Standard Model of Particle Physics

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7. Neutrino Physics

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Neutrino Masses and Oscillations

* In the conventional Standard Model neutrinos are assumed to be massless. But there is no symmetry which enforces this.

* There is now compelling evidence that neutrinos do have a mass.

* In the case of massive neutrinos the eigenstates of the weak interaction, ν_α $(\alpha = e, \mu, \tau)$, can in general be different from the mass eigenstates ν_i $(i = 1, 2, 3)$. As was the case for quarks, there can be neutrino mixing:

$$\nu_\alpha = \sum_{i=1}^{3} U^*_{\alpha i} \, \nu_i$$

U is the leptonic mixing matrix (the analogue of the CKM matrix), often referred to as the Maki-Nakagawa-Sakata (-Pontecorvo) or MNS(P) matrix. It can be parametrized exactly in the same way as the CKM matrix (for Dirac neutrinos).
↳ see below

* In order to have massive neutrinos we have to add right-handed neutrino fields to the Standard Model fields.

* Since neutrinos are electrically neutral there are two possibilities for their mass terms in the Lagrange density.

* The Dirac mass term

$$\mathcal{L}_D = - m_D \bar{\nu}_R \nu_L + \text{h.c.}$$

is similar to the other fermion mass terms with m_D generated via a Higgs-Yukawa coupling $m_D = g_\nu \frac{v}{\sqrt{2}}$. Pictorially,

* For neutrinos one can also have Majorana mass terms, which are possible as left handed

$$\mathcal{L}_{m_L} = - \frac{m_L}{2} \overline{\nu_L^c} \nu_L + \text{h.c.}$$

or right handed

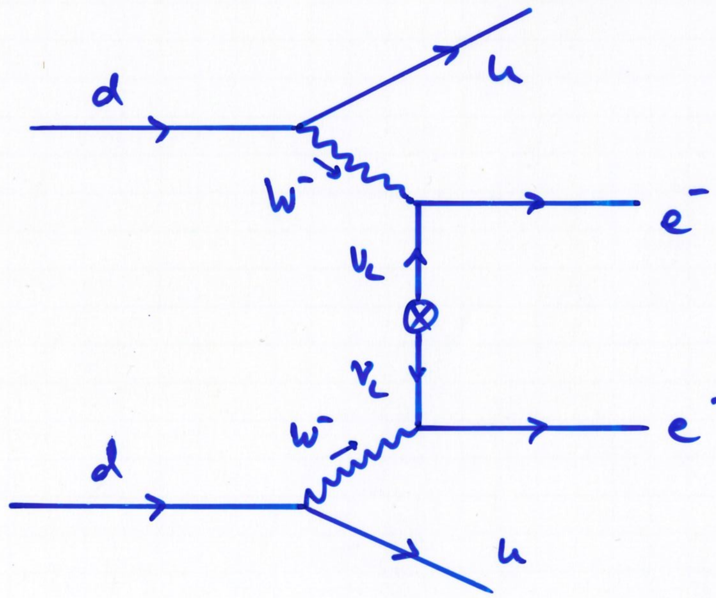
$$\mathcal{L}_{m_R} = - \frac{m_R}{2} \overline{\nu_R^c} \nu_R + \text{h.c.}$$

terms, where $\psi^c = C \bar{\psi}^T$. Pictorially,

* In general the above mass terms can be present simultaneously. If no Majorana mass term is present the neutrinos are called Dirac neutrinos, otherwise Majorana neutrinos.

* The Dirac mass term conserves lepton number L, whereas the Majorana mass term violates lepton number conservation.

* If neutrinos are Majorana particles there can hence be neutrinoless double beta decay ($0\nu\beta\beta$) via the diagram



The observation of such a decay would prove the Majorana nature of neutrinos.

* Neutrino masses are very small. In the case of a Dirac neutrino, a mass of say $0.05\,\text{eV}$ would require a Yukawa coupling g_ν of the order 10^{-13}. Such a fine-tuning appears unnatural.

