

QUARK-GLUON PLASMA PHYSICS

SET 2:

$$P_1 = (E_b, 0, 0, E_b) \quad P_2 = (E_b, 0, 0, -E_b)$$

$$\hat{P}_1 = x_1 P_1 \quad \hat{P}_3 = (P_T \cosh y_3, \bar{P}_T, P_T \sinh y_3)$$

$$\hat{P}_2 = x_2 P_2 \quad \hat{P}_4 = (P_T \cosh y_4, -\bar{P}_T, P_T \sinh y_4)$$

Starting from $\hat{P}_1 + \hat{P}_2 = \hat{P}_3 + \hat{P}_4$ we get

$$((x_1 + x_2)E_b, 0, 0, (x_1 - x_2)E_b) = (P_T(\cosh y_3 + \cosh y_4), 0, 0, P_T(\sinh y_3 + \sinh y_4))$$

$\hookrightarrow \equiv \cosh(y_3)$

$$\hookrightarrow \begin{cases} (x_1 + x_2)E_b = P_T(\cosh y_3 + \cosh y_4) \\ (x_1 - x_2)E_b = P_T(\sinh y_3 + \sinh y_4) \end{cases}$$

$$\Rightarrow 2x_1 E_b = P_T(\cosh y_3 + \cosh y_4 + \sinh y_3 + \sinh y_4)$$

\rightarrow we recall that

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\Rightarrow \cosh x + \sinh x = e^x$$

$$\cosh x - \sinh x = e^{-x}$$

We also know that $E_b = \frac{\sqrt{s}}{2} \Rightarrow \sqrt{s} = 2E_b$

$$\Rightarrow x_1 = \frac{P_T}{\sqrt{s}} (e^{y_3} + e^{y_4})$$

$$\text{and similarly } x_2 = \frac{P_T}{\sqrt{s}} (e^{-y_3} + e^{-y_4})$$

⑤ Show that $\chi_{CM} = \frac{1}{2} \log\left(\frac{x_1}{x_2}\right)$

For the perturbed system, the sum of the two momenta is

$\vec{P} = ((x_1 + x_2)\vec{E}, 0, 0, (x_1 - x_2)\vec{E})$. We know that

$$\beta = \frac{|\vec{P}|}{E}$$

$$\Rightarrow \beta = \frac{x_1 - x_2}{x_1 + x_2}$$

We also know that

$$\chi_{CM} = \frac{1}{2} \log\left(\frac{1 + \beta}{1 - \beta}\right) =$$

$$= \frac{1}{2} \log\left(\frac{1 + \frac{x_1 - x_2}{x_1 + x_2}}{1 - \frac{x_1 - x_2}{x_1 + x_2}}\right) =$$

$$= \frac{1}{2} \log\left(\frac{2x_1}{2x_2}\right) = \frac{1}{2} \log\left(\frac{x_1}{x_2}\right) \quad \square$$