3. Phenomenology of Neutral Meson Mixing

In the following neutral “flavored” meson systems (K^0, D^0, B^0, B_s^0) in which particle and anti-particle are distinguished by the flavor quantum number will be discussed in a general way: P^0 and $\overline{P}^0$.

3.1 Flavor- and CP-eigenstates

A generic neutral flavored meson system P^0 and $\overline{P}^0$ is described by:

$$F\left|P^0\right\rangle = +\left|P^0\right\rangle \quad \text{and} \quad F\left|\overline{P}^0\right\rangle = -\left|\overline{P}^0\right\rangle$$

F = Flavor number operator

CP conjugation transforms particle into anti-particle but introduces in general a phase η_{CP}:

$$CP\left|P^0\right\rangle = \eta_{CP}\left|\overline{P}^0\right\rangle \qquad CP\left|\overline{P}^0\right\rangle = \eta_{CP}^*\left|P^0\right\rangle$$

Double application of the CP operator results into the original state.

(via decays to CP eigenstates)

As these states mix , they cannot be the physical/mass eigenstates. The physical states which propagate in time are mixtures of the flavor states.

If CP is conserved in mixing*) the physical states are CP eigenstates:

$$P_{\substack{1\\+}} = \tfrac{1}{\sqrt{2}}\left(\left|P^0\right\rangle + \left|\bar{P}^0\right\rangle\right) \qquad P_{\substack{2\\-}} = \tfrac{1}{\sqrt{2}}\left(\left|P^0\right\rangle - \left|\bar{P}^0\right\rangle\right)$$

with $\mathrm{CP}\left|P_1\right\rangle = +\left|P_1\right\rangle$ $\qquad$ $\mathrm{CP}\left|P_2\right\rangle = -\left|P_2\right\rangle$

if the phase η_{CP} is chosen to be +1 (convention).

*) we will discuss this condition later.

3.2 Effective Lagrangian and the physical states

In the most general case the time dependence of a physical state can be described by the flavor states using coeff. p(t) and q(t) in the following way:

$$\psi(t) = p(t)\left|P^0\right\rangle + q(t)\left|\bar{P}^0\right\rangle$$

$\psi(t)$ should fulfill the Schrödinger-Eq. with the non-hermitian effective Hamiltonian $\mathcal{H}$ (non-hermitian, because ψ decays outside the (P^0, $\overline{P^0}$) subspace)*):

$$i\hbar \tfrac{d}{dt}\psi(t) = \mathcal{H}\psi(t)$$

One usually splits $\mathcal{H}$ which consists of a flavor conserving part $\mathcal{H}_0$ and a weak flavor-violating part $\mathcal{H}_w$ $\left(\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_w\right)$ into a hermitian and anti-hermitian part:

$$\mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma}$$

Where both $\mathbf{M}$ (mass matrix) and $\mathbf{\Gamma}$ (decay matrix) are hermitian.

*) V.F. Weisskopf and E.P. Wigner, Z. Phys. **63** (1930) 54; **65** (1930) 18; T. D. Lee, R. Oehme and C. N. Yang, Phys. Rev. **106** (1957) 340.

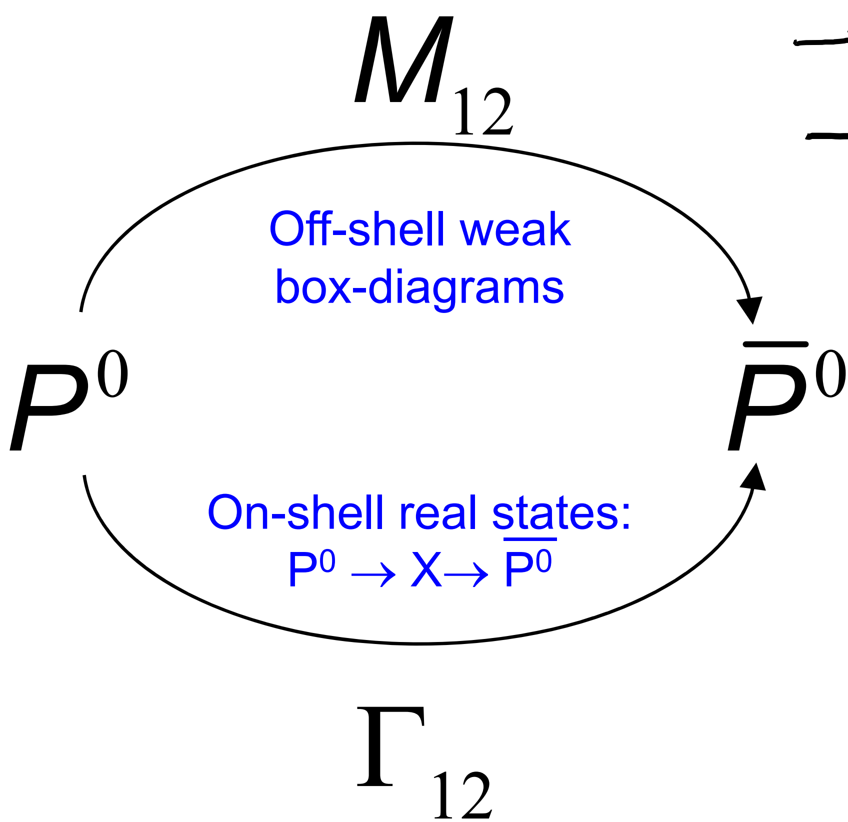
With the representation $|\psi\rangle = \begin{pmatrix} p \\ q \end{pmatrix}$ one can write $\mathcal{H}$ as 2-dim. matrix:

