4.4 Dirac vs Majorana: Neutrinoless Double-beta Decay

The problem of the nature of massive neutrinos v_i (Dirac or Majorana?) is one of the most fundamental problems of neutrino physics. The answer to this question will have an important impact on the understanding of the origin of neutrino masses.

The Majorana mass term breaks lepton number by two units - the Majorana mass term is the lowest dimension operator which uses SM fields and obeys SM gauge symmetries and which breaks lepton number at tree-level. In order to reveal the nature of neutrinos with definite masses it is necessary to study processes in which the total lepton number L is violated by two units (neither neutrino oscillations nor CC interactions can reveal the neutrino nature).

Lepton flavor violation experiments:

• In case of Majorana particle $v = v^{C}$ the following process becomes possible:

$$\pi^{+} \to \mu^{+} + \nu_{i}; \quad \nu_{i} + N \to \mu^{+} + p \quad \text{with } \mathcal{A}\left(\nu_{i}N \to \mu^{+}N\right) \sim \frac{m_{\nu}}{E_{\nu}} \to \sigma \sim \left(\frac{m_{\nu}}{E_{\nu}}\right)$$

neutrino beam

Thus the cross section for the observation of this reaction in a collider experiment (E_v larger than typ. 1 MeV, $m_v < 1 \text{ eV}$) is suppressed by (×10⁻¹²); Much too small for an observation with current experiments.

 $\searrow 2$

• Decays of B or K-mesons. E. g.: $K^+ \rightarrow \pi^- \mu^+ \mu^+$

Experimental bounds:
$$\frac{\Gamma(K^{+} \to \pi^{-} \mu^{+} \mu^{+})}{\Gamma(K^{+} \to \text{all})} \leq 3 \cdot 10^{-9} \quad \Box \Rightarrow$$

Limit on the effective mass $|m_{\mu\mu}| < 4 \cdot 10^4 \text{MeV}$ (not very strong) (meaning of "effective mass" : see below)

• Processes such as $\mu^- + (A,Z) \rightarrow (A,Z-2) + e^+$

Experimental bounds:
$$\frac{\Gamma(\mu^{-}\mathsf{Ti} \to e^{-}\mathsf{Ca})}{\Gamma(\mu^{-}\mathsf{Ti} \to \mathsf{all})} \leq 1.7 \cdot 10^{-12} \quad \Longrightarrow$$

Limit on the effective mass $|m_{\mu e}| < 82 \,\text{MeV}$ (not very strong) (meaning of "effective mass" : see below)

• The most sensitive probe to whether neutrinos are Dirac or Majorana states is the neutrinoless double-beta decay $(0v\beta\beta)$ of a nucleus.

$$(A,Z) \rightarrow (A,Z+2) + e^{-} + e^{-}$$

chirality flip

$$\mathcal{A}_{0\nu2\beta} \sim \sum_{i} m_{i} U_{ei}^{2}$$

$$m_{ee} \quad \text{(similar definition for } m_{\mu\mu} \text{ and } m_{\mu e}$$

Under the assumption that the Majorana neutrino mass is the only source of lepton number violation at low energies, the decay half-life is given by:

$$\Gamma_{1/2}^{0\nu} \sim \left(T_{1/2}^{0\nu}\right)^{-1} = \mathbf{G}^{0\nu} \left|\mathcal{M}^{0\nu}\right|^2 \left(\frac{m_{ee}}{m_e}\right)^2$$

 $G^{0\nu}$ is the phase space integral taking into account the final atomic state; $\mathcal{M}^{0\nu}$ is the nuclear matrix element of the transition;

 m_{ee} is the effective Majorana mass of v_e :

$$m_{ee} = \left| \sum_{i} m_{i} U_{ei}^{2} \right|$$

Note that the term $\sum m_i U_{ei}^2$ is in general complex and depends on the phases of the PMNS elements (δ_{CP} and the two Majorana phases $\eta_{1,2}$)

