CP-Violation

Recap:

- CP transformation is the combination of parity transformation (P) and charge conjugation (C).
- CP violation was discovered in 1964 in the system of neutral K-mesons by Christenson, Cronin, Fitch, Turlay where it occurs in the mixing of K^0 and K^0 (effect only ~10^{-3})
- Many years later, also direct CPV has been discovered in the decay of neutral kaons (even smaller effect ~10^{-6}).
- More recently CPV has been discovered in neutral B-mesons (B^0_d in 2001, and B_s mesons – very large effect O(10-20%)) and more recently (2019) also in neutral D mesons (small).

CP-Violation in Standard Model

In the Standard Model CP violation can arise from 3 different sources:

- in the quark (hadron) sector via the CKM matrix ⇨ UU
- for massive neutrinos via the PMNS matrix (the neutrino mixing matrix, analogue to CKM matrix)
- strong interaction allow a CP violating θ-term to the Lagrangian

$$\mathcal{L}_\theta = \theta \cdot \frac{g_s^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{A,\mu\nu}$$

with the dual field strength tensor $\tilde{G}^{A,\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$

A non –zero value of θ would imply CP-violation and would lead to an electric dipole moment of the neutron. Experimental bounds on the neutron EDM limits $\theta < 10^{-10}$. No convincing mechanism for the absence of θ has been established so far: strong CP problem.

⇨ SD

Baryon Asymmetry

CP violation together with a departure from thermal equilibrium and the violation of Baryon number conservation are necessary conditions for the Baryon Asymmetry of the Universe (BAU).

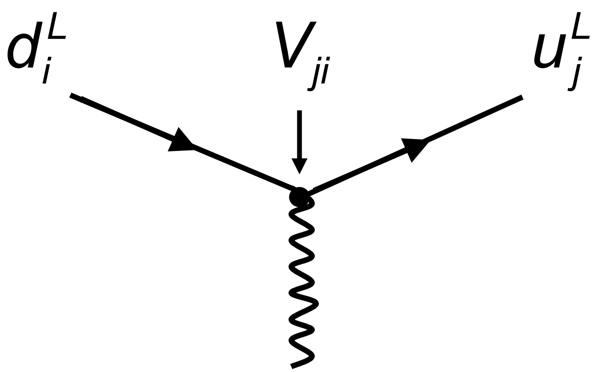
Conditions formulated by Andrei Sakharov in 1967

Quark Mixing Matrix

Weak and mass eigenstates of the quarks differ – described by the quark mixing (Cabibbo-Kobayashi-Maskawa CKM) matrix:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Complex, unitary matrix with 3+1 free parameters: Often chosen as 3 mixing angles and 1 phase

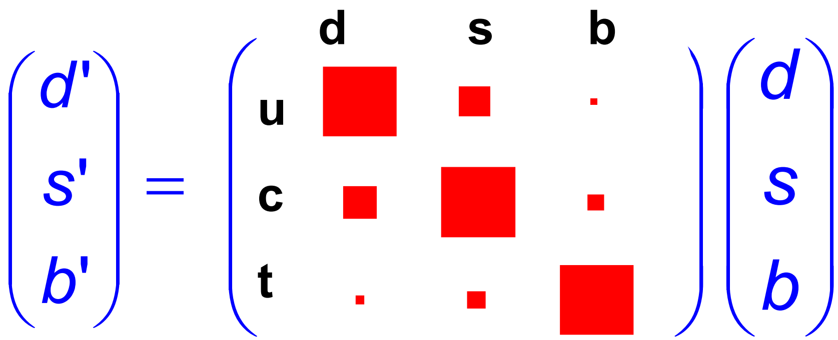


CKM matrix elements only enter in the weak charged currents (neutral currents are flavor diagonal).

Unitarity of matrix : $\sum_i V_{ij} V_{ik}^* = \delta_{jk}$ $\sum_j V_{ij} V_{kj}^* = \delta_{ik}$

Wolfenstein Parametrization

The structure of the matrix cannot be predicted.
From data we find that the matrix has a hierarchical structure



which is very well reflected by the Wolfenstein parametrization:

λ, A, ρ, η with $\lambda = 0.22$

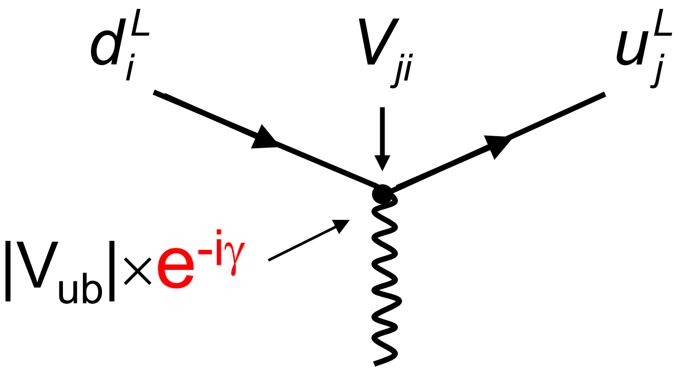
$\approx |V_{ub}| \times e^{-i\gamma}$

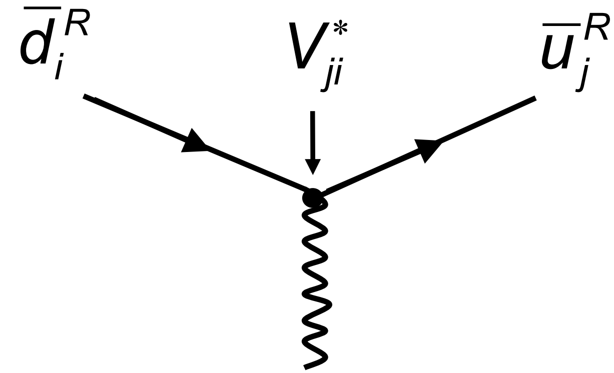
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

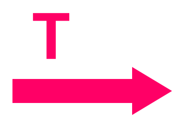
$\approx |V_{td}| \times e^{-i\beta}$

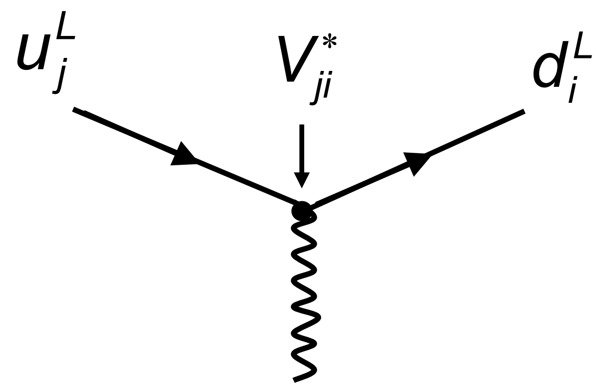
(exact definition of β and γ - see below)

CP Violating Phase









CP (T) violation $\Leftrightarrow$ $V_{ji} \neq V_{ji}^*$

i.e. complex elements

Remark: For 2 quark generations the mixing is described by the **real 2x2** Cabibbo matrix $\rightarrow$ **no CP violation**! To explain **CPV** in the SM Kobayashi and Maskawa have predicted a **third quark generation**.

