1. Lepton masses and mixing in the Standard Modell

The flavor structure of the lepton sector in the Standard Model can be analyzed similarly to the quark sector (assuming massless neutrinos).

Recall that the masses of the charged leptons are generated by their interaction with Higgs VEV:

Symmetry breaking:

$$\mathcal{L}_{Yukawa}^{Lepton} = -\sum_{i,j} \left\{ Y_{E}^{ij} \overline{L}_{L}^{i} E_{R}^{j} H + h.c. \right\} \xrightarrow{\langle H \rangle} \mathcal{L}_{Mass}^{Lepton} = -\frac{v}{\sqrt{2}} \sum_{i,j} \left\{ \overline{E}_{L}^{i} Y_{E}^{jj} E_{R}^{j} + h.c. \right\} \xrightarrow{Y_{E} \text{ in general not diagonal}} Leptons: \mathcal{L}_{L}^{i} == \begin{pmatrix} N_{L}^{i} \\ E_{L}^{i} \end{pmatrix} \xrightarrow{E_{R}^{i}}$$

Yukawa matrix \mathbf{Y}_{E} is diagonalized by: $E_{L} = \mathbf{V}_{e_{L}} \mathbf{e}_{L}$ and $E_{R} = \mathbf{V}_{e_{R}} \mathbf{e}_{R}$ (Remark: different notation than in Chap. I, \mathbf{e}_{L} and \mathbf{e}_{R} are here linear combinations of $\mathbf{E}_{L,R}$)

$$\mathbf{\hat{Y}}_{E} = \mathbf{V}_{e_{L}}^{\dagger} \mathbf{Y}_{E} \mathbf{V}_{e_{R}} = \begin{pmatrix} \mathbf{y}_{e} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{y}_{\mu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{y}_{\tau} \end{pmatrix}$$

In the mass eigenbasis of the charged leptons, the charged weak interaction (W exchange) of the leptons is given:

$$\mathcal{L}_{weak}^{cc} = \frac{g}{\sqrt{2}} \overline{N}_L \gamma^{\mu} V_{e_L} e_L W_{\mu}^+ + h.c. = \frac{g}{\sqrt{2}} \overline{v}_L \gamma^{\mu} e_L W_{\mu}^+ + h.c. \qquad \checkmark$$

- For massless neutrinos, the transformation $N_L \rightarrow V_{\nu_L} V_L$ with $V_{\nu_L} = V_{e_L}$ renders the charged current flavor-diagonal $(V_{\nu_l}^{\dagger}V_{e_l}) = V_{e_l}^{\dagger}V_{e_l} = 1$).
- For massive neutrinos, the flavor mixing among left-handed leptons is physical: $\int_{cc}^{cc} = \frac{g}{V} \overline{V} \gamma^{\mu} V^{\dagger} V e W^{\dagger} + hc$

$$\mathcal{L}_{weak}^{cc} = \frac{J}{\sqrt{2}} \overline{v}_{L} \gamma^{\mu} V_{v_{L}}^{\dagger} V_{e_{L}} e_{L} W_{\mu}^{\dagger} + h.c$$

$$U_{PMNS}^{\dagger}$$

CC does not remain diagonal in mass states

We work in a basis where Y_E is diagonal. The neutrino mass (v_i) and flavor (N_i) eigenstates are then related by:

$$N_{\alpha} = U_{PMNS}^{\alpha i} \quad V_{i} \qquad U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

The physical (mass) neutrino states are thus mixtures of different flavor eigenstates (see below).

Like the CKM matrix the PMNS^{*)} matrix can be parametrized by 3 angles θ_{12} , θ_{23} , θ_{13} and 1 phase δ (if neutrinos are Majorana particles the lepton sector has two additional phases – see below). From neutrino oscillation experiments, we know that

$$\sin^2 \theta_{12} \approx 0.3$$
, $\sin^2 \theta_{23} \approx 0.4$, $\sin^2 \theta_{13} \approx 0.02$ (for $m_1 < m_2 < m_3$)

Flavor mixing in the lepton sector is thus sizeable. The PMNS matrix does not exhibit a hierarchical structure as the CKM matrix does.

*) PMNS matrix is unitary by construction

In the Standard Model there are no FCNC with leptons at tree-level. At oneloop level, FCNC are induced by weak interactions. They can (in principle) be observed in flavor-changing processes involving charged leptons, such as

 $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma, \mu \rightarrow 3e$ (unmeasurable small in SM, see below)

2.1 $\mu \rightarrow e\gamma$ decay:

With massive neutrinos this process can be induced at loop-level:



$$O_{\gamma} \sim m_{\mu} \overline{e}_L \sigma_{\mu\nu} F^{\mu\nu} \mu_R$$

The amplitude is given by

$$\mathcal{A} \sim \sum_{i=1}^{3} U_{\mu i}^{*} U_{ei} f\left(\frac{m_{i}^{2}}{M_{W}^{2}}\right)$$
 with

 \dot{m}_i neutrino mass

$$f\left(\frac{m_i^2}{M_W^2}\right) \approx f(0) + f'(0)\frac{m_i^2}{M_W^2} + \dots$$

Loop function (see above)

Applying the unitarity condition of the PMNS matrix $\sum_{i} U_{2i}^* U_{1i} = 0$ one obtains

$$\mathcal{A} \sim \sum_{i=2}^{3} U_{\mu i}^{*} U_{e i} \frac{\Delta m_{i1}^{2}}{M_{W}^{2}} \quad \text{with} \quad \Delta m_{i1}^{2} = m_{i}^{2} - m_{1}^{2} \quad \text{GIM suppression.}$$
Very small!