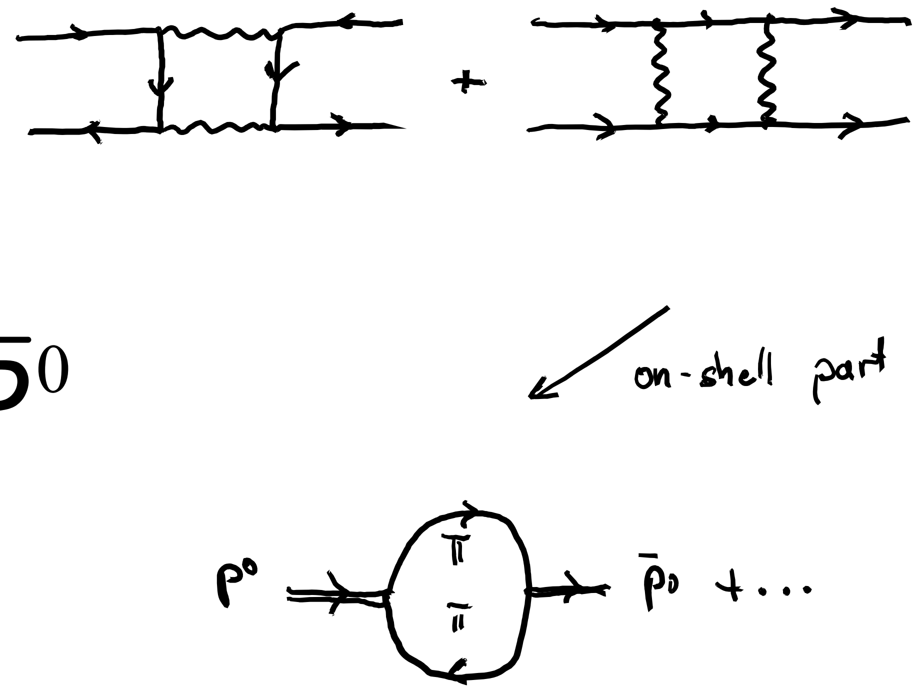
$$\mathcal{H} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} = \begin{pmatrix} m - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix} = \begin{pmatrix} H & H_{12} \\ H_{21} & H \end{pmatrix}$$

Hermiticity of $\mathbf{M}$ and $\mathbf{\Gamma}$:	$M_{12} = M_{21}^*$	$\Gamma_{12} = \Gamma_{21}^*$
CPT conservation:	$M_{11} = M_{22} \equiv m$	$\Gamma_{11} = \Gamma_{22} \equiv \Gamma$

- States P^0 and $\overline{P^0}$ are eigenstates of $\mathcal{H}_0$ $\rightarrow$ no mixing, no decay
  $\mathcal{H}_0$ therefore contributes only to $\mathrm{diag}(\mathbf{M})$
- M_{12} dispersive part of the $P^0 \leftrightarrow \bar{P}^0$ transition describes mixing via virtual intermediate states (off-shell parts of the mixing diagrams)
- Γ_{12} absorptive part of the $P^0 \leftrightarrow \bar{P}^0$ transition describes mixing via real intermediate states (on-shell part of the mixing diagrams)
- Γ_{11} , Γ_{22} describe the decay to real final states

Mixing phenomenology





The physical states $|P_a>$ and $|P_b>$ are obtained by diagonalizing the matrix **H** $\rightarrow$ eigenvalues λ_a , λ_b and eigenstates $P_{a,b}$:

$$\mathcal{H}\left|P_{a,b}\right\rangle = \lambda_{a,b}\left|P_{a,b}\right\rangle$$

with
$$\lambda_{a,b} = H \pm \sqrt{H_{12}H_{21}}$$

$$\lambda_{a,b} = m_{a,b} - \tfrac{i}{2}\Gamma_{a,b}$$

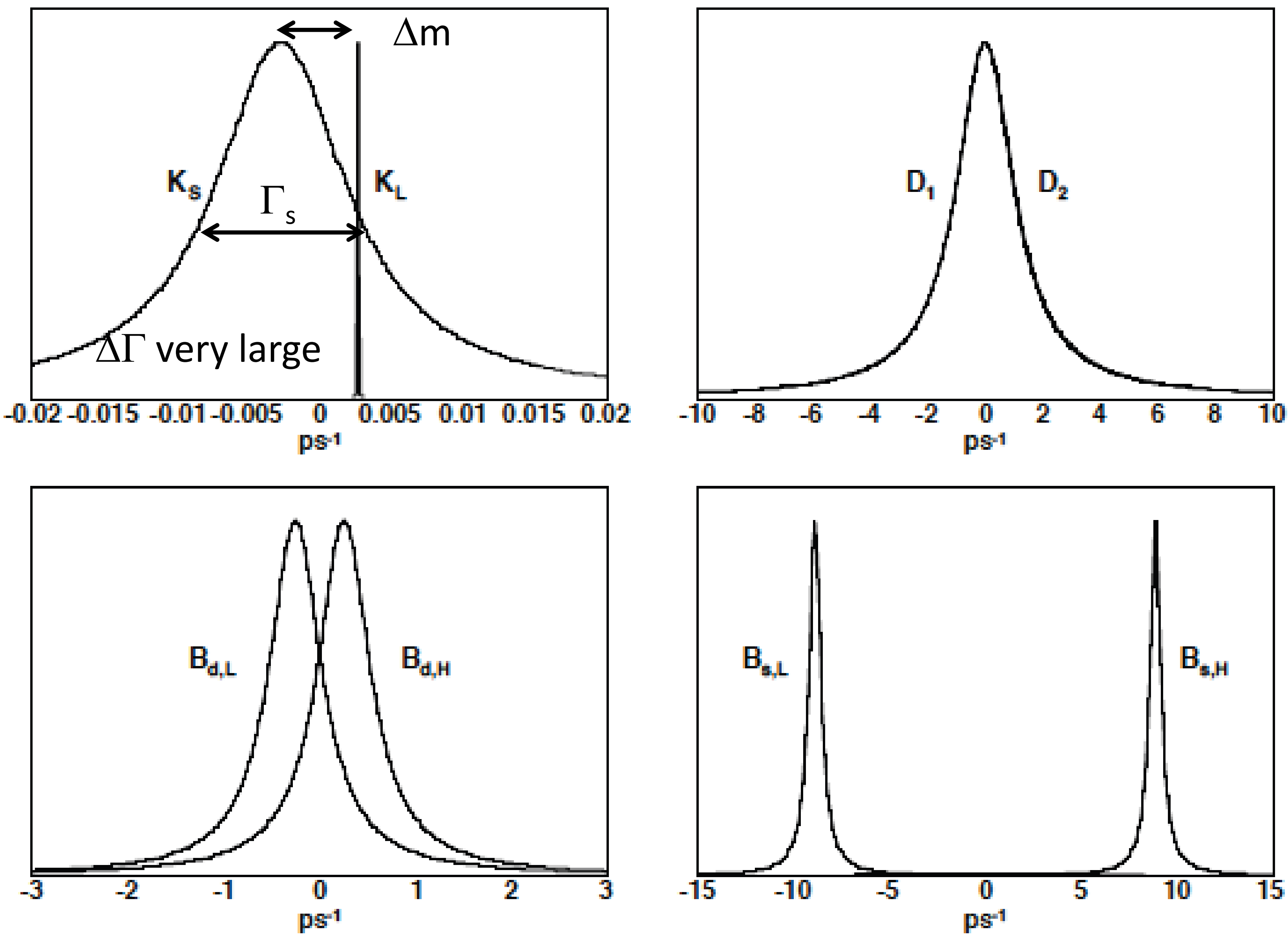
where $m_{a,b}$ are the masses and $\Gamma_{a,b}$ are the decay widths of P_a,P_b:

$$\left|P_{a,b}(t)\right\rangle = e^{-im_{a,b}t}e^{-\Gamma_{a,b}\frac{t}{2}}\left|P_{a,b}(t=0)\right\rangle$$

