
total SM prediction	1.001 159 652 180 252 (011)(012)(093)
measured $g/2$ (2022)	1.001 159 652 180 593 (134)



Fan *et al.*, Phys. Rev. Lett. **130**, 071801 (2023)



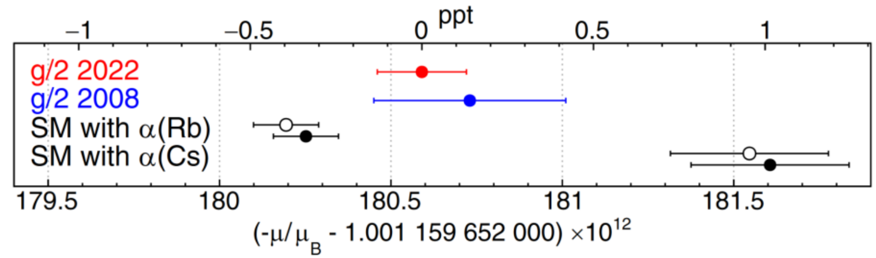


Xing Fan, 2022. Doctoral dissertation (Harvard)

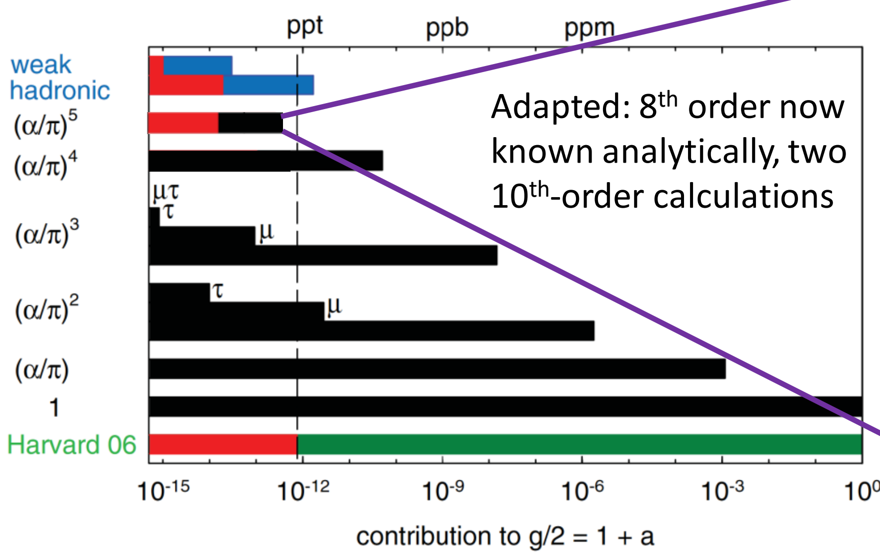
Electron: orders and uncertainties

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\mu,\tau} + a_{\text{hadron}} + a_{\text{weak}}$$

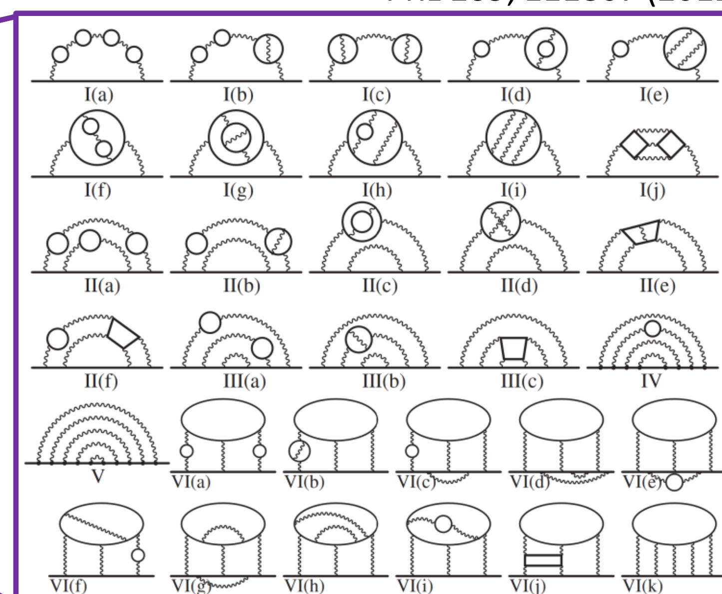
term	contribution
tree level	1.000 000 000 000 000
$C_2\left(\frac{\alpha}{\pi}\right)$	0.001 161 409 731 851 (000)(093)
$C_4\left(\frac{\alpha}{\pi}\right)^2$	−0.000 001 772 305 060 (000)(000)
$C_6\left(\frac{\alpha}{\pi}\right)^3$	0.000 000 014 804 204 (000)(000)
$C_8\left(\frac{\alpha}{\pi}\right)^4$	−0.000 000 000 055 668 (000)(000)
$C_{10}\left(\frac{\alpha}{\pi}\right)^5$	0.000 000 000 000 456 (011)(000)
$a_{\mu,\tau}$	0.000 000 000 002 748 (000)
a_{hadron}	0.000 000 000 001 693 (012)
a_{weak}	0.000 000 000 000 031 (000)
total SM prediction	1.001 159 652 180 252 (011)(012)(093)
measured $g/2$ (2022)	1.001 159 652 180 593 (134)



Fan *et al.*, Phys. Rev. Lett. **130**, 071801 (2023)



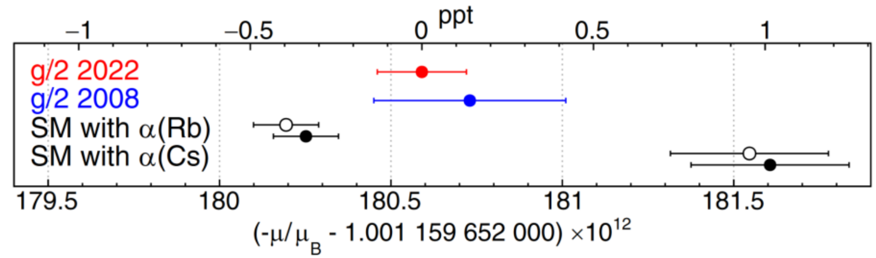
PRL **109**, 111807 (2012)



Electron: adding it all up

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\mu,\tau} + a_{\text{hadron}} + a_{\text{weak}}$$

term	contribution
tree level	1.000 000 000 000 000
$C_2\left(\frac{\alpha}{\pi}\right)$	0.001 161 409 731 851 (000)(093)
$C_4\left(\frac{\alpha}{\pi}\right)^2$	−0.000 001 772 305 060 (000)(000)
$C_6\left(\frac{\alpha}{\pi}\right)^3$	0.000 000 014 804 204 (000)(000)
$C_8\left(\frac{\alpha}{\pi}\right)^4$	−0.000 000 000 055 668 (000)(000)
$C_{10}\left(\frac{\alpha}{\pi}\right)^5$	0.000 000 000 000 456 (011)(000)
$a_{\mu,\tau}$	0.000 000 000 002 748 (000)
a_{hadron}	0.000 000 000 001 693 (012)
a_{weak}	0.000 000 000 000 031 (000)
total SM prediction	1.001 159 652 180 252 (011)(012)(093)
measured $g/2$ (2022)	1.001 159 652 180 593 (134)



Fan *et al.*, Phys. Rev. Lett. **130**, 071801 (2023)