In order to find a possible explanation for small neutrino masses consider the case of a Dirac and a righthanded Majorana mass term

$$\mathcal{L}_{m_\nu} = -m_D \overline{\nu_R} \nu_L - \frac{m_R}{2} \overline{\nu_R^c} \nu_R + h.c.$$

$$= -\frac{1}{2} \left(\overline{\nu_L^c}, \overline{\nu_R} \right) \underbrace{\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}}_{= M_\nu} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

where M_ν is the neutrino mass matrix.

It seems natural to assume that $m_D \sim \mathcal{O}(m_e)$ since it arises due to the Higgs mechanism like the electron mass. On the other hand nothing requires m_R to be small. We therefore assume $m_R \gg m_D$.

Then to order $\left(\frac{m_D}{m_R}\right)^2$ the mass matrix has the eigenvalues

$$m_1 \simeq m_D^2/m_R$$

$$m_2 \simeq m_R$$

With $m_D \sim O(m_e)$ and $m_R \gg m_D$ the mass of ν_1 can be very small. The emergence of this very small mass scale is known as the <u>see-saw mechanism</u>.

The mass m_R is assumed to reflect some high mass scale where new physics resides that is responsible for neutrino masses. ν_2 is then a hypothetical very heavy lepton.

Assuming for example $m_R \sim 10^{15}$ GeV (just below a typical grand unification scale) and $m_D \sim m_t = 175$ GeV one finds $m_1 \sim 3 \cdot 10^{-2}$ eV, in a realistic range.

* ## Neutrino Oscillations

Neutrino mixing implies neutrino flavour oscillations. For simplicity consider only two neutrino species, ν_e and ν_μ. Assume that ν_μ's are created in a weak interaction process at time $t=0$ with momentum p. The weak eigenstates are then

$$\nu_\mu(0) = \nu_1(0)\cos\alpha + \nu_2(0)\sin\alpha$$
$$\nu_e(0) = -\nu_1(0)\sin\alpha + \nu_2(0)\cos\alpha$$

The mass eigenstates vary with time as

$$\nu_i(t) = e^{-iE_it}\,\nu(0)$$

since they are freely propagating with energies $E_i = \sqrt{m_i^2+p^2}$. Thus

$$\begin{aligned}\nu_\mu(t) &= e^{-iE_1t}\,\nu_1(0)\cos\alpha + e^{-iE_2t}\,\nu_2(0)\sin\alpha \\ &= \left(e^{-iE_1t}\cos^2\alpha + e^{-iE_2t}\sin^2\alpha\right)\nu_\mu(0) \\ &\quad + \sin\alpha\cos\alpha\left(e^{-iE_2t} - e^{-iE_1t}\right)\nu_e(0)\end{aligned}$$

The second term is in general non-zero if $m_1 \neq m_2$. The state hence has an admixture of ν_e after time t.

The probability that an initial beam of ν_μ later contains some ν_e is

$$P(\nu_\mu \to \nu_e) = |\langle \nu_e(0) | \nu_\mu(t) \rangle|^2$$
$$= \sin^2\alpha \cos^2\alpha \, |e^{-iE_2 t} - e^{-iE_1 t}|^2$$
$$= \frac{1}{2} \sin^2 2\alpha \left[1 - \cos(E_2 - E_1)t \right]$$

For small masses m_1, m_2

$$E_2 - E_1 \simeq \frac{m_2^2 - m_1^2}{2p}.$$

After travelling a distance $L \simeq ct$ hence

$$P(\nu_\mu \to \nu_e) = \frac{1}{2} \sin^2 2\alpha \left[1 - \cos \frac{2\pi L}{L_{osc}} \right]$$

with the oscillation length

$$L_{osc} = \pi \frac{4p}{\Delta m^2} \quad \text{with} \quad \Delta m^2 = m_2^2 - m_1^2.$$

Note that oscillations between neutrino flavours are only sensitive to Δm^2 but not to m_1 and m_2 independently.

The above considerations are readily generalized to the case of three neutrino flavours.