The physical states (P_a ,P_b) are usually labeled (indexed) by the property which distinguish them best:

- lifetime for kaons: K_S and K_L (short and long)
- mases for the B mesons: B_H and B_L (heavy and light)
- CP values for D mesons: D_1 and D_2 (assuming no direct CPV)

Neutral Mesons: physical states



The physical states can be written as:

$$\left|P_a\right\rangle = p\left|P^0\right\rangle + q\left|\bar{P}^0\right\rangle$$
$$\left|P_b\right\rangle = p\left|P^0\right\rangle - q\left|\bar{P}^0\right\rangle$$

with $\left|p\right|^2 + \left|q\right|^2 = 1$

(we used here CPT invariance from which follows: $q_a/p_a = q_b/p_b \equiv q/p$)

<u>For B mesons:</u>

$$\left|B_L\right\rangle = p\left|B^0\right\rangle + q\left|\bar{B}^0\right\rangle$$
$$\left|B_H\right\rangle = p\left|B^0\right\rangle - q\left|\bar{B}^0\right\rangle$$

One further defines the following quantities for (B_H , B_L):

$$\Delta m = m_H - m_L \qquad \Delta\Gamma = \Gamma_L - \Gamma_H$$

(convention for B $\rightarrow \Delta\Gamma > 0$)

$$m = \tfrac{1}{2}\left(m_H + m_L\right) \qquad \Gamma = \tfrac{1}{2}\left(\Gamma_H + \Gamma_L\right)$$

For q/p one finds:

$$\frac{q}{p} = \pm\sqrt{\frac{H_{21}}{H_{12}}} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

The sign $\pm$ of q/p determines whether m_a or m_b is heavier:
The usual choice is $\Delta m > 0$: $q/p > 0 \Leftrightarrow$ "+" sign

Attention: this choice is not fixing the sign of $\Delta\Gamma$
(experiment has to tell whether $\Delta\Gamma$ < / > 0, i.e. whether the CP even/odd state lives longer.)

Remark:
While P^0 and $\overline{P^0}$ as well as P_1 and P_2 (as defined above) are orthogonal, P_a and P_b are in general not orthogonal:

$$\xi = \langle P_a | P_b \rangle = |p|^2 - |q|^2 \neq 0$$

If CP symmetry is conserved and P_a and P_b are CP eigenstates:
$|q/p| = 1 \rightarrow \xi = 0$ (states are orthogonal)

Observables Δm and ΔΓ (for experts)

$$m = \tfrac{1}{2}\left(m_H + m_L\right) \qquad \Gamma = \tfrac{1}{2}\left(\Gamma_H + \Gamma_L\right)$$

$$m = M_{11} = M_{22} \qquad \Gamma = \Gamma_{11} = \Gamma_{22}$$

Mass difference: dispersive part of the box-diagrams

$$\Delta m = m_H - m_L \qquad \Delta m = 2\Re\left(\sqrt{H_{12}H_{21}}\right)$$

$$\approx 2\left|M_{12}\right|\left(1 - \tfrac{1}{8}\frac{\left|\Gamma_{12}\right|^2}{\left|M_{12}\right|^2}\sin^2\phi_{\Gamma/M} + \ldots\right) \qquad \phi_{\Gamma/M} = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Remark:
In the Standard Model $\dfrac{\left|\Gamma_{12}\right|}{\left|M_{12}\right|}$ for the B_d and the B_s system is small ($O(5\times10^{-3})$):

$$\Delta m \approx 2\left|M_{12}\right|$$

(⇔ dispersive "off-shell" contributions dominate in mixing)

M_{12} describes the virtual part of the box-diagrams → very sensitive to new effects

<u>Decay rate difference:</u> absorptive part of the box-diagrams

$$\Delta\Gamma = \Gamma_L - \Gamma_H \qquad \Delta\Gamma = -4\Im\left(\sqrt{H_{12}H_{21}}\right)$$

$$\approx 2\left|\Gamma_{12}\right|\cos\phi_{\Gamma/M}\left(1-\tfrac{1}{8}\frac{\left|\Gamma_{12}\right|^2}{\left|M_{12}\right|^2}\sin^2\phi_{\Gamma/M}+\ldots\right)$$

Remark:
In the Standard Model $\phi_{\Gamma/M}$ for the B_d and the B_s system is close to zero:

$$\cos\phi_{\Gamma/M} \approx 1$$

Related to the on-shell part of the box-diagrams w/ internal u and c-quarks.
$\rightarrow$ CKM favored tree-level decays w/ almost no new physics contributions.
Only way to affect $\Delta\Gamma$ is the phase $\phi_{\Gamma/M}$.
As the phase is close to zero, new physics can only lower the value of $\Delta\Gamma$ w/r to SM. Γ_{12} not affected by new “virtual” effects (on-shell effects only).

3.3 Time-evolution of flavor states

Physical states:
(ignore the norm)

$$\left|P_a\right\rangle = p\left|P^0\right\rangle + q\left|\bar{P}^0\right\rangle$$

$$\left|P_b\right\rangle = p\left|P^0\right\rangle - q\left|\bar{P}^0\right\rangle$$

(Depending on the property which distinguish the states best one could call them $P_{H,L}$, $P_{L,S}$ or $P_{1,2}$)

For the flavor states one obtains correspondingly:

$$\left|P^0\right\rangle = \tfrac{1}{2p}\left[\left|P_a\right\rangle + \left|P_b\right\rangle\right]$$

$$\left|\bar{P}^0\right\rangle = \tfrac{1}{2q}\left[\left|P_a\right\rangle - \left|P_b\right\rangle\right]$$