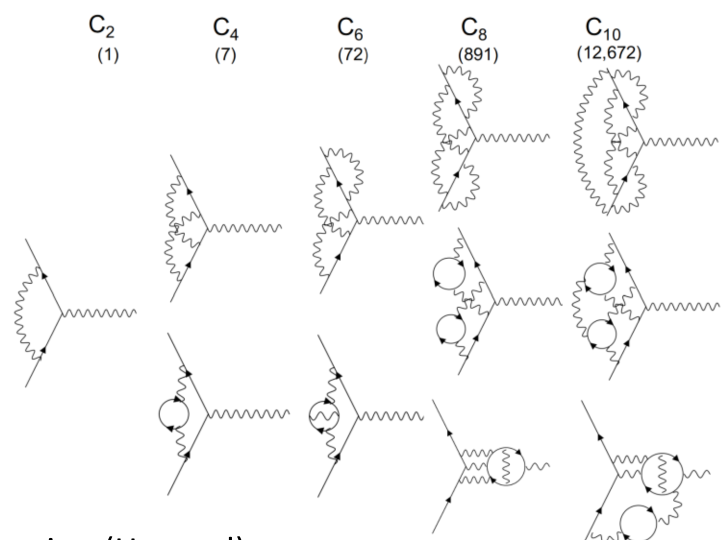
$$C_2 = \frac{1}{2} = 0.5 \quad [3]$$

$$C_4 = \frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3) - \frac{1}{2}\pi^2 \ln 2$$
$$= -0.328\,478\,965\,579\,193\ldots \quad [9,10]$$

$$C_6 = \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5)$$
$$+ \frac{100}{3}\left[\left(\sum_{n=1}^{\infty}\frac{1}{2^n n^4} + \frac{1}{24}\ln^4 2\right) - \frac{1}{24}\pi^2\ln^2 2\right] - \frac{239}{2160}\pi^4 + \frac{139}{18}\zeta(3)$$
$$- \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} = 1.181\,241\,456\,587\ldots \quad [11]$$

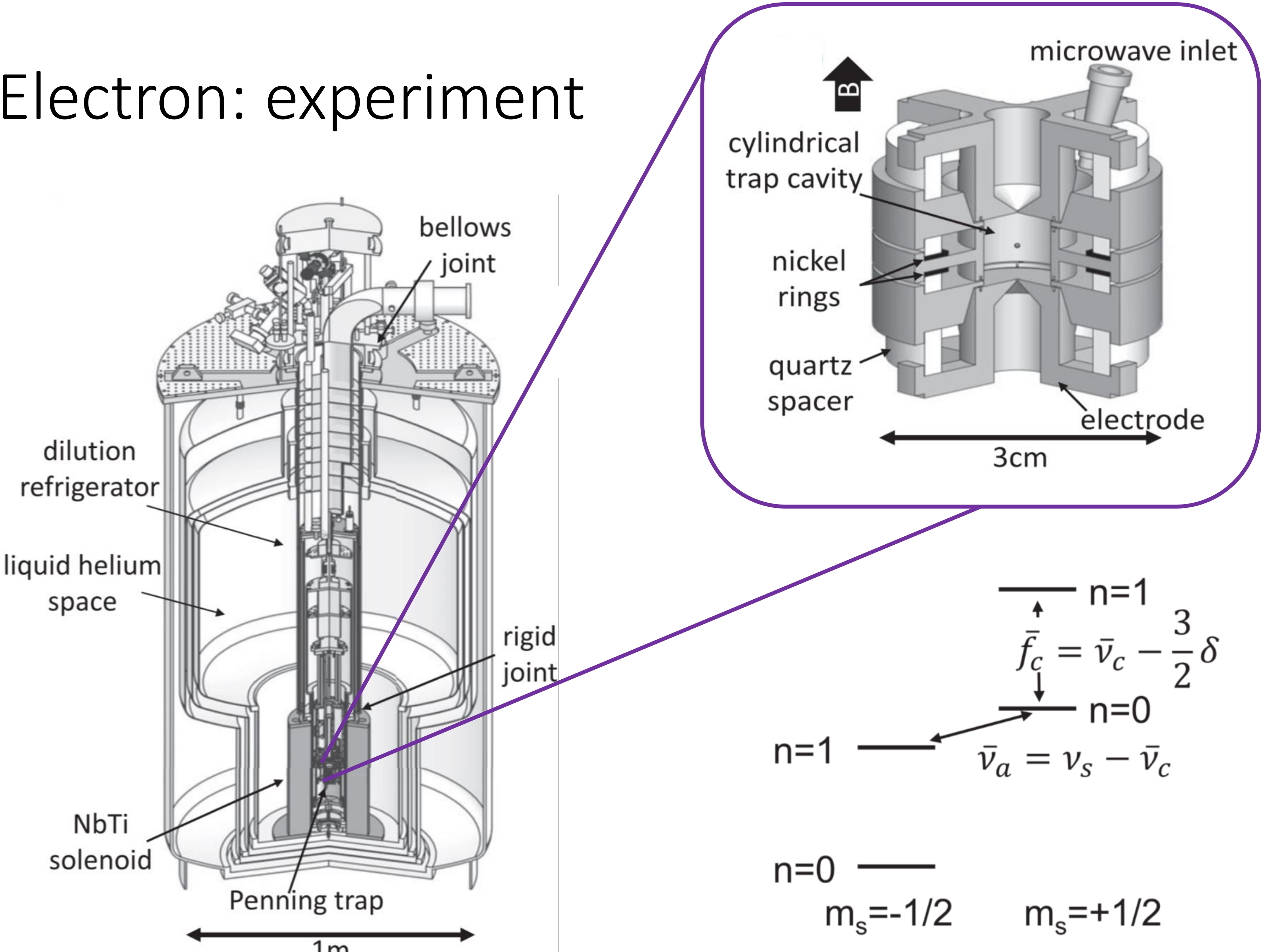
$$C_8 = -1.912\,245\,764\,926\ldots \text{ (evaluated up to 1100 digits)} \quad [12,13]$$

$$C_{10} = 6.737\,(159) \quad [8,14],$$



Xing Fan, 2022. Doctoral dissertation (Harvard)

Electron: experiment



Fan *et al.*, Phys. Rev. Lett. **130**, 071801 (2023)

Electron: cryostat and Penning trap

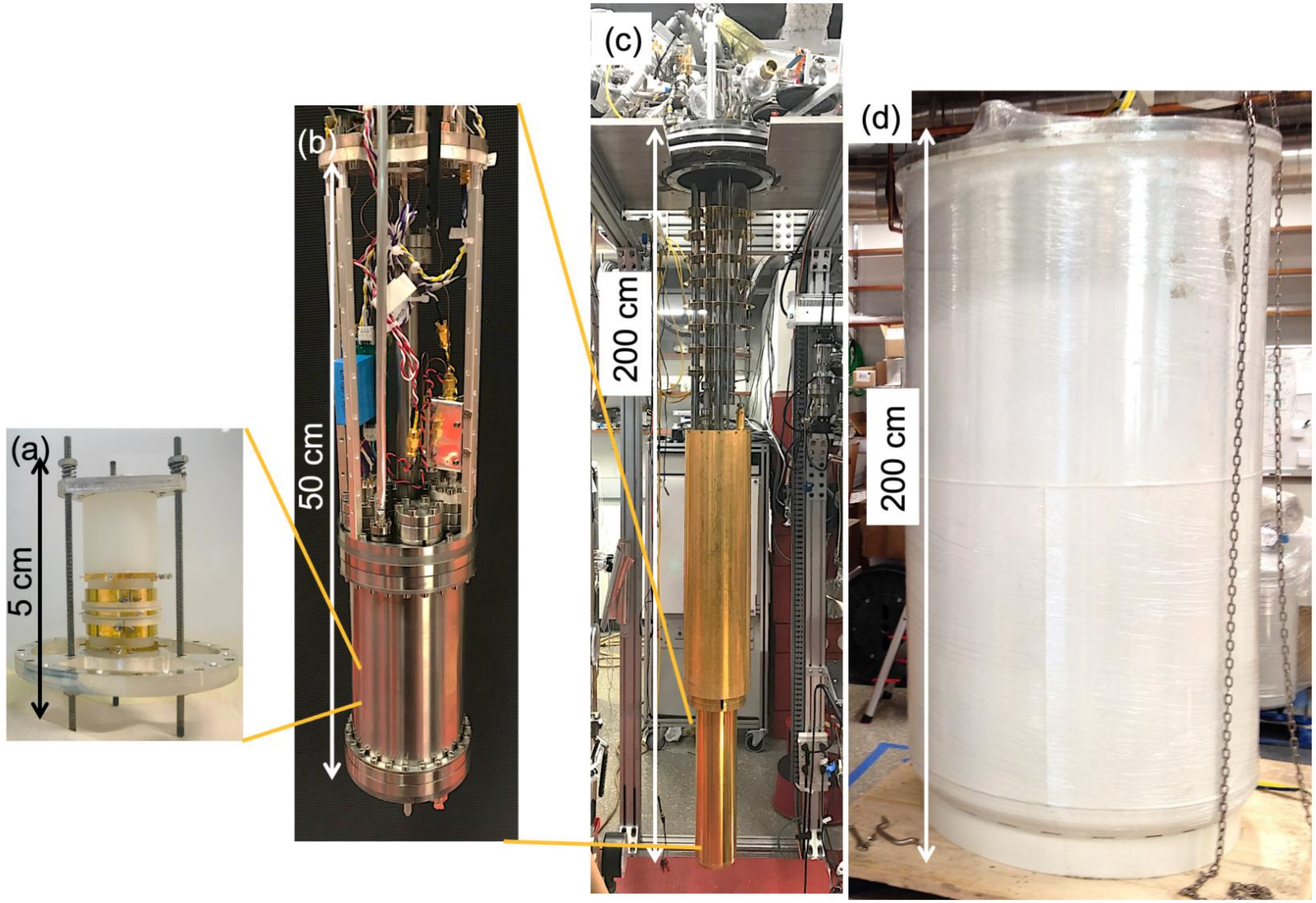
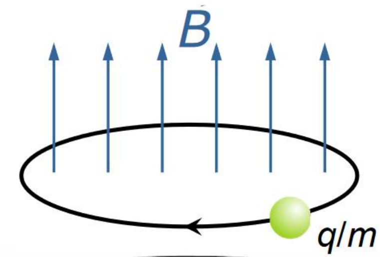