Thus, in addition to the masses and mixing parameters the decay spectrum depends also on the leptonic CP violating phases (\rightarrow allows determination):

$$m_{ee} = \left| \sum_{i} m_{i} U_{ei}^{2} \right| = \left| m_{1} c_{13}^{2} c_{12}^{2} e^{i2\eta_{1}} + m_{2} c_{13}^{2} s_{12}^{2} e^{i2\eta_{2}} + m_{3} s_{13}^{2} e^{-i2\delta_{CP}} \right|$$

arXiv:1811.05487

One can discuss two different mass orderings:

(inspired by experimental data)

- 1. Normal ordering (NO): $m_1 < m_2 < m_3$; $\Delta m_{12}^2 \ll \Delta m_{23}^2$; $\Rightarrow m_1 < m_2 \ll m_3$
- 2. Inverted ordering (IO): $m_3 < m_1 < m_2$; $\Delta m_{12}^2 \ll |\Delta m_{13}^2|$; $\Rightarrow m_3 \ll m_1 < m_2$

$$\begin{split} m_{ee} &= \left| \sum_{i} m_{i} U_{ei}^{2} \right| \qquad \text{with } m_{0} = m_{1} \text{ (NO), } m_{3} \text{ (IO), smallest mass} \\ &= \left\{ \begin{array}{l} \left| m_{0} c_{12}^{2} c_{13}^{2} + \sqrt{\Delta m_{21}^{2} + m_{0}^{2}} s_{12}^{2} c_{13}^{2} e^{2i(\eta_{2} - \eta_{1})} + \sqrt{\Delta m_{32}^{2} + \Delta m_{21}^{2} + m_{0}^{2}} s_{13}^{2} e^{-2i(\delta_{\text{CP}} + \eta_{1})} \right| \quad \text{in NO }, \\ m_{0} s_{13}^{2} + \sqrt{m_{0}^{2} - \Delta m_{32}^{2}} s_{12}^{2} c_{13}^{2} e^{2i(\eta_{2} + \delta_{\text{CP}})} + \sqrt{m_{0}^{2} - \Delta m_{32}^{2} - \Delta m_{21}^{2}} c_{12}^{2} c_{13}^{2} e^{2i(\eta_{1} + \delta_{\text{CP}})} \right| \quad \text{in IO }, \end{split}$$

$$\begin{bmatrix} \leq \left(\sin^2 \theta_{12} \sqrt{\Delta m_{12}^2} + \sin^2 \theta_{13} \sqrt{\Delta m_{23}^2}\right) & \text{in NC} \\ \simeq \sqrt{|\Delta m_{13}^2|} & (1 - \sin^2 2 \theta_{12} \sin^2 \alpha)^{\frac{1}{2}}, & \text{in IO} \\ \text{S.Bilenky (2010)} & \alpha \text{ is N} \end{bmatrix}$$

in NO: m_{ee} can me arbitrarily small

in IO: there is a lower bound on m_{ee}_{α} is Majorana phase diff. 72

Neutrinoless double beta decay can help to resolve the neutrino mass hierarchy (of course only if neutrinos are Majorana particles):



Searching for neutrinoless double-beta decay:

2β decay:

mass parabola from Weizsäcker formula



Possible 2β candidates:

Transition	$T_0 = Q_{\beta\beta} \; (\text{KeV})$
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	$2,039.6 \pm 0.9$
100 Mo \rightarrow 100 Ru	$3,934 \pm 6$
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	$2,533 \pm 4$
136 Xe \rightarrow 136 Ba	$2,479 \pm 8$
150 Nd \rightarrow 150 Sm	$3,367.1 \pm 2.2$
82 Se $\rightarrow ^{82}$ Kr	$2,995 \pm 6$
$^{48}Ca \rightarrow ^{48}Ti$	$4,271 \pm 4$

Normal β -decay energetically forbidden for ⁷⁴Ge. Double β -decay allowed: even-even nuclei. $T_{1/2}^{2\nu}(^{76}\text{Ge}) =$ (1.929 ± 0.095) · 10²¹ yr
arXiv:1501.02345

Two-neutrino double β decay is a process of second order in the Fermi constant G_F, which is governed by the standard CC Hamiltonian of the weak interaction. This decay was observed in more than ten different nuclei with half-lives in the range $(10^{18}-10^{24})$ years.