But: Absolut phases not observable, and what about rephrasing of CKM matrix?

CP Violating Invariants

Independent of a specific parameter choice or a possible rephasing of the CKM matrix is the so called Jarlskog invariant:

$$\mathrm{Im}\left\{ V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \right\} = J_{CP} \sum_{\gamma} \varepsilon_{\alpha\beta\gamma} \sum_{k} \varepsilon_{ijk}$$

$$J_{CP} = \lambda^6 A^2 \eta = O\left(10^{-5}\right)$$ is relatively small

Unitarity condition: $\sum_i V_{ij} V^*_{ik} = \delta_{jk}$ leads to 6 triangle equations in the complex plane. The most important relation for experimental tests is:

$$V_{ud} V^*_{ub} + V_{cd} V^*_{cb} + V_{td} V^*_{tb} = 0$$

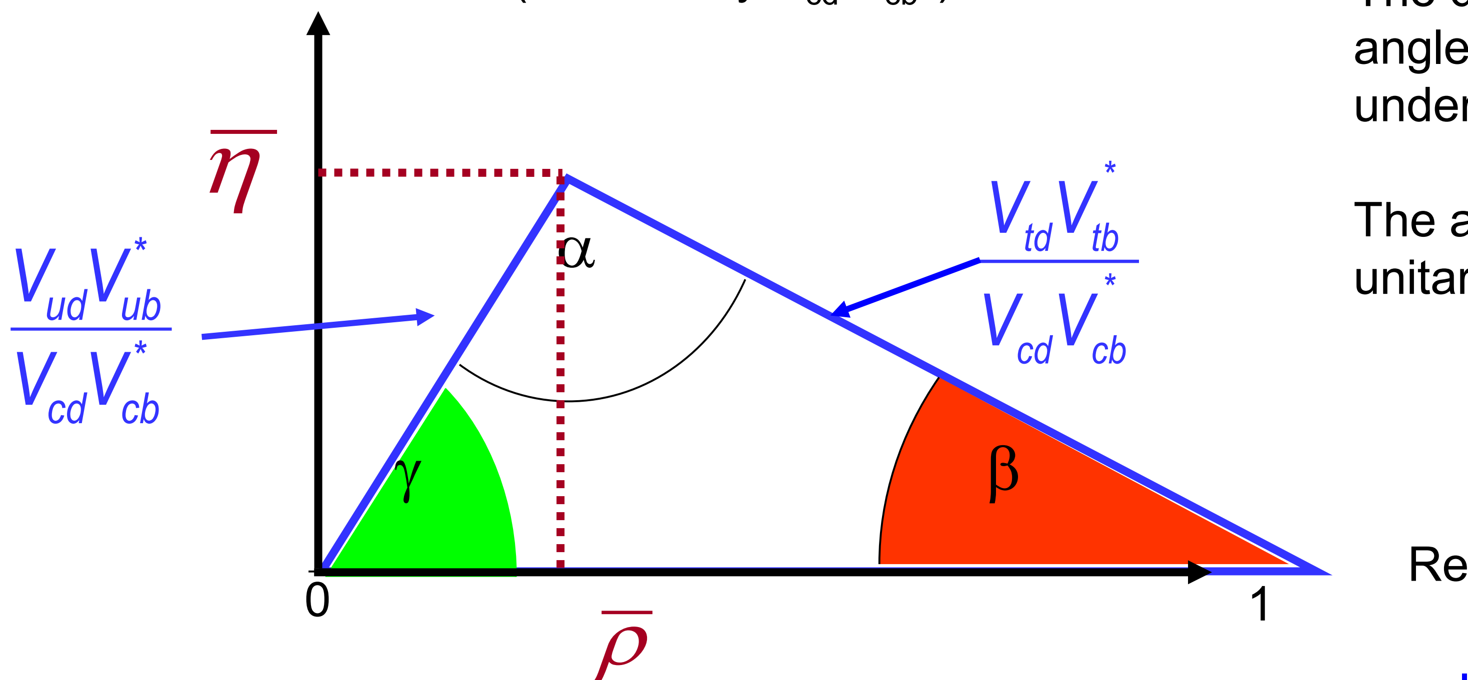
Angles and sides experimentally accessible using B decays,
All sides have similar length ~A λ^3 - other triangles are squashed.

“The Unitarity triangle”

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

Rescaled unitarity condition

(Division by $V_{cd}\, V_{cb}^{*}$)



The definition of the angle are invariant under rephasing.

The area of all unitarity triangles is

$$\frac{J_{CP}}{2}$$

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right] \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right] \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right]$$

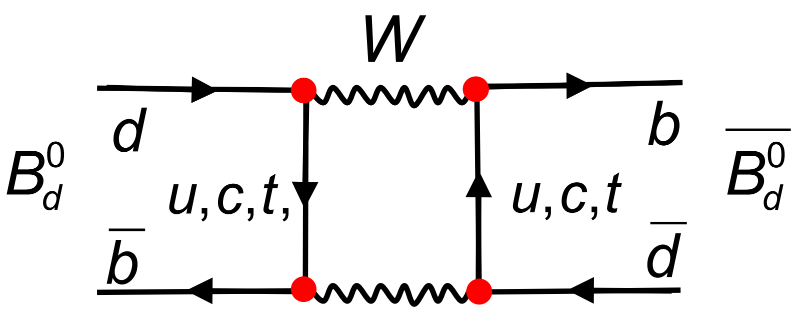
Invariant under rephasing of the CKM matrix → measureable

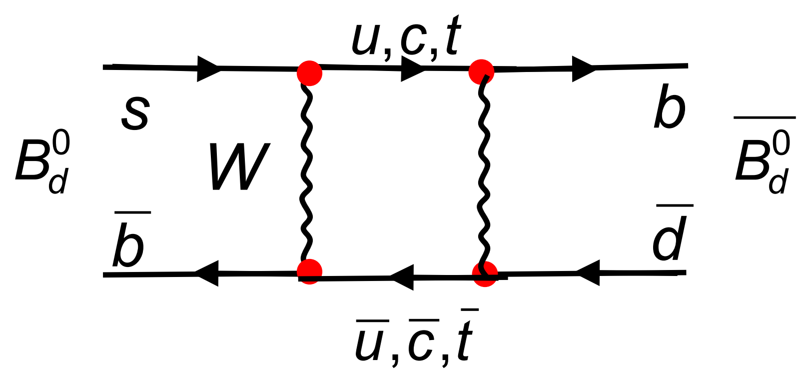
Mixing of neutral mesons

The quark mixing results into several interesting "loop" effects:
Standard Model predicts at loop-level: Flavor Changing Neutral Currents

(forbidden at tree-level)