Using the time-dependence of $|P_a>$ and $|P_b>$ as above:

$$\left|P_{a,b}(t)\right\rangle = e^{-im_{a,b}t} e^{-\Gamma_{a,b}\frac{t}{2}} \left|P_{a,b}(t=0)\right\rangle$$

$$\left|P^0(t)\right\rangle = \tfrac{1}{2p}\Big\{e^{-im_a t}e^{-\frac{1}{2}\Gamma_a t}\underbrace{\left|P_a(0)\right\rangle}_{=p\left|P^0\right\rangle + q\left|\bar{P}^0\right\rangle} + e^{-im_b t}e^{-\frac{1}{2}\Gamma_b t}\underbrace{\left|P_b(0)\right\rangle}_{=p\left|P^0\right\rangle - q\left|\bar{P}^0\right\rangle}\Big\}$$

$$= \quad g_+(t)\left|P^0\right\rangle \quad + \quad \frac{q}{p}g_-(t)\left|\bar{P}^0\right\rangle$$

$$\left|\bar{P}^0(t)\right\rangle = \quad g_+(t)\left|\bar{P}^0\right\rangle \quad + \quad \frac{p}{q}g_-(t)\left|P^0\right\rangle$$

with

$$g_+(t) = \tfrac{1}{2}\left(e^{-im_a t-\frac{1}{2}\Gamma_a t} + e^{-im_b t-\frac{1}{2}\Gamma_b t}\right)$$

$$= \tfrac{1}{2}e^{-imt}\left(e^{-\frac{i}{2}\Delta m t-\frac{1}{2}\Gamma_a t} + e^{+\frac{i}{2}\Delta m t-\frac{1}{2}\Gamma_b t}\right)$$

$$m = \tfrac{1}{2}\left(m_a + m_b\right)$$

$$\Delta m = m_a - m_b$$

$$g_-(t) = \tfrac{1}{2}e^{-imt}\left(e^{-\frac{i}{2}\Delta m t-\frac{1}{2}\Gamma_a t} - e^{+\frac{i}{2}\Delta m t-\frac{1}{2}\Gamma_b t}\right)$$

Starting from a pure sample of P^0 mesons (e.g. unambiguously produced in strong interaction) the probability to measure the flavor state $\bar{P}^0$ at time t is given by:

$$\left|\left\langle \bar{P}^0 \middle| P^0(t) \right\rangle\right|^2 = \left|g_-(t)\right|^2 \cdot \left|\frac{q}{p}\right|^2$$

and correspondingly for a pure $\bar{P}^0$ sample at t=0:

$$\left|\left\langle P^0 \middle| \bar{P}^0(t) \right\rangle\right|^2 = \left|g_-(t)\right|^2 \cdot \left|\frac{p}{q}\right|^2$$

Possibility for "CP violation in mixing"

$\mathcal{P}(P^0 \to \bar{P}^0) \neq \mathcal{P}(\bar{P}^0 \to P^0)$

With the above expression for $g_\pm(t)$ one finds:

$$\left|g_\pm(t)\right|^2 = \tfrac{1}{4}\left(e^{-\Gamma_a t} + e^{-\Gamma_b t} \pm e^{-\Gamma t}\left(e^{-i\Delta m t} + e^{+i\Delta m t}\right)\right)$$

$$= \tfrac{1}{2} e^{-\Gamma t}\left(\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm \cos\left(\Delta m t\right)\right)$$

For $\Delta\Gamma \approx 0$ (as e.g. in B_d mesons) and $|p/q| \approx 1$ (CPV in mixing very small):

$$\text{Prob}\left(P^0_{t=0} \rightarrow P^0\right)(t) = \tfrac{1}{2} e^{-\Gamma t}\left(1 + \cos\left(\Delta m t\right)\right)$$

$$\text{Prob}\left(P^0_{t=0} \rightarrow \bar{P}^0\right)(t) = \tfrac{1}{2} e^{-\Gamma t}\left(1 - \cos\left(\Delta m t\right)\right)$$

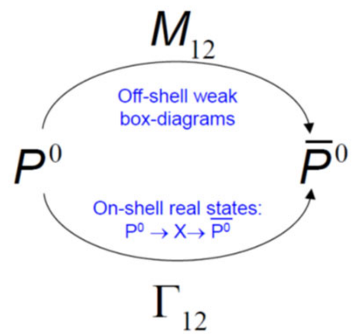
$$\begin{tikzcd} & P^0 \\ P^0 \arrow[ur, bend left, "\mathcal{M} \sim g_+(t)"] \arrow[dr, bend right, "\mathcal{M} \sim \frac{q}{p} g_-(t)"'] & \\ & \bar{P}^0 \end{tikzcd}$$

What did we learn?

- P^o and P^0 – two state system, Flavor states not equal to mass states.
- Effective time dependent Schrödinger equation to get mass staes

$$i\hbar \tfrac{d}{dt}\psi(t) = \mathcal{H}\psi(t) \qquad \mathcal{H} = \mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma} \qquad \text{(non-hermitian Hamiltonian)}$$

$$\mathcal{H} = \begin{pmatrix} m - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix}$$



$$\left|P_a\right\rangle = p\left|P^0\right\rangle + q\left|\bar{P}^0\right\rangle$$
$$\left|P_b\right\rangle = p\left|P^0\right\rangle - q\left|\bar{P}^0\right\rangle$$

$$\left|P_{a,b}(t)\right\rangle = e^{-im_{a,b}t} e^{-\Gamma_{a,b}\frac{t}{2}} \left|P_{a,b}(t=0)\right\rangle$$

$$\left|P^0(t)\right\rangle = \tfrac{1}{2p}\left\{e^{-im_a t} e^{-\frac{1}{2}\Gamma_a t}\left|P_a(0)\right\rangle + e^{-im_b t} e^{-\frac{1}{2}\Gamma_b t}\left|P_b(0)\right\rangle\right\}$$
$$= \quad g_+(t)\left|P^0\right\rangle \quad + \quad \frac{q}{p} g_-(t)\left|\bar{P}^0\right\rangle$$

$$\left|P^0\right\rangle = \tfrac{1}{2p}\left[\left|P_a\right\rangle + \left|P_b\right\rangle\right]$$
$$\left|\bar{P}^0\right\rangle = \tfrac{1}{2q}\left[\left|P_a\right\rangle - \left|P_b\right\rangle\right]$$

$$\left|\bar{P}^0(t)\right\rangle = \quad g_+(t)\left|\bar{P}^0\right\rangle \quad + \quad \frac{p}{q} g_-(t)\left|P^0\right\rangle$$

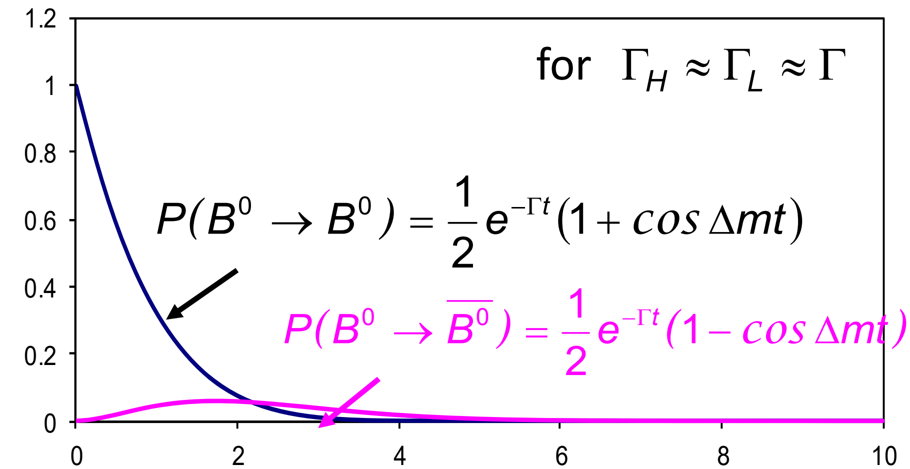
Time-dependent mixing $(\Delta\Gamma \approx 0)$

