2. Cavities / waveguides

rectangular & cylindric waveguides

from Maxwell equations

$$\vec{\nabla}^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} \qquad \vec{\nabla}^2 \vec{H} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{H}$$

assuming periodic dependency $\vec{E} \pm \vec{E}(\vec{r}) e^{i(\omega t - \underset{\uparrow}{k_z} z)}$ $\qquad \vec{H} = \vec{H}(\vec{r}) e^{i(\omega t - k_z z)}$

wave number $k_z = \frac{\omega_z}{c}$

$$\left(\vec{\nabla}^2 - \frac{\partial^2}{\partial z^2}\right) \underset{\vec{H}}{\vec{E}} \ldots + \left(\frac{\omega^2}{c^2} - k_z^2\right) \underset{\ldots \vec{H}}{\vec{E}} = 0 \quad \ldots$$

solution from ~~border~~ boundary conditions : $\vec{E}$ - parallel to the conducting wall
$\vec{H}$ - orthogonal to the conducting wall

- small field component producing Eddy currents
- field components in z-direction

Slide

E - waves → $E_z \neq 0$ $H_z = 0$ TM (transverse H)

H - waves → $E_z = 0$ $H_z \neq 0$ TE (transverse E)

(L3)(7) TE_{mn} / TM_{mn} where m - number of zero-crossings in x/r direction

n - .. in y/φ direction

- the k_z is smaller than $k = \omega/c$ (free EM - wave)

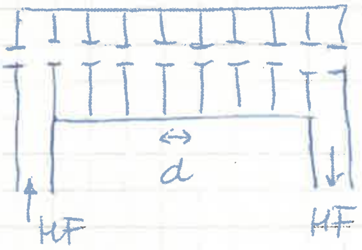
wave travels through the waveguide only when $\frac{\omega}{c} > \underset{\substack{\uparrow \\ \text{smallest } k}}{k_c}$

$$k_c^2 + k_z^2 = k^2 = \frac{\omega^2}{c^2} \qquad \text{If } \frac{\omega}{c} < k_c \rightarrow k_z \text{ - Im.}$$

- phase velocity $v_{ph} = \frac{\omega}{k_z} = c \frac{1}{\sqrt{1 - k_c^2/k^2}} > c$

- group velocity (E-velocity) $v_g = \frac{d\omega}{dk_z} = \frac{c^2 k_z}{\omega} = c\sqrt{1 - k_c^2/k^2} < c$

- Waveguide with Iris-holes



Example: SLAC-structure

$v_{ph} \simeq c$; $k_z = \frac{2\pi}{3d}$; $\lambda_z = 3d$

one can change the size of holes / d ..

- resonator cavities

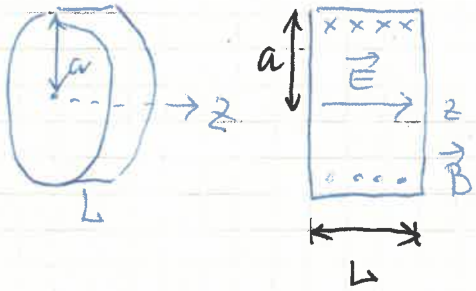
  - standing wave ⇒ superposition of direct & reflected waves
  - resonance $\underset{\text{length}}{\underline{L}} = q \frac{\lambda_z}{2}$ ⇒ $k_z = \frac{q\pi}{L}$ with $q = \overset{\nearrow}{0}, 1, 2 ..$

    only for TM

    $q = 0$ (TM-only) ⇒ $\lambda_z = \infty$, $k_z = 0$ fields independent of z

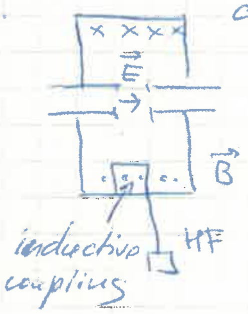
  - resonant frequency

    for a given q: $\omega = c\sqrt{k_z^2 + k_c^2}$



TM_{010}
xyz

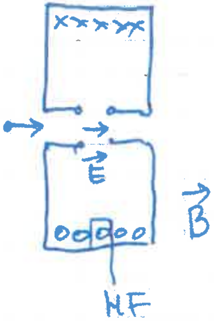
- accelerating cavity



from lecture 2. - Single resonator
- Alvarez structure

25.1

- Single resonator (Einzelresonator)

