Introduction to Accelerator Physics

Yuri A. Litvinov
y.litvinov@gsi.de

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Physikalisches Institut der Universität Heidelberg

Lecture Dates

https://uebungen.physik.uni-heidelberg.de/vorlesung/20222/1611/lecture

Date	Topic
19.10.2022	Introduction and basic definitions
26.10.2022	Accelerating structures
02.11.2022	Accelerator Components
09.11.2022	Optics with magnets (1)
16.11.2022	Optics with magnets (2)
23.11.2022	Equations of motion
30.11.2022	Phase ellipses and magneto-optical system / Transverse beam dynamics
07.12.2022	Transverse beam dynamics, beam stability / Longitudinal beam dynamics
14.12.2023	Longitudinal beam dynamics and a summary
11.01.2023	Phase space and beam cooling (Invitation)
18.01.2023	Space charge and beam-beam dynamics
25.01.2023	Physics at Storage Rings and Colliders
01.02.2023	New accelerator technologies and final summary
08.02.2023	Student seminar
15.02.2023	Visit GSI
22.02.2023	Visit MPIK

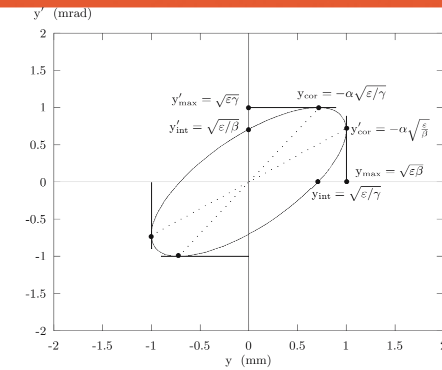
Wednesdays, 14:15-16:00

Summary of the previous lecture

Describe Machine

Courant-Snyder Invariant

Machine parameters



Dispersion

Dispersion function

$$D(s) = \frac{\sqrt{\beta}}{2\sin(\mu/2)} \int_s^{s+C} h(\bar{s})\sqrt{\beta(\bar{s})}\cos[\Psi(\bar{s}) - \Psi(s) - \mu/2]d\bar{s}$$

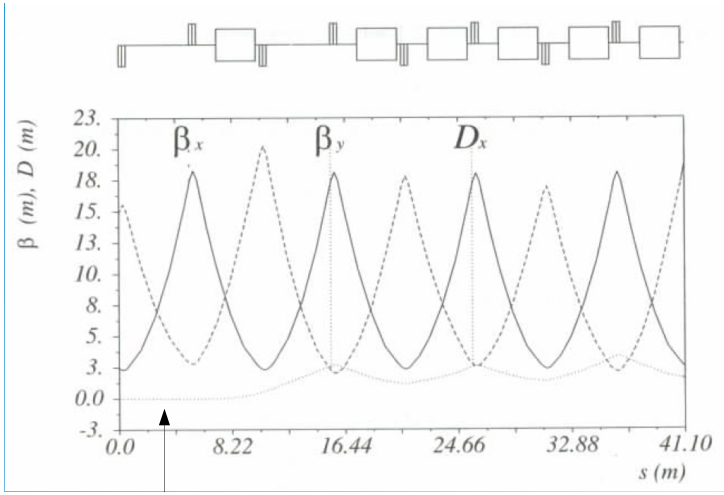
$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{16} \\ M_{21} & M_{22} & M_{26} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D \\ D' \\ 1 \end{pmatrix}$$

Distortion and Resonances

Distortion function

Stop-bands 1st, 2nd, 3rd order

$$D = \frac{M_{16}}{1 - M_{11}}$$



Summary of the previous lecture

Describe Machine

Courant-Snyder Invariant

Machine parameters

$$x_c(s) = \frac{\sqrt{\beta(s)}}{2\sin(Q\pi)} \int_s^{s+C} F(\bar{s})\sqrt{\beta(s)}\cos[\Psi(\bar{s}) - \Psi(s) - Q\pi]d\bar{s}$$

Dispersion

Dispersion function

Consequences: $x_c(s) \propto \Delta x'$

$x_c(s) \propto \sqrt{\beta(s_0)}$
$x_c(s) \propto \sqrt{\beta(s)}$
} Correction magnets (trial & error)

$x_c(s) \propto 1/\sin(Q\pi) \Leftarrow$ Q can not be integer

Distortion and Resonances

Distortion function

Stop-bands 1st, 2nd, 3rd order

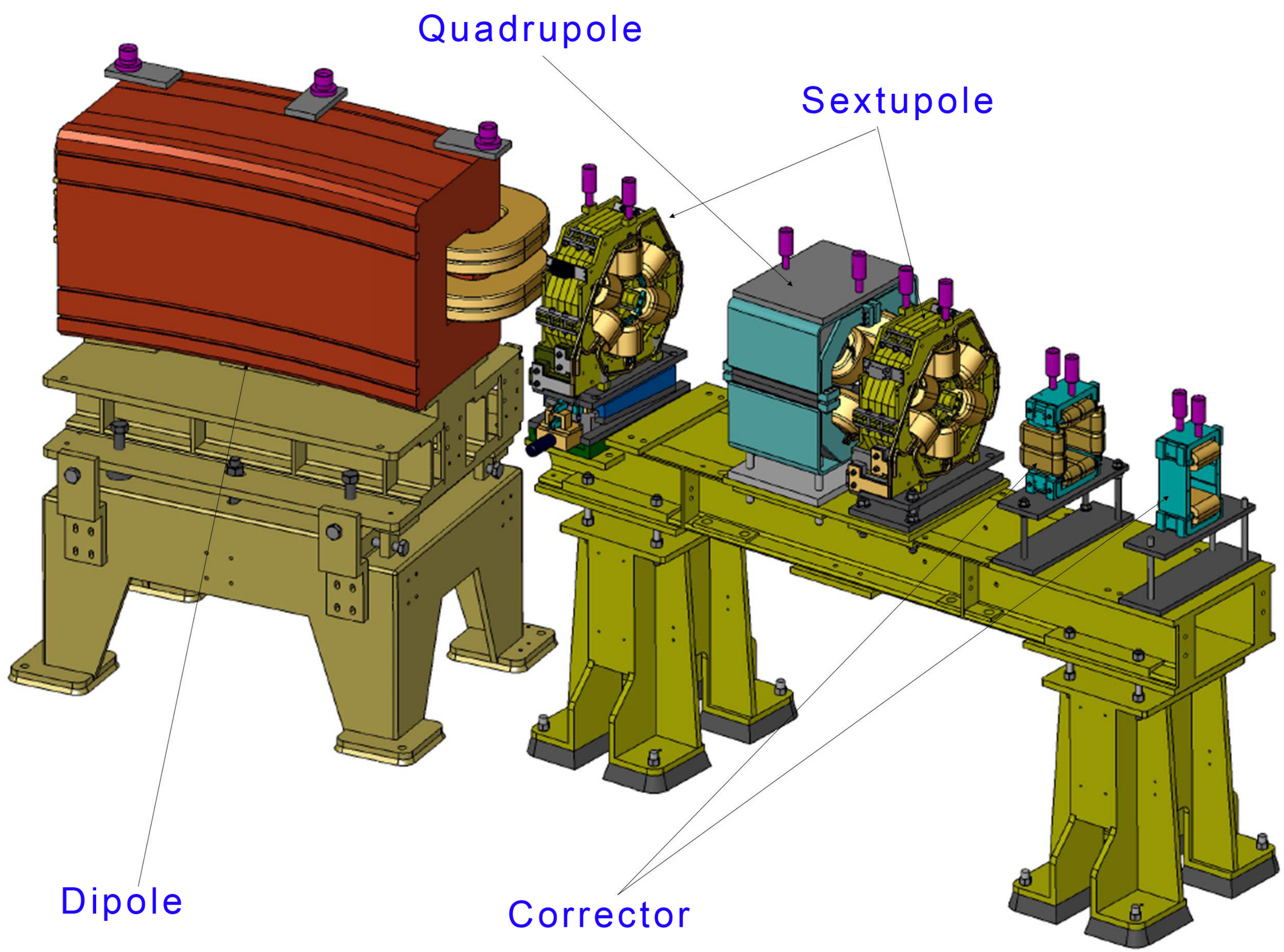
$$\Delta\beta(s) = \frac{\beta(s)}{2\sin^2(2Q\pi)} \int_s^{s+C} \delta K(\bar{s})\beta(\bar{s})\cos 2[\Psi(\bar{s}) - \Psi(s) - Q\pi]d\bar{s}$$

Important: $2Q\pi$ -> Q=half-integer -> Resonance
Stop-band of second order

$$\delta Q = \frac{\beta^{3/2}}{16\pi}\left(\frac{\partial^2 B_y}{\partial x^2}\right)\frac{\Delta s}{B\rho}a\cos(3\Psi)$$

Resonance of $3Q$-integer
Stop-band of 3rd order:

Summary of previous lectures



13. Distortions and Resonances

13.1 Floquet transformation (used to discuss resonances and instabilities)

Goal: tranform tilted ellipse into a circular diagram

$$y(s) \longleftrightarrow \eta(\psi) \text{ mit } \eta = \frac{y}{\sqrt{\beta}}\,,$$

$$y'(s) \longleftrightarrow \frac{\mathrm{d}\eta}{\mathrm{d}\psi} \text{ mit } \frac{\mathrm{d}\eta}{\mathrm{d}\psi} = \alpha\frac{y}{\sqrt{\beta}} + \sqrt{\beta}y'\,.$$

$$\begin{pmatrix} \eta \\ \frac{\mathrm{d}\eta}{\mathrm{d}\psi} \end{pmatrix} = \begin{pmatrix} \beta^{-1/2} & 0 \\ \alpha\beta^{-1/2} & \beta^{1/2} \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix} \qquad \begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \beta^{1/2} & 0 \\ -\alpha\beta^{-1/2} & \beta^{-1/2} \end{pmatrix} \begin{pmatrix} \eta \\ \frac{\mathrm{d}\eta}{\mathrm{d}\psi} \end{pmatrix}$$

„Dimensionless“ units

$$\sqrt{\mathrm{mm\ mrad}}$$

$$\eta^2 + \left(\frac{\mathrm{d}\eta}{\mathrm{d}\psi}\right)^2 = a^2$$

13. Distortions and Resonances

13.1 Floquet transformation (used to discuss resonances and instabilities)

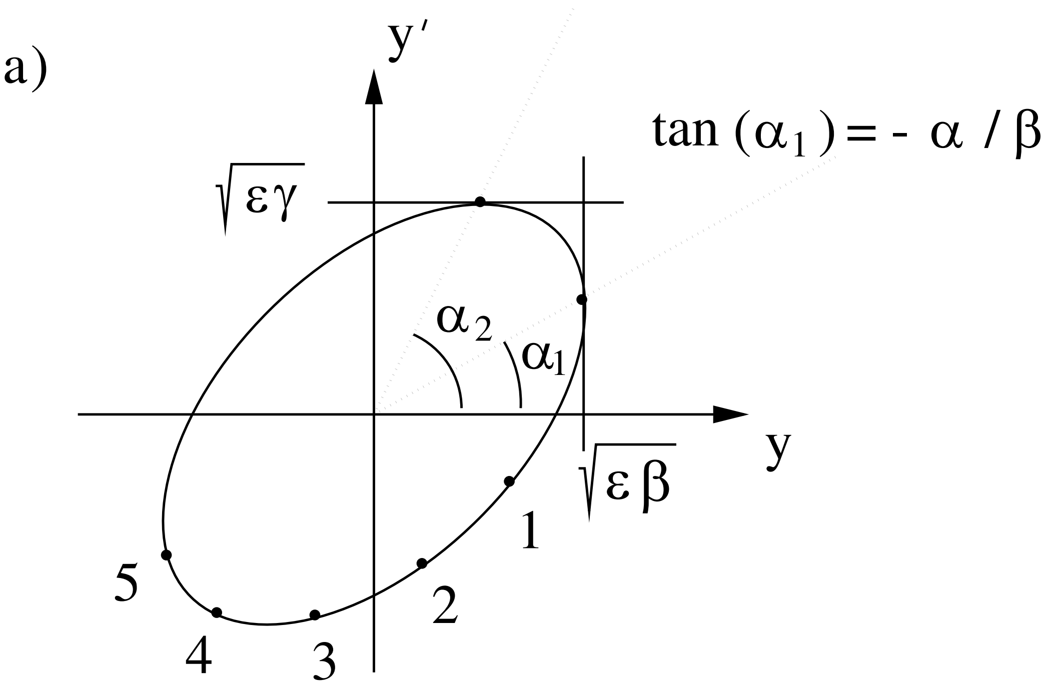
Goal: tranform tilted ellipse into a circular diagram

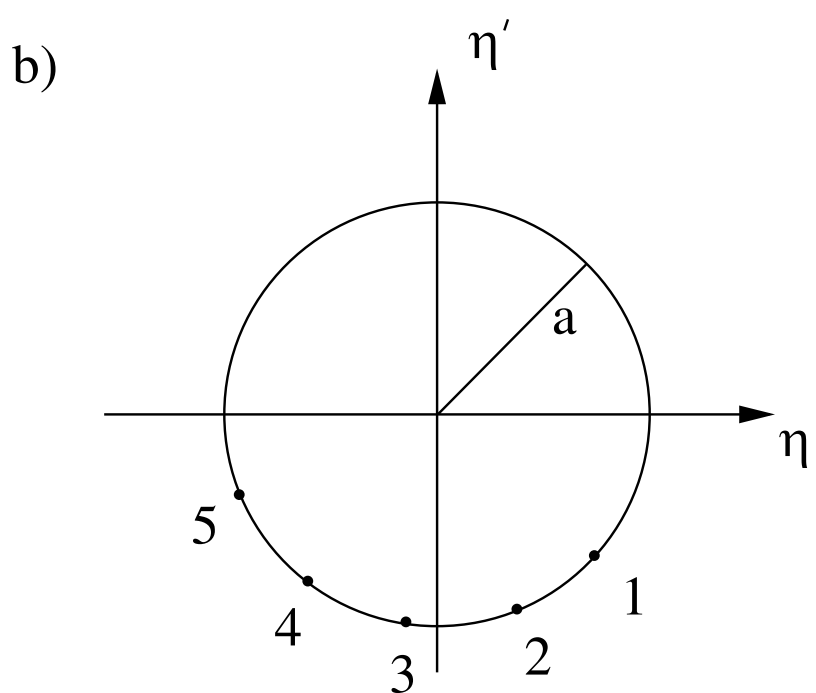
$$\frac{y^2}{\beta} + \frac{(\alpha y + \beta y')^2}{\beta} = a^2 = \epsilon$$