Figure 2.4: The entire setup of the experiment. A Penning trap (a) is housed in a titanium vacuum chamber (b), and the vacuum chamber is suspended at the bottom of a dilution refrigerator (c). The dilution refrigerator is inserted into the dewar (d), which has the superconducting magnet at its bottom. See also Fig. 2.7.

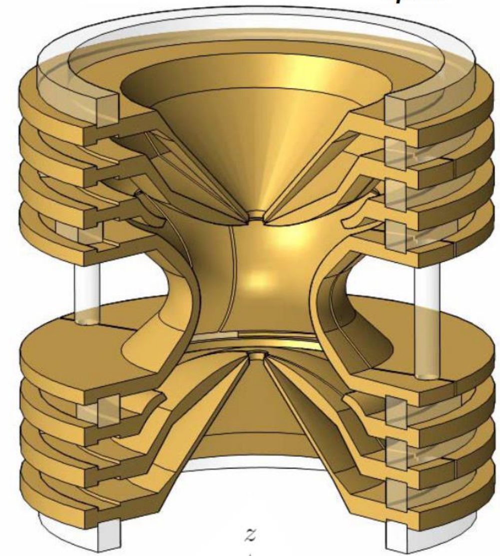
Xing Fan, 2022. Doctoral dissertation (Harvard)

Electron: Penning trap



$$\omega_c = \frac{e}{m} B$$

cyclotron frequency



Principle of a penning trap …

Combination of B-field and cylindric electric quadrupole field …

Trap single electrons; electron & trap constitute 'atom' with macroscopic dimensions …

Motion of electron characterized by 3 frequencies:

$$\omega_z = \frac{qV}{md^2}$$

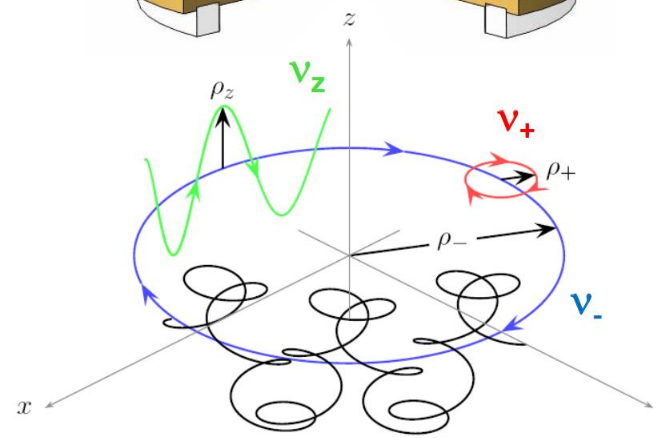
axial frequency
[oscillations in E-field]

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$

xy-oscillation
[perturbed ω_c]

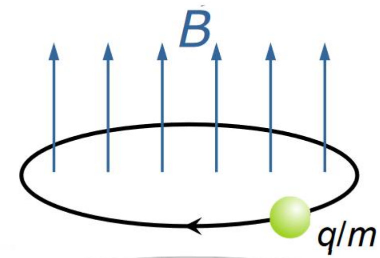
$$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$

magnetron oscillation
[small frequency]



slide courtesy of Hans-Christian Schultz-Coulon

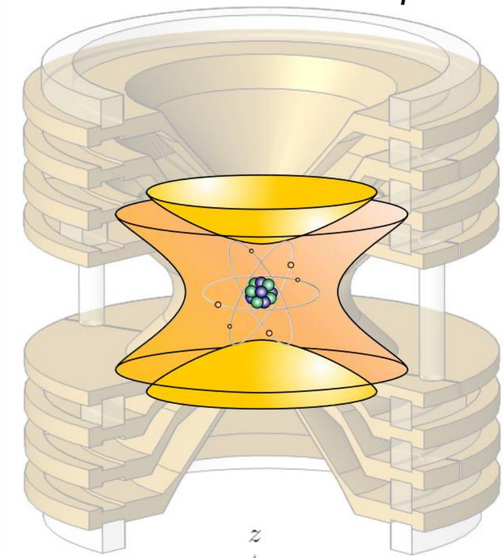
Electron: Penning trap



Principle of a penning trap …

$$\omega_c = \frac{e}{m}B$$

cyclotron frequency



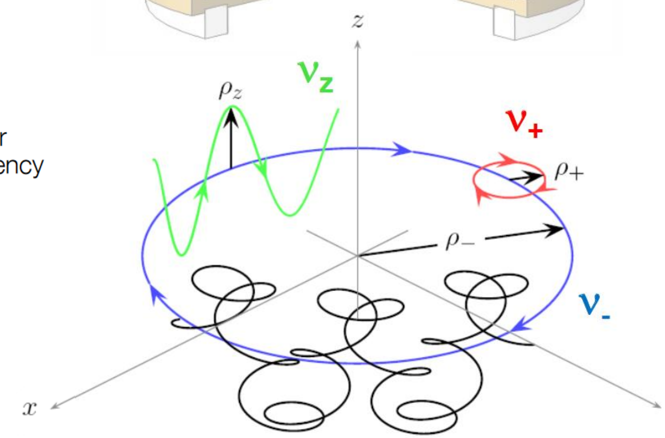
Combination of B-field and cylindric electric quadrupole field …

Trap single electrons;
electron & trap constitute 'atom' with macroscopic dimensions …

Energy levels quantized …
[harmonic oscillators; include spin]

$$E = (n_+ + \tfrac{1}{2})\hbar\omega_+ + (n_z + \tfrac{1}{2})\hbar\omega_z - (n_- + \tfrac{1}{2})\hbar\omega_- + m_s\hbar\omega_s$$