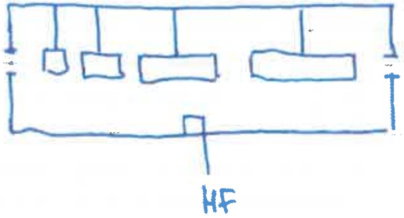


flexibility since synchronisation between individual SR can be specially tuned
a sequence of SR allows for accurate and simple energy tuning

↳ details later

L3 8

- Alvarez structure



a series of SR without separating walls

↳ details later

Skip to lecture 3

L3 9

(3) Accelerator components

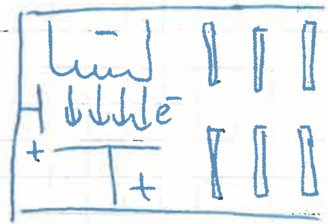
after Alvarez

1. Ion sources

- Electron-impact ionisation

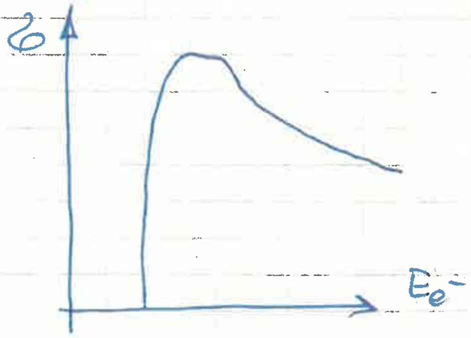
$$e^- + A \rightarrow 2e^- + A^+$$

$E_{e^-} \geq$ ionisation energy of A



Ⓛ3⑤

slide Ⓛ3④



ionisation

$$\sigma_i = N\pi p^2 = N\pi \frac{e^4}{(E^i)^2} f\left(\frac{E}{E_0}\right)$$

number of e⁻ in the outer shell

collision parameter

ionisation energy

energy of the electron

energy transfer

If we increase E_{e^-} → higher ionisation ~~of sev~~ ⇒ highly charged ions

slide Ⓛ3⑥ EBIT/EBIS

Example of sources slide

- Surface ionisation / Contact ionisation

If E_i small (Alkali $E_i < 5\,eV$) / E_a (Electron affinity) (Halogens) } ⇒ material with large Work function W, Re …

- long enough contact $t \sim 10^{-5} - 10^{-3}\,s$
- surface – high temperature

$$\underset{\text{probability}}{P_i} = \frac{n_i}{n_0 + n_i} = \left(1 + \frac{g_0}{g_i} e^{e(\phi_i - \phi_s)/kT}\right)^{-1}$$