Courant-Snyder Invariant

$$\eta^2 + \left(\frac{d\eta}{d\psi}\right)^2 = a^2$$

„Kreisdiagramm“





13.3 Quadrupole errors (stop-band second-order)

$$\mu' = \mu_0 + \Delta\mu \quad (\Delta\mu << 1)$$

Stability condition:

$$\cos\mu = \frac{1}{2}Tr(M) = \cos(\mu_0) - \frac{1}{2}\frac{\beta_0}{f}\sin(\mu_0)$$

An additional term

$$\Delta\mu = 2\pi\Delta Q = \frac{1}{2}\frac{\beta_0}{f}$$

$$\Delta Q = \frac{1}{4\pi}\frac{\beta_0}{f}$$

Can be used to measure β_0

13. Distortions and Resonances

13.3 Quadrupole errors (stop-band second-order)

Result: shift of **operation point**

$$\Delta Q = \frac{1}{4\pi} \oint \beta(\bar{s}) \delta K(\bar{s}) d\bar{s}$$

Similar as in the case of the dispersion, but β instead of $\sqrt{\beta}$

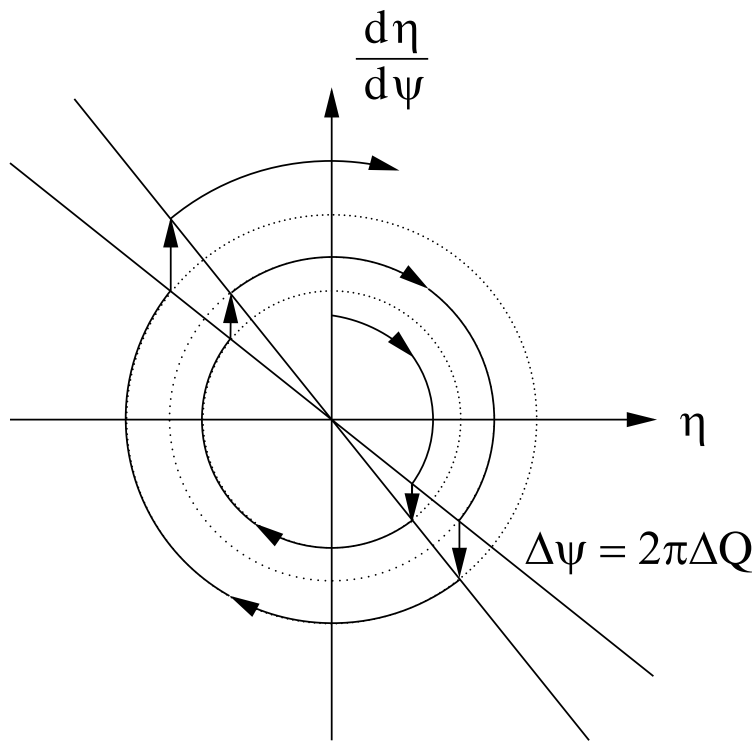
$$\Delta\beta(s) = \frac{\beta(s)}{2\sin^2(2Q\pi)} \int_s^{s+C} \delta K(\bar{s})\beta(\bar{s}) \cos 2[\Psi(\bar{s}) - \Psi(s) - Q\pi] d\bar{s}$$

Important: $2Q\pi$ -> Q=half-integer -> Resonance
Stop-band of second order

13.3 Quadrupole errors (stop-band second-order)

Floquet transformation

Kick: $$\Delta y' = -\frac{y}{f} = -\frac{1}{f} a\sqrt{\beta}\cos\psi$$



Amplitude enlargement

$$\Delta a = \Delta\left(\frac{d\eta}{d\psi}\right)\sin\psi = -\frac{a\beta}{f}\cos\psi\sin\psi\,,$$

$$\Delta\psi = -\frac{1}{a}\Delta\left(\frac{d\eta}{d\psi}\right)\cos\psi = \frac{\beta}{f}\cos^2\psi\,.$$

Phase shift

13.3 Quadrupole errors (stop-band second-order)

$$\Delta Q = \frac{1}{2\pi}\frac{\beta}{f}\cos^2\psi = \frac{1}{4\pi}(1 - 2\cos(2\Psi))$$

Average shift of the working point by $$\Delta\bar{Q} = \frac{1}{4\pi}\frac{\beta}{f}$$

With superimposed modulation $$\delta\bar{Q} = \frac{1}{4\pi}\frac{\beta}{f}\cos(2\Psi)$$

Modulations and shift are small if β-small and f-large

13. Distortions and Resonances

13.4 Sextupole errors (stop-band third-order)

Similar to quadrupoles

Resonance of *3Q*-integer

Stop-band of 3rd order:

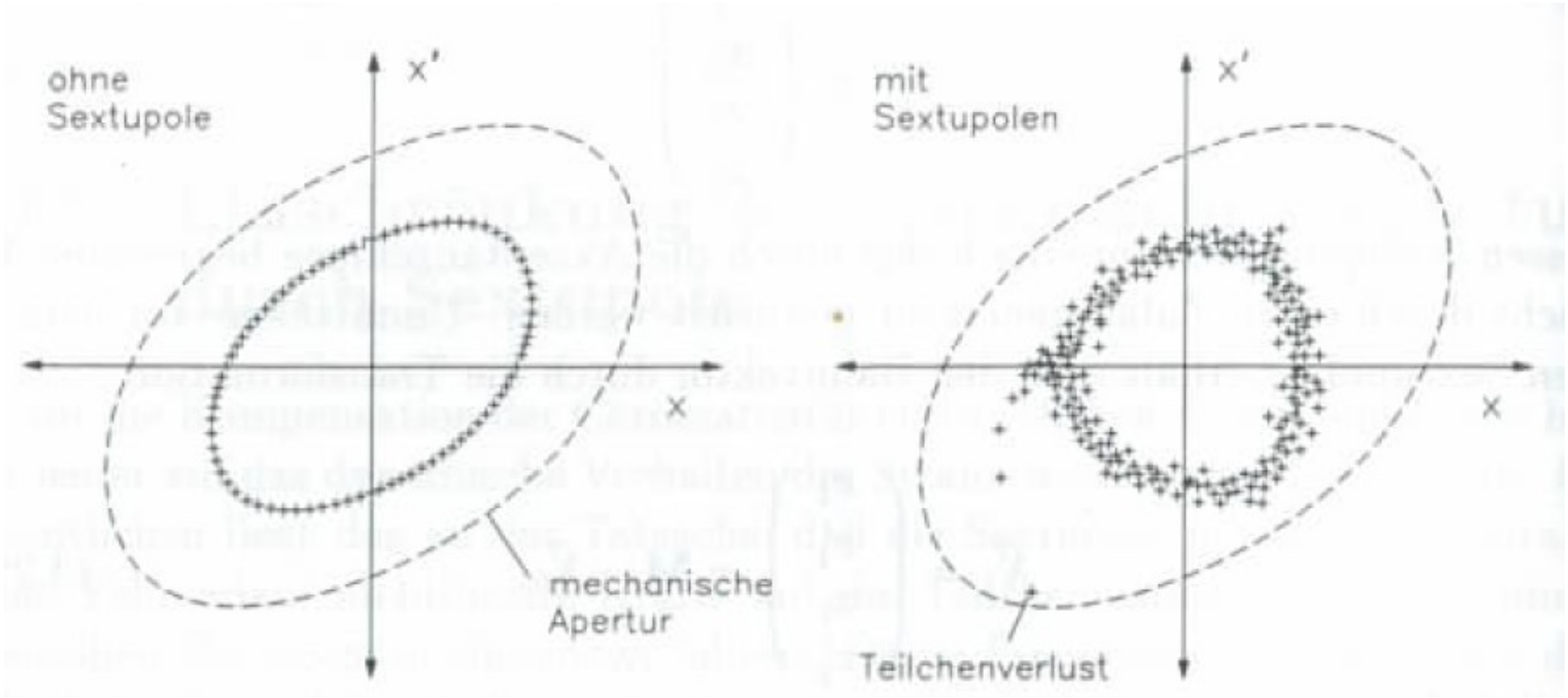
$$\delta Q = \frac{\beta^{3/2}}{16\pi} \left(\frac{\partial^2 B_y}{\partial x^2} \right) \frac{\Delta s}{B\rho} a \cos(3\Psi)$$

Amplitude of betatron oscillations!!!

13. Distortions and Resonances

13.4 Sextupole errors (stop-band third-order)

- Non-linear effect (fast grows: $\Delta a \propto a^2$)
- Dynamic aperture



13.5 Stability conditions

Stay away from resonances!!!

1st order – dipoles
2nd order – quadrupoles
3rd order – sextupoles
…

Resonance conditions:

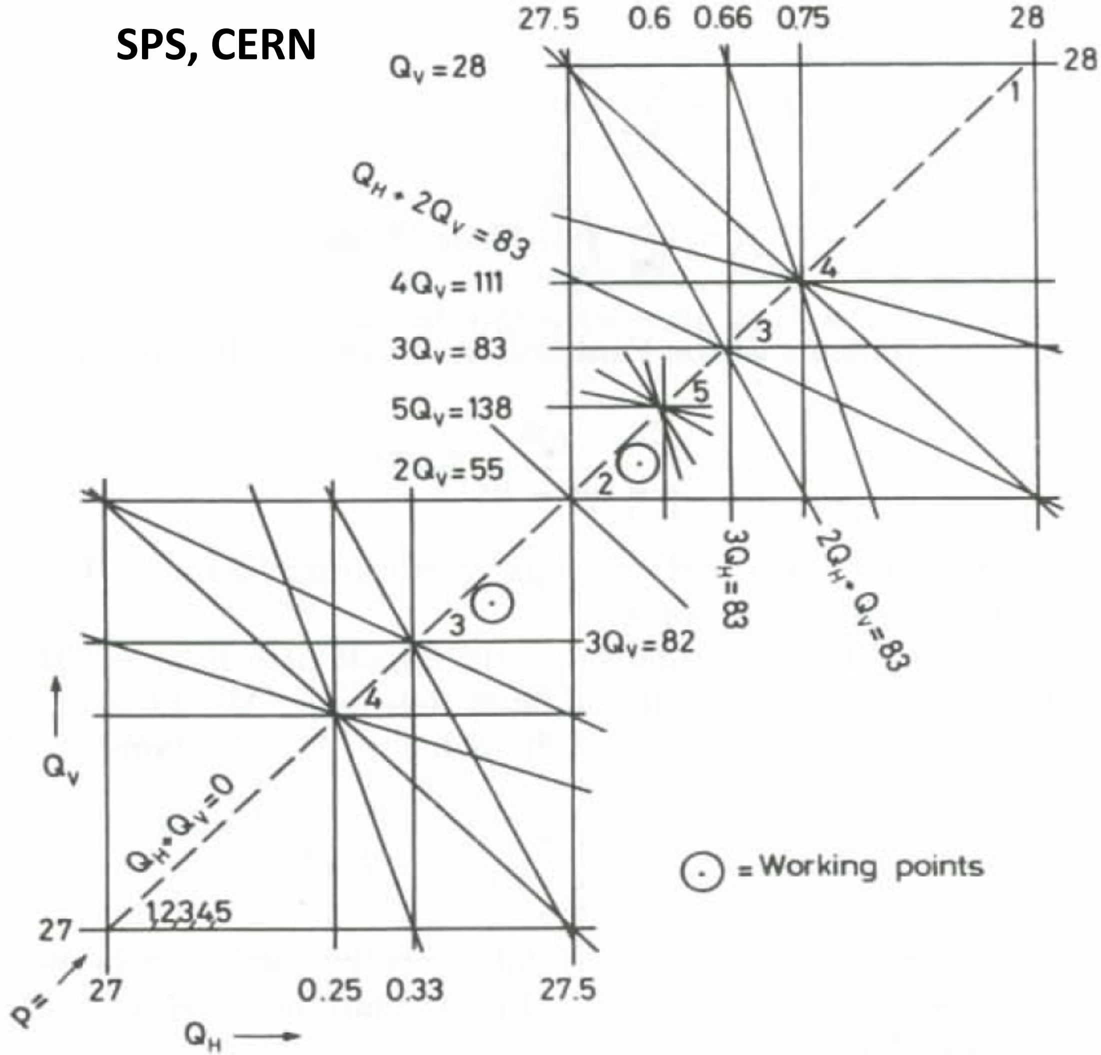
$$p = nQ_x$$
$$p = nQ_y$$

$$\text{(coupled)}$$
$$p = lQ_x + mQ_y$$
$$n = |l| + |m|$$

These are bands:
depending on β (QP)
emittance (SP)