* ## Neutrino Oscillations in Matter

When neutrinos travel through matter (e.g. in the Sun, Earth, supernova ...) their coherent forward scattering from matter particles can significantly modify their propagation. This is the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

For simplicity consider again the case of only two neutrino flavours. We can describe the time evolution of neutrino flavour oscillations in vacuum by a Schrödinger equation with Hamiltonian

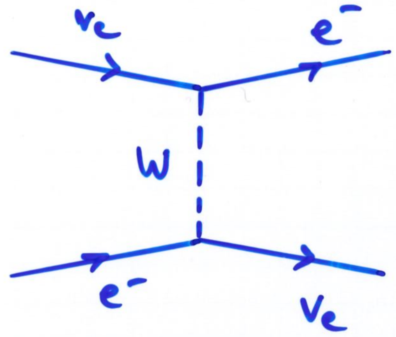
$$\mathcal{H}_{vac} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}.$$

The action of $\mathcal{H}_{vac}$ on a neutrino state (ν_e, ν_μ) then gives for the transition probability

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \quad \sin^2\left(\Delta m^2 \frac{L}{4E}\right)$$

as above.

In matter the ν_e can scatter off electrons via W-exchange:



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but this process is not possible for ν_μ due to the absence of muons in matter. This gives a contribution to the $\nu_e - \nu_e$ element of the Hamiltonian (but none to the $\nu_\mu - \nu_\mu$ element):

$$\mathcal{H} = \mathcal{H}_{vac} + \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}$$

with $V = \sqrt{2}\, G_F N_e$, where N_e is the number of electrons per unit volume. In addition, there is an identical contribution to the $\nu_e - \nu_e$ and $\nu_\mu - \nu_\mu$ elements of the Hamiltonian from Z-exchange. Since it is diagonal it does not affect the probability $P(\nu_e \to \nu_\mu)$, and can hence be neglected here.

The above Hamiltonian can be written in the equivalent form

$$\mathcal{H} = \frac{\Delta m_M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

with the effective mass splitting in matter,

$$\Delta m_M^2 = \Delta m^2 \left[\sin^2 2\theta + (\cos 2\theta - x)^2 \right]^{1/2}$$

and the effective mixing angle in matter,

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

where

$$x = V \cdot \frac{2E}{\Delta m^2} .$$

Note that neutrino oscillations in vacuum cannot distinguish between a mixing angle θ and an angle $\theta' = \frac{\pi}{2} - \theta$. But if neutrinos propagate through matter (e.g. the Sun) these cases can be distinguished since the $\nu_e - \nu_e$ element of $\mathcal{H}$ is different in matter for the two mixing angles.

Note also that matter interaction can change a small mixing into an effectively maximal one.