Mixing of neutral mesons, e.g.: $B_d^0 \Leftrightarrow \overline{B_d^0}$





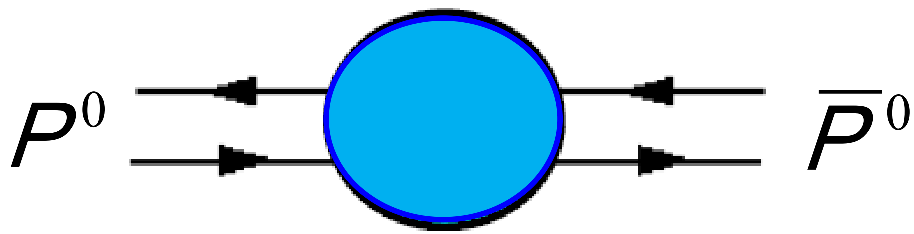
Mixing present in neutral mesons:

$$\left|P^0\right\rangle: \quad K^0 = \left|d\bar{s}\right\rangle \quad D^0 = \left|\bar{u}c\right\rangle \quad B_d^0 = \left|d\bar{b}\right\rangle \quad B_s^0 = \left|s\bar{b}\right\rangle$$

$$\left|\overline{P^0}\right\rangle: \quad \overline{K^0} = \left|\bar{d}s\right\rangle \quad \overline{D^0} = \left|\bar{u}c\right\rangle \quad \overline{B_d^0} = \left|d\bar{b}\right\rangle \quad \overline{B_s^0} = \left|s\bar{b}\right\rangle$$

Discovery:	1960	2007	1987	2006

Mixing Phenomenology



$$i\frac{d}{dt}\begin{pmatrix}|P^0(t)\rangle\\ |\bar{P}^0(t)\rangle\end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix}|P^0(t)\rangle\\ |\bar{P}^0(t)\rangle\end{pmatrix}$$

No mass eigenstates

Non-hermitian $\rightarrow$ P^0 and $\bar{P}^0$ decay

CPT

$$m_{11} = m_{22} = m$$

$$\Gamma_{11} = \Gamma_{22} = \Gamma$$

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

M and **Γ** hermitian:

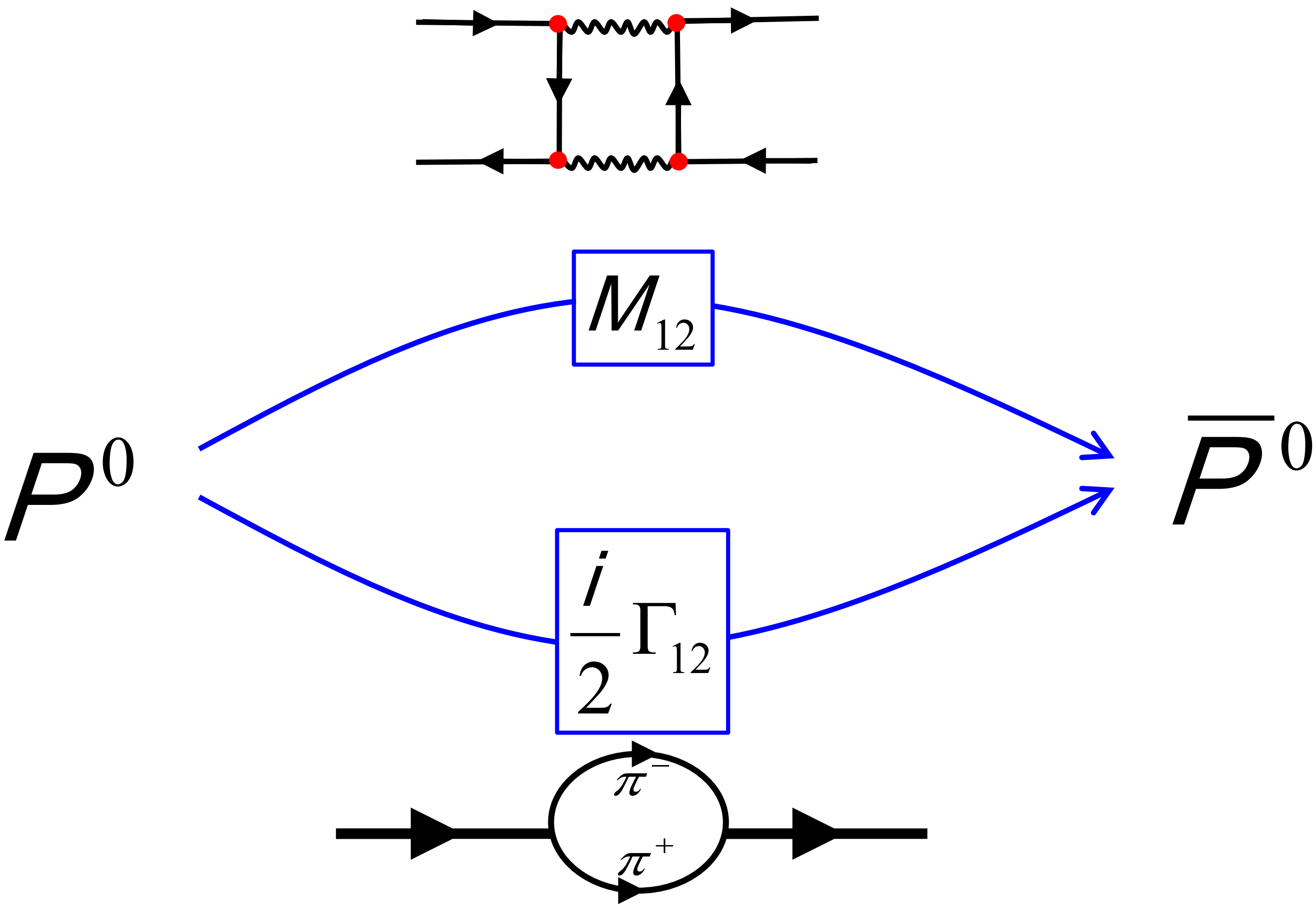
$$m_{21} = m_{12}^*$$

$$\Gamma_{21} = \Gamma_{12}^*$$

Off – diagonal elements describe the mixing.