Resonance diagram

SPS, CERN

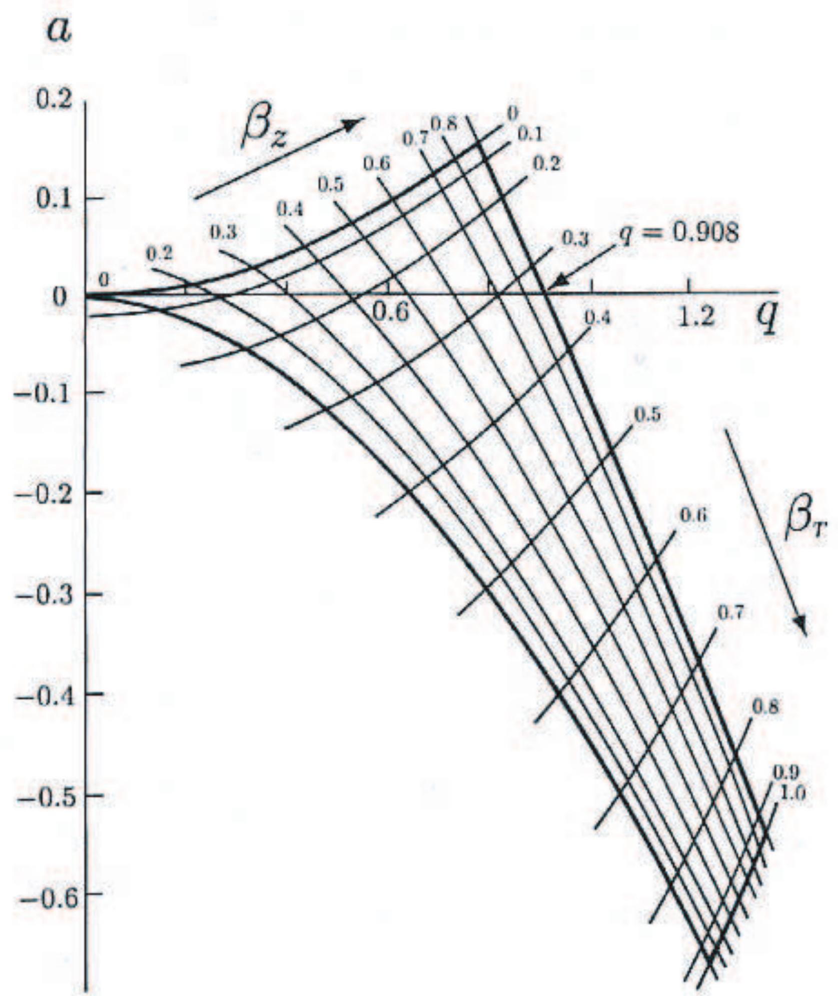


13.5 Resonance Diagram

3D Paul trap

$$\frac{d^2x}{d\zeta^2} + (a - 2q\cos(2\zeta))x = 0$$

$$\frac{d^2y}{d\zeta^2} - (a - 2q\cos(2\zeta))y = 0 \ .$$



Lost planet or lost beam – all the same physics

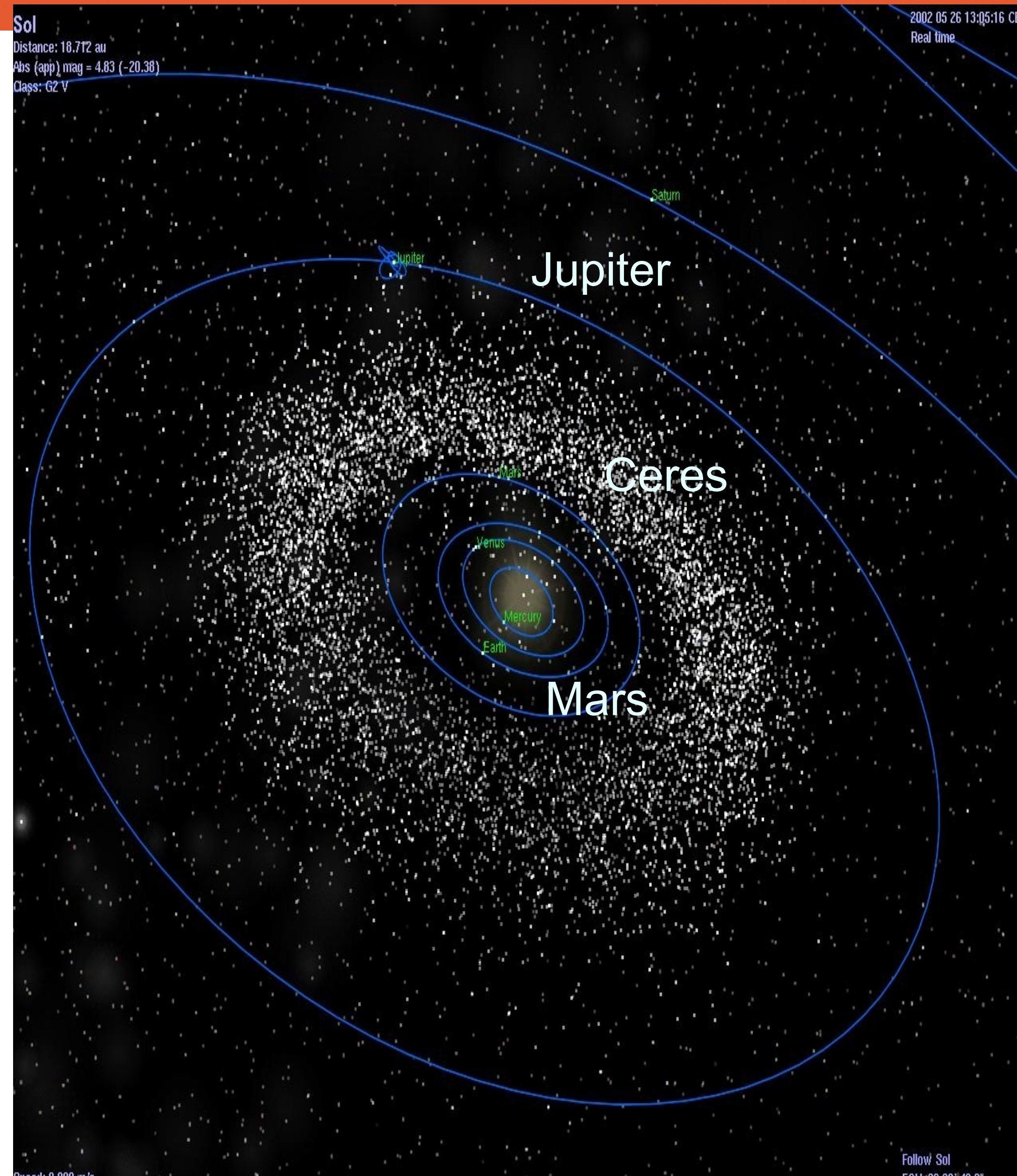
Orbit of a former planet Ceres was periodically influenced by Jupiter

3.Kepler law: $T^2/a^3 = const.$

$1\ AU = 1.5 \cdot 10^8$ km, $T_E = 1$ y

$a_M = 1.52$, $a_J = 5.20$, $R_{MJ} = 0.29$

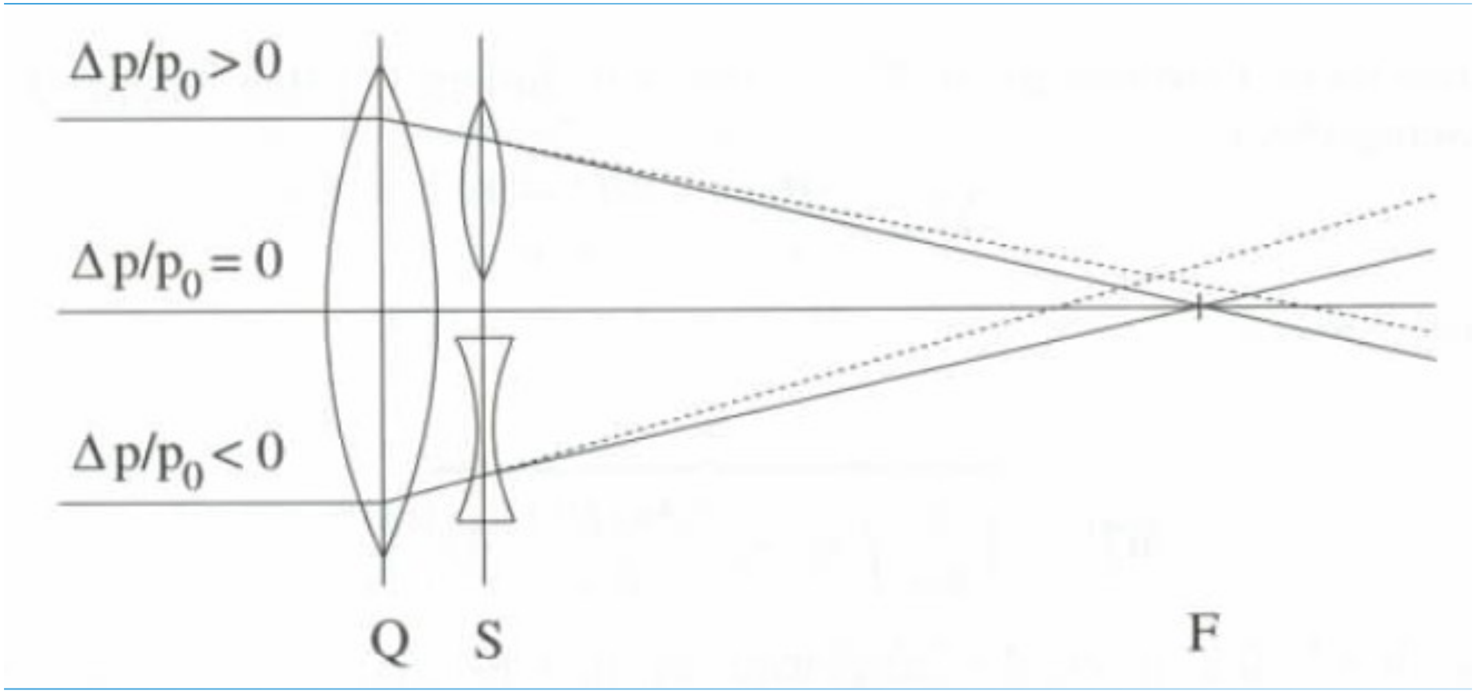
$a_C = 2.77$, " $R_{CJ} = 0.53$



13. Distortions and Resonances

13.6 Chromaticity

Problem: particles with $\delta \neq 0$ experience different bending and focusing in the machine



13. Distortions and Resonances

13.6 Chromaticity

Quadrupole:

$$\Delta Q_x = \xi_x \frac{\Delta p}{p_0}, \qquad \Delta Q_y = \xi_y \frac{\Delta p}{p_0}$$

ξ - (natural) chromaticity (ξ^n)

The focusing functions in thte Hill's equations: $\Delta K_x = -K_x \frac{\Delta p}{p_0}$

Always negative

Tune shift

Larger difference for slower particles

$$\Delta Q_x = \overbrace{\left[-\frac{1}{4\pi} \oint \beta(\bar{s}) K_x(\bar{s}) \mathrm{d}\bar{s} \right]}^{\xi_x^{\mathrm{n}}} \frac{\Delta p}{p_0}$$

13.6 Chromaticity

Additional chromaticity comes from sextupole fileds

QP SP

$$\xi_x = \xi_x^n + \xi_x^s$$

$$\xi_y = \xi_y^n + \xi_y^s$$

QP SP

13.6 Chromaticity

Chromaticity correction

Employing additional sextupoles in dispersion regions

$$\xi_x^{\mathrm{s}} = \frac{1}{4\pi} \oint g_{\mathrm{s}}(\bar{s}) \frac{\beta_x(\bar{s}) D(\bar{s})}{B\rho} \mathrm{d}\bar{s} \,,$$

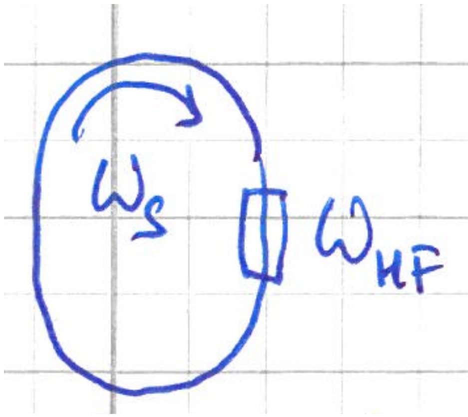
$$\xi_y^{\mathrm{s}} = -\frac{1}{4\pi} \oint g_{\mathrm{s}}(\bar{s}) \frac{\beta_y(\bar{s}) D(\bar{s})}{B\rho} \mathrm{d}\bar{s} \,.$$

$$g_s = \frac{\partial^2 B_y}{\partial x^2}$$

Tune shift

$$\Delta Q_x^{\mathrm{s}} = \overbrace{\left[\frac{1}{4\pi} \oint g_{\mathrm{s}}(\bar{s}) \frac{\beta_x(\bar{s}) D(\bar{s})}{B\rho} \mathrm{d}\bar{s} \right]}^{\xi_x^{\mathrm{s}}} \frac{\Delta p}{p_0}$$

13. Logitudinal beam dynamics



$$\omega_{\mathrm{HF}} = h\omega_{\mathrm{s}}$$

h – integer, harmonic number

Synchronous particle

Change of energy per revolution of synchronous particle

$$[\Delta E_{\mathrm{s}}]_{\mathrm{U}} = C_{\mathrm{s}} \frac{\mathrm{d}p_{\mathrm{s}}}{\mathrm{d}t}$$

Circumference

Change of momentum
per time unit

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

13. Logitudinal beam dynamics

- The energy gain of a particle per revolution is a function of the phase ψ with which the particle passes the HF unit

$$\Delta E^{\mathrm{HF}} = qU_0 \sin\varphi$$

Charge (q)

HF amplitude (U_0)

Relative to the middle of the acceleration unit (φ)

- The „zero" is defined as $\varphi = 0,\ \Delta E^{\mathrm{HF}} = 0$

$\varphi > 0$ Later particle

$\varphi < 0$ Earlier particle

13. Logitudinal beam dynamics

- Energy losses have to be taken into account

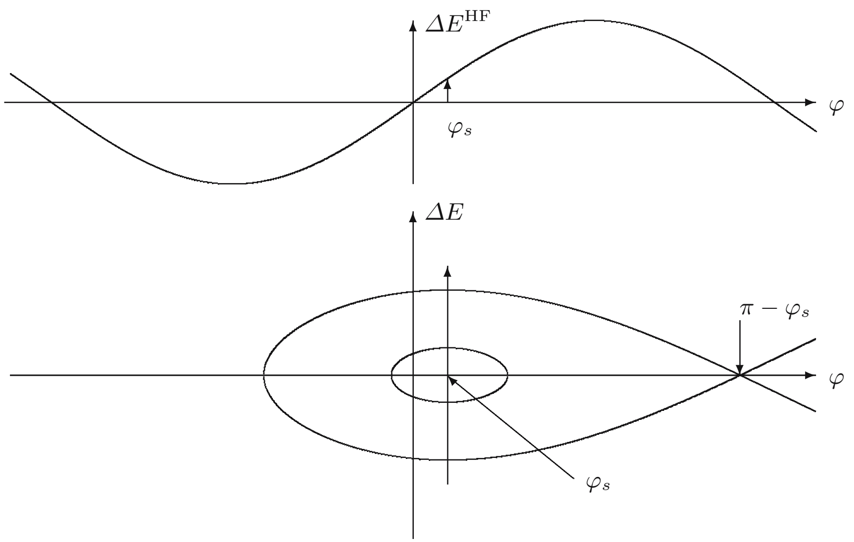
$$\Delta E^{\text{loss}} = -\Delta E_{\text{rad}}(E) - \Delta E_{\text{loss}}(E) - \ldots..$$

radiation — Collisions with rest gas — etc

$$[\Delta E]_U = qU_0 \sin\varphi + \Delta E^{\text{loss}}$$

$$[\Delta E_S]_U = qU_0 \sin\varphi_S + \Delta E_S^{\text{loss}}$$

13. Logitudinal beam dynamics



$$\Delta\varphi \rightarrow \Delta E \rightarrow \Delta p \rightarrow \Delta T \rightarrow \Delta\varphi$$

