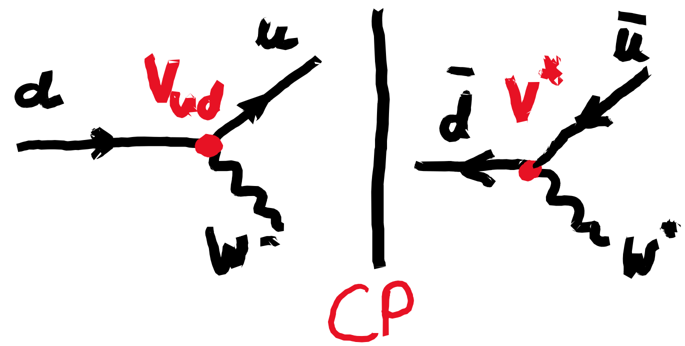
4. CP Violation in meson decays

4.1 General remarks

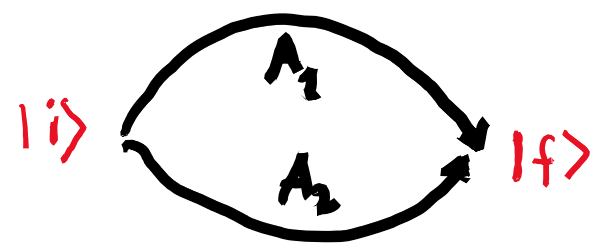
(see part I)

CP violation in mesons is linked to the CKM phases in the transition amplitude.



But: All observable quantities are in general "squares" of the amplitudes. Phases do not lead easily to observable effects (absolute phases are not observable!)

Only phase differences are observable via interference effects: At lease two interfering amplitudes are required to observe a phase difference related to a CKM phase and to study CP violation.



$$\mathcal{A}(i \to f) = \mathcal{A}_1 + \mathcal{A}_2 = |\mathcal{A}_1| e^{i\phi_1} + |\mathcal{A}_2| e^{i\phi_2}$$

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⇩

$$|\mathcal{A}(i \to f)|^2 = |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 + 2|\mathcal{A}_1||\mathcal{A}_2|\cos(\phi_1 - \phi_2) \qquad (*)$$

However, this is also not sufficient to observe CPV:
CP-conjugation is reversing the sign of both weak (CKM) phases ϕ_1 and ϕ_2 such that one obtains for the CP conjugated version of Eq. (*):

$$|\mathcal{A}(\bar{i} \to \bar{f})|^2 = |\mathcal{A}_1|^2 + |\mathcal{A}_1|^2 + 2|\mathcal{A}_1||\mathcal{A}_2|\cos(\phi_1 - \phi_2)$$

Which is exactly the same equations as for i $\to$ f.

In order to observe CP violation additional "strong" phases δ_1 and δ_2 which do not change sign under CP conjugation must be present:

$$\mathcal{A}(i \to f) = |\mathcal{A}_1| e^{i\phi_1} e^{i\delta_1} + |\mathcal{A}_2| e^{i\phi_2} e^{i\delta_2}$$
$$\mathcal{A}(\bar{i} \to \bar{f}) = |\mathcal{A}_1| e^{-i\phi_1} e^{i\delta_1} + |\mathcal{A}_2| e^{-i\phi_2} e^{i\delta_2}$$

For the difference of the transition rate one finds:

$$\left|\mathcal{A}(\bar{i} \rightarrow \bar{f})\right|^2 - \left|\mathcal{A}(i \rightarrow f)\right|^2 = 2\left|\mathcal{A}_1\right|\left|\mathcal{A}_2\right| \sin\left(\phi_1 - \phi_2\right) \sin\left(\delta_1 - \delta_2\right)$$

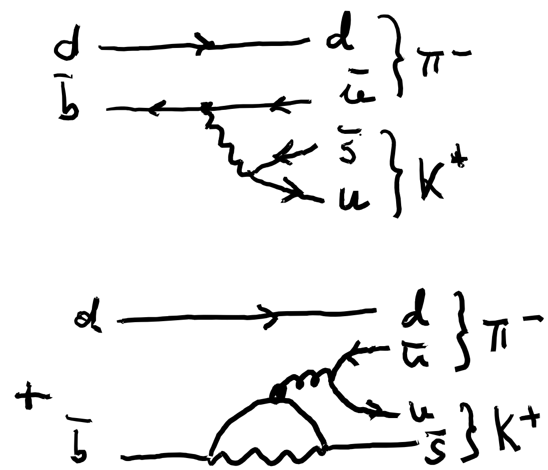
i.e.: CP violation in the decay if the weak phase difference and the strong phase difference is different from zero!