Mixing Phenomenology

„short distant, virtual states“



„long distant, on-shell states“

for K^0 very important, for B^0 small

Mass eigenstates

Mass eigenstates are obtained by diagonalizing the matrix:

$$|P_a\rangle = p|P^0\rangle + q|\overline{P^0}\rangle \quad \text{with}\, m_a, \Gamma_a \quad \Longrightarrow \quad |P_a(t)\rangle = e^{-im_a t} \cdot e^{-\frac{1}{2}\Gamma_a t}|P_a(0)\rangle$$

$$|P_b\rangle = p|P^0\rangle - q|\overline{P^0}\rangle \quad \text{with}\, m_b, \Gamma_b \quad \Longrightarrow \quad |P_b(t)\rangle = e^{-im_b t} \cdot e^{-\frac{1}{2}\Gamma_b t}|P_b(0)\rangle$$

complex coefficients $|p|^2 + |q|^2 = 1$

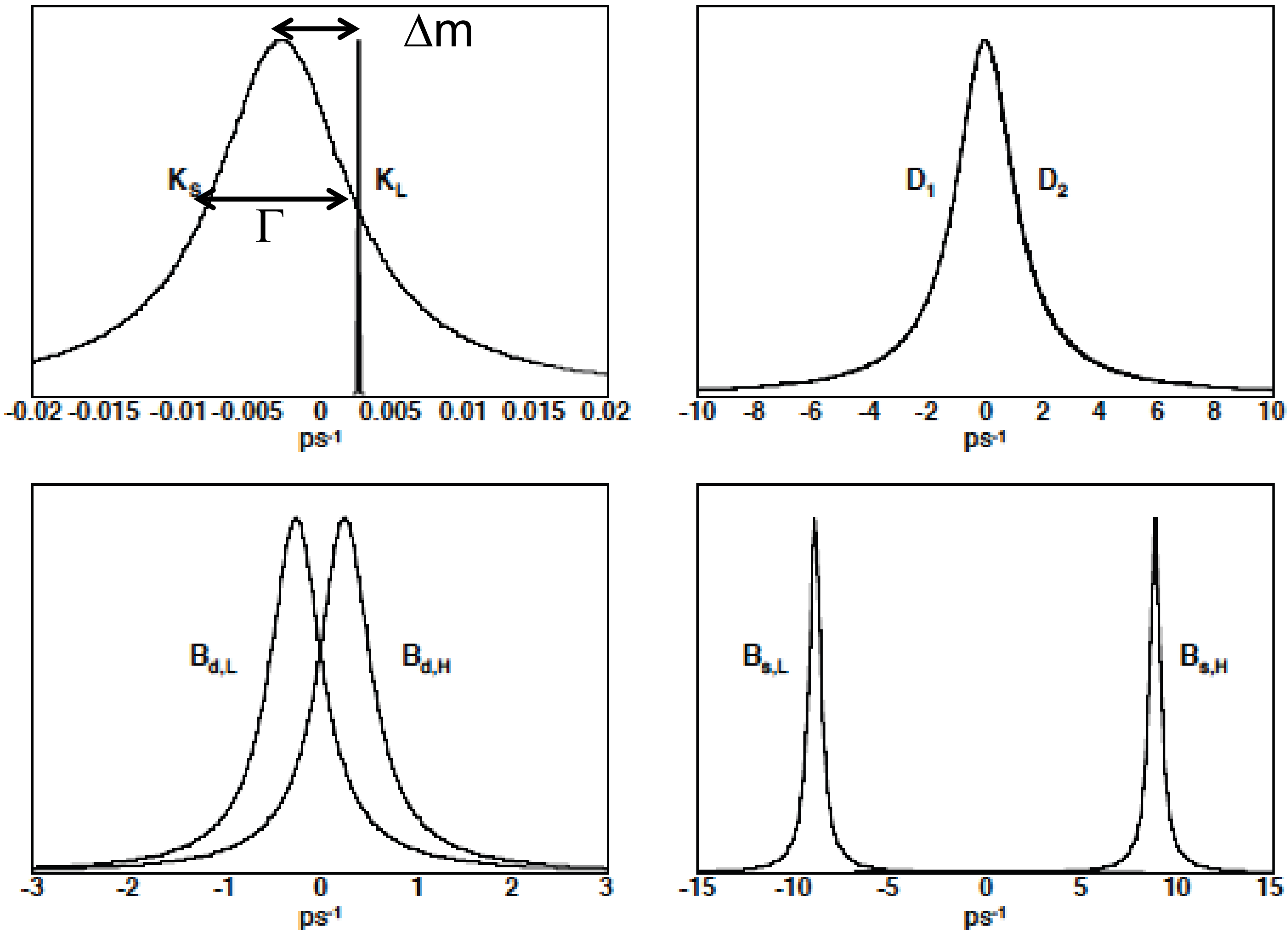
For p = q = 1/√2: $P_a = P_1$ (CP +) , $P_b = P_2$ (CP -) are CP eigenstates!

The mass (physical) states P_a and P_b are usually labeled by the properties which distinguish them the best:

- lifetime for kaons: K_S and K_L (short and long)
- mases for the B mesons: B_H and B_L (heavy and light)
- CP values for D mesons: D_1 and D_2 (assuming no direct CPV)

Neutral Mesons K^0, D^0, B^0, B_s

Labeling of physical states: heavy/light, short/long, CP-even/CP-odd



Parameter: $\Delta m = m_b - m_a \qquad m = \frac{1}{2}(m_b + m_a) \qquad \Delta\Gamma = \Gamma_b - \Gamma_a \quad \Gamma = \frac{1}{2}(\Gamma_a + \Gamma_b)$

Mixing of neutral mesons

Example: B_d^0 Mesons $\Delta\Gamma = \Gamma_H - \Gamma_L \approx 0$ → simplifies mixing formulae

Use time evolution of B_H, B_L and insert into B^0 and B^0 …. with some algebra

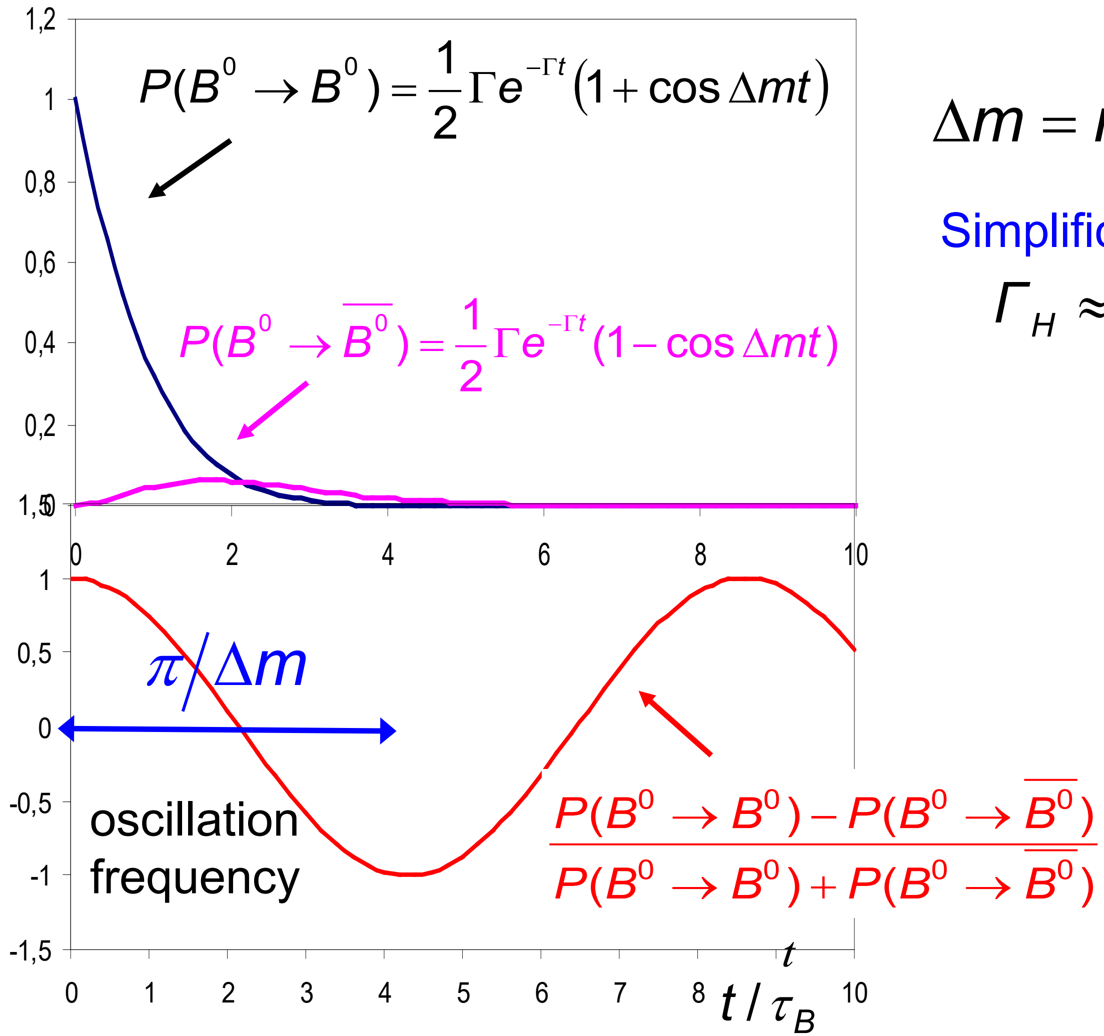
$$\underbrace{P(B^0 \to B^0) = P(\overline{B^0} \to \overline{B^0})}_{\text{CPT}} = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{4}\left|\frac{q}{p}\right|^2\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right] \qquad \Delta m = m_H - m_L$$