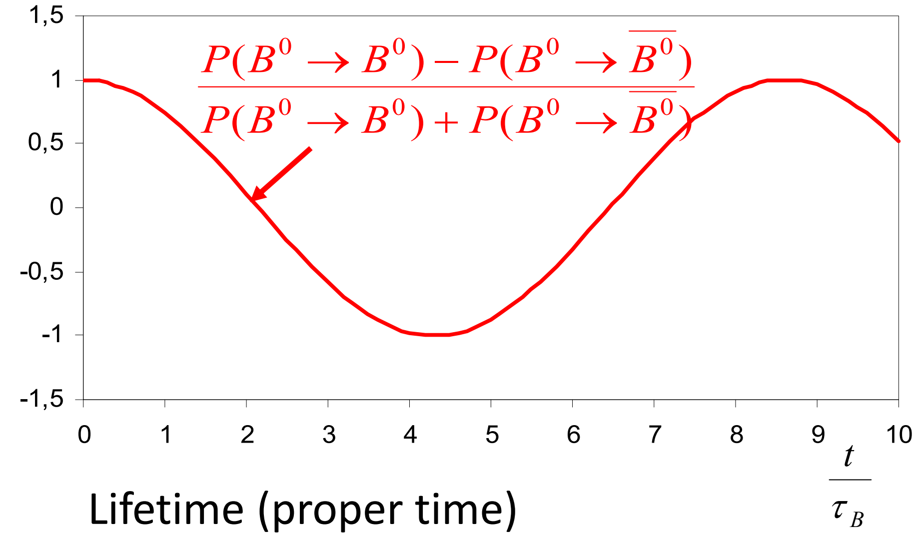
Mixing probability:

$$B_d \rightarrow f = X\mu^+ \quad \text{unmixed}$$

$$B_d \rightarrow \overline{B}_d \rightarrow \overline{f} = \overline{X}\mu^- \quad \text{mixed}$$

$$A(t) = \frac{unmixed(t) - mixed(t)}{unmixed(t) + mixed(t)} = \cos(\Delta m t)$$





B^0

$\updownarrow$

$\overline{B^0}$

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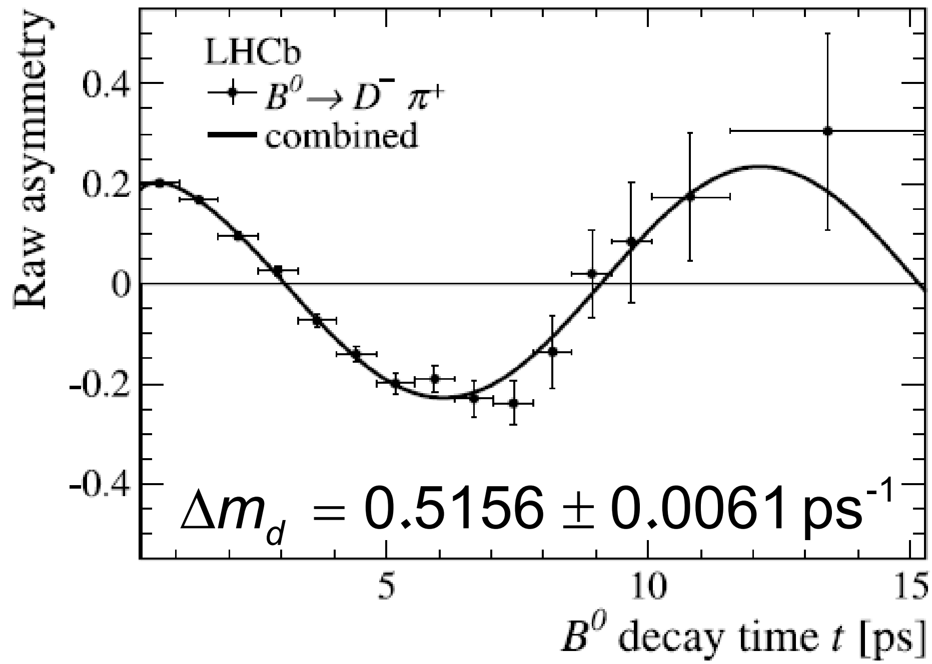
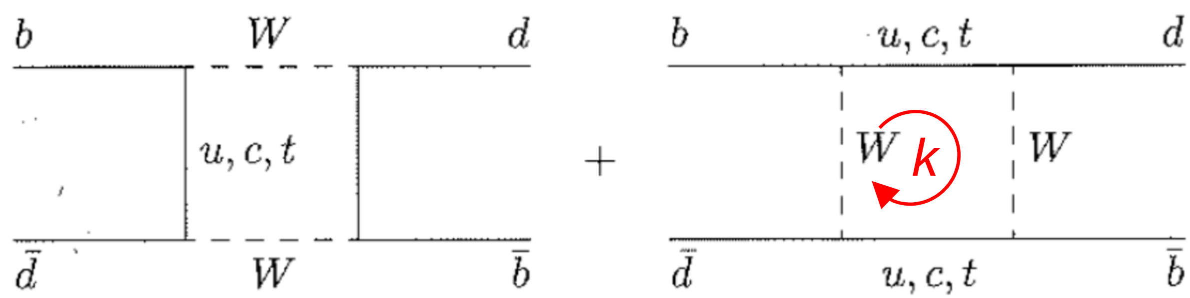


3.4 Standard Model prediction

B-mixing contribution from on-shell terms (decay to common states X) is small and H_{12} (H_{21}) are dominated by the off-shell box-diagrams:



$$\mathcal{M} \sim \int_k dk \; V_{tb}V_{td}^*\Pi_t \cdot V_{tb}V_{td}^*\Pi_t$$
$$+ V_{cb}V_{cd}^*\Pi_c \cdot V_{tb}V_{td}^*\Pi_t$$
$$+ V_{ub}V_{ud}^*\Pi_u \cdot V_{tb}V_{td}^*\Pi_t$$
$$+ V_{tb}V_{td}^*\Pi_t \cdot V_{cb}V_{cd}^*\Pi_c$$
$$+ V_{tb}V_{td}^*\Pi_t \cdot V_{ub}V_{ud}^*\Pi_u$$
$$+ V_{cb}V_{cd}^*\Pi_c \cdot V_{cb}V_{cd}^*\Pi_c$$
$$+ V_{ub}V_{ud}^*\Pi_c \cdot V_{ub}V_{ud}^*\Pi_u$$
$$+ V_{cb}V_{cd}^*\Pi_c \cdot V_{ub}V_{ud}^*\Pi_u$$
$$+ V_{ub}V_{ud}^*\Pi_u \cdot V_{cb}V_{cd}^*\Pi_c$$