Problem:
The strong phases are a result of interactions between the hadronic final state particles $\rightarrow$ difficult to calculate

4.2 Classification of CP violation

The observed CP violating effects in meson decays are usually classified in the following way:

(I) CPV in decay:



$$\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})$$

This implies

$$\left| \frac{\mathcal{A}(\bar{P} \to \bar{f})}{\mathcal{A}(P \to f)} \right| = \left| \frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_f} \right| \neq 1$$

e.g.:

$$\Gamma\left(B^0 \to K^+\pi^-\right) \neq \Gamma\left(\bar{B}^0 \to K^-\pi^+\right)$$

$$\Gamma\left(B^+ \to \bar{D}K^+\right) \neq \Gamma\left(B^- \to DK^-\right)$$

Interference between b→u tree amplitudes and so called penguin amplitudes

In charged mesons where no mixing is possible, CPV in decay is the only possible type of CPV which can occur.

$B^0 \rightarrow K^+ \pi^-$

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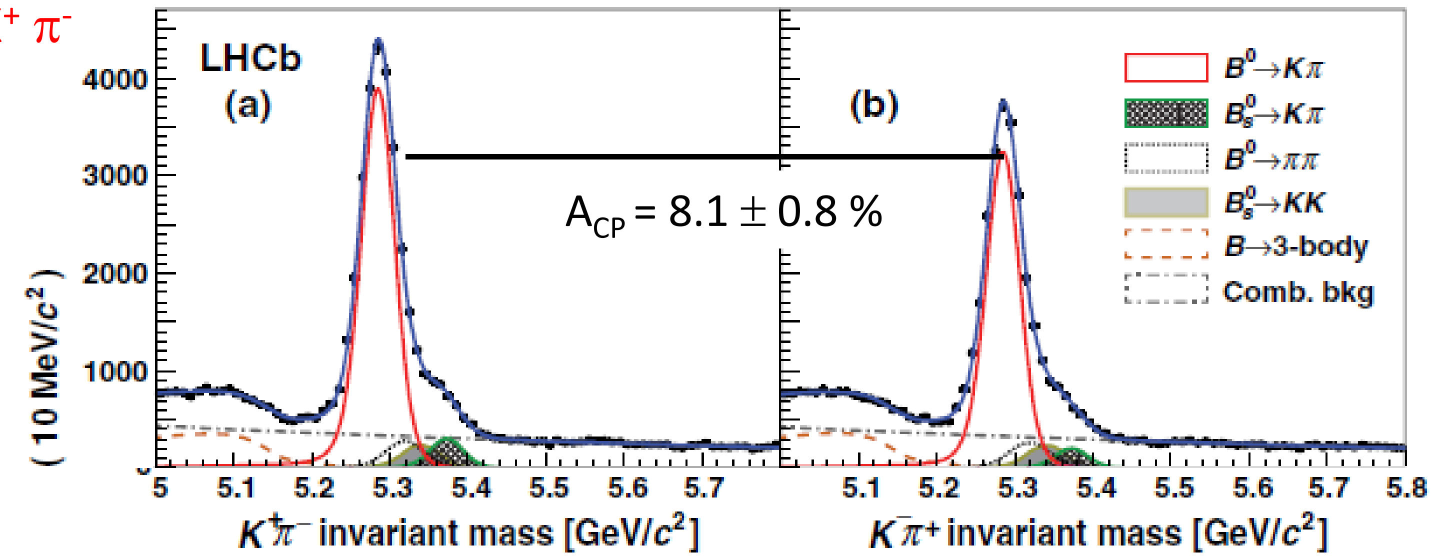
$\bar{B}^0 \rightarrow K^- \pi^+$