$$P(\overline{B^0} \to B^0) = \frac{1}{4}\left|\frac{p}{q}\right|^2\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

CP - violation in mixing: $P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0) \Rightarrow \left|\frac{q}{p}\right| \neq 1$

B^0-B^0 Mixing - illustration



$$\Delta m = m_H - m_L$$

Simplification for

$$\Gamma_H \approx \Gamma_L \approx \Gamma$$

Mixing
asymmetry

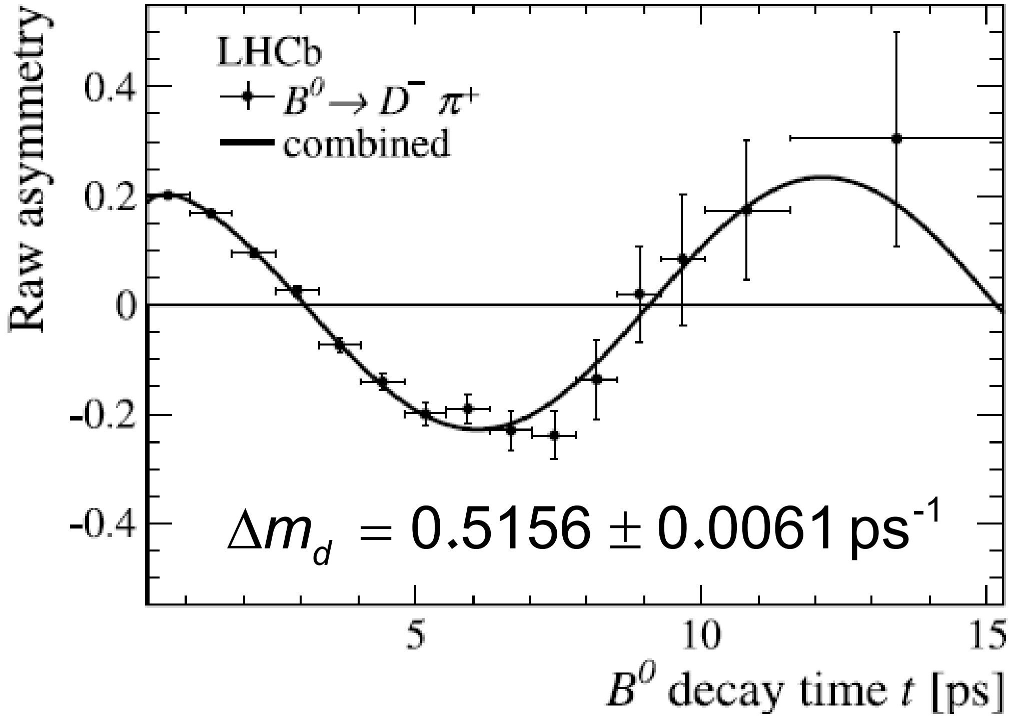
Measurement of B_d^0 - Mixing

Phys. Lett. B 719 (2013) 318.