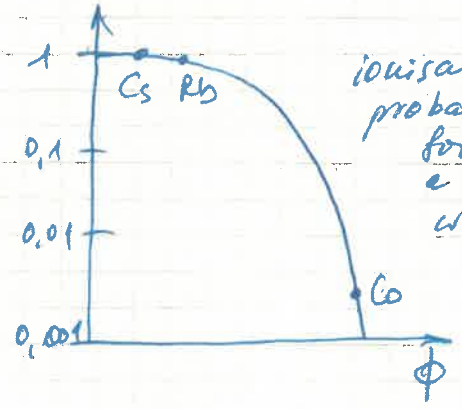
atoms ions

statistical weight

ionisation potential

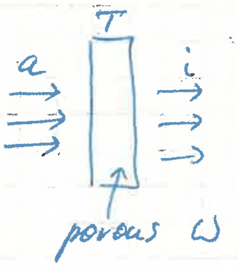
work function

Statistical ~Sacha-like equation



ionisation probability for a surface with $\Phi_s = 5.25$

T = 1000 - 2500 K

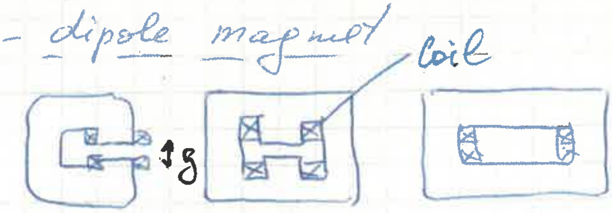


- Field ionisation
- Resonant laser ionisation

Start here 15.11

3. Magnets

- dipole magnet



"C"	"H"	Window frame
asymmetric	symmetric	compact
open - easy access	compact	less iron to achieve B than for H-magnet
	highest B-homogenity	+ large apertures possible
	for $\frac{\Delta B}{B} \sim 10^{-4}$	- complicated
	$\frac{\Delta g}{g(4cm)} \leq 4\mu m$	

Fe: L
$B \leq 2T$
Steele with a few Carbon
small remanence
small coercivity
Laminated to reduce Eddy currents

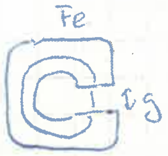
Coil: Copper water cooled

(L3) (10)

g - gap

- Static magnetic field

$\vec{\nabla} \times H = 0$ field $\quad \vec{\nabla} \cdot \vec{B} = 0$ flux $\quad (\vec{B} = \underset{\uparrow}{\mu} \vec{H})$ permeability



$\oint H ds = H_0 g + H_{Fe} l_{Fe} = \underset{\uparrow}{nI}$ number of wirings

$\mu_{Fe} \gg 1 \Rightarrow H_{Fe} \overset{\ll}{\cancel{\gg}} H_0$

$\hookrightarrow H_0 \approx nI/g \Rightarrow B = \mu_0 \frac{nI}{g}$ $\quad \mu_0 = 4\pi \, 10^{-7} \frac{Tm}{A}$

Example: $nI = 50\,000A \Rightarrow B \simeq 1{,}57\,T$
$g = 4cm$

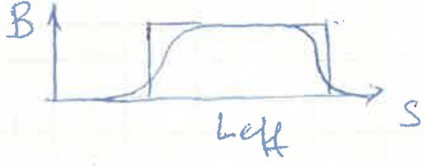
Problems

B(H) → saturation (complicated calculations)

hysteresis / remanence

↳ reproducibility, stability

- Effective lenght



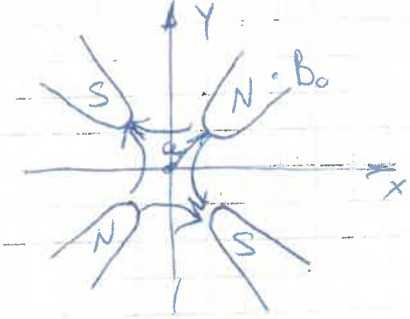
$$L_{eff} = \frac{1}{B_0} \int_{-\infty}^{\infty} B(s) ds$$

for dipole: $L_{eff} \simeq L_{Fe} + 1{,}3g$

Continue with magnets after ionsources

- Quadrupole magnet

purpose - focusing / defocusing



center - $B_\theta = 0$

$\vec{B} = -\vec{\nabla}\Phi$ $\quad \Phi(x,y)$ - scalar potential

g - field gradient

$\Phi(x,y) = -gxy$

$g = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = \frac{B_0}{a} \approx 2\mu_0 \frac{nI}{a^2}$

a - aperture

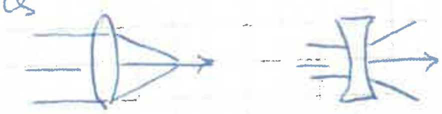
B_0 - flux at the pole tip

[Lorentz force $F = -e\vec{v}\times\vec{B}$

$F_x = -egcx$ focus $y=0$

$F_y = egcy$ defocus $x=0$] skip

like optics



$L_{eff} = L_{Fe} + a$

slide

- Sextupole magnet

6 - poles typicaly used to correct for aberration - chromaticity - slide

$L_{eff} = L_{Fe} + \frac{a}{2}$ mix x & y planes

- Multipole expansion → errors e.g. QP poles not exactly hyperbolic, mechanical errors etc

slide

- Superconducting magnets

s.c. $B \leq 10T$ conventional $B \leq 2T$

$g \leq 100T/m$ $\quad g \leq 20T/m$