- Phase focusing

13. Logitudinal beam dynamics

- Phase focusing

Only possible if momentum change leads to change in ω

$$\frac{\Delta\omega}{\omega} = \eta\frac{\Delta p}{p} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right)\frac{\Delta p}{p}$$

Synchrotron principle

Dispersion of particle frequencies

$$\alpha_p = \frac{-1}{\gamma_t^2}$$

Momentum compaction factor

13. Logitudinal beam dynamics

- Synchrotron oscillations with small amplitude

$$\Delta\varphi = \varphi - \varphi_S$$

$$\Delta p = p - p_S$$

$$\Delta E = E - E_S$$

$$\Delta\omega = \omega - \omega_S$$

Change of $\Delta\varphi$ and ΔE per revolution

$$\delta(\Delta\varphi) = -\eta_S \frac{\Delta p}{p_S} \underbrace{2\pi h}$$

Phase change of synchronous particle per revolution

$$\delta(\Delta E) = qU_0(\sin\varphi - \sin\varphi_S)$$

13. Logitudinal beam dynamics

- Period of synchronous particle $T_S = \frac{2\pi}{\omega_S}$

- Time derivative

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta\varphi = -\frac{1}{T_\mathrm{s}}\eta_\mathrm{s}\frac{\Delta p}{p_\mathrm{s}}h2\pi = -\frac{h\eta_\mathrm{s}\omega_\mathrm{s}}{p_\mathrm{s}v_\mathrm{s}}\Delta E\,, \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta E = \frac{1}{T_\mathrm{s}}qU_0(\sin\varphi - \sin\varphi_\mathrm{s}) = \frac{\omega_\mathrm{s}}{2\pi}qU_0(\sin\varphi - \sin\varphi_\mathrm{s})\,. \quad (2)$$

φ_S is the phase at which energy gain (acceleration) and energy losses equalize

- Adiabatic approximation $\eta_S,\ \omega_S,\ p_S,\ v_S,\ U_0,\ \varphi_S$ change little

Set them constant

- Insert (2) into (1)

13. Logitudinal beam dynamics

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta\varphi = -\frac{h\eta_\mathrm{s}\omega_\mathrm{s}}{p_\mathrm{s}v_\mathrm{s}}\frac{\mathrm{d}}{\mathrm{d}t}\Delta E$$

$$= -\frac{h\eta_\mathrm{s}\omega_\mathrm{s}^2}{2\pi p_\mathrm{s}v_\mathrm{s}}qU_0(\sin\varphi - \sin\varphi_\mathrm{s})$$

3

For small $\Delta\varphi$ $\quad \sin\varphi - \sin\varphi_\mathrm{s} \approx \cos\varphi_\mathrm{s}\Delta\varphi$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Delta\varphi + \overbrace{\frac{h\eta_\mathrm{s}\omega_\mathrm{s}^2}{2\pi p_\mathrm{s}v_\mathrm{s}}qU_0\cos\varphi_\mathrm{s}}^{\omega_\mathrm{syn}^2}\,\Delta\varphi = 0$$

Harmonic oscillations

Analytically not solvable

13. Logitudinal beam dynamics

$$\omega_{\mathrm{syn}} = \omega_{\mathrm{s}} \sqrt{\frac{h\eta_{\mathrm{s}}}{2\pi p_{\mathrm{s}} v_{\mathrm{s}}} q U_0 \cos\varphi_{\mathrm{s}}} \,.$$

$$Q_{\mathrm{syn}} = \frac{\omega_{\mathrm{syn}}}{\omega_{\mathrm{s}}} = \sqrt{\frac{h\eta_{\mathrm{s}}}{2\pi p_{\mathrm{s}} v_{\mathrm{s}}} q U_0 \cos\varphi_{\mathrm{s}}}$$

Number of synchrotron oscillations per revolution

13. Logitudinal beam dynamics

- If we assume $\Delta\varphi = \Delta\varphi_0 \cos\omega_{\mathrm{syn}} t$ and set it in (1)

$$\Delta E = \overbrace{\frac{\omega_{\mathrm{syn}}}{\omega_{\mathrm{s}}} \frac{p_{\mathrm{s}} v_{\mathrm{s}}}{h\eta_{\mathrm{s}}} \Delta\varphi_0}^{\Delta E_0} \sin\omega_{\mathrm{syn}} t$$

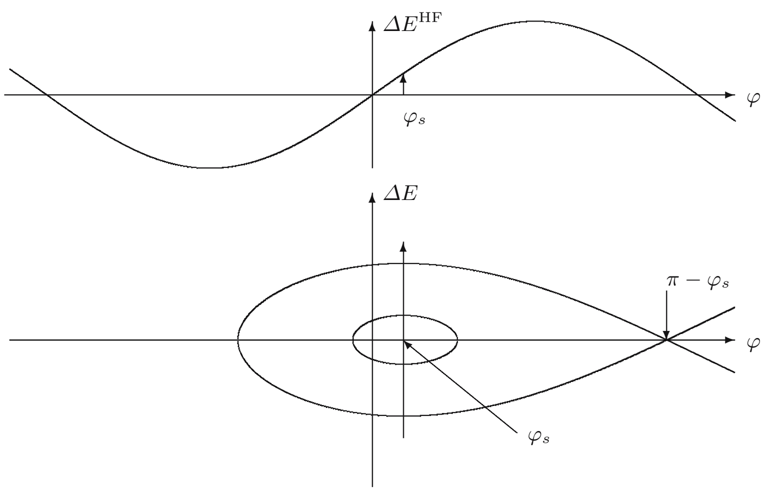
Equation of ellipse

Oscillations in $(\Delta E,\ \Delta\varphi)$ plane

$$\left(\frac{\Delta\varphi}{\Delta\varphi_0}\right)^2 + \left(\frac{\Delta E}{\Delta E_0}\right)^2 = 1\,.$$

$$\Delta E_0 = Q_{\mathrm{syn}} \frac{p_{\mathrm{s}} v_{\mathrm{s}}}{h\eta_{\mathrm{s}}} \Delta\varphi_0\,.$$

Connection between two amplitudes



13. Logitudinal beam dynamics

- Stability condition

$$\eta_\mathrm{s}\cos\varphi_\mathrm{s} > 0$$

$$\cos\varphi_\mathrm{s} > 0 \ \text{ für } \ \eta_\mathrm{s} > 0 \ \ (\gamma_\mathrm{s} < \gamma_\mathrm{tr})$$

$$\cos\varphi_\mathrm{s} < 0 \ \text{ für } \ \eta_\mathrm{s} < 0 \ \ (\gamma_\mathrm{s} > \gamma_\mathrm{tr})$$

4 regions:

$\varphi_S > 0$ - acceleration

$\varphi_S < 0$ - deceleration/slowing down

$$\eta_\mathrm{s} > 0 \ (\gamma_\mathrm{s} < \gamma_\mathrm{tr})\,,\ \cos\varphi_\mathrm{s} > 0\,,\ \sin\varphi_\mathrm{s} > 0:\ 0 < \varphi_\mathrm{s} < \pi/2\,,$$

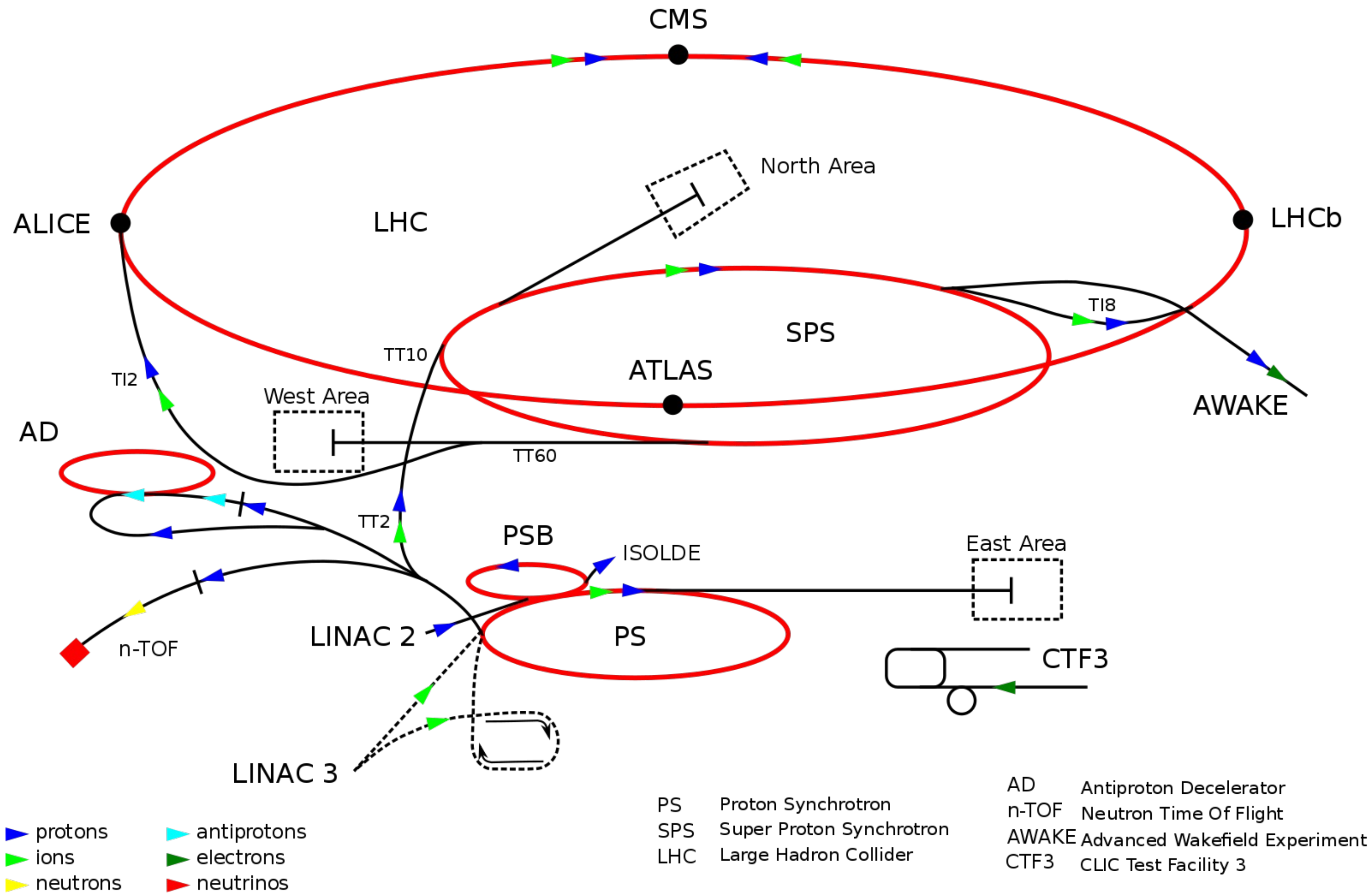
$$\eta_\mathrm{s} < 0 \ (\gamma_\mathrm{s} > \gamma_\mathrm{tr})\,,\ \cos\varphi_\mathrm{s} < 0\,,\ \sin\varphi_\mathrm{s} > 0:\ \pi/2 < \varphi_\mathrm{s} < \pi\,,$$

$$\eta_\mathrm{s} > 0 \ (\gamma_\mathrm{s} < \gamma_\mathrm{tr})\,,\ \cos\varphi_\mathrm{s} > 0\,,\ \sin\varphi_\mathrm{s} < 0:\ -\pi/2 < \varphi_\mathrm{s} < 0\,,$$

$$\eta_\mathrm{s} < 0 \ (\gamma_\mathrm{s} > \gamma_\mathrm{tr})\,,\ \cos\varphi_\mathrm{s} < 0\,,\ \sin\varphi_\mathrm{s} < 0:\ -\pi < \varphi_\mathrm{s} < -\pi/2\,.$$

For: $\gamma_S = \gamma_t \quad \eta = 0$ and for $\cos\varphi_S = 0$ no stable solutions

CERN in Geneva



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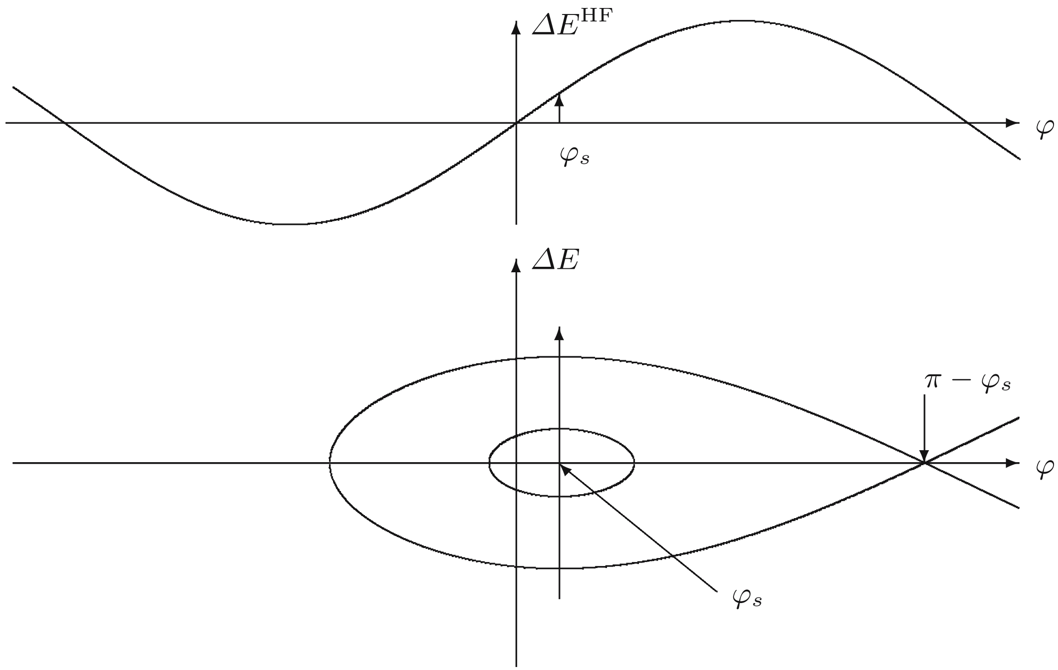
14. Optics in longitudinal plane