B^0

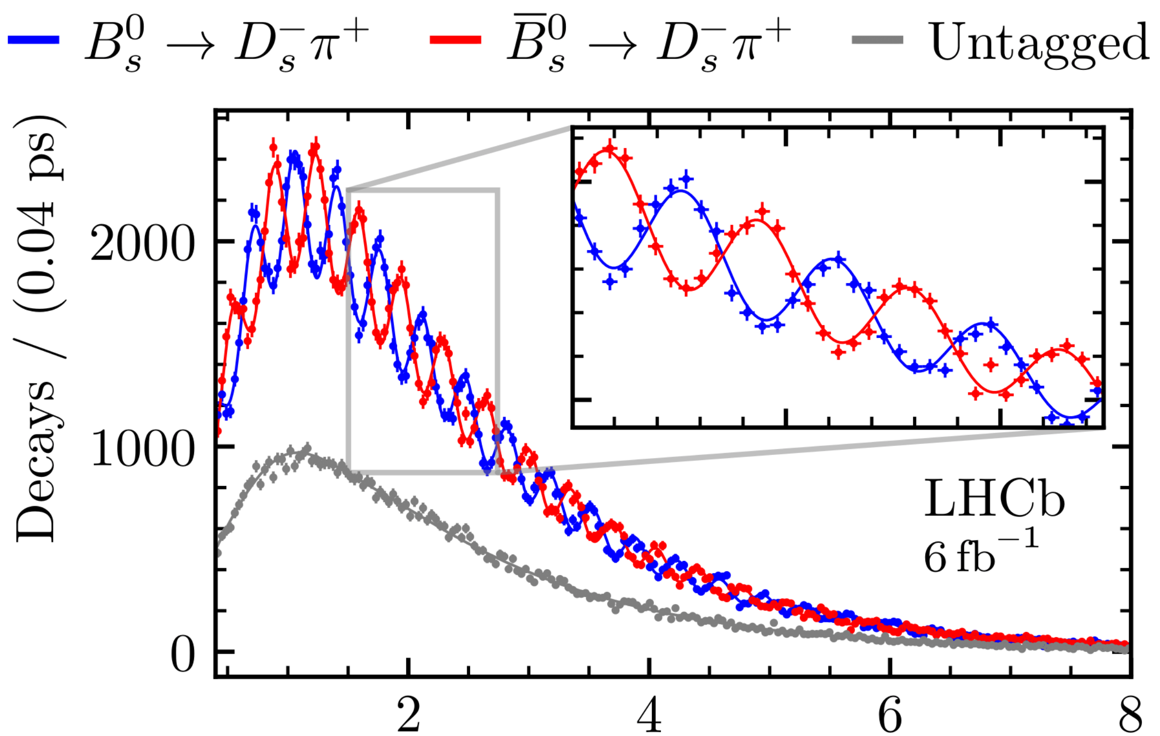
↕

$\bar{B}^0$



$$\Delta m_d = 0.5156 \pm 0.0061\,\text{ps}^{-1}$$

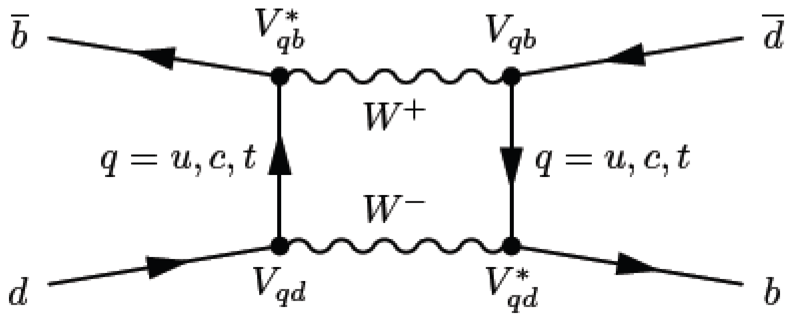
B_s-Mixing



Δm_s = 17.7656± 0.0057 ps^{-1}

(B$_s$ oscillation is much faster than B$_d$ because $V_{ts} >> V_{td}$)

Theoretical predictions Δm for B mesons



$t-\bar{t}: \quad \propto m_t^2 |V_{tb}V_{td}^*|^2 \quad \propto m_t^2\lambda^6$

$c-\bar{c}: \quad \propto m_c^2 |V_{cb}V_{cd}^*|^2 \quad \propto m_c^2\lambda^6$

$c-\bar{t}, \bar{c}-t: \quad \propto m_c m_t V_{tb}V_{td}^* V_{cb}V_{cd}^* \quad \propto m_c m_t \lambda^6$

u quark is very small – can be neglected

$$M_{12} \approx \frac{G_F^2}{12\pi^2} \left(V_{td}^* V_{tb}\right)^2 M_W^2 \, m_B B_B f_B^2 \, S_0\left(\;\right) \eta_B$$

$$\Delta m \approx 2|M_{12}|$$

B_s: $(V^*_{ts}V_{tb})^2 \sim \lambda^4$ about ×20 larger

$$\Delta m_s / \Delta m_d \sim |V_{ts}|^2 / |V_{td}|^2 \approx \left(\frac{0.0404}{0.0087}\right)^2$$

$S_0\left(m_t^2/m_W^2\right)$ = Loop-function = result of box diagram.

$B_B f_B^2$ = non-perturbative hadronic effects

η_B = *perturbative QCD corrections*

Question: What happens if quarks have the same mass?

Unitarity makes the mixing diagram to vanish!

CP Violation in meson decays

General remarks:

(see part I)

CP violation in mesons is linked to the CKM phases in the transition amplitude.

But: All observable quantities are in general “squares” of the amplitudes. Phases do not lead easily to observable effects (absolute phases are not observable!)

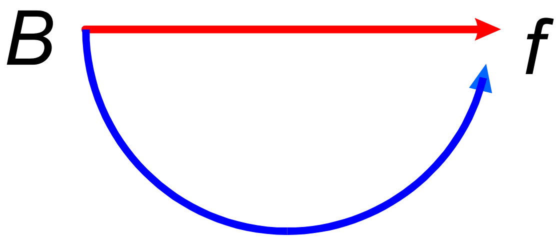
Only phase differences are observable via interference effects: At lease two interfering amplitudes are required to observe a phase difference related to a CKM phase and to study CP violation:

The interference term should be a product of 4 matrix elements.

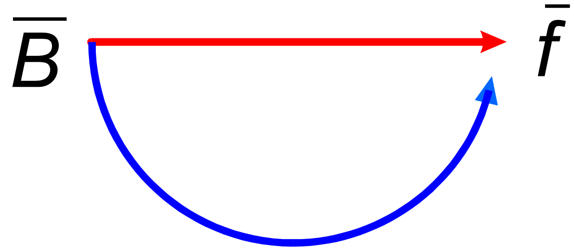
CP Violation in meson decays

Observation of CP violation needs at least two amplitudes with different weak (sign flip under CP) and different strong (invariant under CP) amplitudes:

$$\mathcal{A}_1 = A_1 e^{i\phi_1} e^{i\delta_1} \qquad \overline{\mathcal{A}}_1 = A_1 e^{-i\phi_1} e^{i\delta_1}$$



CP



$$\mathcal{A}_2 = A_2 e^{i\phi_2} e^{i\delta_2} \qquad \overline{\mathcal{A}}_2 = A_2 e^{-i\phi_2} e^{i\delta_2}$$

$$\mathcal{A}(B \to f) = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2} \qquad \overline{\mathcal{A}}(\overline{B} \to \bar{f}) = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}$$

CPV:

$$\left|\overline{\mathcal{A}}\right|^2 - \left|\mathcal{A}\right|^2 = 4\, A_1 A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)$$

<u>Problem:</u> The strong phases are a result of interactions between the hadronic final state particles → difficult to calculate

CP Violation in meson decays

The observed CP violating effects in meson decays are usually classified in the following way:

(I) CPV in decay:

$$\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})$$

This implies

$$\left| \frac{\mathcal{A}(\bar{P} \to \bar{f})}{\mathcal{A}(P \to f)} \right| = \left| \frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_f} \right| \neq 1$$

e.g.:

$$\Gamma\left(B^0 \to K^+\pi^-\right) \neq \Gamma\left(\bar{B}^0 \to K^-\pi^+\right)$$

$$\Gamma\left(B^+ \to \bar{D}K^+\right) \neq \Gamma\left(B^- \to DK^-\right)$$

Interference between b→u tree amplitudes and so called penguin amplitudes with different weak and strong phased