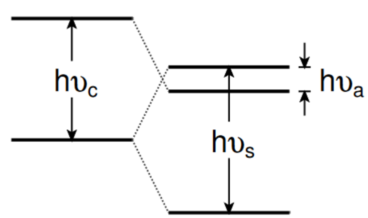
Lamor frequency



For simplicity consider only ω_c and ω_s …
[ω_c can be extracted from ω_z, ω_+, ω_-]

$$E = (n_c + \tfrac{1}{2} + m_s + a m_s)\,\hbar\omega_c$$

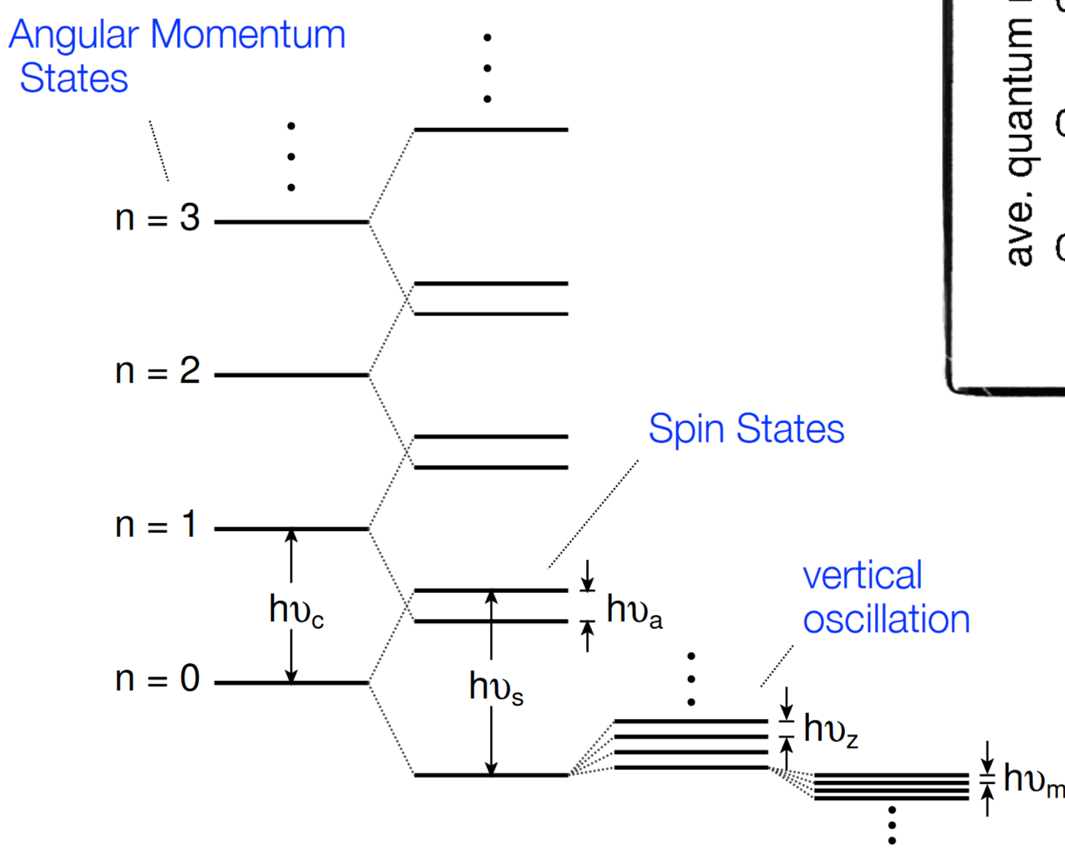
using $\omega_s - \omega_c = a\omega_c$

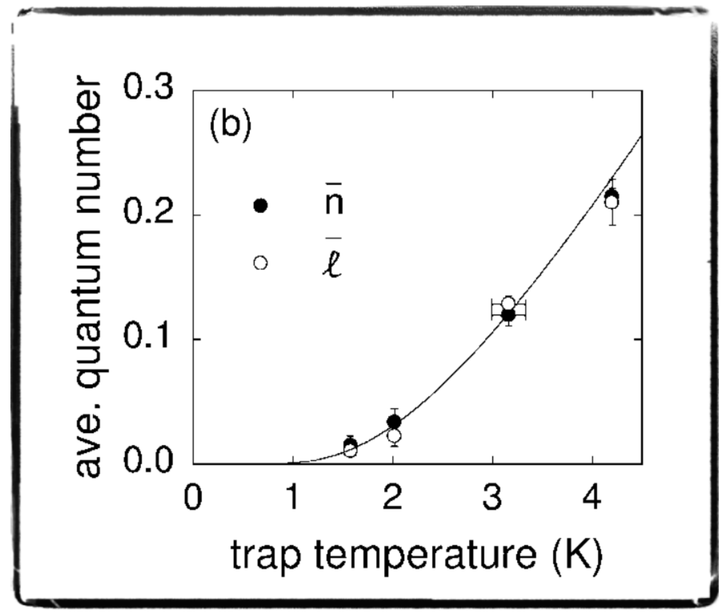


slide courtesy of Hans-Christian Schultz-Coulon

Electron: Harmonic motion and temperature

Idea: resolve
cyclotron and spin levels





T = 80 mK

Vertical oscillation coupled to cyclotron and Larmor motion

Observe Quantum Jumps

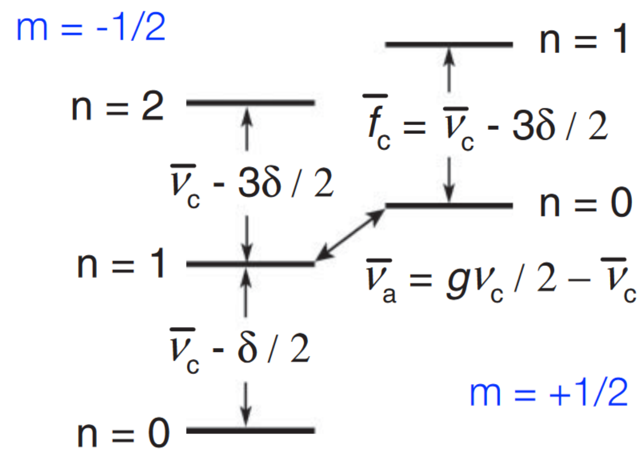
[QND: Quantum-non-demolition meas.]

slide courtesy of Hans-Christian Schultz-Coulon

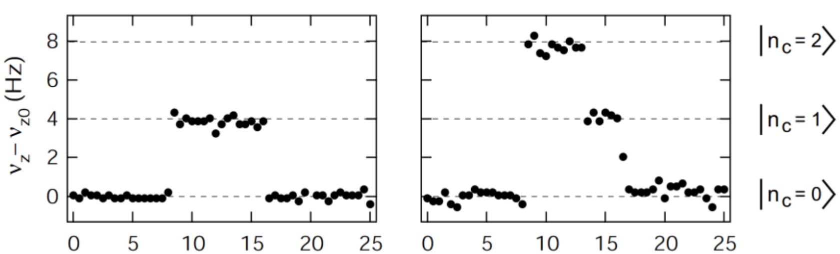
Electron: quantum jump spectroscopy

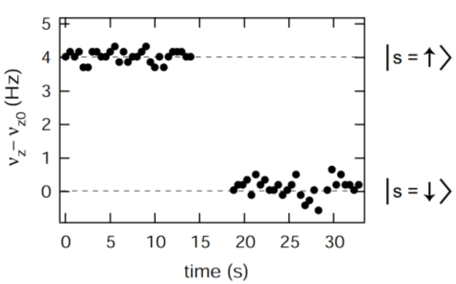
1. Prepare
$|\, n=0, m=½ \rangle$ State

2. Drive with frequencies
$\bar{f}_c$ or $\bar{\nu}_a$ and observe possible quantum jump



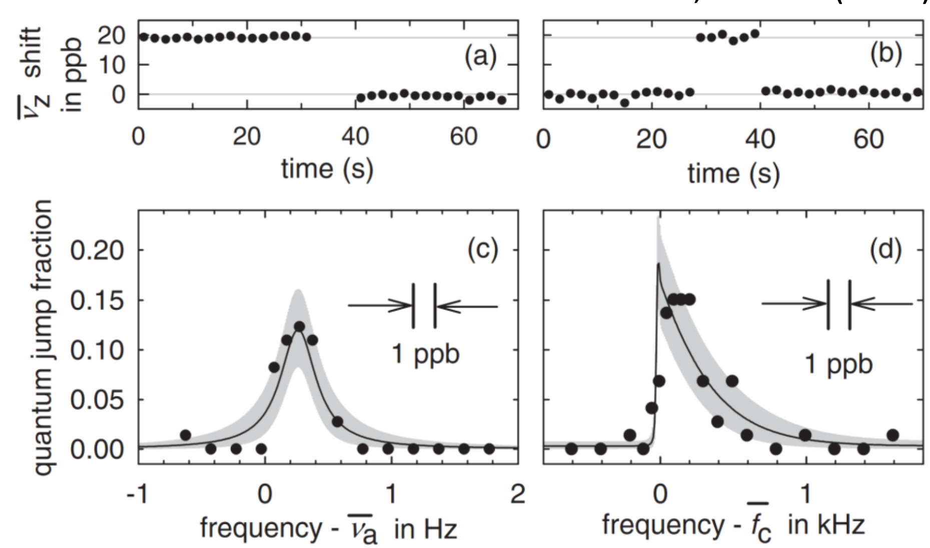
PRL **97**, 030801 (2006)



