3.1 Equation of state from lattice QCD

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a) from figure: $\frac{\varepsilon}{T^4}(T_c) \simeq 4$ $\Rightarrow \varepsilon = 4T_c^4/(\hbar c)^3 = 4\cdot(156\,\text{MeV})^4/(197\,\text{MeV fm})^3$
$\simeq 300\,\text{MeV}/\text{fm}^3$

density of normal nuclear matter via radius $R \simeq 1.2\,\text{fm}\cdot A^{1/3}$

$\to$ number density $n = \frac{A}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi\cdot(1.2\,\text{fm})^3} \approx 0.14\,\text{fm}^{-3}$

$\to$ energy density $\varepsilon = m_{nucleon}\cdot n \approx 940\,\text{MeV}\cdot 0.14\,\text{fm}^{-3} = 130\,\text{MeV}/\text{fm}^3$ $\to$ not so different

b) from figure: $3P/T^4 \simeq 2 \Rightarrow p = \frac{2}{3}(156\,\text{MeV})^4/(197\,\text{MeV fm})^3 \simeq 50\,\frac{\text{MeV}}{\text{fm}^3}$

unit: $[p] = \frac{[F]}{[A]} = \frac{[W]/[L]}{[L]^2} = \frac{\text{MeV}}{\text{fm}^3}$ $= \frac{50\cdot 10^6\cdot \overset{1.6\cdot 10^{-19}\,C}{e}}{(10^{-15})^3}\,\frac{V}{m^3} = 8\cdot 10^{33}\,\text{Pa}$

c) black dashed line: value expected from non-interacting phase of quarks & gluons
$\to$ valid when asymptotic freedom is reached ($\to$ at much higher energies)

from lecture: $\varepsilon_{qg} = \frac{\pi^2}{30}\left(g_g + \frac{7}{8}g_q\right)T^4 + \underset{\substack{\| \\ 0.5\,\frac{\text{GeV}}{\text{fm}^3}}}{B} = \frac{\pi^2}{30}\underbrace{\left(16 + \frac{21}{2}N_f\right)}T^4 + B$

for $N_f = 2$, I get a value of 12,
figure reads ≈ 16

- as $p = \frac{1}{3}\varepsilon$ and $s = \frac{4}{3}\varepsilon/T$ $\to$ one value for $\frac{3p}{T^4}$, $\frac{\varepsilon}{T^4}$, $\frac{3s}{4T^3}$