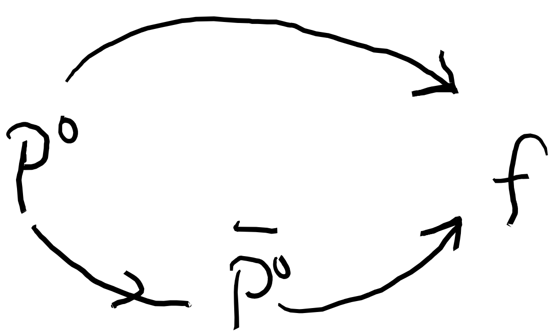
(II) CPV in mixing:

$$\mathcal{P}\left(P^0 \to \bar{P}^0\right) \neq \mathcal{P}\left(\bar{P}^0 \to P^0\right)$$

This implies $\left|\frac{q}{p}\right| \neq 1$ (see the mixing equation section 3)

While for B-mesons $|q/p| = 1 + O(10^{-5}...10^{-5}) \approx 1$
CPV in mixing is the dominating effect for kaons $O(10^{-3})$

(III) CPV in interference between a decay w/ and w/o mixing:



→ time-dependent effect (see below)
→ No effect in time integrated measuremts!

Can only occur if $\Im\left(\frac{q}{p}\frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_{\bar{f}}}\right) \neq 0$. I.e. if either q/p or the amplitude ratio has a non-trivial phase.

An alternative classification distinguishes between **direct** and **indirect** CPV:

Direct CP violation: $\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})$

Indirect CP violation: CPV that involves the mixing phenomenon in any way.

Final remarks:

CP Violating effects all depend on J_{CP} (Jarlskog invariant) and should therefore be in the same order in the Standard Model. The observable asymmetries = ratio between CP violating to CP conserving quantities are enhanced for suppressed quantities. Observable CP asymmetries are in general larger in B decays than in kaon:

$\rightarrow$ B decays have smaller CKM couplings (suppressed compared to kaons), sizable contributions from V_{ub} and V_{td} possible

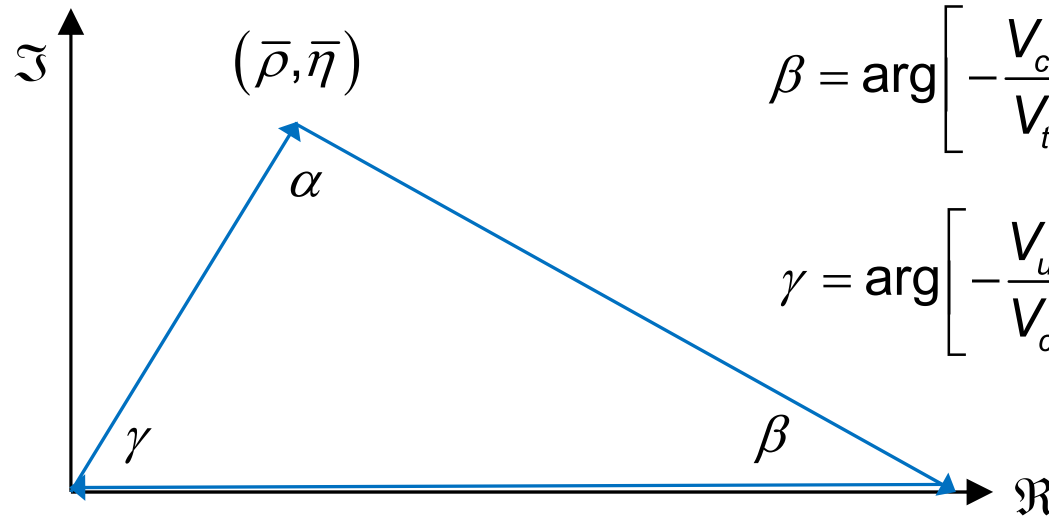
To exhibit a CP violating phase the interference term must involve at least 4 CKM matrix elements (see definition of J_{CP}).