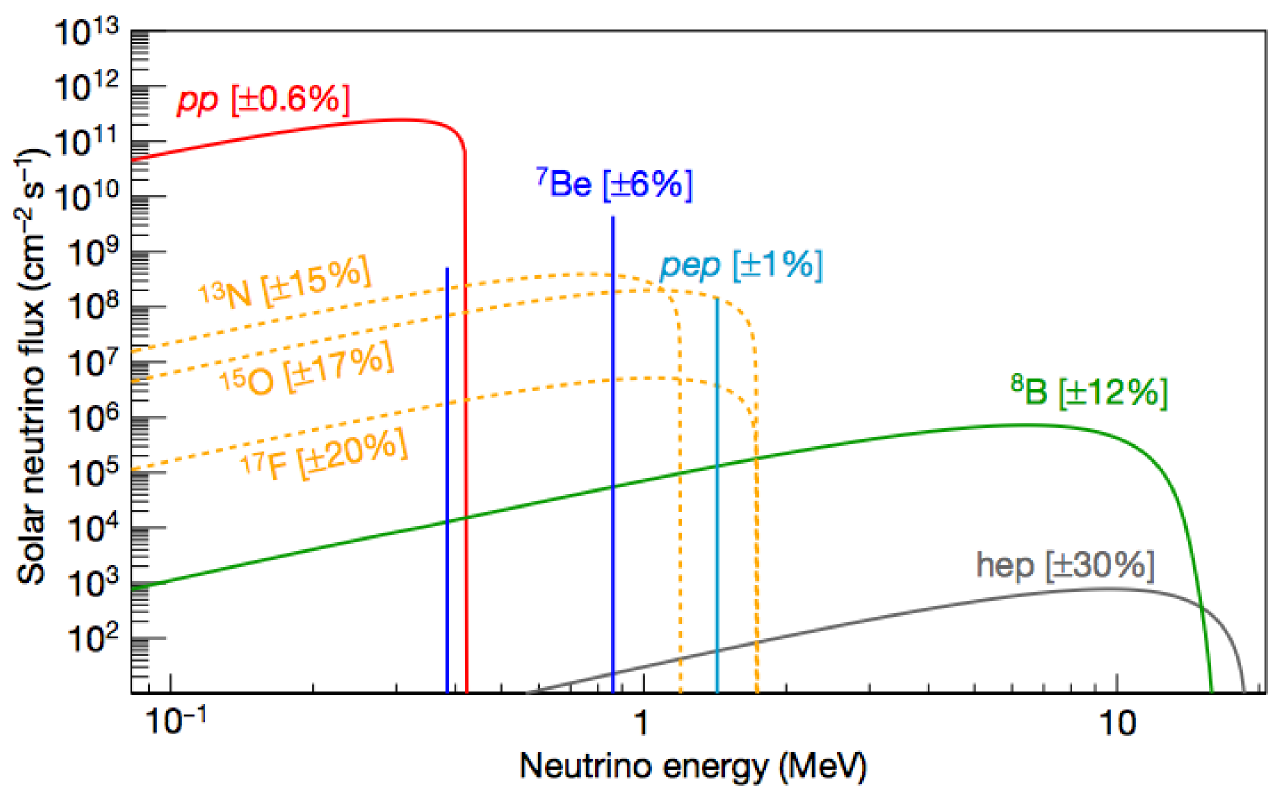
Neutrino Oscillations - Experiments

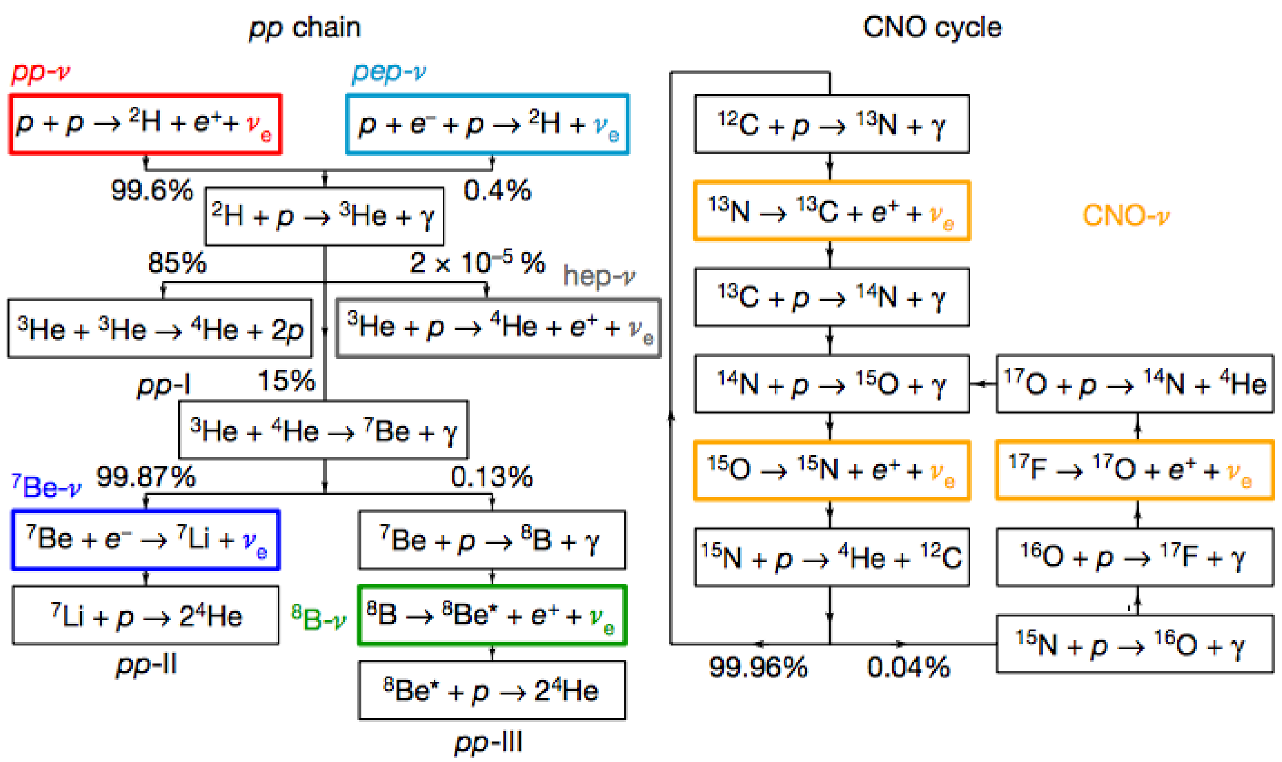
Variety of experiments, sensitive to different neutrino flavors, energies and oscillation lengths:

- Solar neutrinos
- Atmospheric neutrinos
- Reactor neutrinos
- Beam neutrinos

Solar Neutrinos

Solar model predicts flux:



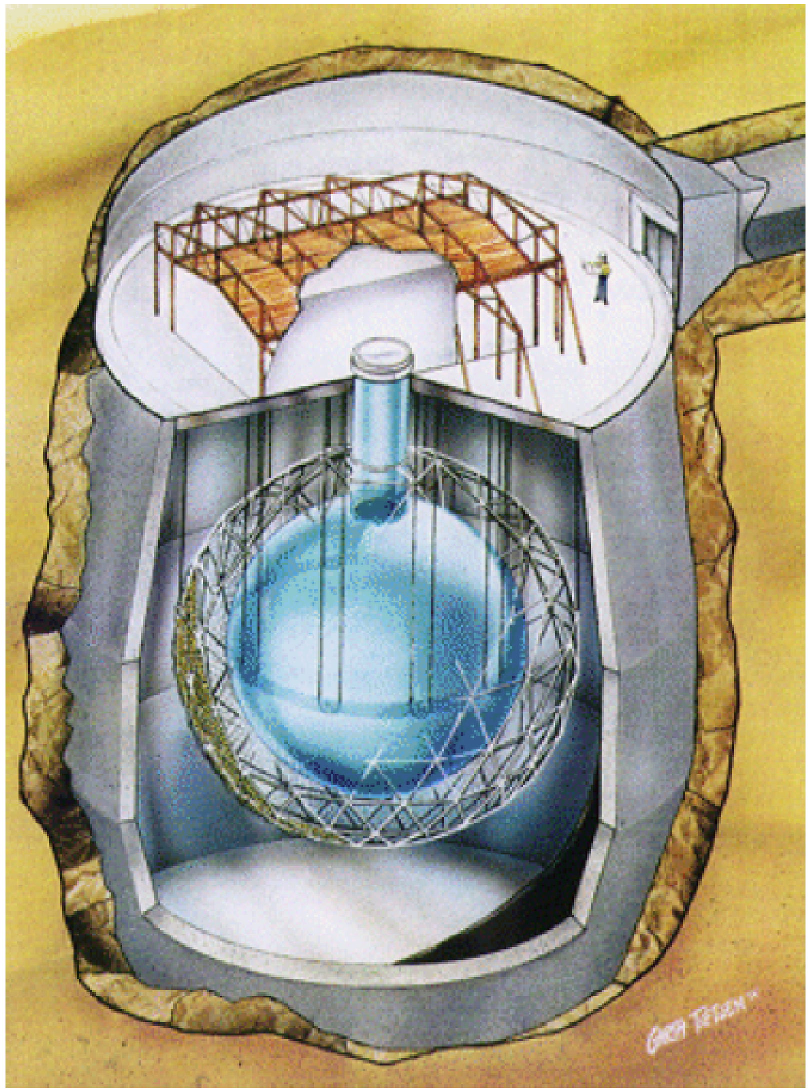


Neutrinos Experiments

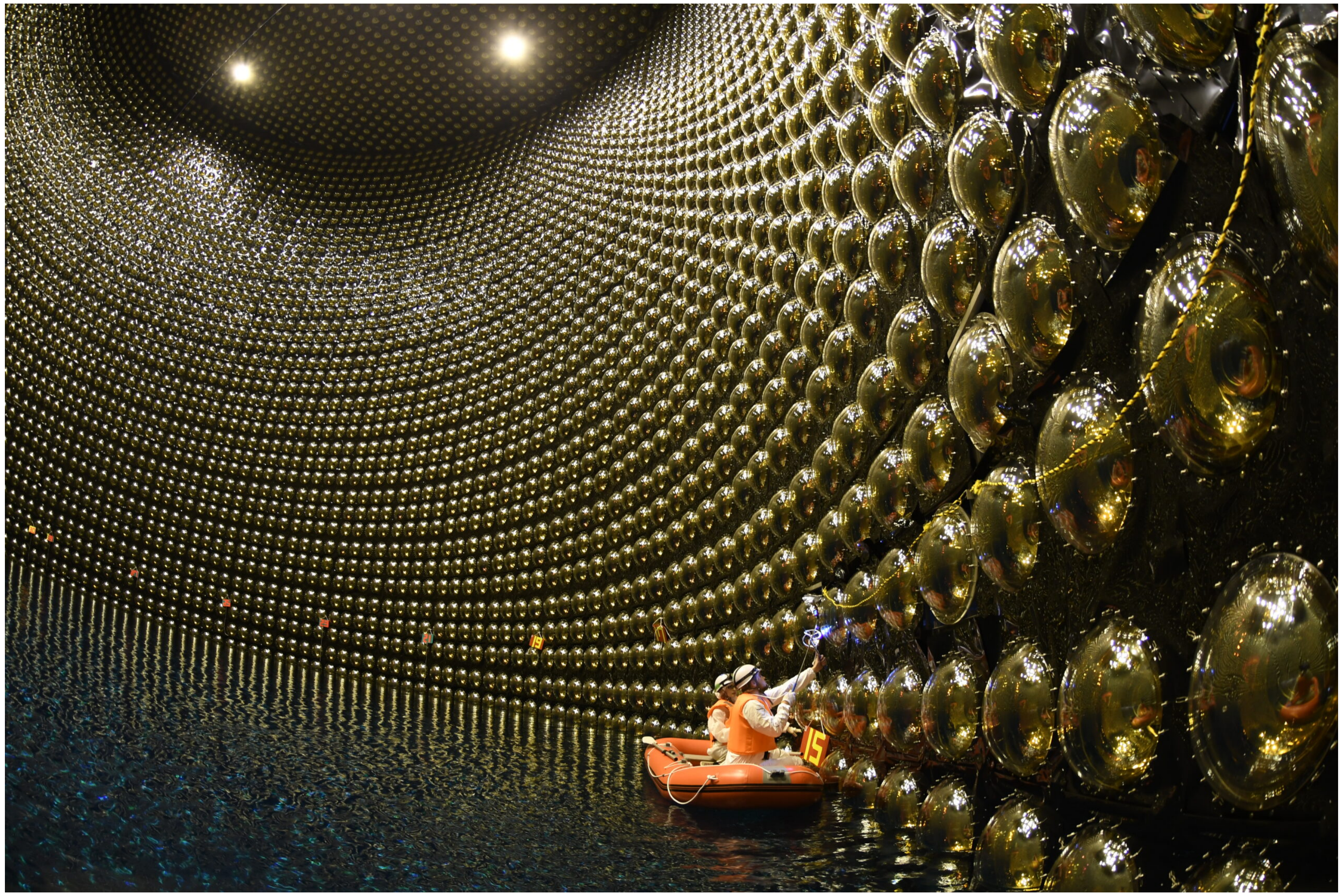
Solar neutrino experiments:
Homestake exp ($^{37}Cl \rightarrow {}^{37}Ar$),
GALLEX, GNO, SAGE ($^{71}Ge \rightarrow {}^{71}Ga$),
(Super-)Kamiokande (water, Cerenkov),
Sudbury Neutrino Observatory SNO
(heavy water, Cerenkov),
Borexino (liquid organic scintillator)