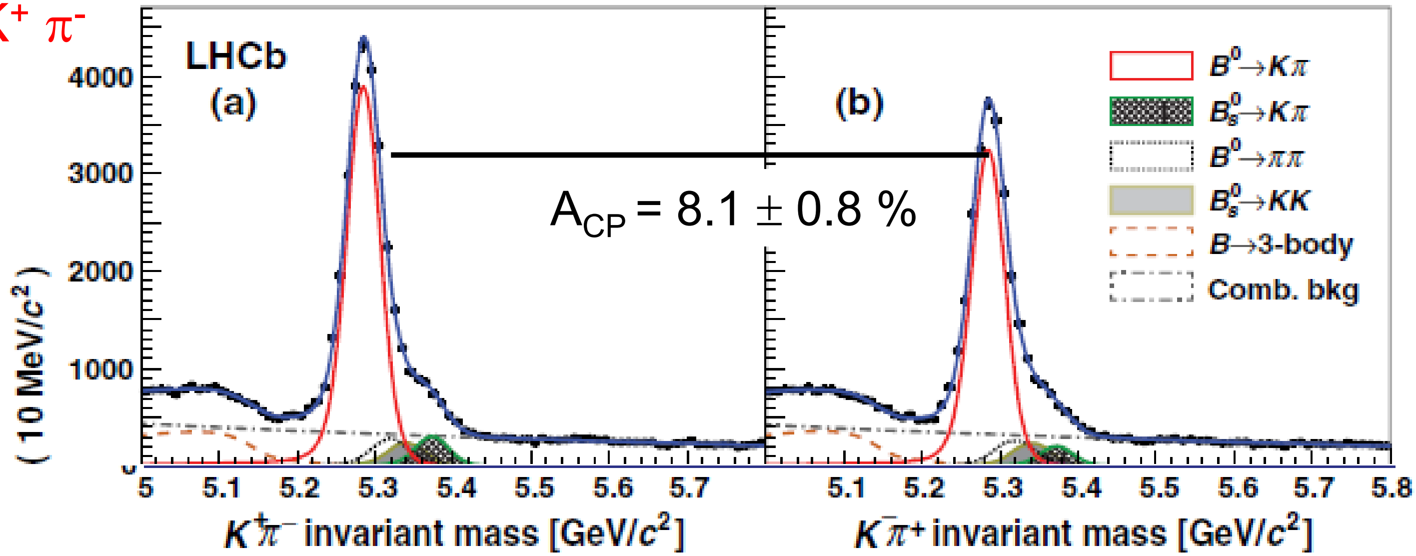
In charged mesons where no mixing is possible, CPV in decay is the only possible type of CPV which can occur.

Direct CP Violation in B decays

PRL 110 (2013) 221601

$B^0 \to K^+ \pi^-$

$\bar{B}^0 \to K^- \pi^+$



(direct CPV is a huge effect in B decays)

Indirect CP Violation

(II) CPV in mixing:

$$\mathcal{P}\left(P^0 \to \bar{P}^0\right) \neq \mathcal{P}\left(\bar{P}^0 \to P^0\right)$$

This implies $\left|\frac{q}{p}\right| \neq 1$ (see the mixing equation section 3)

While for B-mesons $|q/p| = 1 + O(10^{-6}...10^{-5}) \approx 1$
CPV in mixing is the dominating effect for kaons $O(10^{-3})$

(III) CPV in interference between a decay w/ and w/o mixing:

For experts only.

$\rightarrow$ time-dependent effect (see below)
$\rightarrow$ No effect in time integrated measuremts!

Can only occur if $\Im\left(\frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}\right) \neq 0$. I.e. if either q/p or the amplitude ratio has a non-trivial phase.

CP violation in mixing: Discovery of CPV in K^0 decays

Reminder:

Kaon ,ass eigenstates assuming no CP violation (p=q, no direct CPV):

$$|K_L\rangle = "|K_2\rangle" \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right)$$

$$|K_S\rangle = "|K_1\rangle" \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right)$$

Decay: $K_L \rightarrow 3\pi$ CP= -

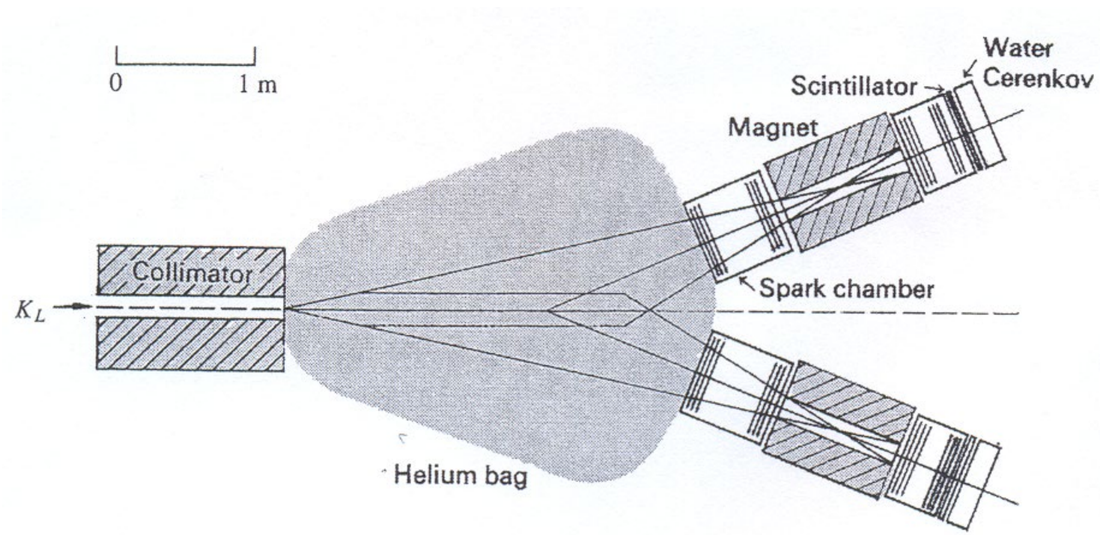
Decay: $K_L \rightarrow 2\pi$ CP=+

} Decays obey CP

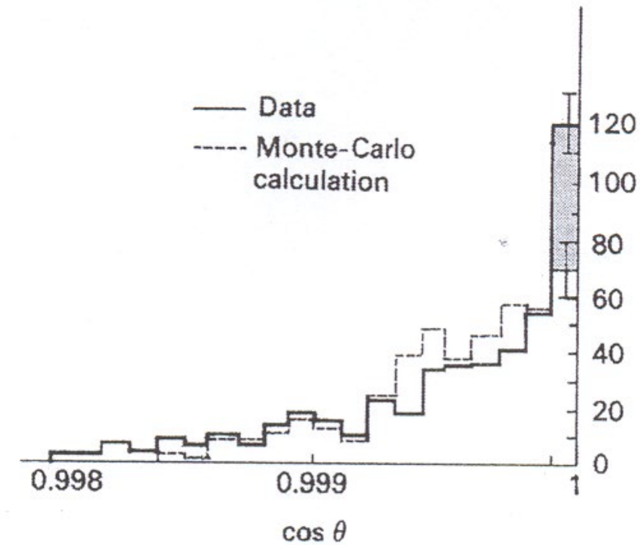
One can show that the 2π final state has always CP=+1:
The observation of K_L (CP=-) $\rightarrow \pi\pi$ (CP=-) thus violates CP.