- Below the charm threshold on-shell processes cannot violate CP as only V_{ud} and V_{us} are involved (no phases).

- CPV in Kaon sector only through virtual processes to which also heavier quarks can contribute: K^0 mixing diagrams or penguin decays.

4.3 Measurement of direct CP violation and determination of γ

Reminder 1: **UT**



$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

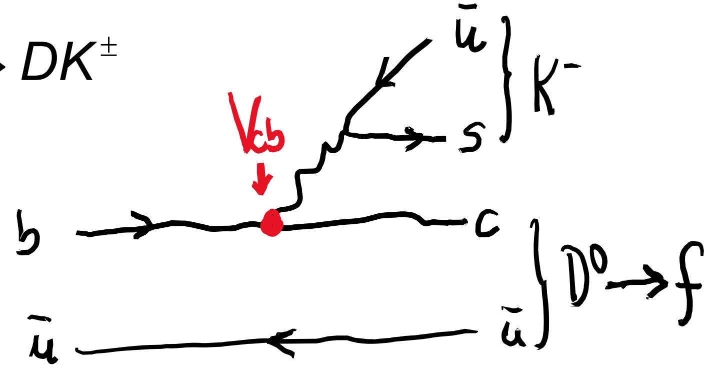
$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

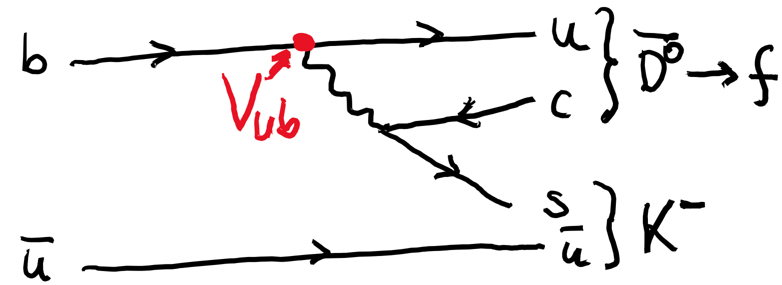
Reminder 2: CKM-elements
To $O(\lambda^3)$ only the matrix elements V_{ub} and V_{td} are complex.

One can simplify: $V_{ub} = |V_{ub}| \cdot e^{-i\gamma}$

Measurement of γ proceeds via decays where the interference of two tree-level amplitude contribute: e.g. $B^\pm \rightarrow DK^\pm$

$B^{\pm} \rightarrow DK^{\pm}$





favored b→c transition

$\mathcal{A} \sim 1 \cdot \mathcal{A}_0$

CKM suppressed b→u transition

$\mathcal{A} \sim r_B e^{i\theta} \cdot \mathcal{A}_0$

Interference of the two amplitudes possible if D^0 and $\bar{D}^0$ decay to the same final state. E.g.: CP eigenstates, as for example $D^0, \bar{D}^0 \rightarrow K^+K^-, \pi^+\pi^-$

Phase θ contains 2 parts: δ_B a strong phase and the CKM (weak) phase:

$$\theta = \delta_B \pm \gamma$$

Weak phase is to very good approximation equal to γ:
Non-tree SM processes (e.g. penguin amplitudes) contribute $\leq O(10^{-7})$

Since the weak phase γ changes sign under CP conjugation one obtains for the decay rate:

$$\Gamma\left(B^- \to DK^-\right) \propto 1 + r_B^2 + 2r_B \cos\left(\delta_B - \gamma\right)$$

$$\Gamma\left(B^+ \to DK^+\right) \propto 1 + r_B^2 + 2r_B \cos\left(\delta_B + \gamma\right)$$

$\rightarrow$ Allows the measurement of the angle γ (one needs to exploit simultaneously several different channels to also determine the channel dependent r_B and δ_B in a common fit).