Source = Detector

Decay & detection material ⁷⁶Ge:

- Ge is a 2β decay isotope
- Source material = detector material
- Germanium detectors (=semiconductor) have excellent energy resolution: FWHM ~ 1.5 10⁻³ @ 2 keV
- Enrichment of ⁷⁶Ge up to 86%



Ge diode w/ reverse biasing

GERDA Experiment



Problem: Shielding against natural radioactivity and possible radioactive pollution producing background around the end-point.

- Located in Hall A at Laboratori Nazionali del Gran Sasso of INFN
- ▶ 3800 mwe overburden (μ flux ~ 1 m⁻²h⁻¹))
- Array of bare Ge detectors 86% enriched in ⁷⁶Ge directly inserted in liquid argon (LAr)

mwe = meter water equivalent

Material	Activity $[\mu Bq/Kg]$
Rock, concrete Stainless steel Cu (NOSV), Pb	$3000000\ \sim 5000\ < 20$
Purified water LN ₂ , LAr	$< 1 \ \sim 0$



Y. KERMAIDIC at Neutrino 2020



Future: Large Enriched Germanium Experiment for neutrinoless 2β decay LEGEND

The collaboration aims to develop a phased, ⁷⁶Ge based double-beta decay experimental program with discovery potential at a half-life beyond 10²⁸ years, using existing resources as appropriate to expedite physics results."



⁷⁶Ge (88% enr.)

CUORE - ¹³⁰Te Bolometer Experiment



CUORE uses bolometers to search for neutrinoless double beta ($0\nu\beta\beta$) decay and other rare processes. The bolometers are ultra-cold tellurium dioxide (TeO₂) crystals containing the candidate $0\nu\beta\beta$ isotope ¹³⁰Te. Every time a tellurium nucleus decays or a particle interacts in the crystal, it releases a minute amount of energy (less than a few MeV), causing the temperature of the crystal to rise slightly.



This rise in temperature is then converted into an electrical signal using temperaturedependent resistors (thermistors). For this temperature rise to be measurable, the baseline temperature of the crystals must be very low. We use ultra-cold cryogenic temperatures: a few thousandths of a degree above absolute zero. 80

arXiv:1912.10966.



No evidence for $0\nu\beta\beta$ decay. A 90% CL lower limit of 3.2×10^{25} yr on the ¹³⁰Te half-life for this process is set.



Constraints on mass hierarchy



In the hypothesis that $0\nu\beta\beta$ decay is mediated by light Majorana neutrinos, the Cuore limit results in an upper limit on the effective Majorana mass of 75–350 meV, depending on the nuclear matrix elements used.

4.5 Neutrino mass scale determination

In case of massive neutrinos the flavor states are linear combinations of the mass states. Mass limits can only be put on the effective mass of a neutrino with lepton flavor I:

$$m_{\nu_\ell, eff}^2 = \sum_i \left| \boldsymbol{U}_{\ell i} \right| m_i^2$$

Upper bounds on neutrino masses can be deduced from weak decays:

(³ H decay)	PDG 2019	
$n \rightarrow p + e^- + \overline{v}_e$	$m_{\overline{v}_{e}, eff} < 2 eV $ (update)	Study energy distribution of visible
$\mu^{\pm} \rightarrow v_{\mu} + \mathbf{e}^{\pm} + v_{\mathbf{e}}$	$m_{_{\nu_{\mu}},\mathrm{eff}} < 0.19\mathrm{MeV}$	 final state particles: "missing" invariant mass
$\tau^{\pm} \rightarrow \mathbf{n} \cdot \pi + v_{\tau}$	$m_{\nu_{\tau}, eff} < 18.2 \mathrm{MeV}$	\rightarrow neutrinos mass