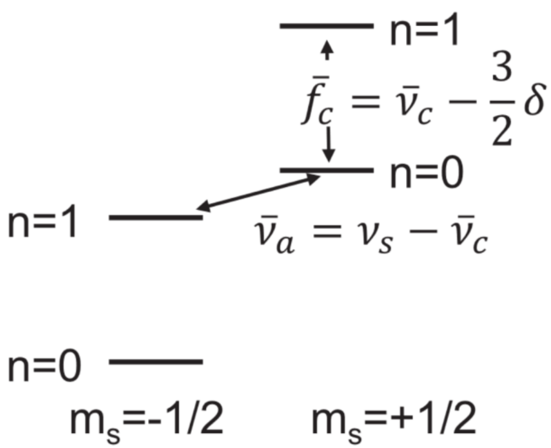


Idea:

Quantum Jump Spectroscopy



Electron: The latest measurements



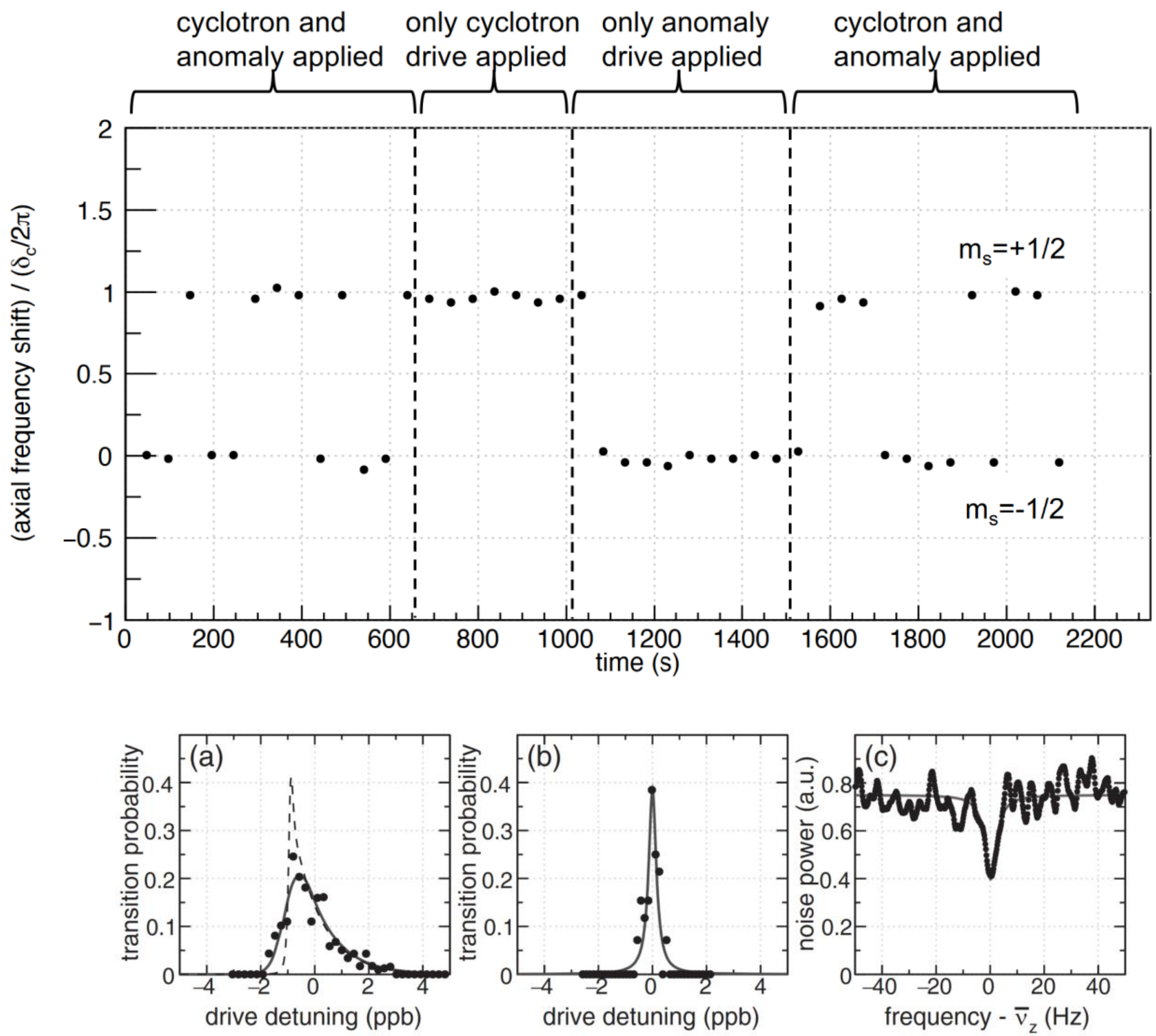
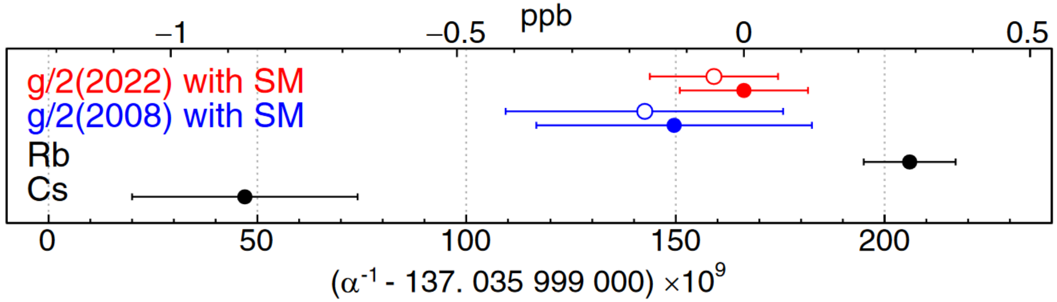


FIG. 3. Quantum jump cyclotron (a) and anomaly (b) line shapes that are measured (points), predicted (dashed line) and fit (solid line) vs fractional drive detunings from $\bar{f}_c(1+\epsilon)$ and $\bar{\nu}_a(1+\epsilon)$ (defined later in the text). (c) A dip in Johnson noise reveals $\bar{\nu}_z$.

Xing Fan, 2022. Doctoral dissertation (Harvard)

Electron: 2023 result and cavity corrections

$$-\frac{\mu}{\mu_B} = \frac{g}{2} = 1.001\,159\,652\,180\,59(13) \quad [0.13\text{ ppt}]$$