$\rightarrow$ LHCb has published a combined measurement of γ several different decays and obtained a value of

$$\gamma = \left(76.8^{+5.1}_{-5.7}\right)^o \quad \text{(LHCb 2017)}$$

4.4 CP violation in mixing

$$\mathcal{P}\left(P^0 \to \bar{P}^0\right) \neq \mathcal{P}\left(\bar{P}^0 \to P^0\right) \quad \Longleftrightarrow \quad \left|\frac{q}{p}\right| \neq 1$$

"The example": CP violation in K^0 decays

With $\left|\frac{q}{p}\right| \neq 1$ it is usual to introduce a complex number ε and to rewrite $\frac{q}{p}$:

$$\frac{q}{p} = \frac{1-\varepsilon}{1+\varepsilon} \quad \text{with} \quad \varepsilon = \frac{p-q}{p+q} \qquad (\varepsilon \text{ is a complex number})$$

The physical states of the neutral Kaons are no longer equal to CP eigenstates

$$K_{1,2} = \tfrac{1}{\sqrt{2}}\left(\left|K^0\right\rangle \pm \left|\bar{K}^0\right\rangle\right) \quad \text{with p=q=1 but:}$$

$$K_{S,L} = \frac{1}{\sqrt{2\left(1+|\varepsilon|^2\right)}}\left(\left(1+\varepsilon\right)\left|K^0\right\rangle \pm \left(1-\varepsilon\right)\left|\bar{K}^0\right\rangle\right)$$

Or equivalently using $K_{1,2}$ defined as the CP eigenstates:

$$K_S = \frac{1}{\sqrt{1+\varepsilon^2}}\left(\left|K_1\right\rangle - \varepsilon\left|K_2\right\rangle\right)$$

$$K_L = \frac{1}{\sqrt{1+\varepsilon^2}}\left(\left|K_2\right\rangle + \varepsilon\left|K_1\right\rangle\right)$$

Small admixture of the "wrong" CP component in the physical eigenstates

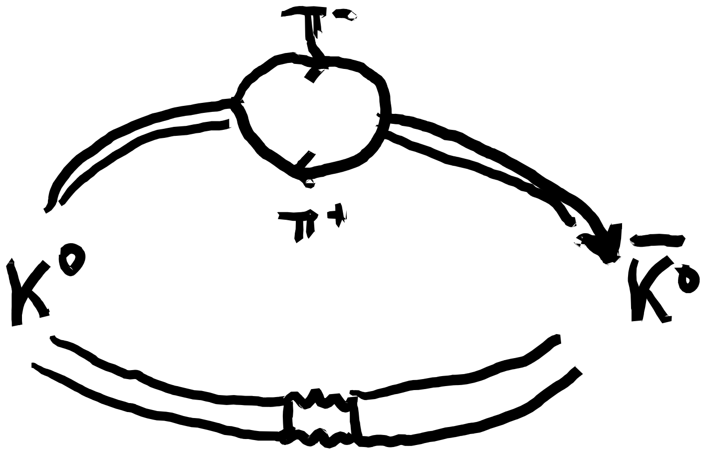
One can rewrite the CP violation parameter η_{+-} :

$$\eta_{+-} = \frac{\mathcal{A}\left(K_L \to \pi\pi\right)}{\mathcal{A}\left(K_S \to \pi\pi\right)} = \frac{1 - q/p}{1 + q/p} = \frac{p-q}{p+q} = \varepsilon$$

Measurement:

$$\left|\eta_{+-}\right| = \left|\varepsilon\right| = \left(2.232 \pm 0.025\right)\cdot 10^{-3} \quad \Longleftrightarrow \quad \left|q/p\right| \approx 0.9956$$

Question: What are the amplitudes (w/ different weak and strong phases) which contribute to the CPV in mixing of neutral kaons?



No weak phases because π consists of only u and d quarks.

Interfering diagrams with weak phases (internal heavy quarks)

different strong phase

→ CP violating weak phases from internal c and t quarks.

