Accelerator Physics (L1)

① History of accelerators

1862 Theory of electromagnetism Maxwell
1887 Discovery of electromagnetic waves Hertz
1886 Positive-charged beams in gas-discharge tubes Goldstein
1894 First electron beam Lenard
1895 Discovery of X-rays Röntgen
→ 1911 First reaction $^{197}Au(\alpha,\alpha)^{197}Au$, $\alpha_{7.7}$ from ^{214}Po Rutherford
1919 First nuclear reaction $^{14}N(\alpha,p)^{17}O$ Rutherford
1928 - 1932 Linear, circular, electrostatic accelerators

Slide 1

1932 p (linear accelerator) @ 400 keV + ^7Li → $^4He + {}^4He$
Cockcroft & Walton ↘ $^7Be + n$
1932 p (cyclotron) @ 1,25 MeV
Livingston
1938 p (cyclotron) @ 9 MeV & d @ 19 MeV . . . . .

1950s p+p @ 5.63 GeV → production of $\bar{p}$
1980s p @ 100 - 1000 GeV W^+-W^- pair ~~90,2 GeV~~ 160,7 GeV
$Z^0 \sim 90$ GeV

Resolution

$$\frac{\lambda}{2\pi} = \frac{\hbar c}{pc} = \frac{197{,}3 \text{ MeV fm}}{pc}$$

0,1 fm → $p \geq 10$ GeV/c ⇒ $E > 10$ GeV

Quality of beam

momentum/energy spread
geometrical size

$\Delta p/p$
$\Delta E/E$
$\Delta x, \Delta y$

Beam Intensity (number of particles per time unit)
Luminosity
target = 10^{10} part/cm²
intensity = 10^{10} part/c
luminosity = 10^{20} cm^{-2}s^{-1}

Particle-type p . . . . U
Polarisation $P = (N_+ - N_-)/(N_+ + N_-)$ (e^-, p, ^3He) with $s = \pm 1/2\,\hbar$
Secondary Beams
Synchrotron radiation & production of radionuclides

2 Relativistic Kinematics

to characterize motion of a particle it is sufficient to know momentum vector ($\vec{p}$) and the rest mass (m)

- total energy

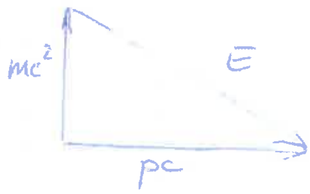
$$E^2 = (mc^2)^2 + (\vec{p}c)^2$$

natural units $c = 1$

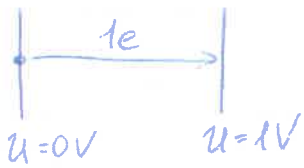
$$E^2 = m^2 + \vec{p}^{\,2} \quad \Rightarrow \quad (E \geq m)$$

- kinetic energy

$$T = E - \underbrace{mc^2}_{\text{rest energy}}$$



- 1eV is the kinetic energy which 1e ($1{,}60217653(14) \cdot 10^{-19}$ C) gains in 1V potential difference



μeV, meV, eV, keV, MeV, GeV, TeV ...

- particle velocity $\beta = \frac{v}{c}$ in units of c

$$\vec{\beta} = \frac{\vec{v}}{c} = \left(\frac{v_x}{c}, \frac{v_y}{c}, \frac{v_z}{c}\right)$$

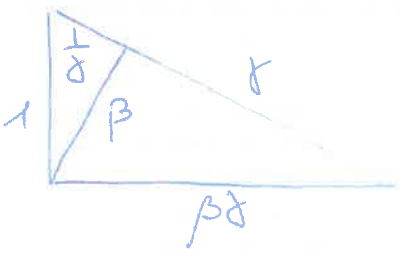
- relativistic Lorentz factor (mass gain)

$$\gamma = \frac{E}{mc^2} = \frac{1}{\sqrt{1-\beta^2}}$$

- in natural units

$E = \gamma m$	total energy	[eV]
$\vec{p} = \vec{\beta}\gamma m$	momentum	[eV/c]
$T = (\gamma - 1)\, m$	kinetic energy	[eV]
$\vec{\beta} = \vec{p}/E$	velocity	[-]
m	rest mass	[eV/c²]
mc^2	rest energy	[eV]
~~velo~~ $\vec{v}$	velocity	[m/s]
γ	Lorentz factor	[-]

- useful relations



$$\beta^2\gamma^2 + 1 = \gamma^2$$
$$\beta^2 + 1/\gamma^2 = 1$$
$$\beta\gamma = \sqrt{\gamma^2 - 1}$$
$$\gamma = 1/\sqrt{1-\beta^2}$$
$$\beta = \sqrt{1 - 1/\gamma^2}$$

- 4- vectors

$$\mathbb{p} = p = (E, p_x, p_y, p_z) = (E, \vec{p})$$

- scalar (multiplication) product

$$p \cdot q = p_0 q_0 - p_x q_x - p_y q_y - p_z q_z = p_0 q_0 - \vec{p} \cdot \vec{q}$$