- Separatix

$$\gamma < \gamma_t$$

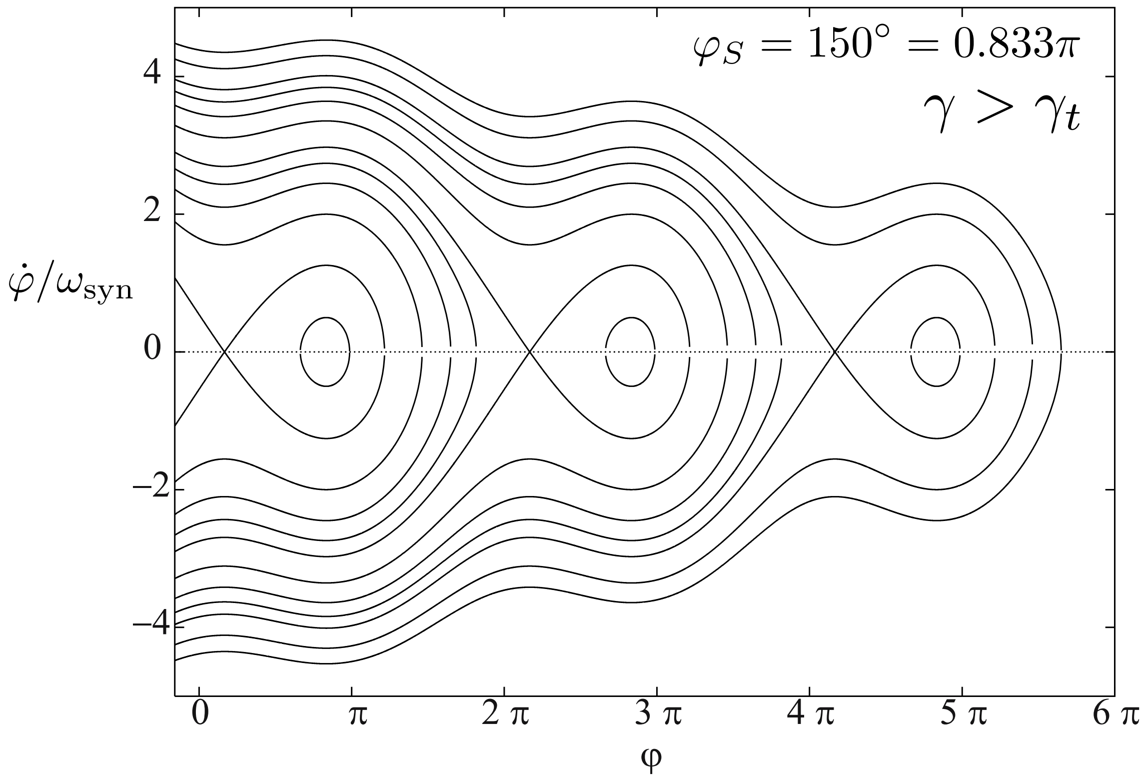
$$\Delta E = E - E_S$$



14. Optics in longitudinal plane

- Large amplitudes $\Delta\varphi$

  Formation of buckets



14. Optics in longitudinal plane

- Longitudinal coordinates

$$\Delta\varphi = \varphi - \varphi_{\mathrm{s}}$$
$$\Delta t = t - t_{\mathrm{s}}$$
$$\delta = \Delta p / p_{\mathrm{s}}$$

$$l = -\frac{v_{\mathrm{s}}}{\omega_{\mathrm{HF}}}\Delta\varphi = -\frac{v_{\mathrm{s}}}{h\omega_{\mathrm{s}}}\Delta\varphi = -\frac{C_{\mathrm{s}}}{h2\pi}\Delta\varphi\,,$$

$$\Delta t = \frac{1}{\omega_{\mathrm{HF}}}\Delta\varphi = \frac{1}{h\omega_{\mathrm{s}}}\Delta\varphi\,.$$

$$\Delta p = \frac{1}{v_{\mathrm{s}}}\Delta E\,,$$

$$\frac{\Delta p}{p_{\mathrm{s}}} = \frac{1}{p_{\mathrm{s}}v_{\mathrm{s}}}\Delta E\,.$$

14. Optics in longitudinal plane

- Invariant logitudinal emittance

Phase space in $(\varphi, \Delta E)$ plane

$$E_\varphi^{\mathrm{n}} = \pi\epsilon_\varphi^{\mathrm{n}} = \pi\Delta\varphi_0\Delta E_0$$

$$E_\varphi^{\mathrm{n}} = 2\int_{\varphi_1}^{\varphi_2} |\Delta E| \mathrm{d}\varphi$$

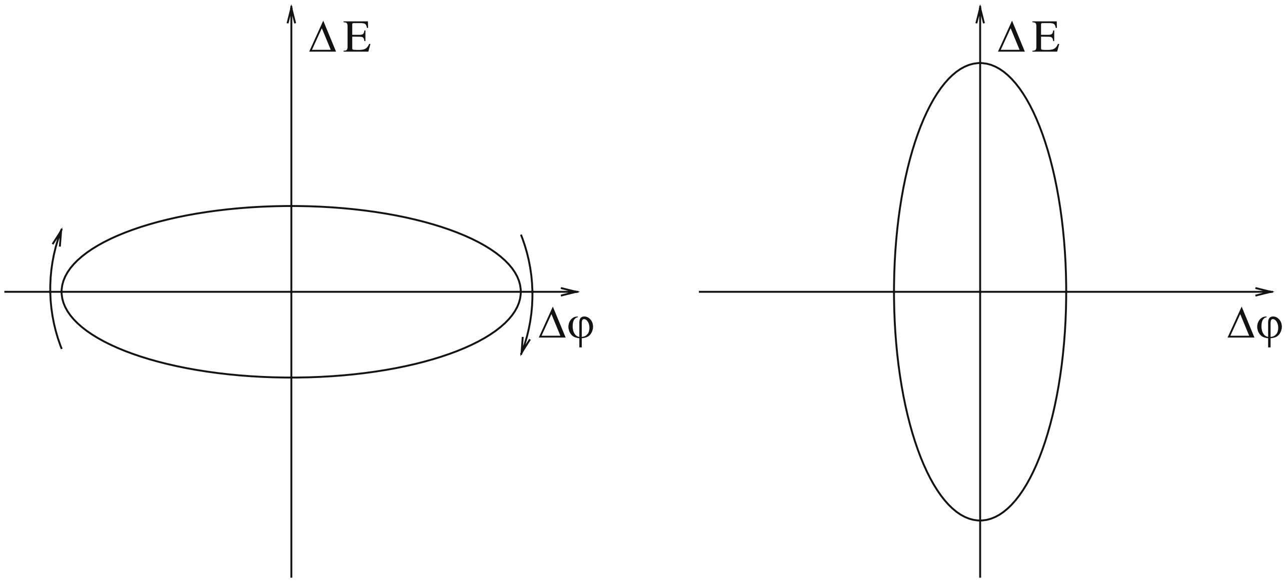
- Normalized longitudinal acceptance

Area of separatix

$$A_\varphi^{\mathrm{n}} = 2\int_{\varphi_{1\mathrm{sep}}}^{\varphi_{2\mathrm{sep}}} |\Delta E_{\mathrm{sep}}| \mathrm{d}\varphi$$

14. Optics in longitudinal plane

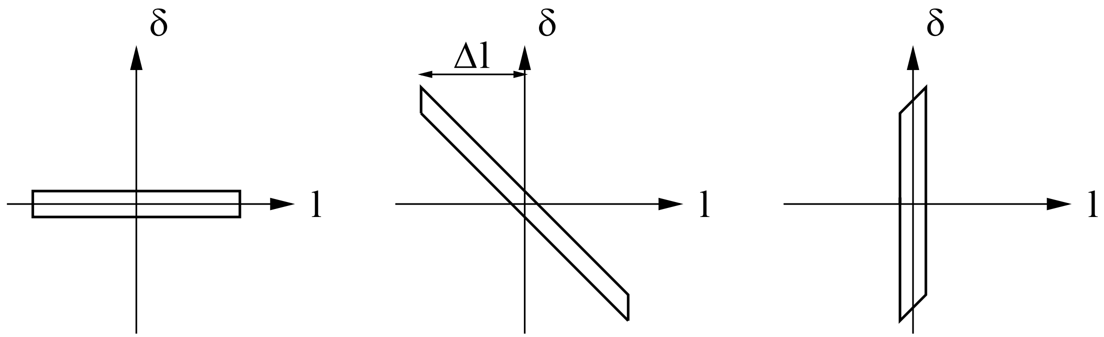
- Transformation of logitudinal phase ellipse

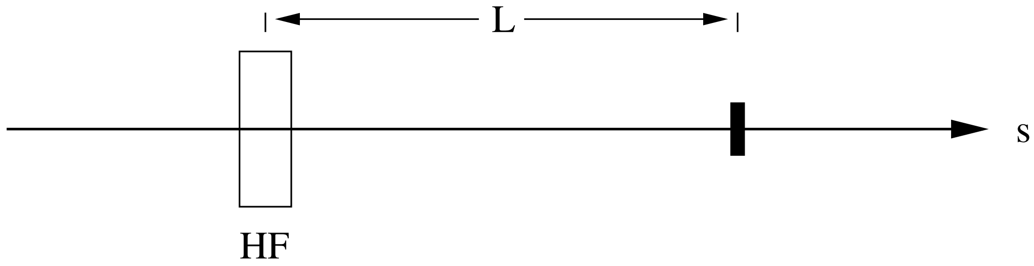


$$\epsilon_{\varphi}^{\mathrm{n}} = \Delta\varphi_0 \Delta E_0 = const$$

14. Optics in longitudinal plane

- Buncher

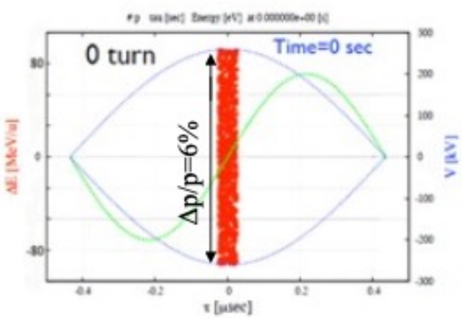


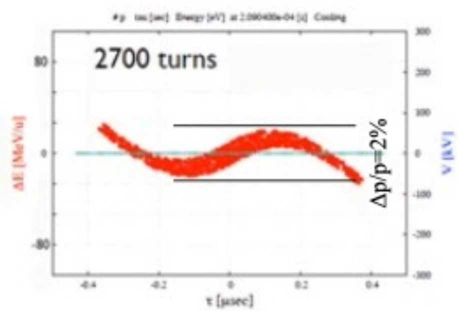


Buch rotation in Collector Ring of FAIR

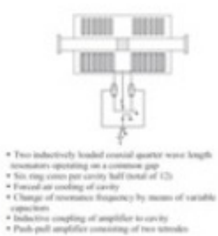
First step of cooling- RF bunch rotation

Using bunch rotation RF cavity the momentum spread is reduced by factor of 3
P-bars (from 6% to less than 2%)
RIBs (from 3% to less then 1 %)
Time of such cooling is about 3 ms





Total voltage (5 cavities)	**200 kV**
Peak voltage per cavity	**40 kV**
Peak dissipation per valve	393 kW
Peak dissipation per valve into ring cores	648 kW
Peak power per valve	1040 kW



A. Dolinskii

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

14. Optics in longitudinal plane

- Ion-optics in longitudinal plane

Relative position deviation

$$\begin{pmatrix} l \\ \delta \end{pmatrix} = M \begin{pmatrix} l_0 \\ \delta_0 \end{pmatrix}$$

Relative momentum deviation

Accleration with a HF field acts as a thin lense with focusing strength

$$\frac{1}{f} = \frac{\mathrm{d}E}{\mathrm{d}t}\frac{1}{pv^2} = \frac{qU_0\omega_{\mathrm{HF}}}{pv^2}$$

$\frac{dE}{dt} > 0$ „focusing“ – „buncher“

$\frac{dE}{dt} < 0$ „defocusing“ – „debuncher“

14. Optics in longitudinal plane

- Ion-optics in longitudinal plane

Drift:

$$\begin{pmatrix} l \\ \delta \end{pmatrix} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} l_0 \\ \delta_0 \end{pmatrix}$$

Exercise....