Theoretical prediction of ε limited (as for kaon mixing) by hadronic uncertainties.

$$\left[|\varepsilon| = \frac{G_F^2 M_W^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} B_K \Big\{ \eta_{cc} S\left(m_c^2/M_W^2\right) \Im\left(V_{cs}V_{cd}^*\right)^2 + \eta_{tt} S\left(m_t^2/M_W^2\right) \Im\left(V_{ts}V_{td}^*\right)^2 + \eta_{ct} S\left(m_c/M_W, m_t/M_W\right) \Im\left(V_{cs}V_{cd}^*V_{ts}V_{td}^*\right) \Big\} \right] \approx (2.3 \pm 0.4) \times 10^{-3}$$

e.g.: arXiv:1606.00731v1 [hep-ph]

Remark:
For B^0 and B_s mesons theoretical prediction only $O(10^{-5} \ldots 10^{-4})$
– why so much smaller?

B – mixing dominated by tt-box diagrams:

- The "on-shell" contributions w/ different strong phases are very small (branching fractions of $B^0 \to f_{CP}$ is very small, typ. $10^{-5} \ldots 10^{-6}$)
- cc, ct "off-shell" diagrams have the same strong phase as tt-diagrams.

4.5 CP Violation through interference w/ and w/o mixing

(only for neutral meson decays to CP eigenstates)

In addition to mixing we consider in the following the subsequent decays of neutral mesons. There exists 4 different amplitudes:

$$\mathcal{A}_f = \mathcal{A}\left(P^0 \to f\right) \qquad \bar{\mathcal{A}}_f = \mathcal{A}\left(\bar{P}^0 \to f\right)$$

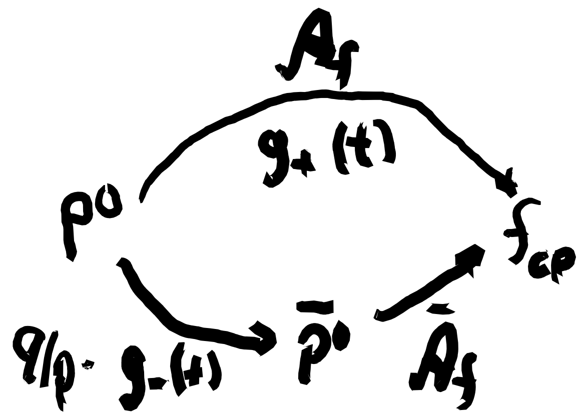
CP

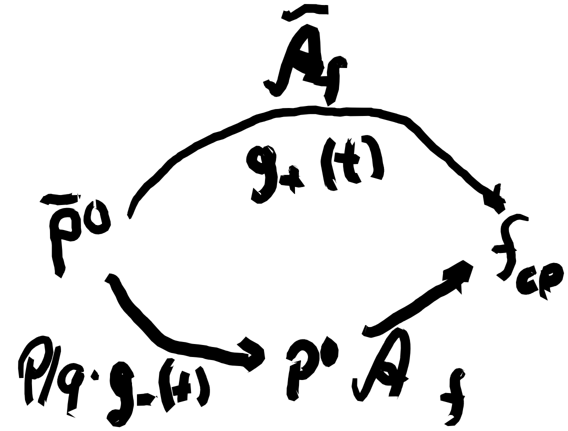
$$\mathcal{A}_{\bar{f}} = \mathcal{A}\left(P^0 \to \bar{f}\right) \qquad \bar{\mathcal{A}}_{\bar{f}} = \mathcal{A}\left(\bar{P}^0 \to \bar{f}\right)$$

We further define the complex parameter λ (observable):

$$\lambda_f = \frac{q}{p} \cdot \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \qquad \bar{\lambda}_f = \frac{1}{\lambda_f} \qquad \lambda_{\bar{f}} = \frac{q}{p} \cdot \frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_{\bar{f}}} \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

The time-dependent decay rate $\Gamma\left(P^0 \to f\right)(t) \sim \left|\mathcal{A}_{tot}\left(P^0 \to f\right)(t)\right|^2$ gives the probability that an initial P^0 decays at time t into f:





$$\left|\mathcal{A}_{tot}\left(P^0 \to f\right)(t)\right|^2 = \Big|\underbrace{\mathcal{A}\left(P^0 \to f\right)(t)}_{g_+(t)\mathcal{A}_f} + \underbrace{\mathcal{A}\left(P^0 \to \bar{P}^0 \to f\right)(t)}_{\frac{q}{p} g_-(t)\bar{\mathcal{A}}_f}\Big|^2$$