- Lorentz transformation

center of mass system - c.m.s.

$$\begin{pmatrix} E^{cms} \\ p_x^{cms} \\ p_y^{cms} \\ p_z^{cms} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

back transformation

$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E^{cms} \\ p_x^{cms} \\ p_y^{cms} \\ p_z^{cms} \end{pmatrix}$$

- Lorentz invariants

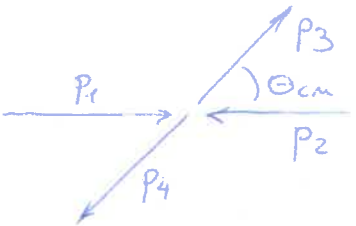
$$\boxed{p^2 = E^2 - \vec{p}^{\,2} = m^2}$$

m_{inv}^2 - invariant mass of 2 particles with $(\vec{p}_1, E_1)$ & $(E_2, \vec{p}_2)$

$$m_{inv}^2 = (p_1 + p_2)^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

$$m_{inv} = E_{tot}^{CM} = \sum_i E_i^{CMS}$$

- Example : collision



$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$

square of invariant mass

$t = (p_3 - p_1)^2 = (p_4 - p_2)^2$

square of momentum transfer from 1 to 3

$u = (p_4 - p_1)^2 = (p_3 - p_2)^2$

4   1 to 4

$$E_{tot}^{CMS} = \sqrt{s} = E_1^{CMS} + E_2^{CMS} = E_3^{CMS} + E_4^{CMS}$$

- target at rest

$p_1 = (E_1, \vec{p}_1)$ $p_2 = (m_2, \emptyset)$

$p = p_1 + p_2 = (E_1 + m_2, \vec{p}_1)$ $\boxed{E^2 - p^2 = m^2}$

$$s = p^2 = E_1^2 + 2E_1 m_2 + m_2^2 - \vec{p}_1^{\,2} = 2E_1 m_2 + m_1^2 + m_2^2$$

if $m \ll E$ $E_{tot} = \sqrt{s} \propto \sqrt{E_1}$

(*) see page 5

- collider   special case - C.M.S. = Laboratory system

$$E_{tot}^{CMS} = \sqrt{(E_1 + E_2)^2 - (p_1 + p_2)^2}$$

if $p_1 = -p_2$ then $E_{tot}^{CMS} = E_1 + E_2$

$$\beta = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$$

- Example: HERA @ DESY [1990 – 2007]

Hadron-Elektron-Ring-Anlage (* Andre)

protons 920 GeV

electrons 27,5 GeV neglect masses of proton & e⁻

$$\left(E_{tot}^{cms}\right)^2 = \overset{E}{(920+27{,}5)^2} - \overset{p}{(920-27{,}5)^2}\ \mathrm{GeV}^2 =$$

$$= 318^2\ \mathrm{GeV}^2$$

corresponds to ~ 54 TeV electrons on protons @ rest

- Example: LEP @ CERN [1989 - 2000]

Large e⁻-e⁺ Collider

electrons - 100 GeV

positrons - 100 GeV

$$\left(E_{tot}^{cms}\right)^2 = \overset{E}{(100+100)^2} - \overset{p}{\big(\underbrace{100 - 100}_{=\emptyset}\big)}\ \mathrm{GeV}^2$$

$$= 200^2\ \mathrm{GeV}^2$$

(*) – Production of $\bar{p}$

$$p + p \to p + p + p + \bar{p}$$

$E_{cm} \geq 4 m_p$ $\quad m = 938{,}27$ MeV

$$4m = \left[2E_1 m + 2m^2\right]^{1/2} \qquad E_1 = 7m = 6567{,}89\ \mathrm{MeV}$$

kinetic energy $T = E_1 - m = 6m = 5629{,}62$ MeV

(3) Acceleration of electrically charged particles in electromagnetic fields

- Lorentz force

$$\vec{F} = q\,(\vec{E} + \vec{v}\times\vec{B})$$

charge — electric field strength — magnetic field strength

$$\vec{E} = \vec{E}(\vec{r},t)$$
$$\vec{B} = \vec{B}(\vec{r},t)$$

$q = \pm 1e$ for most elementary particles

$q = \pm Ne$ for higher-charged ions

- Acceleration (longitudinal)

$\boxed{\text{only } E_{\parallel} \text{ counts in acceleration process}}$ →

$$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt = \vec{p}(t_2) - \vec{p}(t_1) \quad \text{momentum change}$$

change of energy:

$$\Delta E = \int_{r_1}^{r_2} q\,(E + v\times B)\,dr = \int_{r_1}^{r_2} q\,E\,dr$$

since $d\vec{r} \perp \vec{v}\times\vec{B}$

($E_{\perp}$ acts ($\vec{v}\times\vec{B}$ as well) to change the direction)

