Problem set 2 – Quark Gluon Plasma Physics - SS 2023

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2.1 Parton scattering

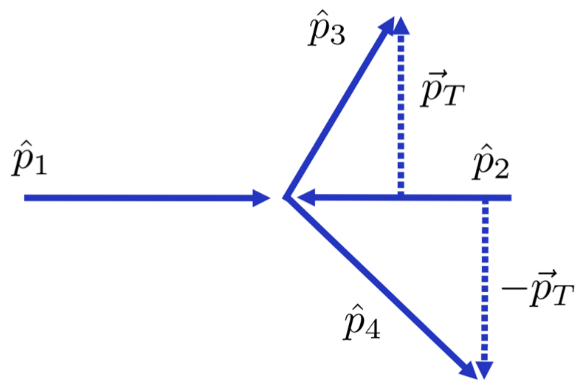

Figure 1: Collision of two partons

a)

We want to calculate the momentum fractions x_1 and x_2 using four-momenta conservation:

$$\begin{aligned}
&\hat{p}_1 + \hat{p}_2 = \hat{p}_3 + \hat{p}_4 \\
\Leftrightarrow\ & x_1(E_b, 0, 0, E_b) + x_2(E_b, 0, 0, -E_b) = (p_T \cosh(y_3), \vec{p_T}, p_T \sinh(y_3)) + (p_T \cosh(y_4), -\vec{p_T}, p_T \sinh(y_4)) \\
\Leftrightarrow\ & x_1 \cdot \begin{pmatrix} E_b \\ 0 \\ 0 \\ E_b \end{pmatrix} + x_2 \begin{pmatrix} E_b \\ 0 \\ 0 \\ -E_b \end{pmatrix} = \begin{pmatrix} p_T \cosh(y_3) + p_T \cosh(y_4) \\ 0 \\ 0 \\ p_T \sinh(y_3) + p_T \sinh(y_4) \end{pmatrix} \\
\Leftrightarrow\ & \begin{pmatrix} E_b \cdot (x_1 + x_2) \\ 0 \\ 0 \\ E_b \cdot (x_1 - x_2) \end{pmatrix} = \begin{pmatrix} p_T \cosh(y_3) + p_T \cosh(y_4) \\ 0 \\ 0 \\ p_T \sinh(y_3) + p_T \sinh(y_4) \end{pmatrix}
\end{aligned}$$

We obtain two equations:

$$E_b \cdot (x_1 + x_2) = p_T \cosh(y_3) + p_T \cosh(y_4) \tag{1}$$

$$E_b \cdot (x_1 - x_2) = p_T \sinh(y_3) + p_T \sinh(y_4) \tag{2}$$

This system of equations can be solved for x_1 and x_2, but first we calculate the center of mass energy:

$$s = (P_1 + P_2)^2 = (2 \cdot E_b)^2 = 4E_b^2 \quad \Rightarrow \quad \sqrt{s} = 2E_b$$

Now we can solve the system of equations, where we use $e^x = \cosh(x) + \sinh(x)$ and $e^{-x} = \cosh(x) - \sinh(x)$:

$$
\begin{aligned}
(1)+(2): \qquad 2E_b x_1 &= p_T \cosh(y_3) + p_T \sinh(y_3) + p_T \cosh(y_4) + p_T \sinh(y_4) \\
&= p_T \cdot (e^{y_3} + e^{y_4}) \\
\Leftrightarrow \qquad x_1 &= \frac{p_T}{\sqrt{s}} \cdot (e^{y_3} + e^{y_4})
\end{aligned}
$$

$$
\begin{aligned}
(1)-(2): \qquad 2E_b x_2 &= p_T \cosh(y_3) - p_T \sinh(y_3) + p_T \cosh(y_4) - p_T \sinh(y_4) \\
&= p_T \cdot (e^{-y_3} + e^{-y_4}) \\
\Leftrightarrow \qquad x_2 &= \frac{p_T}{\sqrt{s}} \cdot (e^{-y_3} + e^{-y_4})
\end{aligned}
$$

b)

We want to show that the center of mass rapidity of the parton system is given by $\frac{1}{2}\ln\left(\frac{x_1}{x_2}\right)$:

$$
\begin{aligned}
y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right) &= \frac{1}{2}\ln\left(\frac{\left(\frac{p_T}{\sqrt{s}}\right)}{\left(\frac{p_T}{\sqrt{s}}\right)} \cdot \frac{(e^{y_3} + e^{y_4})}{(e^{-y_3} + e^{-y_4})}\right) \\
&= \frac{1}{2}\ln\left(\frac{(e^{y_3} + e^{y_4})}{(e^{-y_3} + e^{-y_4})}\right) \\
&= \frac{1}{2}\ln\left(\frac{(e^{y_3} + e^{y_4})}{(e^{-y_3} + e^{-y_4})} \cdot \frac{e^{y_3+y_4}}{e^{y_3+y_4}}\right) \\
&= \frac{1}{2}\ln\left(\frac{(e^{y_3} + e^{y_4}) \cdot e^{y_3+y_4}}{(e^{y_4} + e^{y_3})}\right) \\
&= \frac{1}{2}\ln\left(e^{y_3+y_4}\right) \\
&= \frac{y_3 + y_4}{2} \qquad \square
\end{aligned}
$$