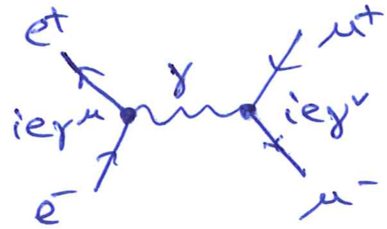
High-energy tests of QED: e^+e^-

Complementarity w/rt low-energy precision measurements
- test at high q^2
- equivalently, short distance scales

Major task: radiative effects at higher orders
* theory but also Monte Carlo for realistic detectors and analysis studies → needed, to make meaningful experimental validations
    - test QED
    - disentangle from weak/QCD
    - limits on possible substructure of elementary fermions

Example: $e^+e^- \to \mu^+\mu^-$

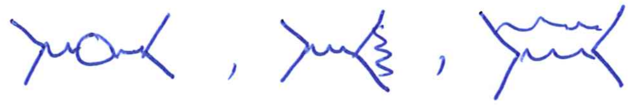


{ spin sums
{ helicity vs. chirality

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}\langle |M_{fi}|^2\rangle$$

leading order: 2 vertices $\Rightarrow |M|^2 \sim e^4 \sim \alpha^2$

next-to-leading order (NLO): 4 vertices $\Rightarrow |M|^2 \sim \alpha^4$

diagrams like  etc

need to sum amplitudes before squaring to get observables like decay rates or cross sections:

$$M_{fi} = \underbrace{M_{LO}}_{\sim\alpha} + \underbrace{\sum M_{NLO}}_{\sim\alpha^2} + \underbrace{\cdots}_{\sim\alpha^3 \text{ etc.}}$$

$$|M_{fi}|^2 = \underbrace{|M_{LO}|^2}_{\sim\alpha^2} + \underbrace{\sum(M_{LO}M^*_{NLO} + c.c.)}_{\sim\alpha^3} + \underbrace{|\sum M_{NLO}|^2}_{\sim\alpha^4 \text{ etc}} + \cdots$$

Each term in $|M_{fi}|^2$ is suppressed by $\alpha \sim \frac{1}{137}$ w/rt the previous one. (But there are more diagrams!)

$\Rightarrow$ naively, LO should be good to $\sim 1\%$ for QED

Spin sums and helicity vs. chirality

LO diagram for $e^+e^- \to \mu^+\mu^-$: 2 possible spin states

$\Rightarrow$ 16 orthogonal helicity combinations

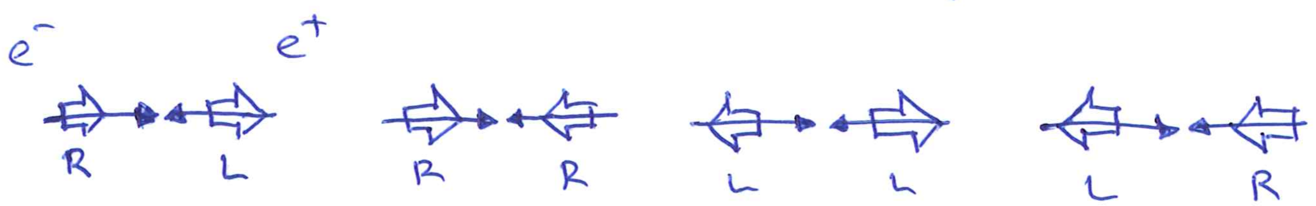
Need to calculate 16 matrix elements?

- trace techniques $\to$ see muon decay also
- direct calculation for each helicity (more laborious)

$$\begin{array}{l}\text{Total annihilation} \\ \text{rate for one } e^+e^- \\ \text{helicity state}\end{array} = \sum_{\substack{\mu^+\mu^- \\ \text{states}}} (\text{rate for each } \mu^+\mu^- \text{ state})$$

Unpolarized beams $\Rightarrow$ equal amounts of all helicities are initially present for e^+e^-

$$\Rightarrow \langle \; \rangle \sim \frac{1}{4} \sum_{\text{spins}}$$



$e^+_\downarrow e^-_\uparrow$ etc $\qquad e^+_\uparrow e^-_\uparrow$

$$\langle |M_{fi}|^2 \rangle = \underbrace{\frac{1}{(2s_e+1)^2} \sum_{e^+e^-}}_{\substack{\text{avg. over} \\ \text{initial spins}}} \underbrace{\sum_{\mu^+\mu^-}}_{\substack{\text{sum over} \\ \text{final spins}}} |M_{fi}|^2$$

helicity: $\dfrac{\vec{\Sigma}\cdot\vec{p}}{2p} \sim \dfrac{\vec{S}\cdot\vec{p}}{\vec{p}}$

- spin projection along the momentum direction
- <u>not</u> Lorentz-invariant, $m>0$
- but <u>is</u> a constant of motion

chirality: $P_R = \frac{1}{2}(1+\gamma^5)$

$P_L = \frac{1}{2}(1-\gamma^5)$

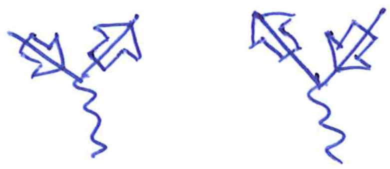
- transformation in LH/RH representation of Poincaré
- <u>is</u> Lorentz-invariant
- <u>not</u> a constant of the motion

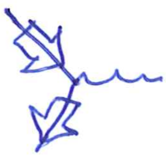
chiral structure of QED: helicity "conserved" (so can also speak of LH/RH helicity) for high-energy QED interactions

$E \gg m$, only 4 of the 16 helicity combinations give $M_{fi} \neq 0$

e.g. $\bar{u}_\downarrow \gamma^\mu u_\uparrow = 0$

nonzero:





etc

helicity and chirality eigenstates are the same for $\begin{cases} \text{massless particles} \\ \text{ultrarelativistic limit} \end{cases}$

Returning to $e^+e^- \to \mu^+\mu^-$ as a concrete example:

$$M_{fi} = -\frac{\overset{\text{coupling}}{e^2}}{\underset{\text{propagator}}{q^2}} \underbrace{(\overset{e^+}{\bar{v}_e}\gamma_\mu \overset{e^-}{u_e})}_{j_e} \underbrace{(\overset{\mu^+}{\bar{v}_\mu}\gamma^\mu \overset{\mu^-}{u_\mu})}_{j_\mu}$$

The helicity spinors can be used to work out the various possibilities explicitly, e.g.