- Useful relations

$$q\vec{E}_{\parallel} = \frac{d\vec{p}_{\parallel}}{dt} = \underbrace{\frac{d\gamma}{dt}\, m\vec{v}}_{(1)} + \gamma m \frac{d\vec{v}_{\parallel}}{dt} \quad (=)$$

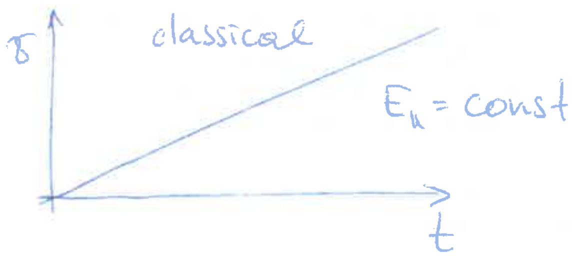
(1) $$\frac{d\gamma}{dt} = \frac{d}{dt}\left(\frac{1}{\sqrt{1-\beta^2}}\right) = \frac{\beta}{(1-\beta^2)^{3/2}}\frac{d\beta}{dt} = \gamma^3\beta\,\frac{d|\vec{v}|}{dt}\,\frac{1}{c}$$

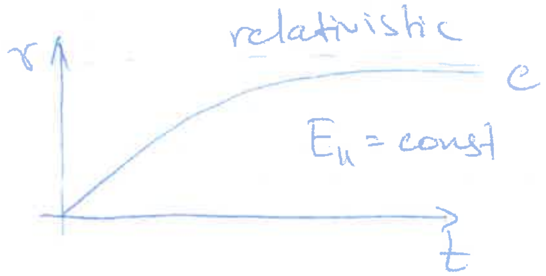
$$q\vec{E}_{\parallel} = \gamma m\left(\frac{d\vec{v}_{\parallel}}{dt} + \frac{\vec{v}_{\parallel}}{c}\gamma^2\beta\,\frac{d|\vec{v}|}{dt}\right) = \gamma m\left(1 + \underbrace{\frac{v}{c}\gamma^2\beta}_{(*)}\right)\frac{d\vec{v}_{\parallel}}{dt}$$

(*) if $v_{\parallel} \approx v$ then $\vec{v}_{\parallel}\frac{dv}{dt} \approx v\,\frac{d\vec{v}_{\parallel}}{dt}$

$$q\vec{E}_{\parallel} = \gamma^3 m \frac{d\vec{v}_{\parallel}}{dt} \qquad \boxed{1+\beta^2\gamma^2 = \gamma^2}$$

$$\Rightarrow \boxed{\frac{d\vec{v}_{\parallel}}{dt} = \frac{q\vec{E}_{\parallel}}{\gamma^3 m}}$$





- orthogonal acceleration

$$q(\vec{E}_{\perp} + \vec{v}\times\vec{B}) = \frac{d\vec{p}_{\perp}}{dt} = \gamma m \frac{d\vec{v}_{\perp}}{dt}$$ centripetal acceleration

$$\left|\frac{d\vec{v}_{\perp}}{dt}\right| = \frac{v^2}{\rho}$$ ← bending radius

⇒ electric field

$$\gamma m \frac{v^2}{\rho} = q\vec{E}_{\perp} \qquad pv = q\underbrace{E_{\perp}\rho}$$

Electrical rigidity

$$E_{\perp}\rho = \frac{pv}{q}$$

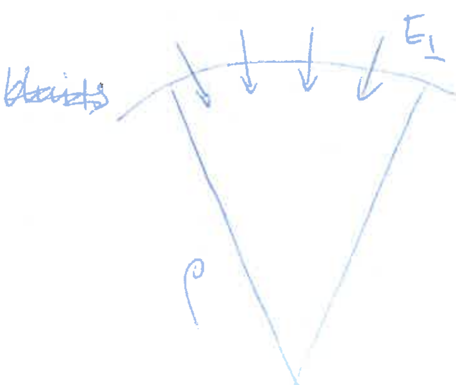
$$1\tfrac{V}{m}\cdot 1m = 1V$$

⇒ magnetic field

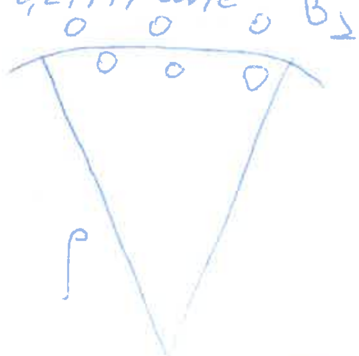
$$\gamma m \frac{v^2}{\rho} = qvB_{\perp} \qquad B\rho = \frac{p}{q}$$ - magnetic rigidity

$1T\cdot 1m = Tm \quad q = 1e$

$1Tm\,e = 0{,}29979\ GeV/c$



„Sollbahn"
reference
orbit



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