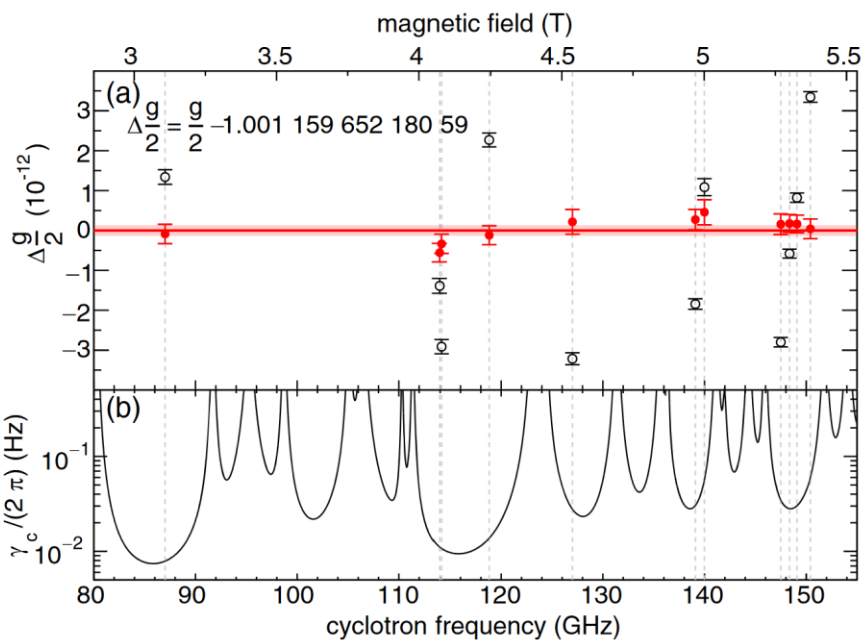
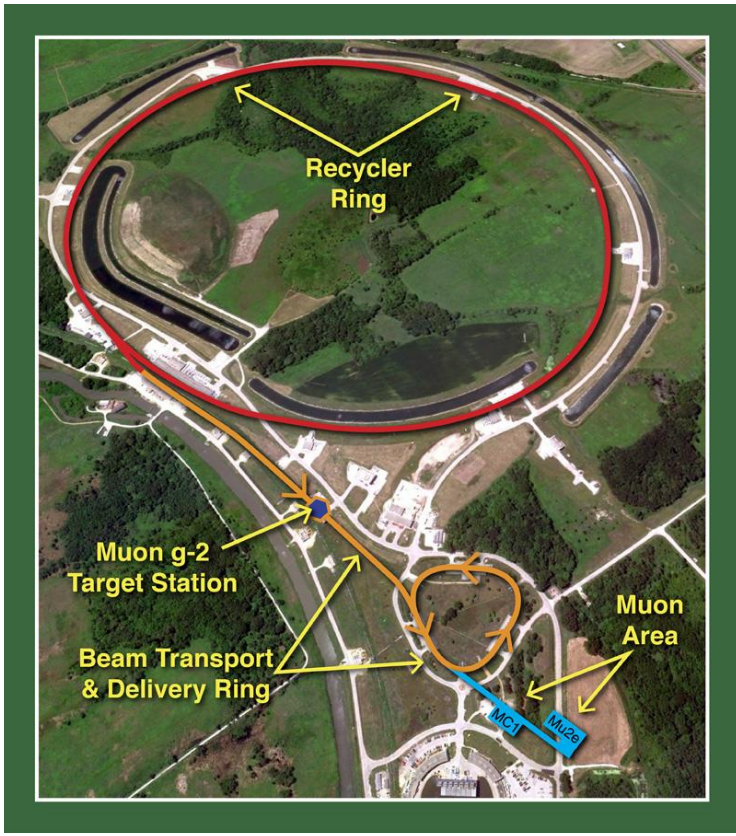


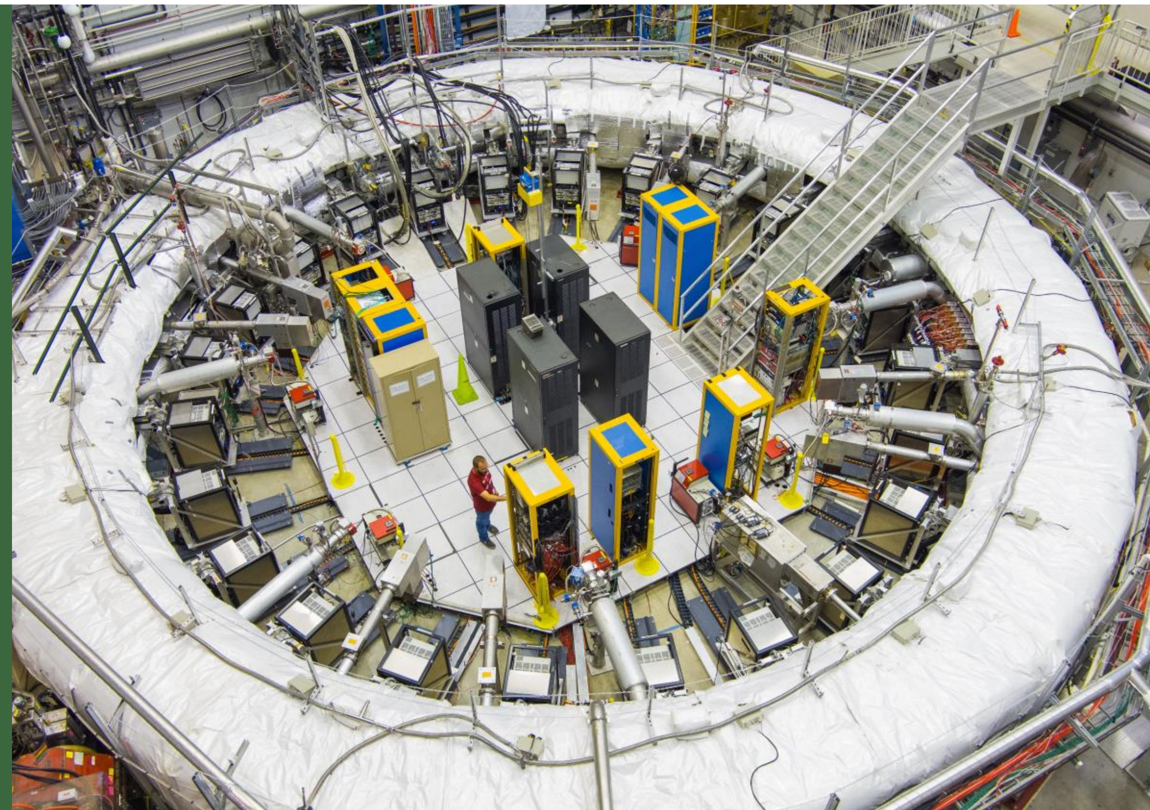
TABLE I. Largest uncertainties for $g/2$.

Source	Uncertainty $\times 10^{13}$
Statistical	0.29
Cyclotron broadening	0.94
Cavity correction	0.90
Nuclear paramagnetism	0.12
Anomaly power shift	0.10
Magnetic field drift	0.09
Total	1.3

Fan *et al.*, Phys. Rev. Lett. **130**, 071801 (2023)

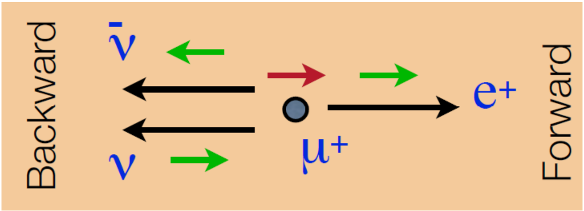
Muon: new experiment (FNAL)