$$\Gamma\left(P^0 \to f\right)(t) = \left|\mathcal{A}_f\right|^2 \left\{\left|g_+(t)\right|^2 + \left|\lambda_f\right|^2 \left|g_-(t)\right|^2 + 2\Re\left(\lambda_f g_+(t) g_-^*(t)\right)\right\}$$

$$\lambda_f = \frac{q}{p} \cdot \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}$$

$$\Gamma\left(\bar{P}^0 \to f\right)(t) = \left|\bar{\mathcal{A}}_f\right|^2 \left\{\left|g_+(t)\right|^2 + \left|\bar{\lambda}_f\right|^2 \left|g_-(t)\right|^2 + 2\Re\left(\bar{\lambda}_f g_+(t) g_-^*(t)\right)\right\}$$

$$\bar{\lambda}_f = \frac{p}{q} \cdot \frac{\mathcal{A}_f}{\bar{\mathcal{A}}_f} \quad \downarrow$$

$$= \left|\mathcal{A}_f\right|^2 \left|\frac{p}{q}\right|^2 \left\{\left|g_-(t)\right|^2 + \left|\lambda_f\right|^2 \left|g_+(t)\right|^2 + 2\Re\left(\lambda_f g_-(t) g_+^*(t)\right)\right\}$$

With (see above): $\left|g_\pm(t)\right|^2 = \frac{1}{2} e^{-\Gamma t}\left(\cosh\left(\frac{1}{2}\Delta\Gamma t\right) \pm \cos\left(\Delta m t\right)\right)$

$$g_\pm^*(t) g_\mp(t) = \frac{1}{2} e^{-\Gamma t}\left(\sinh\left(\frac{1}{2}\Delta\Gamma t\right) \pm i \sin\left(\Delta m t\right)\right)$$

Putting everything together one obtains the Master Equation for time-dependent CP violation in neutral meson decays:

$$\Gamma\left(P^0 \to f\right)(t) = \left|\mathcal{A}_f\right|^2 \left(1 + \left|\lambda_f\right|^2\right) \frac{e^{-\Gamma t}}{2} \left\{\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + D_f \sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) + C_f \cos\left(\Delta m t\right) - S_f \sin\left(\Delta m t\right)\right\}$$

with $D_f = \dfrac{2\Re\mathcal{A}_f}{1+\left|\lambda_f\right|^2}$ $\quad C_f = \dfrac{1-\left|\lambda_f\right|^2}{1+\left|\lambda_f\right|^2}$ $\quad S_f = \dfrac{2\Im\lambda_f}{1+\left|\lambda_f\right|^2}$

and for initial anti-P^0:

$$\Gamma\left(\bar{P}^0 \to f\right)(t) = |\mathcal{A}_f|^2 \left|\frac{p}{q}\right|^2 \left(1+|\lambda_f|^2\right)\frac{e^{-\Gamma t}}{2}\left\{\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + D_f \sinh\left(\tfrac{1}{2}\Delta\Gamma t\right)\right.$$
$$\left. - C_f \cos\left(\Delta m t\right) + S_f \sin\left(\Delta m t\right)\right\}$$

For the asymmetry / difference of the time-dependent decays rates:

$$A_{CP}(t) = \frac{\Gamma\left(P^0 \to f\right)(t) - \Gamma\left(\bar{P}^0 \to f\right)(t)}{\Gamma\left(P^0 \to f\right)(t) + \Gamma\left(\bar{P}^0 \to f\right)(t)}$$

$$= \frac{C_f \cos\left(\Delta m t\right) - S_f \sin\left(\Delta m t\right)}{\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) + D_f \sinh\left(\tfrac{1}{2}\Delta\Gamma t\right)}$$

Produces time-dependent CP violation even if $|q/p|=1$, i.e. no CPV in mixing, and $\bar{A}_f / A_f = 1$, i.e. no direct CP violation, if $\arg(\lambda_f) = \arg(q / p\, \bar{A}_f / A_f) \neq 0$

Remark: For B^0 $\Delta\Gamma \approx 0$ and terms in the denominator disappear.