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Problemset 09

9.1 Deconfinement temperature

a) Determine statistical accuracy, with which 1S and 2S state should be measured, in order to determine deconfinement temperature with 5 MeV precession

We know
$$N(J/\psi) \sim m_{J/\psi}^{\frac{3}{2}}\, e^{-\frac{m_{J/\psi}}{T}}$$
$$N(\psi(2S)) \sim m_{\psi_{2S}}^{\frac{3}{2}}\, e^{-\frac{m_{\psi_{2S}}}{T}}$$

now take ratio
$$r = \frac{N(\psi(2S))}{N(J/\psi)} = \left(\frac{m_{\psi_{2S}}}{m_{J/\psi}}\right)^{\frac{3}{2}} e^{\frac{-m_{\psi_{2S}} + m_{J/\psi}}{T}}$$

for $m_{\psi_{2S}} = 3{,}7\,\text{GeV}$ and $m_{J/\psi} = 3{,}1\,\text{GeV}$ (from VL10b, slide 6)

for $T_1 = 0{,}15\,\text{GeV}$ (lowest value) $\quad T_2 = 0{,}2\,\text{GeV}$ (highest value)

$\Rightarrow \quad r_1 \approx 0{,}0239 \quad\quad r_2 \approx 0{,}065$

determine statistical accuracy $\frac{\delta r}{r}$ in dependence of δT with $\delta T = 5\,\text{MeV}$

we know:
$$\frac{\delta r}{r} = \frac{\partial r}{\partial T} \cdot \delta T\, \frac{1}{r} = \left(\frac{m_{\psi_{2S}}}{m_{J/\psi}}\right)^{\frac{3}{2}} e^{\frac{-m_{\psi_{2S}} + m_{J/\psi}}{T}} \cdot \frac{m_{\psi_{2S}} - m_{J/\psi}}{T^2}\, \delta T \cdot \frac{1}{r}$$
$$\Rightarrow \quad \frac{\delta r}{r} = \frac{m_{\psi_{2S}} - m_{J/\psi}}{T^2}\, \delta T$$

for $T_1 = 0{,}15\,\text{GeV}$: $\frac{\delta r}{r} \approx 0{,}133$

$T_2 = 0{,}2\,\text{GeV}$: $\frac{\delta r}{r} \approx 0{,}075$

b) Estimate how many $\psi(2S)$ should be reconstructed, to obtain the statistical accuracy determined in a).

ratio of signal to combinatorial background: $\frac{N_s}{N_{bkg}} = 5 \cdot 10^{-4}$

we know: $\frac{\delta r}{r} = \sqrt{\left(\frac{\delta N_{J/\psi}}{N_{J/\psi}}\right)^2 + \left(\frac{\delta N(\psi(2S))}{N(\psi(2S))}\right)^2}$

as $m_{\psi(2S)} > m_{J/\psi}$, $N(\psi(2S)) \gg N_{J/\psi}$ and with errors of the same magnitude we can approximate

① $\frac{\delta r}{r} = \frac{\delta N(\psi(2S))}{N(\psi(2S))} \iff N(\psi(2S)) = \delta N(\psi(2S)) \cdot \frac{1}{\frac{\delta r}{r}}$

furthermore we know $\delta N(\psi(2S)) = \sqrt{\delta N_s(\psi(2S))^2 + \delta N_{bkg}(\psi(2S))^2} \approx \delta N_{bkg}(\psi(2S))$

$\delta N(\psi(2S)) = \sqrt{N_{bkg}(\psi(2S))} = \sqrt{\frac{N_s(\psi(2S))}{5 \cdot 10^{-4}}}$

wegen Poissonverteilung

$\frac{N_s}{N_{bkg}} = 5 \cdot 10^{-4} \iff N_{bkg} = \frac{N_s}{5 \cdot 10^{-4}}$

use ①

$$\frac{\delta r}{r} = \frac{1}{N_s(\psi(2S))} \sqrt{\frac{N_s(\psi(2S))}{5 \cdot 10^{-4}}}$$

solve for $N_s(\psi(2S))$: $\left(\frac{\delta r}{r}\right)^2 = \frac{1}{5 \cdot 10^{-4} \cdot N_s(\psi(2S))}$

$$\Rightarrow N_s(\psi(2S)) = \frac{1}{5 \cdot 10^{-4} \cdot \left(\frac{\delta r}{r}\right)^2}$$

$\frac{\delta r}{r}$ determined in a)

therefore we receive:

$T_1 = 0{,}15$ GeV: $N_s(\psi(2S)) = 118300$ $\quad N_{bkg} = 236\ 686\ 390$

$T_2 = 0{,}2$ GeV: $N_s(\psi(2S)) = 355\ 556$ $\quad N_{bkg} = 711\ 111\ 111$

furthermore: $\frac{N_s}{N_{bkg}} = 5 \cdot 10^{-4} \Rightarrow N_{bkg} = \frac{N_s}{5 \cdot 10^{-4}}$