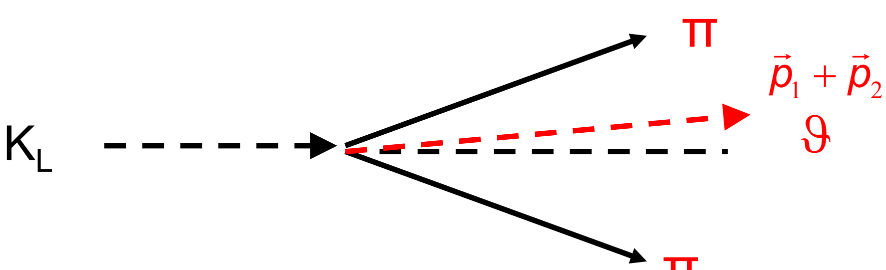
First Observation of CP Violation

Christenson, Cronin, Fitch, Turlay, 1964



(a)





Remark: Experiment was not done to discover CPV but to study unusual “regeneration”. CPV was nor easily accepted by community: difference between a world and a anti-world!

Explanation

K_L is not a pure K_2 state.

$CP = -1 \quad = +1$

$$\left|K_L\right\rangle = \frac{1}{\sqrt{1+\left|\varepsilon\right|^2}}\left(\left|K_2\right\rangle - \varepsilon\left|K_1\right\rangle\right)$$

This is equivalent with $|p/q| \neq 1$

Today's Knowledge

After 35 years of kaon physics:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right)$$

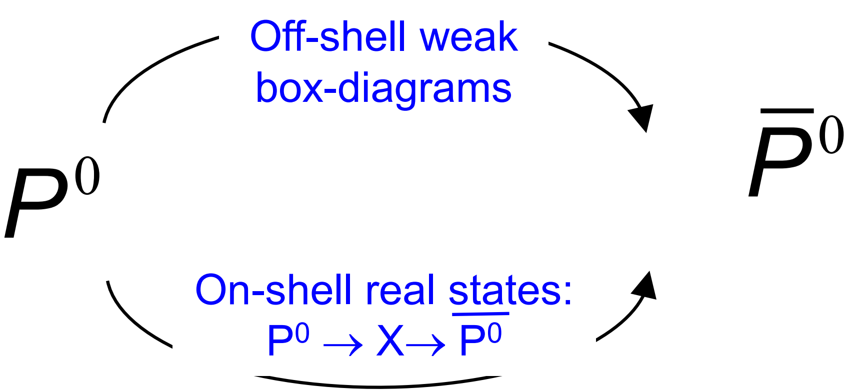
$|K_2\rangle \rightarrow \varepsilon' \rightarrow \pi\pi$ (Direct CPV)

$\varepsilon|K_1\rangle \rightarrow \pi\pi$ (mixing)

$$|\varepsilon| = (2.284 \pm 0.014)\cdot 10^{-3}$$
$$\mathrm{Re}(\varepsilon'/\varepsilon) = (1.67 \pm 0.26)\cdot 10^{-3}$$

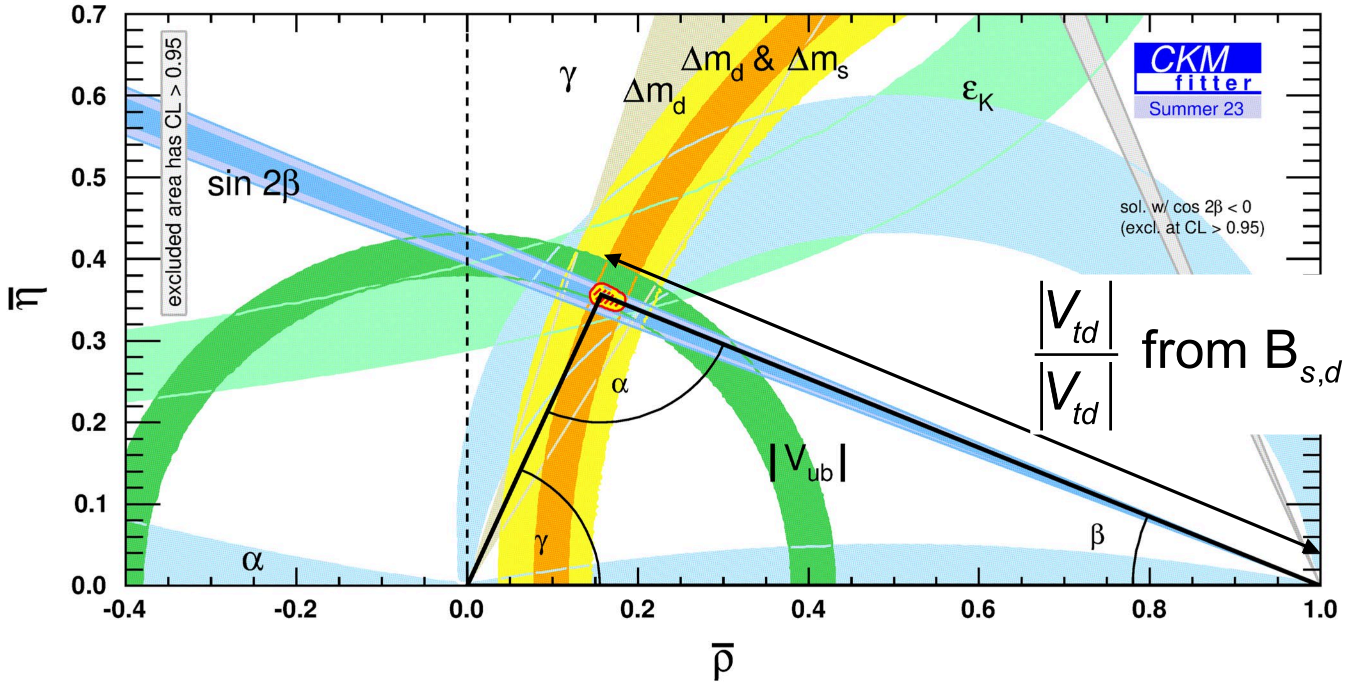
i.e. there is also small a very small (10^{-6}) direct CP violation.

Origin of CPV in kaon mixing:



Experimental Test of the Unitarity Triangle

http://ckmfitter.in2p3.fr



$\frac{|V_{td}|}{|V_{td}|}$ from $B_{s,d}$ mixing

Impressive confirmation of the CKM paradigm of the Standard Model:
Main source of the observed CP violation in mesons is the quark mixing.

Question: Are there contributions from new (unknown) physical effects?
→ rare FCNC decays are excellent testbeds (active research)

CP Violation in Mesons – Final Remarks

CP Violating effects all depend on J_{CP} (Jarlskog invariant) and should therefore be in the same order in the Standard Model. The observable asymmetries = ratio between CP violating to CP conserving quantities are enhanced for suppressed quantities. Observable CP asymmetries are in general larger in B decays than in kaon:

$\rightarrow$ B decays have smaller CKM couplings (suppressed compared to kaons), sizable contributions from V_{ub} and V_{td} possible

To exhibit a CP violating phase the interference term must involve at least 4 different CKM matrix elements (see definition of J_{CP}).

- Below the charm threshold on-shell processes cannot violate CP as only V_{ud} and V_{us} are involved (no phases).

- CPV in Kaon sector only through virtual processes to which also heavier quarks can contribute: K^0 mixing diagrams or penguin decays.