Integral over the possible internal loop momentum k

Quark propagator:

$$\Pi_q(k) \sim \frac{\gamma_\mu k^\mu + m_q}{k^2 - m^2}$$

(all other factors are the same for all diagrams)

Since m_u , m_c << m_t we can treat m_u , $m_c \approx 0$. Use trick as for FCNC calculations :

$$\begin{aligned}\mathcal{M} \sim\ & V_{tb}V_{td}^{*}\Pi_t \cdot V_{tb}V_{td}^{*}\Pi_t \\ & + (V_{cb}V_{cd}^{*} + V_{ub}V_{ud}^{*})\Pi_0 \cdot V_{tb}V_{td}^{*}\Pi_t \\ & + V_{tb}V_{td}^{*}\Pi_t \cdot (V_{cb}V_{cd}^{*} + V_{ub}V_{ud}^{*})\Pi_0 \\ & + (V_{cb}V_{cd}^{*} + V_{ub}V_{ud}^{*})\Pi_0 \cdot (V_{cb}V_{cd}^{*} + V_{ub}V_{ud}^{*})\Pi_0\end{aligned}$$

Exploit the unitarity of CKM matrix:

$$\mathcal{M} \sim (V_{td}V_{tb}^{*})^2[\Pi_t\Pi_t - \Pi_0\Pi_t - \Pi_t\Pi_0 + \Pi_0\Pi_0]$$

Effect of inner-quark propagators are described by the Inami-Lim function

$$S\left({m_q^2}/{M_W^2}\right) \sim \frac{m_q^2}{M_W^2} \quad \Longrightarrow \quad S\left({m_t^2}/{M_W^2}\right) \approx 2.5$$

$$S\left({m_c^2}/{M_W^2}\right) \approx 3.5 \cdot 10^{-4}$$

$\Longrightarrow$ For B-mixing, top quark is dominating the box-diagram (GIM).

Inami-Lim function $S(x) = x\left[\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right] - \frac{3}{2}\left[\frac{x}{1-x}\right]^3 \ln x$

If one concludes the calculation one obtains for the M_{12} part of the Hamiltonian:

$$M_{12} \approx \frac{G_F^2 M_W^2}{12\pi^2} \eta_{QCD} \underbrace{B_B f_B^2} m_B S\left(m_t^2 / M_W^2\right) \left|V_{td} V_{tb}^*\right|^2$$

Perturbative QCD corrections

B_B Bag-parameter
f_B decay constant

Describe the non-perturbative effects of the bound quarks

Remark: for neutral B mesons there exists reliable calculations of the hadronization effects.
For neutral D and K mesons more difficult.

and with $\Delta m \approx 2\left|M_{12}\right|$ one can calculate the mixing frequency.

$\Delta m_d^{SM} = 0.543 \pm 0.091 \text{ps}^{-1}$ exp.: $\Delta m_d \approx 0.515 \text{ps}^{-1}$

$\Delta m_s^{SM} = 17.3 \pm 2.6 \text{ps}^{-1}$ exp.: $\Delta m_s \approx 17.77 \text{ps}^{-1}$

Difference is effect of the CKM elements: $B_s: \sim \left|V_{ts} V_{tb}^*\right|^2$

$$\Delta m_s / \Delta m_s \sim \left|V_{ts}\right|^2 / \left|V_{td}\right|^2 \approx \left(\frac{0.0404}{0.0087}\right)^2$$

B^0

$\updownarrow$

$\overline{B}^0$

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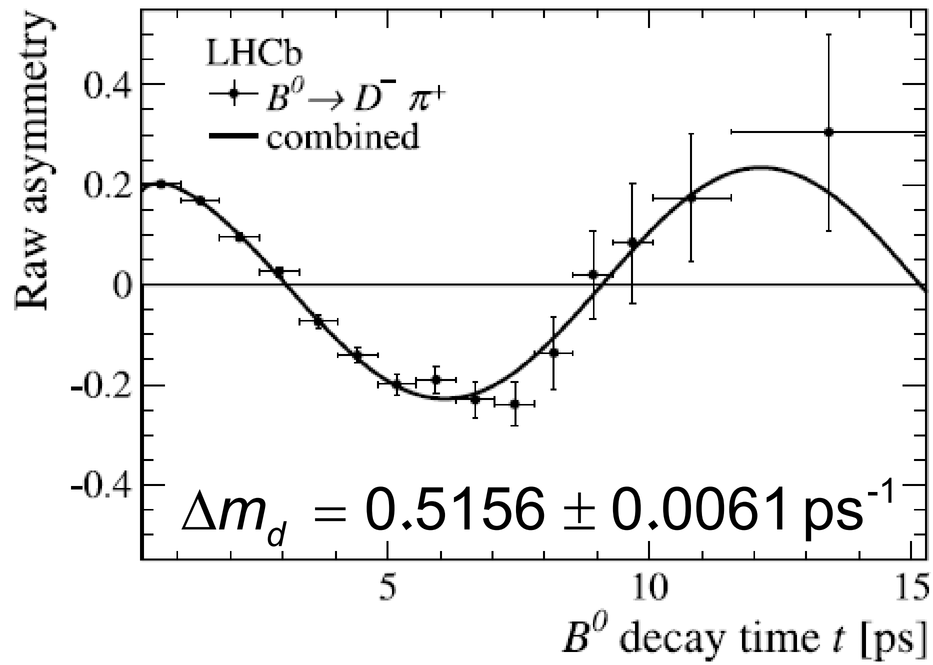
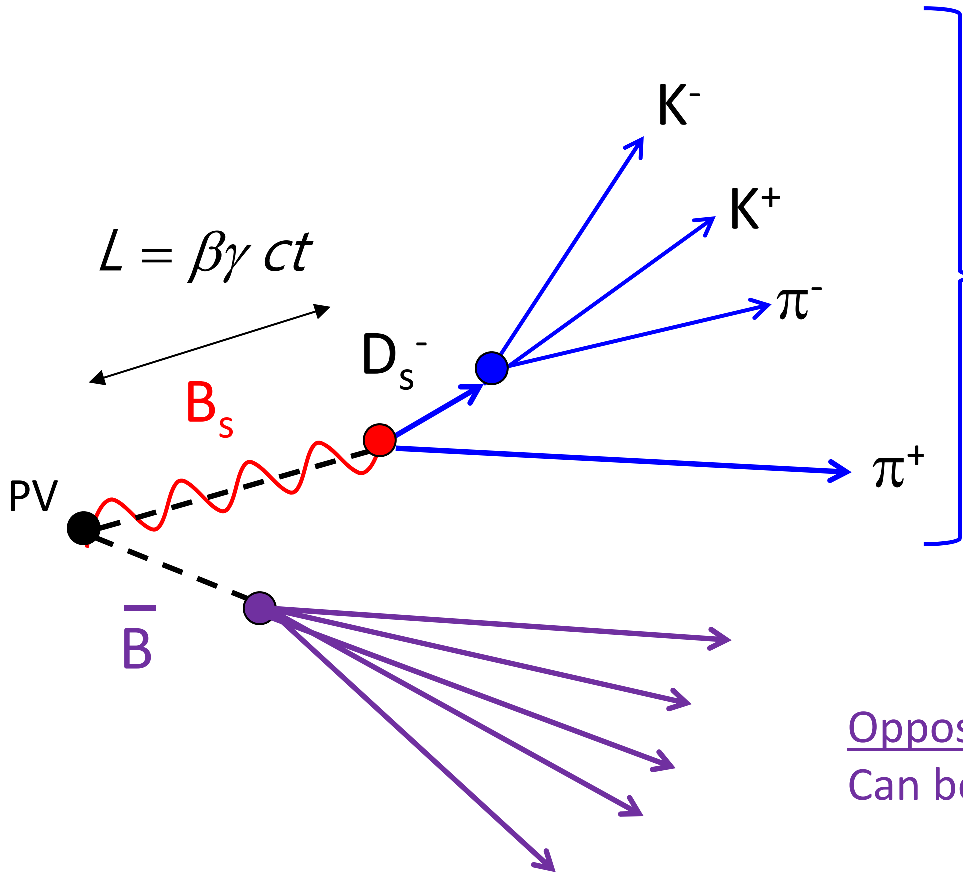
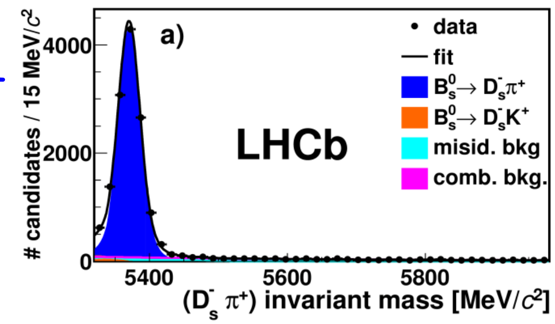