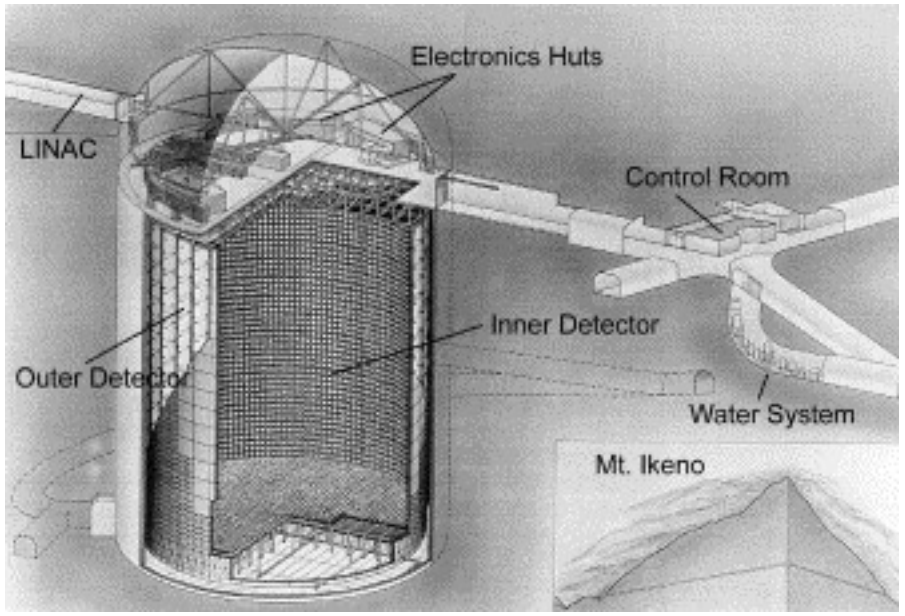
Atmospheric neutrino experiments:
(Super-)Kamiokande, …

SNO



Super-Kamiokande





first observation of oscillations

Neutrinos Experiments

Reactor neutrino experiments:
Double Chooz, Daya Bay, RENO
(all liquid scintillator)

Beam neutrino experiments:
LSND (liquid scintillator),
MiniBooNE (oil, Cerenkov)

with near and far detector:
MINOS (steel-scintillator), NOvA (liquid scint.),
K2K (KEK to Super-K), T2K (J-PARC to Super-K)

Neutrino Oscillation Parameters

$$\begin{aligned}
\sin^2\theta_{12} &= 0.307 \pm 0.013 \\
\Delta m^2_{21} &= (7.53 \pm 0.18) \times 10^{-5}\,\mathrm{eV}^2 \\
\sin^2\theta_{32} &= 0.558^{+0.015}_{-0.021} \\
\Delta m^2_{32} &= 0.002455 \pm 0.000028\,\mathrm{eV}^2 \\
\sin^2\theta_{13} &= 0.0219 \pm 0.0007 \\
\delta &= 1.19 \pm 0.22\,\pi\,\mathrm{rad}
\end{aligned}$$

Two mixing angles large, one small.
CP violating phase depends on assumption about hierarchy of neutrino masses.

Neutrino Masses

Many experiments, only upper limits so far.

Best current upper limit on anti-electron neutrino mass:

Katrin Exp.: $m_\nu < 0.8$ eV