Due to the small neutrino mass differences, the GIM mechanism is extremely effective and strongly suppresses the decay rate. Normalized to the dominant decay process $\mu \rightarrow e v_{\mu} \overline{v}_{e}$ one obtains the branching fraction:

$$BR(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to ev_{\mu}\overline{v}_{e})} = \frac{3\alpha}{32\pi} \left| \sum_{i=2}^{3} U_{\mu i}^{*} U_{ei} \frac{\Delta m_{i1}^{2}}{M_{W}^{2}} \right|^{2} < 10^{-54}$$

The currently strongest experimental limit is provided by the MEG experiment:

 $BR(\mu \rightarrow e\gamma) < 4.2 \cdot 10^{-13}$ (90%CL) Eur. Phys. J. C (2016) 76:434

(many orders above the SM prediction)

Due to the strong suppression of the Standard Model contribution an experimental observation of the decay $\mu \rightarrow e\gamma$ would be a clear sign of New Physics, e.g. from additional SUSY contributions:



Also other Lepton Flavor Violating (LFV) decays are excellent probes for NP.

MEG/MEG-II experiment:

Search for $\mu \rightarrow e\gamma$ requires a high-intensity μ -beam, stopped inside a detector.

PSI (Paul Scherrer Institute, Villingen, Schweiz): μ^+ -beam w/ intensities of up to $10^8 \,\mu$ /sec (produced by a 1.2 MW proton cyclotron). Future plan to build a new beam-line w/ ~10⁹ μ /sec.

Principle:

Muon beam is stopped on target in the detector: back-to-back topology of the e_{γ} .

MEG Detector:





Figure 2: Data and PDFs from the analysis of the 2009-2013 data. Black dots: data. Blue: Total PDF. Magenta: accidental background. Red: RMD background. Green: Expected signal for a BR equal to 100 times the upper limit. See the text for the definition of R_{sig} the bottom-right plot.

$$BR(\mu \to e\gamma) < 4.2 \cdot 10^{-13}$$
 (90%CL)

SM: B($\mu \rightarrow ev \bar{v}\gamma$)=(6.03±0.14(stat.)±0.53(sys.))×10⁻⁸, E_e>45 MeV and E_y>40 MeV, 8



2.2 μ \rightarrow eee decay:



 $O_{\gamma} \sim m_{\mu} \overline{e}_{L} \sigma_{\mu\nu} F^{\mu\nu} \mu_{R}$ $O_{4\ell} \sim \left(\overline{e}_{L} \gamma_{\mu} \mu_{L} \right) \left(\overline{e} \gamma^{\mu} e \right)$

Additional suppression from γ conversion.

Currently strongest experimental limit from SINDRUM experiment:

 $BR(\mu \rightarrow eee) < 1.0 \cdot 10^{-12}$ (90%CL)

Future experiment:

Mu3e at PSI – strong Heidelberg contribution (AG Prof. A. Schöning). Very promising experiment – will be discussed in the following.

Background reduction





Accidentals: Overlay of 2 ordinary decays with another e.

Sensitivity to BR of 10⁻¹⁴ requires experim.tal energy resolution better than 1 MeV! In addition excellent vertex resolution.

Mu3e detector



Tracking detector will consist of very thin CMOS pixel sensors to minimize multiple scattering and to optimize energy / vertex resolution.



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2.3 $\mu \rightarrow e$ conversion: μ -N $\rightarrow e$ -N



be tested in $\tau{\rightarrow}\phi\mu$

Current best limit: $BR(\mu^- + Au \rightarrow e^- + Au) < 7 \times 10^{-13}$ (SINDRUM II)

Future experiments:

Mu2e and COMET, both aiming at

 $BR(\mu^{-} + AI \rightarrow e^{-} + AI) < 10^{-16}$

Mu2e at Fermilab



- Wait for prompt backgrounds to disappear
- Measure the electron spectrum
- Look for the *monoenergetic* conversion electron (for Al E_e~105 MeV)

Expected limits: 10⁻¹⁶



2.4 LFV in τ-decays

 τ -pairs are abundantly produced at the e⁺e⁻ B factories:

 $e^+e^- \rightarrow \tau^+\tau^-$ with $\sigma(\sqrt{s=10.56}) \approx 1 \text{ nb} \approx \sigma_{bb}$

<u>τ→3μ</u>

 $\tau \rightarrow 3\mu$ signature is an easy to trigger and to reconstruct topology:

$$\left(\sum_{i} p_{\mu_{i}}\right) = m_{\tau}^{2}$$

Current limits: $BR < 2 (4) \times 10^{-8}$ by Belle (BABAR) limited by available statistics.

Prospects for SuperKEKB (Belle-2) $BR < 10^{-9}$

LHCb has also performed a search for $\tau \rightarrow 3\mu$: $BR < 4.6 \times 10^{-8}$ (sensitivity is limited by irreducible backgrounds)

<u> $\tau \rightarrow \mu \gamma$:</u> BR < 4 × 10⁻⁸ (Belle)

Prospects for SuperKEKB (Belle-2) $BR < 10^{-9}$

Prospect for LFV measurements in τ decays at Belle-2:



48 different LFV modes were studied at *B* factories

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2.5 Summary Limits on LFV



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Final remark:

For the search for New Physics (see arguments for b \rightarrow sll decays) the complementarity of $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu \rightarrow e$ conversion and τ -decays is interesting: Depending on the effective new interaction, the sensitivity of the respective observables can change significantly.