Measurement of mixing:



Signal B
(flavor specific decay)



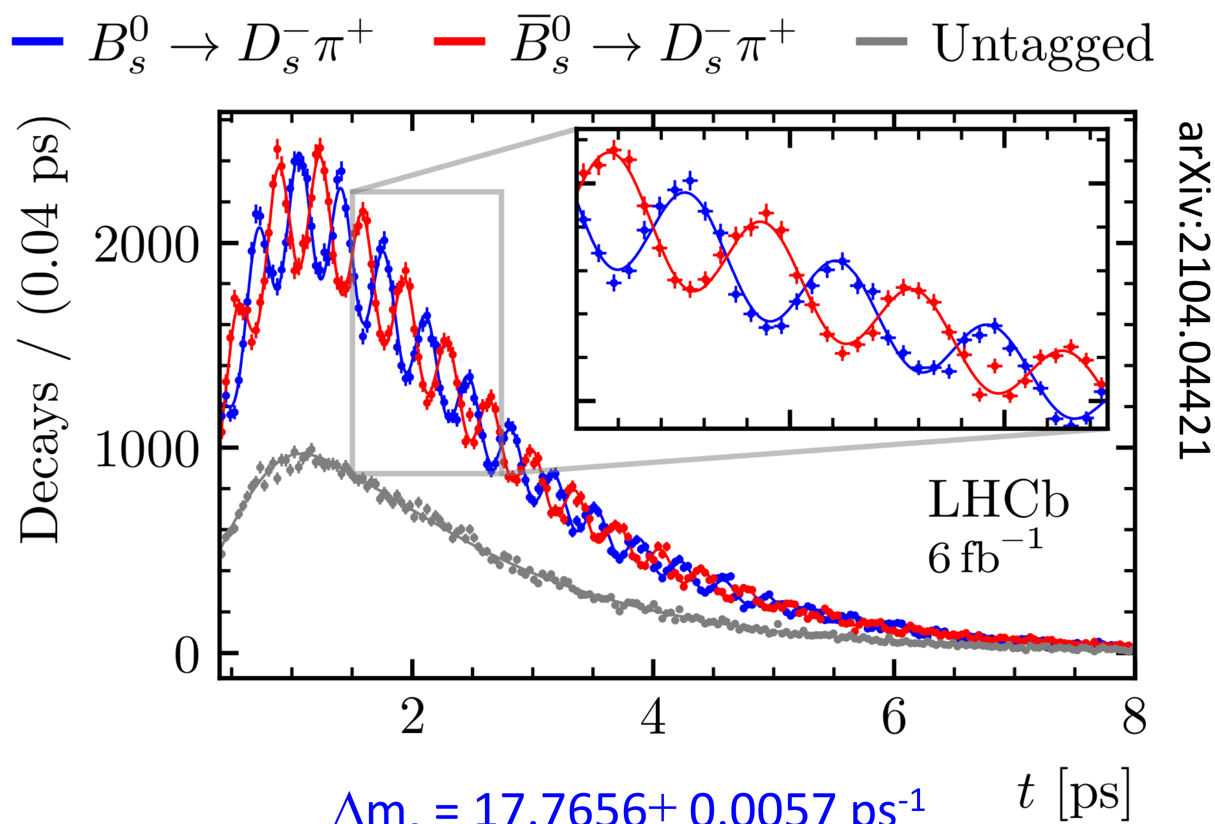
Invariant mass: $m^2 = \left(\sum p_i\right)^2$

Opposite B
Can be used for flavor tagging

$$PDF_{B(t)} \propto \left[e^{-\Gamma t} \cdot \left(1 \underset{B \to \bar{B}}{\overset{B \to B}{\pm}} \cos(\Delta m \cdot t) \right) \right]$$

$$P(B^0 \to B^0) = \frac{1}{2} e^{-\Gamma t} \left(\cosh\left(\frac{\Delta\Gamma}{2} t \right) + \cos \Delta m t \right)$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{2} e^{-\Gamma t} (\cosh\left(\frac{\Delta\Gamma}{2} t \right) - \cos \Delta m t)$$