Upper bounds also exist from cosmology:

Large scale structure of galaxies, cosmic microwave background, type Ia supernovae, and big bang nucleosynthesis: $\sum m_i < 0.26 \text{ eV}$ arXiv:1811.02578v2

a) Effective electron anti-neutrino mass:

End-point method of a β -emitter (tritium ³H)

$$\frac{dN}{dE} = C p(E + m_e) (E_0 - E) \sqrt{(E_0 - E)^2 - (m_{\nu_e}^{eff})^2} \cdot F(Z, E)$$
$$\equiv R(E) \sqrt{(E_0 - E)^2 - (m_{\nu_e}^{eff})^2}$$

 E_0 = Mass diff. of nuclei E = kin energy of electron P = e momentum F Fermi function



Experimental requirements:

- High activity source
- Excellent energy resolution

"Direct" kinetic measurement: spectral distortion measures the "effective" mass squared: Effective neutrino mass – consider mixing:

$$\frac{dN}{dE} = R(E)\sqrt{\left(E_0 - E\right)^2 - \left(m_{v_e}^{\text{eff}}\right)^2} \quad \text{with:} \quad m_{v_e,\text{eff}}^2 = \sum_i \left|U_{\ell i}\right|^2 m_i^2$$

The KATRIN experiment has provided an upper bound for the effective neutrino mass:

$$1.1 \,\mathrm{eV} \geq m_{\nu_e}^{\mathrm{eff}} = \sqrt{\sum_i \left| U_{\ell i} \right|^2 m_i^2}$$

Depending on the neutr hierarchy this leads to a dence on the light neutr

KATRIN = Karlsruhe Tritium Neutrino Experiment

Goal: measure neutrino mass w/ sensitivity of 0.2 eV (90%CL)



MAC-E Filter - Principle

Electrostatic spectrometer:



B fields serves to align the electron directions.



Adiabatic variation of B-field leads to alignment of momentum vector.



arXiv:1909.06048

KATRIN-Results:

First results from a 4 weeks measurement; Source activity 2.45×10^{10} Bq (Tritium density 1/5 of nominal).

Fit in the interval around the kinematic endpoint at 18.57 keV gives an effective neutrino mass square value of

$$m_{\nu,\text{eff}}^2 = (-1.0^{+0.9}_{-1.1}) \text{eV}^2$$

From this an upper limit of

 $m_{v,eff} < 1.1 \,\mathrm{eV} \ (90\% \,\mathrm{CL})$

on the absolute mass scale of neutrinos is derived.

Sensitivity after 1000 days of data-taking and nominal tritium density: 0.2 eV



FIG. 3. a) Spectrum of electrons $R(\langle qU \rangle)$ over a 90 eVwide interval from all 274 tritium scans and best-fit model $R_{\rm calc}(\langle qU \rangle)$ (line). The integral β -decay spectrum extends up to E_0 on top of a flat background $R_{\rm bg}$. Experimental data are stacked at the average value $\langle qU \rangle_l$ of each HV set point and are displayed with 1- σ statistical uncertainties enlarged by a factor 50. b) Residuals of $R(\langle qU \rangle)$ relative to the 1- σ uncertainty band of the best fit model. c) Integral measurement time distribution of all 27 HV set points.

Holmium Electron Capture:

Slide by K. Valerius



Challenges:

- production & purification of isotope ¹⁶³Ho
- incorporation of ¹⁶³Ho into high-resolution detectors
- operation & readout of large calorimeter arrays
- detailed understanding of calorimetric spectrum (nuclear & atomic physics + detector response)

How to measure 2.8 keV w/ high precision?

Micro Calorimeters: MMCs





ECHo Experiment

Uni Heidelberg: C. Enss, L. Gastaldo