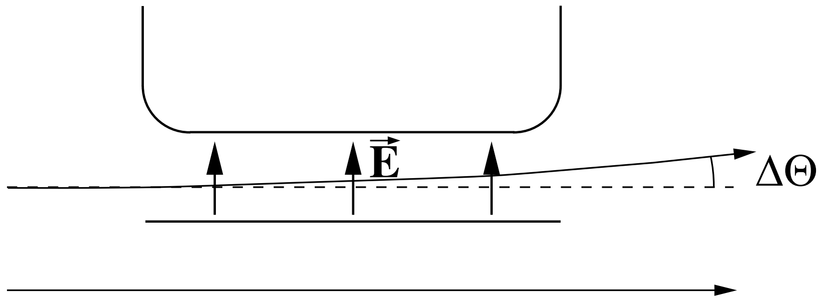
- Longitudinal phase-space ellipse

$$\sigma_l = \begin{pmatrix} \sigma_{55} & \sigma_{56} \\ \sigma_{56} & \sigma_{66} \end{pmatrix}$$

- Transformation of longitudinal phase-space ellipse

$$\sigma_l(s) = R_l(s)\sigma_l(0)R_l^{\mathrm{T}}(s)$$

15. Injection & Extraction

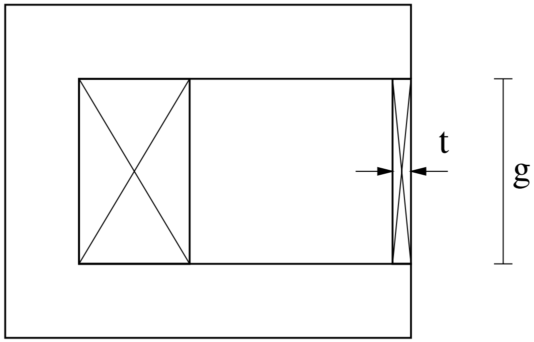


Electrostatic septum

$$\Theta = \frac{q|\boldsymbol{E}|L}{pv} = \frac{|\boldsymbol{E}|L}{(B\rho)v}$$

E~10 MV/m

d~0.1 mm



Magnetic septum

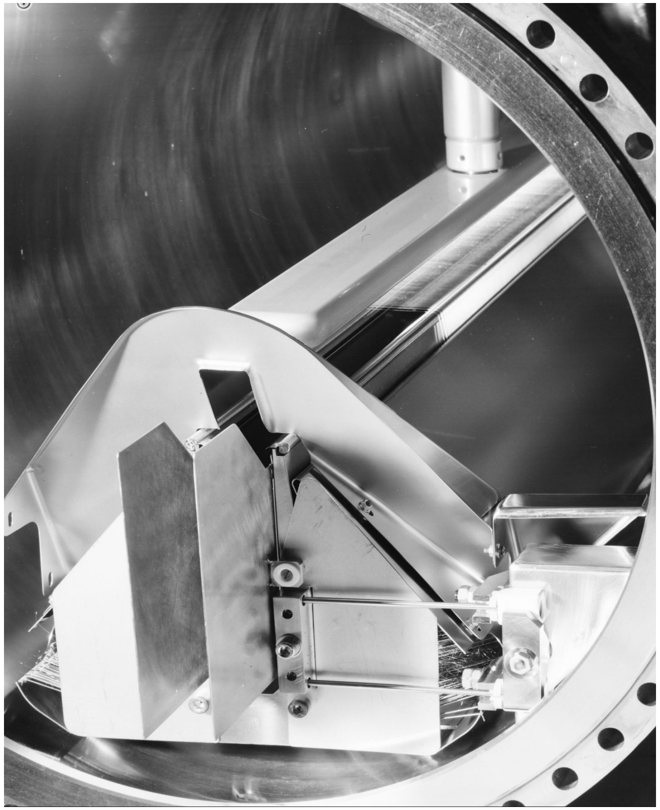
$$\Theta = \frac{|\boldsymbol{B}|L}{(B\rho)}$$

t~10 mm

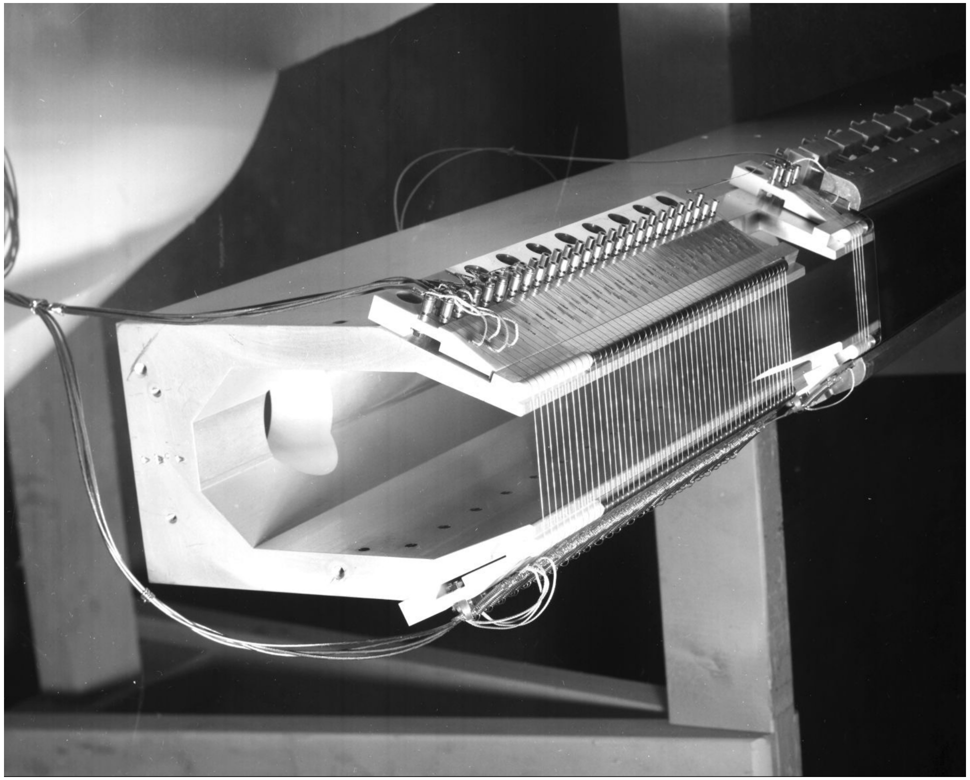
B~1T (pulsed mode: B<2 T)

15. Injection & Extraction

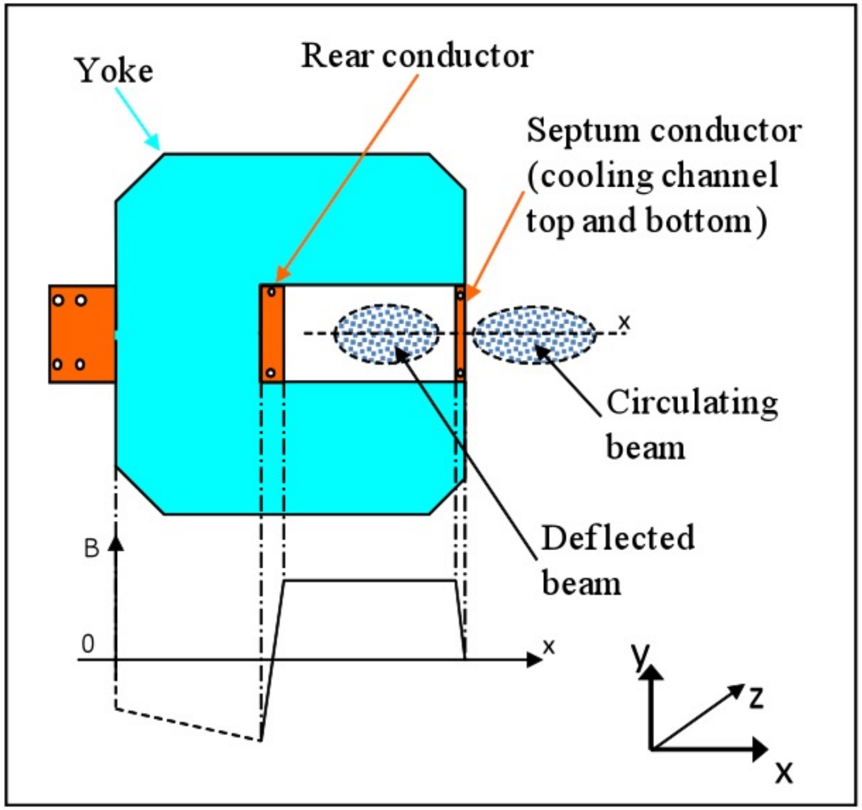
Electrostatic septum SPS, CERN

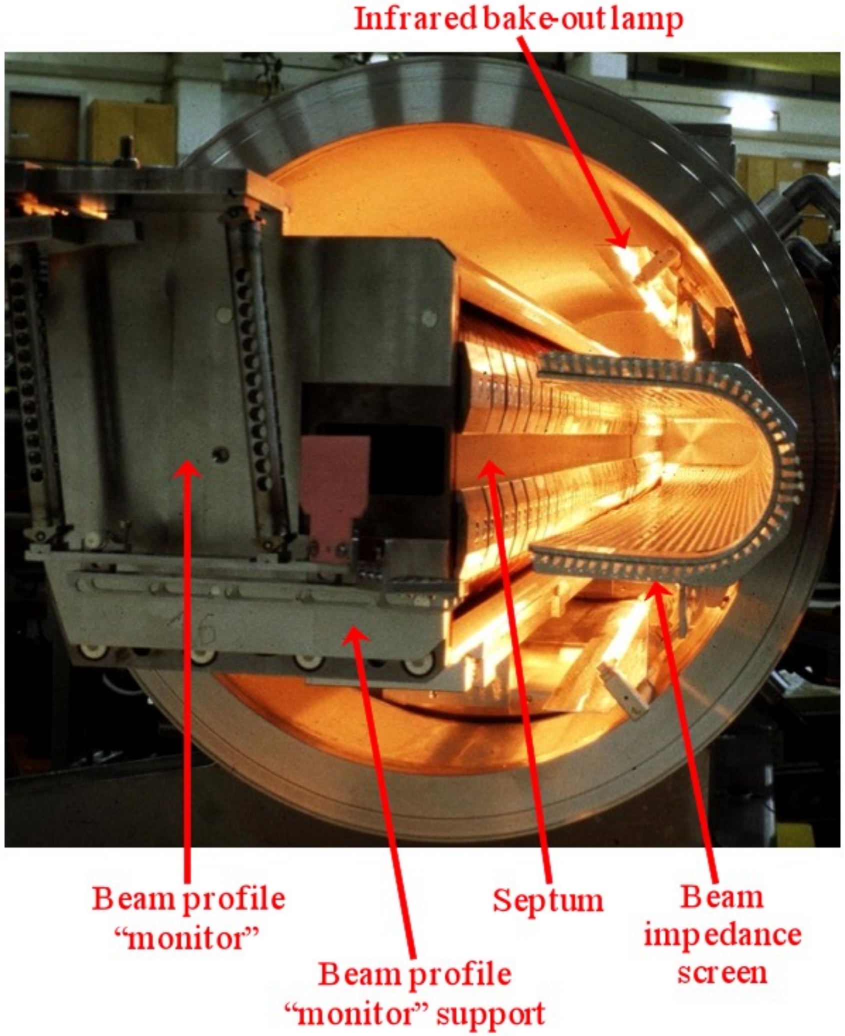


Electrostatic septum PS, CERN (towards SPS)



15. Injection & Extraction





Magnetic septum

15. Injection & Extraction

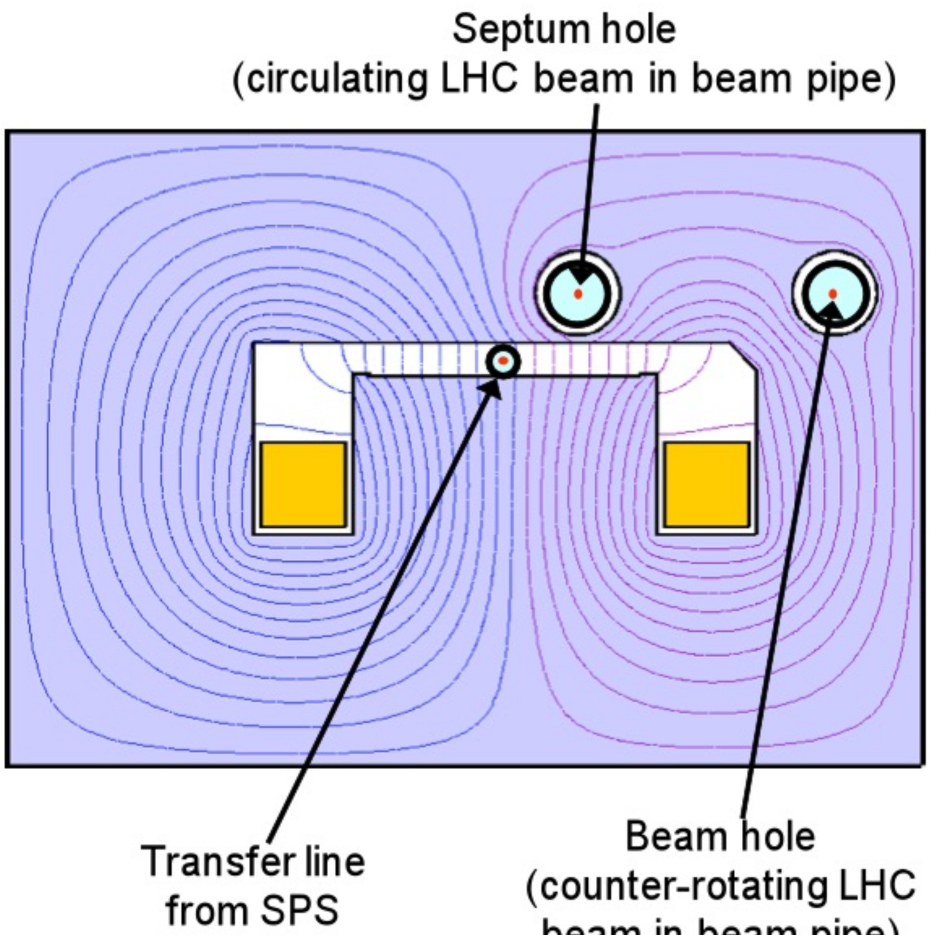
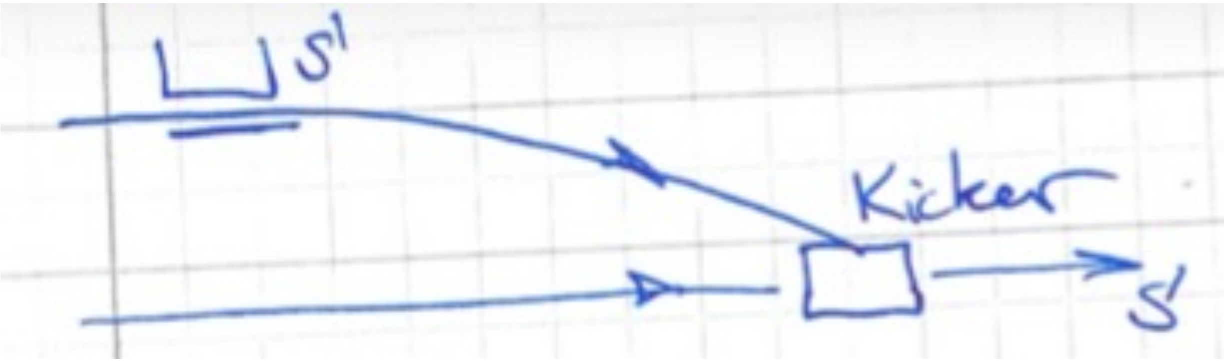