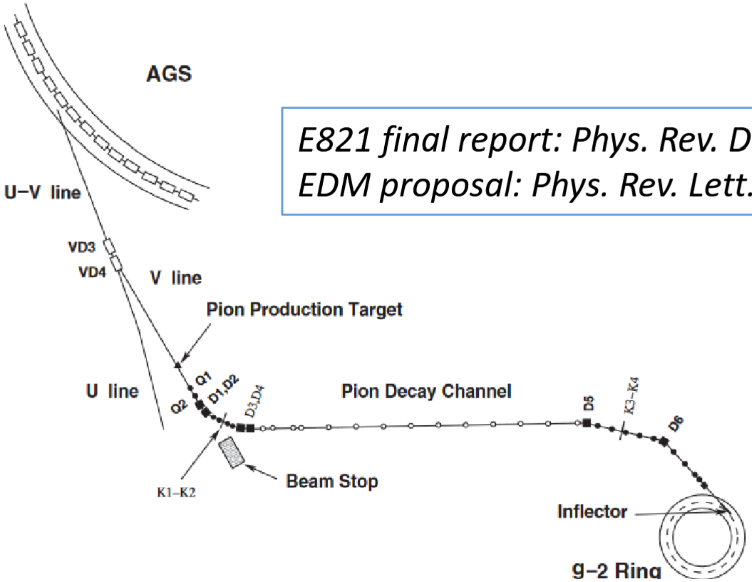


slide courtesy of Hans-Christian Schultz-Coulon

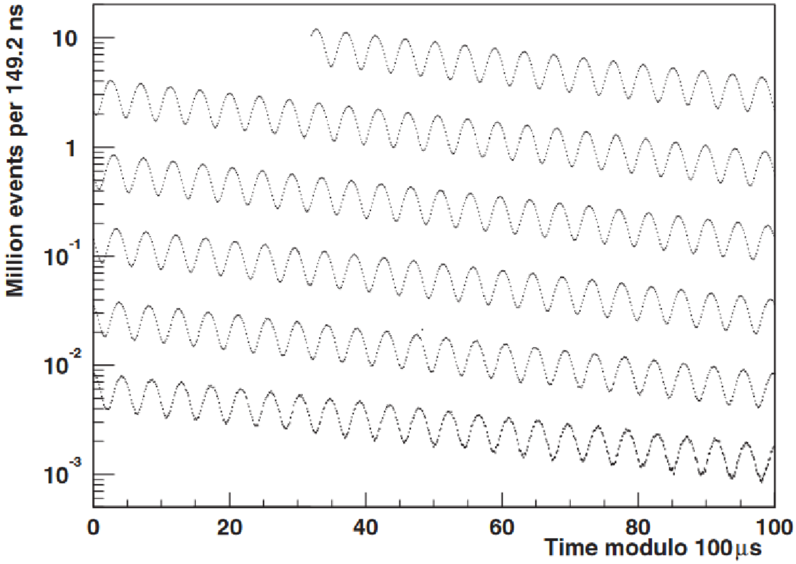
Muon: concept and history



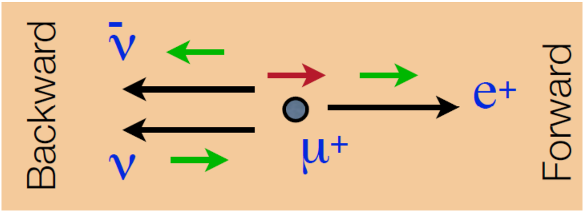
- ***Background:*** E821 at Brookhaven, measurement of the muon's anomalous magnetic moment: $a_\mu = \frac{1}{2}(g_\mu - 2)$
- Relativistic evolution of a charged particle's momentum and spin, observed with *static and uniform* electric and magnetic fields applied



E821 final report: Phys. Rev. D ***73****, 072003 (2006)*
EDM proposal: Phys. Rev. Lett. ***93****, 052001 (2004)*



Muon: wiggle plot



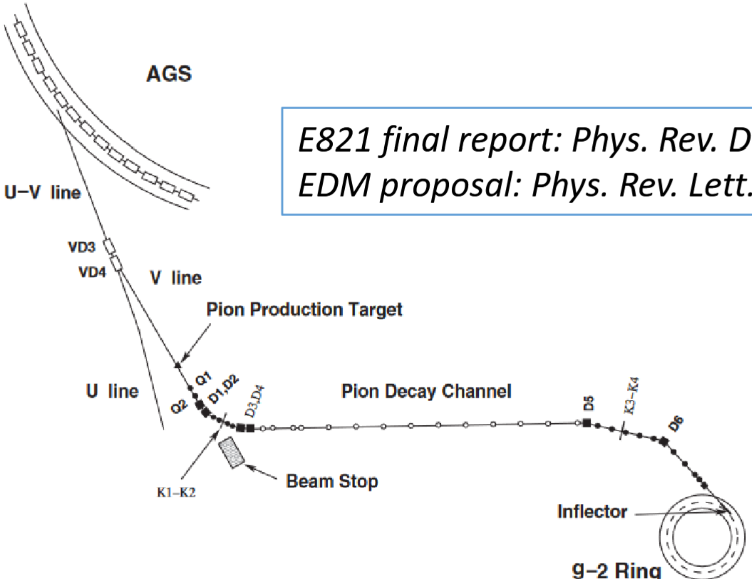
Master equation:

$$\boldsymbol{\omega}_a = -\frac{q}{m}\left[a_\mu \boldsymbol{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\boldsymbol{\beta} \times \boldsymbol{E}}{c}\right]$$

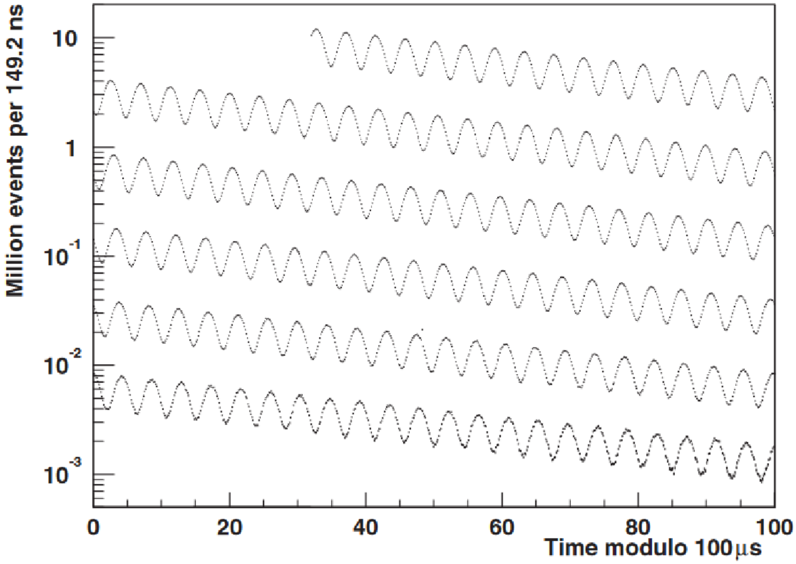
vanishes at the "magic" value $\gamma = 29.3$

Result: $a_\mu(\text{Expt}) = 11659208.0(5.4)(3.3) \times 10^{-10}$

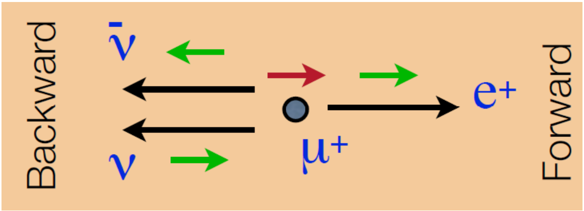
- ***Background:*** E821 at Brookhaven, measurement of the muon's anomalous magnetic moment: $a_\mu = \frac{1}{2}(g_\mu - 2)$
- Relativistic evolution of a charged particle's momentum and spin, observed with *static and uniform* electric and magnetic fields applied



*E821 final report: Phys. Rev. D **73**, 072003 (2006)*
*EDM proposal: Phys. Rev. Lett. **93**, 052001 (2004)*



Muon: full equation



Master equation:

$$\boldsymbol{\omega}_a = -\frac{q}{m}\left[a_\mu \boldsymbol{B} - \underbrace{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)}_{\text{vanishes at the ``magic'' value } \gamma = 29.3} \frac{\boldsymbol{\beta} \times \boldsymbol{E}}{c}\right]$$

Result: $a_\mu(\mathrm{Expt}) = 11659208.0(5.4)(3.3) \times 10^{-10}$

Spin evolution in the lab frame:

$$\frac{d\boldsymbol{S}}{dt} = \frac{q\boldsymbol{S}}{m\gamma} \times \left[\boldsymbol{B} - \frac{\gamma}{\gamma+1}\left(\frac{\boldsymbol{v} \times \boldsymbol{E}}{c^2}\right)\right] + a\frac{q\boldsymbol{S}}{m} \times \left[\boldsymbol{B} - \frac{\boldsymbol{v} \times \boldsymbol{E}}{c^2} - \frac{\gamma}{\gamma+1}\frac{\boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{B})}{c^2}\right] + \eta\frac{q\boldsymbol{S}}{2mc} \times \left[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - \frac{\gamma}{\gamma+1}\frac{\boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{E})}{c^2}\right]$$

