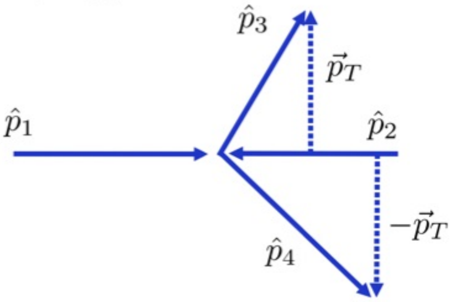
Problem set 2 – Quark Gluon Plasma Physics – SS 2023

Discussion in the lecture: Friday May 5

2.1 Parton scattering

In a collision of two protons, the interaction of two (approximately massless) partons with momentum fractions x_1 and x_2 results in two outgoing partons 3 and 4:



The proton four-momenta can be written as $P_1 = (E_b, 0, 0, E_b)$ and $P_2 = (E_b, 0, 0, -E_b)$ where E_b is the beam energy. The parton four-momenta are given by

$$
\begin{aligned}
\hat{p}_1 &= x_1 P_1, \\
\hat{p}_2 &= x_2 P_2, \\
\hat{p}_3 &= (p_T \cosh y_3, \vec{p}_T, p_T \sinh y_3), \\
\hat{p}_4 &= (p_T \cosh y_4, -\vec{p}_T, p_T \sinh y_4).
\end{aligned}
$$

a) Show that $x_1 = \frac{p_T}{\sqrt{s}}(e^{y_3} + e^{y_4})$ and $x_2 = \frac{p_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4})$.
(hint: $\hat{p}_1 + \hat{p}_2 = \hat{p}_3 + \hat{p}_4$)

Use hint: $\hat{p}_1 + \hat{p}_2 = \hat{p}_3 + \hat{p}_4$

Left side: $\begin{pmatrix} (x_1+x_2)E_b \\ 0 \\ 0 \\ (x_1-x_2)E_b \end{pmatrix} =$ Right side: $\begin{pmatrix} p_T(\cosh y_3 + \cosh y_4) \\ 0 \\ 0 \\ p_T(\sinh y_3 + \sinh y_4) \end{pmatrix}$

$\Rightarrow$ 2 equations

$$(x_1 + x_2) = \frac{p_T}{E_b}(\cosh y_3 + \cosh y_4) \qquad (\mathrm{I})$$

$$(x_1 - x_2) = \frac{p_T}{E_b}(\sinh y_3 + \sinh y_4) \qquad (\mathrm{II})$$

I + II: $x_1 = \frac{p_T}{2E_b}\Big(\underbrace{\cosh y_3 + \sinh y_3}_{e^{y_3}} + \underbrace{\cosh y_4 + \sinh y_4}_{e^{y_4}}\Big)$

$\Rightarrow x_1 = \frac{p_T}{\sqrt{s}}\left(e^{y_3} + e^{y_4}\right)$ using $\sqrt{s} = 2E_b$

I − II: $x_2 = \frac{p_T}{2E_b}\left(\cosh y_3 - \sinh y_3 + \cosh y_4 - \sinh y_4\right)$

Use symmetries: $\cosh x = \cosh(-x)$ and $-\sinh x = \sinh(-x)$

$$\Rightarrow x_2 = \frac{p_T}{2E_b}\Big(\underbrace{\cosh(-y_3) + \sinh(-y_3)}_{e^{-y_3}} + \underbrace{\cosh(-y_4) + \sinh(-y_4)}_{e^{-y_4}}\Big)$$

$$x_2 = \frac{p_T}{\sqrt{s}}\left(e^{-y_3} + e^{-y_4}\right)$$

b) Show that the center-of-mass rapidity of the parton system is given by $\frac{1}{2}\ln\frac{x_1}{x_2}$.

Rapidity: $y = \frac{1}{2}\ln\left(\frac{1+\beta_{cm}}{1-\beta_{cm}}\right)$

To get β_{cm}, consider $p_z^{(1)*} = -p_z^{(2)*}$

$$p_z^{(1)*} = \cancel{\gamma_{cm}}\left(p_z^{(1)} - \beta_{cm}p_0^{(1)}\right) = -\cancel{\gamma_{cm}}\left(p_z^{(2)} - \beta_{cm}p_0^{(2)}\right) = -p_z^{(2)*}$$

$$p_z^{(1)} - \beta_{cm}p_0^{(1)} = -p_z^{(2)} + \beta_{cm}p_0^{(2)}$$

$$p_z^{(1)} + p_z^{(2)} = \beta_{cm}\left(p_0^{(2)} + p_0^{(1)}\right)$$

$$\Rightarrow \beta_{cm} = \frac{p_z^{(1)} + p_z^{(2)}}{p_0^{(2)} + p_0^{(1)}}$$

$$\left.\begin{aligned} p_z^{(1)} &= x_1 E_b \qquad & p_0^{(1)} &= x_1 E_b \\ p_z^{(2)} &= -x_2 E_b \qquad & p_0^{(2)} &= x_2 E_b \end{aligned}\right\} \beta_{cm} = \frac{E_b(x_1 - x_2)}{E_b(x_1 + x_2)} = \frac{x_1 - x_2}{x_1 + x_2}$$

$$\Rightarrow y = \frac{1}{2}\ln\left(\frac{1 + \frac{x_1 - x_2}{x_1 + x_2}}{1 - \frac{x_1 - x_2}{x_1 + x_2}}\right) \overset{1 = \frac{x_1+x_2}{x_1+x_2}}{=} \frac{1}{2}\ln\left(\frac{x_1 + x_2 + x_1 - x_2}{\cancel{x_1 + x_2}}\;\frac{\cancel{x_1 + x_2}}{x_1 + x_2 - x_1 + x_2}\right)$$

$$= \frac{1}{2}\ln\left(\frac{2x_1}{2x_2}\right)$$

$$\Rightarrow y = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right)$$