Fig. 16: Lambertson septum used in LHC injection beam-line

15. Injection & Extraction

Injection

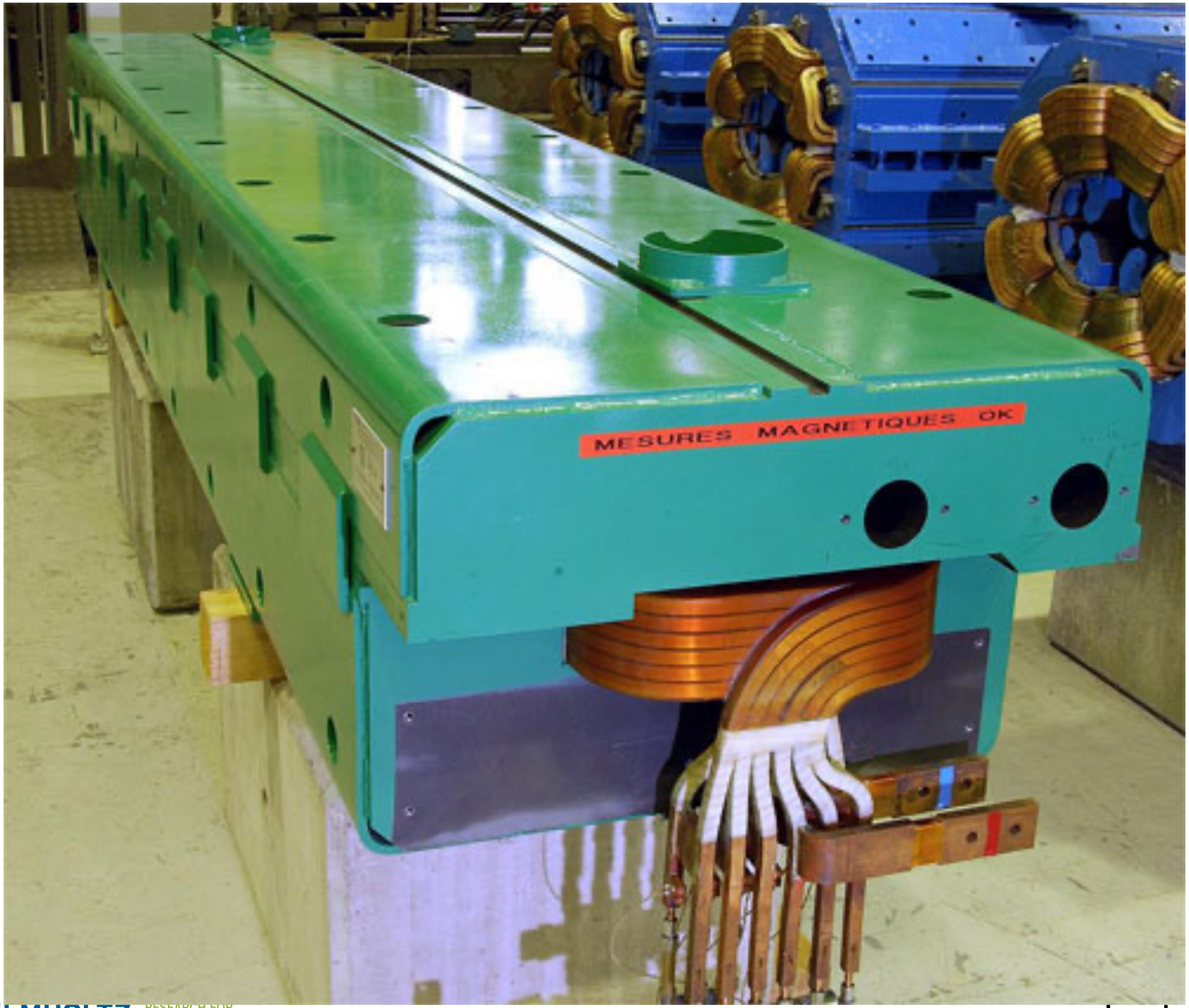


Kickermagnete sind spezielle Dipolmagnete und müssen je nach Aufgabe, innerhalb von 0.0000001 Sekunden das Ablenkmagnetfeld erzeugen