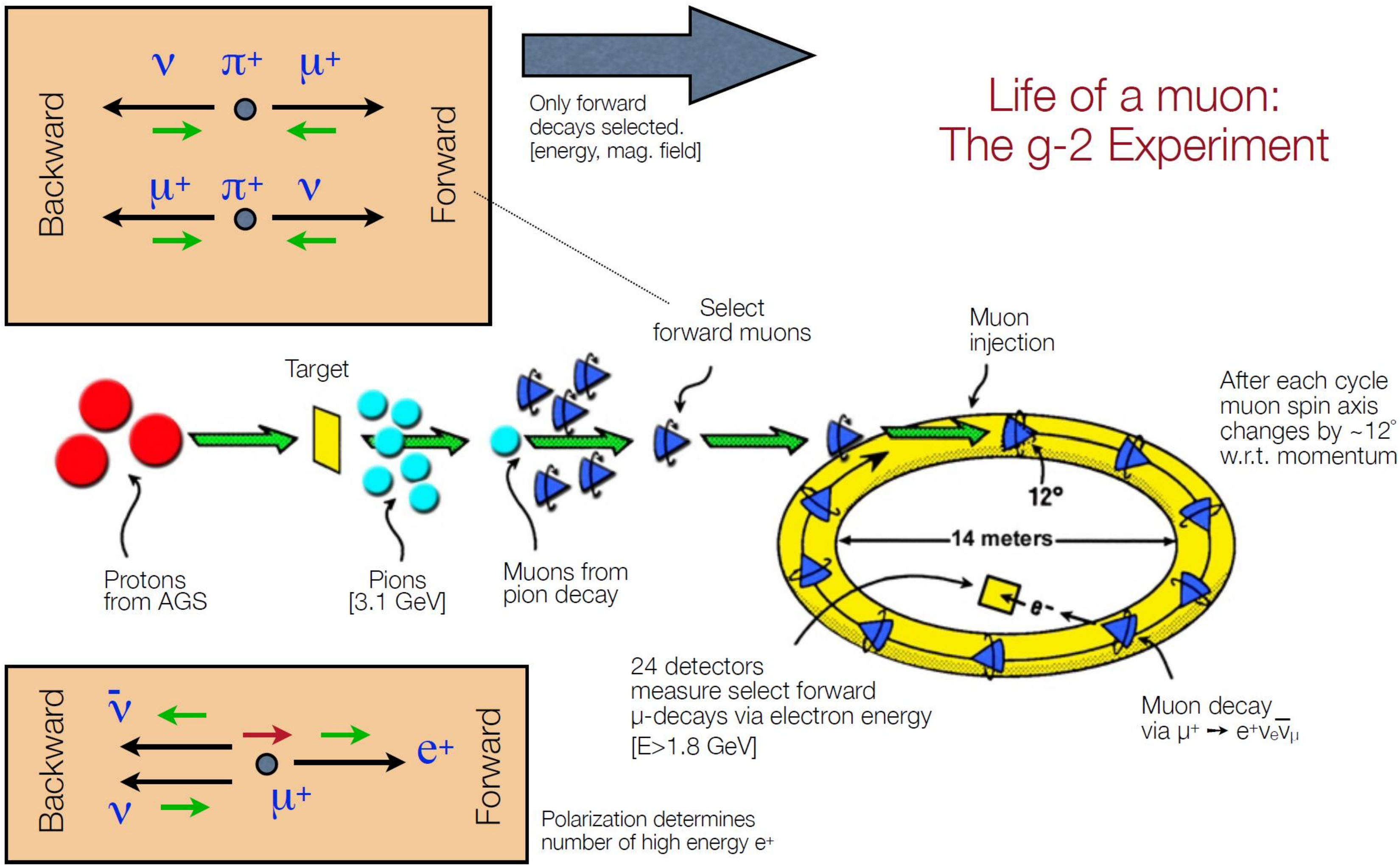
"BMT equation" for torque on MDM in lab frame; see Phys. Rev. Lett. 2, 435 (1959)

EDM term, when $\eta \neq 0$

Muon EDM limit from the Brookhaven experiment: $|d_\mu| < 1.8 \times 10^{-19} e\,\mathrm{cm}$

Phys. Rev. D **80**, 052008 (2009)

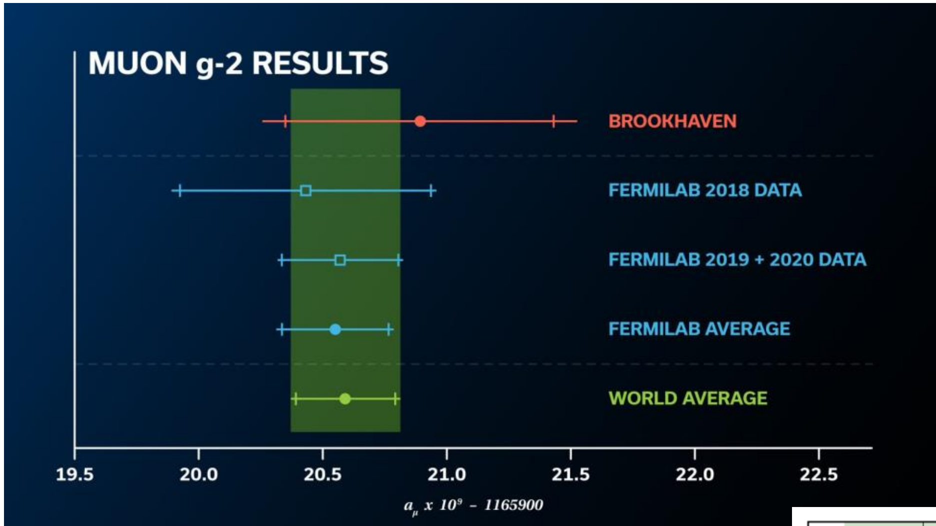
Muon: from production to measurement

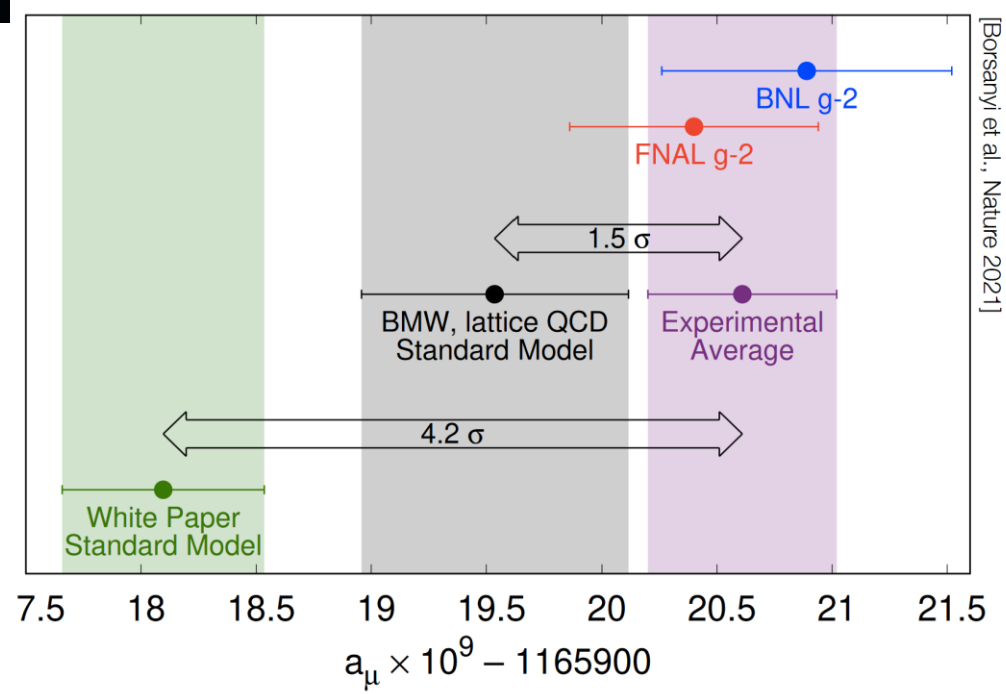


slide courtesy of Hans-Christian Schultz-Coulon


.785
.786

Muon: experiments seem to agree…





…but what about theory?

More recently: issues with form factors.