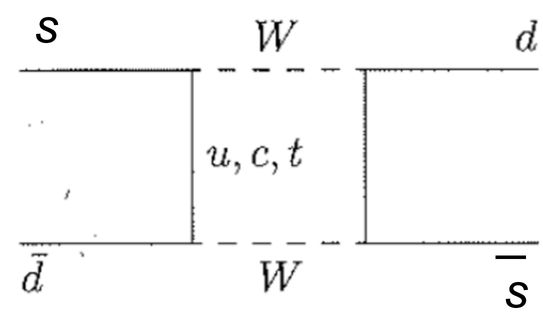
Δm_s = 17.7656± 0.0057 ps^{-1}

<u>For neutral Kaons:</u>

$$\left|V_{cd}V_{cs}^*\right|^2 \gg \left|V_{td}V_{ts}^*\right|^2$$

$$\sim \lambda^2 \qquad \sim \lambda^{10}$$

$$\sim 2.7 \cdot 10^{-2} \qquad \sim 1.1 \cdot 10^{-7}$$



$$\Rightarrow \quad M_{12} \sim S\left(m_c^2/M_W^2\right)\left|V_{cd}V_{cs}^*\right|^2$$

(everything else as above)

Experimentally: $\Delta m_K \approx 5.29 \cdot 10^{-3}\,\text{ps}^{-1} = 5.29 \cdot \text{ns}^{-1}$

Very slow mixing: visible only because of the very long life-times of the K_s and K_L

<u>Remark:</u>
Large contribution from real intermediate states is modifying Γ_{12} and thus Δm_K - difficult to calculate.

For D-mesons: (up-type quark system)

Mass of the most heaviest internal quark (d-type: b-quark) is not large enough to compensate the large CKM suppression ~ $\left|V_{ub}V_{cb}^{*}\right|^{2}$

As a result, the light s-quark dominates the short range mixing:

$$\Delta m_D \sim \left|V_{us}V_{cs}^{*}\right|^{2} S\left(m_s^2/M_W^2\right) \sim \lambda^2 \cdot m_s^2/M_W^2$$

exp.: $\Delta m_D \approx 0.0024\,\mathrm{ps}^{-1}$

Mixing parameters of the neutral D-meson are very small (very slow mixing): most of the D mesons decay before they mix (lifetime of D mesons ~1 ps is much shorter than the one of neutral Kaons).

D mixing was observed with high significance by LHCb – interpretation is difficult.

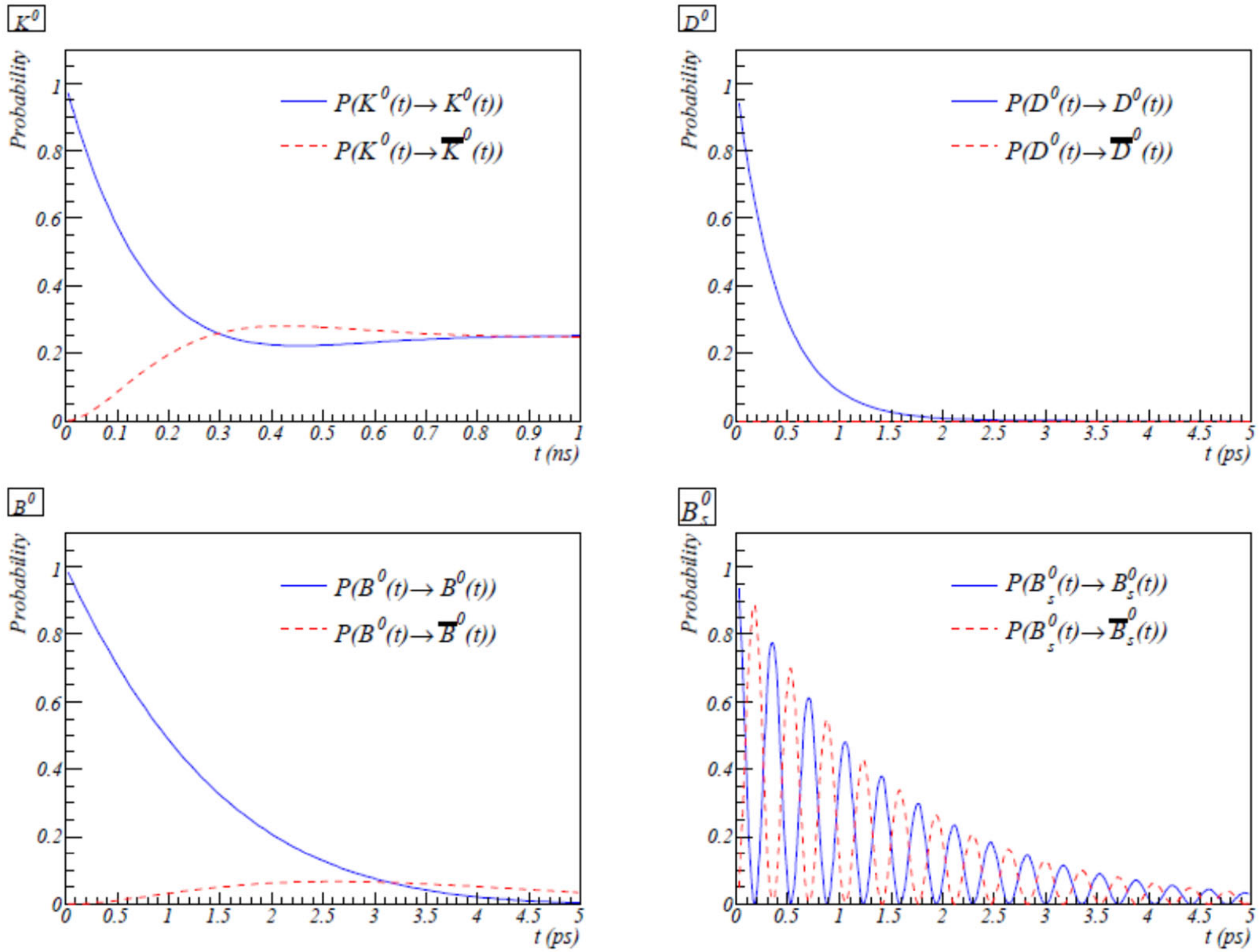


Figure 3.3: *If one starts with a pure P^0-meson beam the probability to observe a P^0 or a $\bar{P}^0$-meson at time t is shown,* $\mathrm{Prob}(t) = \frac{e^{-\Gamma t}}{2}\left(\cosh \frac{1}{2}\Delta\Gamma t \pm \cos \Delta m t\right)$.

Lecture note “CP Violation”, P. Kooijman & N. Tuning