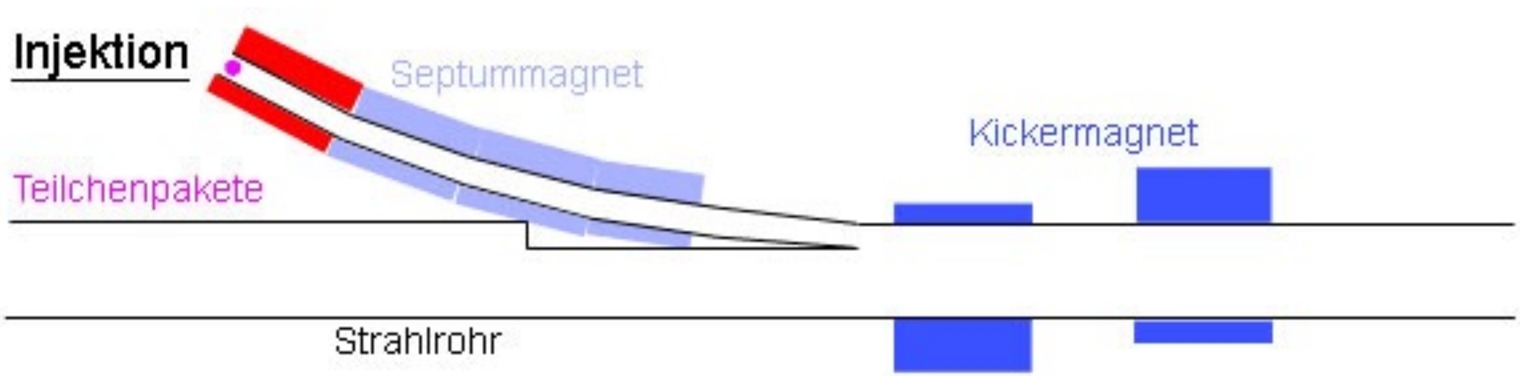
15. Injection & Extraction

Injection

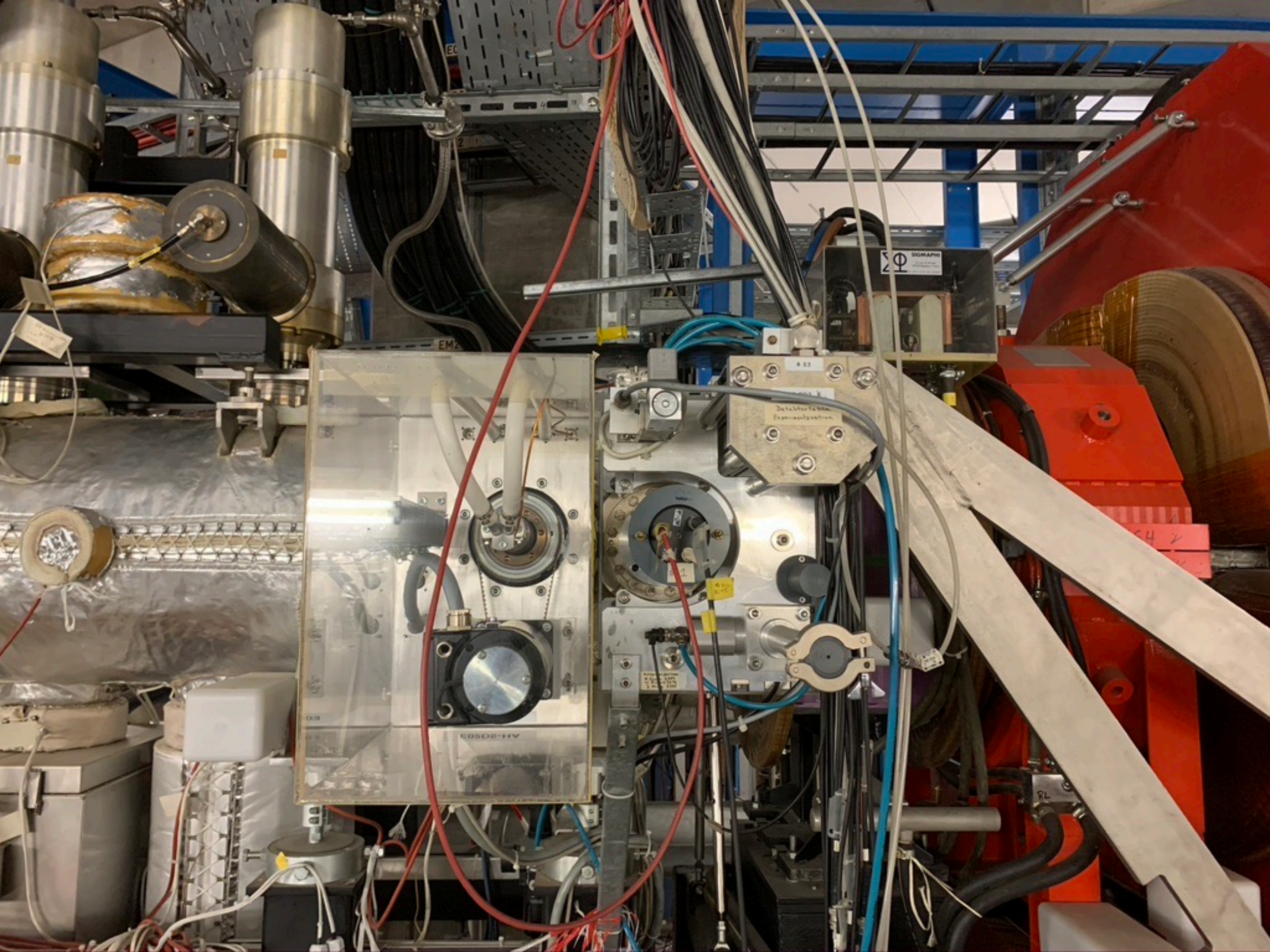
Kicker magnet
CERN



15. Injection & Extraction



LHC, CERN


SIGMAPHI
RL

High-voltage pulse

Lecture 9

15. Injection & Extraction

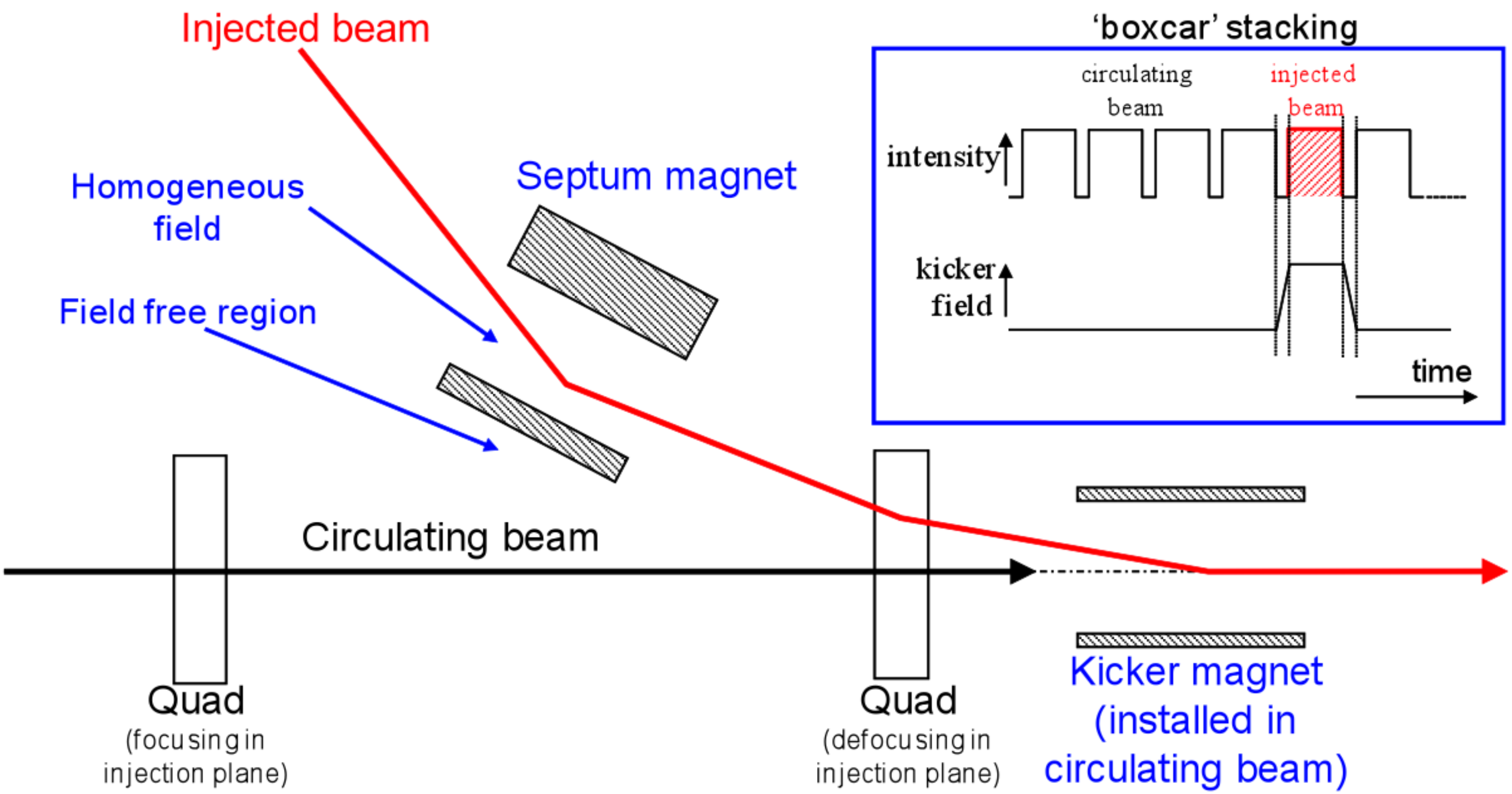


Fig. 1: Fast single-turn injection in one plane

15. Injection & Extraction

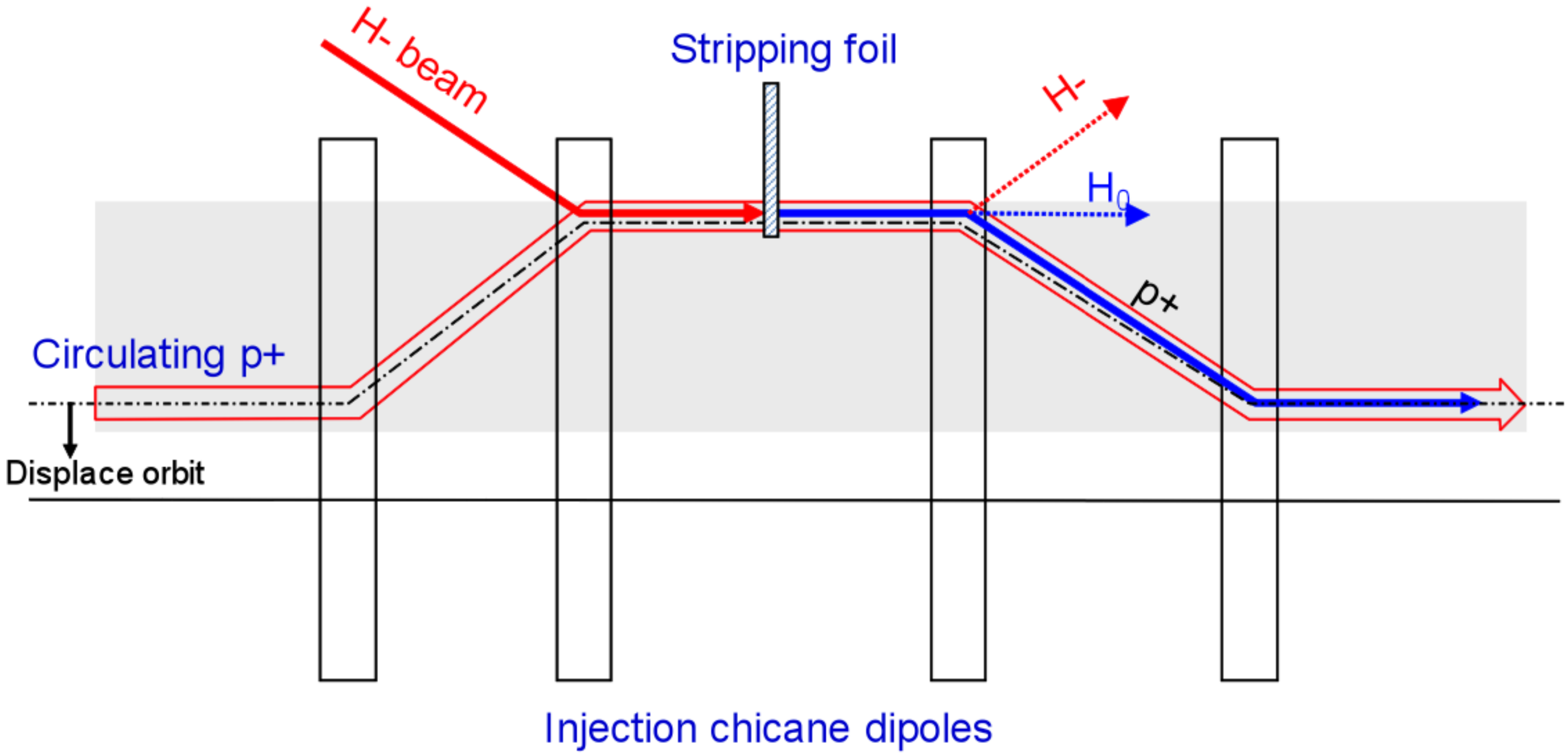
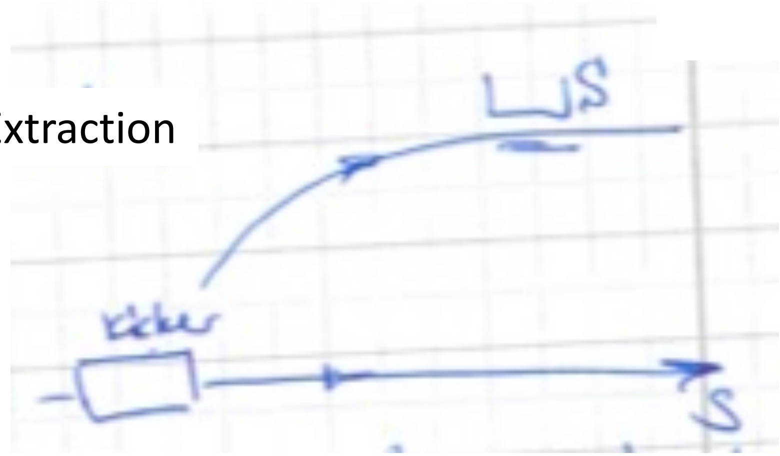


Fig. 4: Charge exchange – start of injection process

15. Injection & Extraction

Extraction

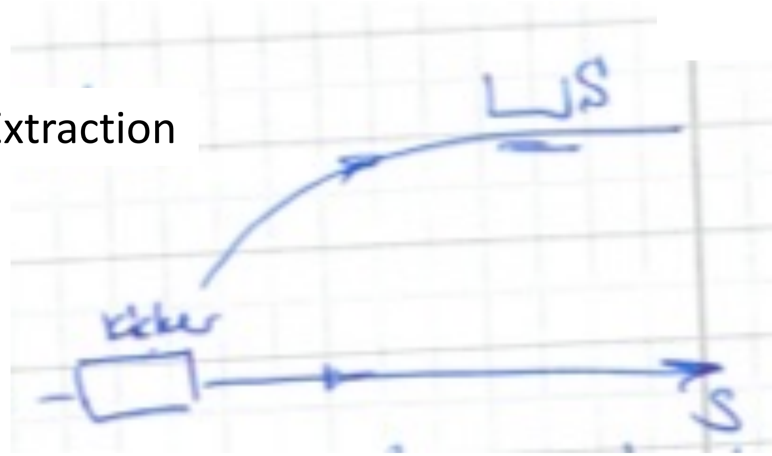


Fast-extraction

Slow-extraction?

15. Injection & Extraction

Extraction



Fast-extraction

Slow-extraction?

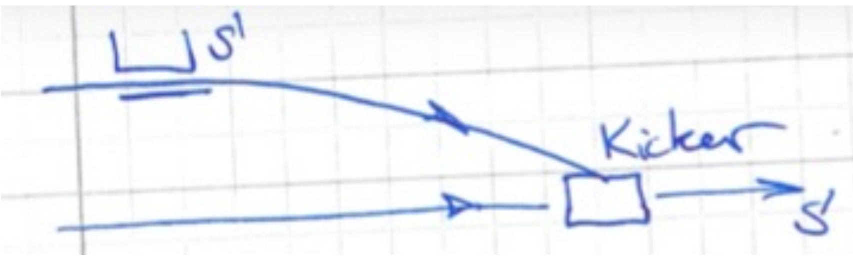
Sextupole errors

$$\delta Q = \frac{\beta^{3/2}}{16\pi}\left(\frac{\partial^2 B_y}{\partial x^2}\right)\frac{\Delta s}{B\rho} a \cos(3\Psi)$$

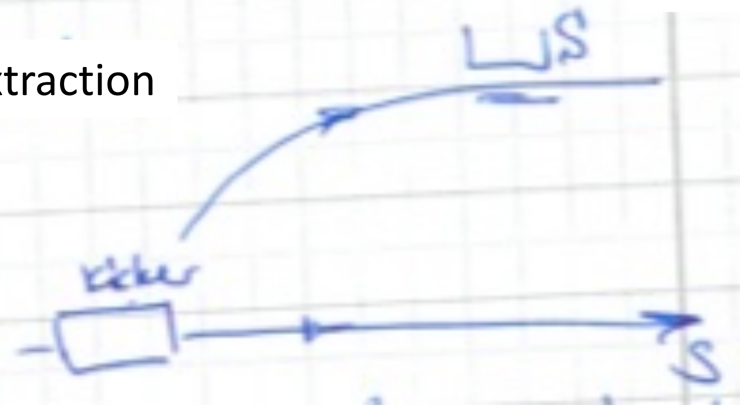
Amplitude of betatron oscillations!!!

15. Injection & Extraction

Injection



Extraction



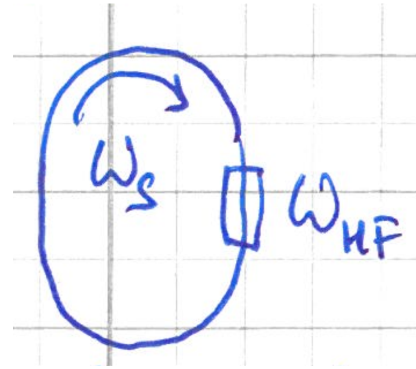
- Single-turn injection
- Multi-turn injection (stacking)
- Charge exchange injection

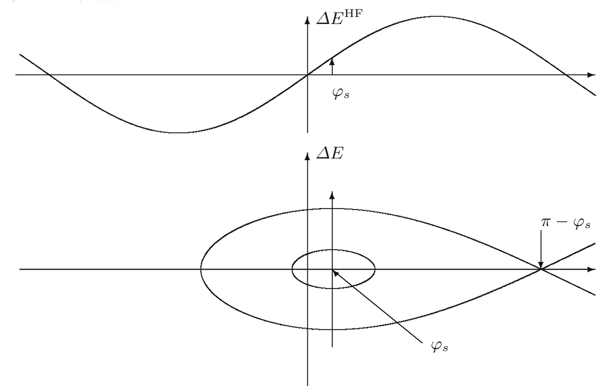
- Single-turn extraction
- Slow resonant extraction (3Qx resonance)
- „Super-slow“ extraction (ESR)

Summary of the lecture

Acceleration

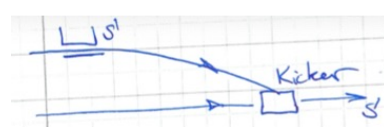
synchrotron principle
phase focusing
transition energy
momentum compaction factor
synchrotron oscillations
separatix
longitudinal acceptance/emittance

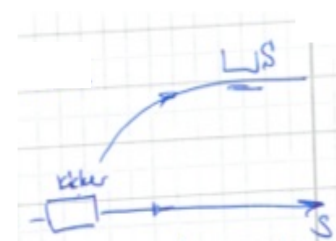




Injection & Extraction

Septa magnets
Kicker magnets
Slow/fast injection/extraction





Lecture Dates

https://uebungen.physik.uni-heidelberg.de/vorlesung/20222/1611/lecture

Date	Topic
19.10.2022	Introduction and basic definitions
26.10.2022	Accelerating structures
02.11.2022	Accelerator Components
09.11.2022	Optics with magnets (1)
16.11.2022	Optics with magnets (2)
23.11.2022	Equations of motion
30.11.2022	Phase ellipses and magneto-optical system / Transverse beam dynamics
07.12.2022	Transverse beam dynamics, beam stability / Longitudinal beam dynamics
14.12.2023	Longitudinal beam dynamics and a summary
11.01.2023	**Phase space and beam cooling (Invitation)**
18.01.2023	**Space charge and beam-beam dynamics**
25.01.2023	**Physics at Storage Rings and Colliders**
01.02.2023	**New accelerator technologies and final summary**
08.02.2023	**Student seminar**
15.02.2023	**Visit GSI**
22.02.2023	**Visit MPIK**

GSI HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

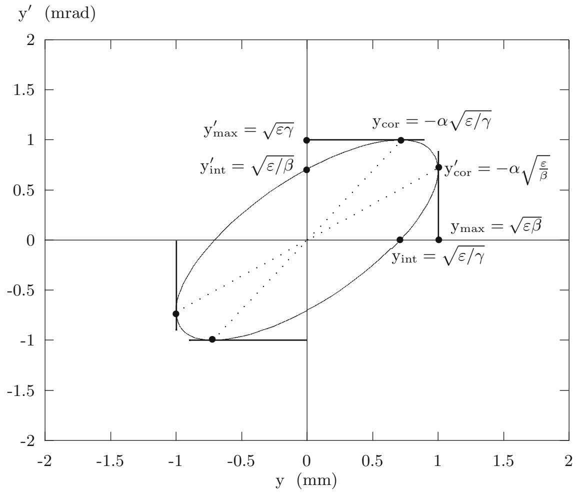
Wednesdays, 14:15-16:00

Summary of the lecture

Describe Machine

Courant-Snyder Invariant

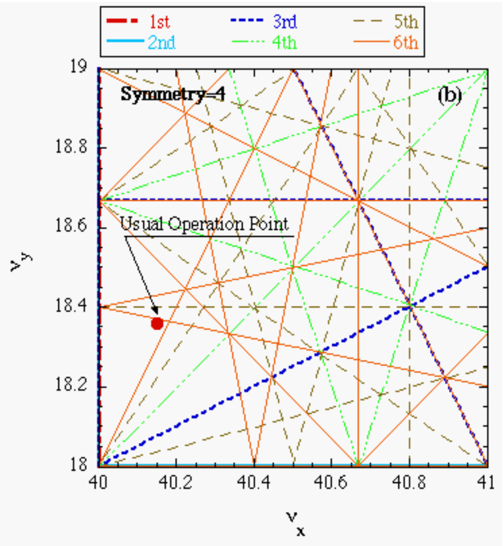
Machine parameters



Matchning Machine and Beam

Machine acceptance

Ellipse matching



Dispersion

Dispersion function

Distortion and Resonances

Floquet transformation

Distortion function

Stop-bands 1st, 2nd, 3rd order

Resonance diagram

Chromaticity

Chromaticity correction