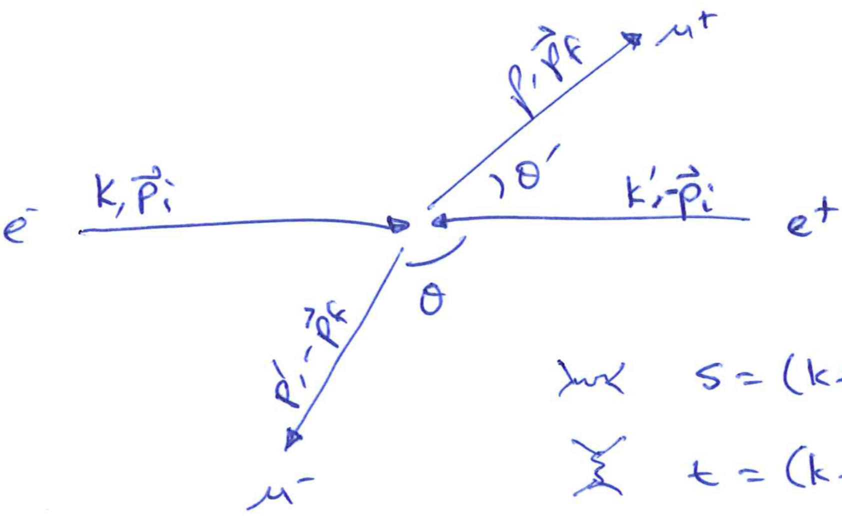
$$M_{RL\to RL} = e^2(1+\cos\theta) = 4\pi\alpha(1+\cos\theta)$$

$$M_{RL\to LR} = e^2(1-\cos\theta)$$

$$\langle |M|^2\rangle = e^4\left[(1+\cos\theta)^2 + (1-\cos\theta)^2\right] = e^4(1+\cos^2\theta)$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \underbrace{\frac{e^4}{64\pi^2 s}}_{\alpha^2/4s}(1+\cos^2\theta) = \frac{e^4}{32\pi^2 s}\,\frac{t^2+u^2}{s^2}$$

Integrating, $\sigma_{tot} = \dfrac{4\pi\alpha^2}{3s} \simeq \dfrac{87\,nb}{s}\,GeV^2$



When is $\theta' \neq \pi - \theta$?

$s = (k+k')^2 = 4E_i^2$

$t = (k-p)^2 \simeq -2E_i^2(1+\cos\theta)$

$= -\frac{s}{2}(1+\cos\theta)$

 $u = (k-p')^2 \simeq -2E_i^2(1-\cos\theta)$

$= -\frac{s}{2}(1-\cos\theta)$

When is a u-channel diagram relevant?

Tests of cut-off scales in QED

- are fundamental fermions point-like?
- could there be a heavy photon w/ modified propagator?

Let Λ ~ mass of heavy photon w/ coupling strength still α

$\Rightarrow$ modified propagator and vertices

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2-\Lambda^2} = \frac{1}{q^2}\left(1 - \frac{q^2}{q^2-\Lambda^2}\right)$$

$\Rightarrow$ change of EM potential: $\frac{1}{r} \rightarrow \frac{1}{r}(1-e^{-\Lambda r})$ Yukawa
modifies point-like nature of the <u>eγ interaction</u>

$\Rightarrow$ modified $e^+e^- \rightarrow \gamma\gamma$ form factor:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s}\frac{1+\cos^2\theta}{\sin^2\theta}\left(1 \pm \frac{s^2}{2\Lambda_\pm^4}\frac{\sin^4\theta}{1+\cos^2\theta}\right)$$

if switched to "−" then this is a heavy (excited?) electron w/ coupling via the same charge e

$\Downarrow$

set limits on its mass via Λ

- lower sign (Λ_-) allows a possibly lower cross-section
- note $\frac{s^2}{\Lambda^4}$ scaling and form at $\theta = \frac{\pi}{2}$ (maximal sensitivity at 90°)
- QED allows coupling via magnetic moments – need to conserve the electromagnetic current

Sensitivity typically rises with COM energy
$\Rightarrow$ limited by luminosity measurement

Second example: $e^+e^- \to e^+e^-$ (Bhabha)

total cross-section: $\sigma = \frac{4\pi\alpha^2}{3s}\left(1 \mp \frac{s}{s-\Lambda_\pm}\right)^2$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left[\left(\frac{s}{t}\right)^2 F(t)^2 + \left(\frac{t}{s}\right)^2 F(s)^2 + \left(1+\frac{s}{t}\right)^2\left(1+\frac{t}{s}\right)^2\left(\frac{s}{t}F(t)+F(s)\right)^2\right]$$

with $F(q^2) = 1 \mp \frac{q^2}{q^2-\Delta_\pm^2}$

timelike: $q^2 = s$

spacelike: $q^2 = t = -\frac{s}{2}(1+\cos\theta)$

(Note different scaling w/rt $e^+e^- \to \gamma\gamma$)

Bhabha: $\sim \frac{q^2}{\Delta^2}$