4. Neutrino masses and Majorana Neutrinos

From neutrino oscillation experiments we know that neutrinos are massive fermions. Cosmological observations yield an upper bound on the sum of masses:

$$\sum_{i=1,2,3} m_{\nu_i} \leq 0.2\,\text{eV}$$

Do neutrinos obtain their mass through the Higgs-mechanism? If yes, why are the Yukawa couplings (masses) so much smaller than for all other SM fermion?

Closer look to the possible mass terms that respect Lorentz and gauge invariance. For charged fermions in the SM the only possible mass term is the so-called Dirac mass:

$$\mathcal{L} = -m\bar{\psi}\psi = -m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \quad \text{with} \quad \bar{\psi} = \psi^\dagger\gamma^0$$

Mass terms mix LH and RH chiral states

For neutral particles (they could be their own anti-particles) other Lorentz-invariant combinations are also possible as mass terms : $\bar{\psi}^c\psi$ and $\bar{\psi}\psi^c$

(with ψ^c is the C-conjugated state)

Charge-conjugation operator C (particle-anti-particle transformation):

To derive the charge-conjugation operator C one examines the Dirac Eqs. of an electron and of its anti-particle (positron) in an electric field:

Electron $$\left[\gamma^\mu\left(i\partial_\mu + eA_\mu\right) - m\right]\psi = 0$$

Positron $$\left[\gamma^\mu\left(i\partial_\mu - eA_\mu\right) - m\right]\psi^C = 0$$

From these Eqs one finds (see text books) for ψ^c and the C-operator:

$$\psi^c = i\gamma_2\psi^* = i\gamma_2\gamma_0\bar{\psi}^T \equiv C\bar{\psi}^T$$

with $\bar{\psi} = \psi^\dagger\gamma^0$

The C-operator flips all charge-like quantum-numbers. One finds:

C is real, anti-symmetric and unitary

$$C^\dagger = C^T = C^{-1} = -C$$

$$C\gamma_\mu C^{-1} = \gamma_\mu^T$$

$$C\gamma_5 C^{-1} = \gamma_5^T$$

$$C\gamma_\mu\gamma_5 C^{-1} = \left(\gamma_\mu\gamma_5\right)^T$$

$$\left(\psi^c\right)^c = \psi$$

$$\bar{\psi}^c = \psi^T C$$

$$\left(\bar{\psi}_1\psi_2^c\right)^\dagger = \bar{\psi}_2^c\psi_1$$

And further: $$\left(\psi_{L,R}\right)^C = \left(\frac{\left(1\pm\gamma_5\right)}{2}\psi\right)^C = \frac{\left(1\mp\gamma_5\right)}{2}\psi^C = \left(\psi^C\right)_{R,L}$$

(see e.g. K. Zuber, Neutrino Physics)

C-operator flips the chirality!

4.1 Dirac mass terms

Dirac masses for neutrinos can be created in the SM by extending the particle content and by adding a RH neutrino singlet ν_R:

$$\mathcal{L}_{Yukawa}^{Neutrino} = -\sum_{i,j} \left\{ Y_\nu^{ij} \overline{L}_L^i \nu_R^j H + h.c. \right\}$$

ν_R is a singlet under all SM gauge transformations: $\rightarrow$ no interaction = sterile

Resulting in a Dirac mass term $m\bar{\nu}\nu$ after symmetry breaking:

$$\mathcal{L}_{Mass}^{D} = -\sum_{i,j} \left\{ \overline{\nu}_L^i M_D^{ij} \nu_R^j + h.c. \right\}$$

i, j are the flavor indices: e, μ, τ
M_D is 3x3 complex matrix, in general non-diagonal.

Mass terms are invariant under a global phase transformation:
$\rightarrow$ From invariance follows the conservation of lepton-number.

$$\nu'^i_{L,R} = e^{i\Lambda} \nu^i_{L,R}$$
$$\ell'^i = e^{i\Lambda} \ell^i$$

The diagonalization of the mass term follows the procedure we applied for quarks:

$$M_D = U_L^\dagger m V_R$$

with two unitary matrices to transform the LH and RH chiral components independently:

$$\nu_L^\ell = \sum_{i=1}^{3} U_L^{\ell i} \nu_L^i \quad \text{and} \quad \nu_R^\ell = \sum_{i=1}^{3} V_R^{\ell i} \nu_R^i \qquad \ell = e, \mu, \tau$$

Expressed in the mass eigenstates ν^i the mass term takes the form:

$$\mathcal{L}^D_{Mass} = -\sum_{i=1}^{3}\left\{m_i \bar{\nu}^i_L \nu^i_R + h.c.\right\} = -\sum_{i=1}^{3} m_i \bar{\nu}^i \nu^i$$

- $\nu_{1,2,3}$ are the neutrino mass eigenstates with masses $m_{1,2,3}$
- The LH flavor states $\nu_{e,\mu,\tau}$ which enter into the standard charged and neutral currents are linear combinations of the mass states.
- The unitary matrix U is called PMNS matrix (see above) – V does not enter
- The Lagrangian is invariant under global phase transformation: lepton number conservation.

The smallness of the neutrinos masses are a result of very tiny Yukawa couplings

$$M_{ij} = \frac{\upsilon}{\sqrt{2}} Y_{ij} \rightarrow Y_{ij} \sim O\left(10^{-12}\right)$$

It is not clear why compared to the quark sector the differences between the ν masses and the charged leptons masses are so large.

The RH neutrino singlet have weak hypercharge Y=0 and weak isospin I=0-
They do not interact with anything: sterile neutrinos.

Massive Dirac neutrinos ν and their anti-particles ν^C are described by four independent chiral components:

$$\nu_L, \nu_R, \nu_L^C, \nu_R^C$$

In the “original” formulation of the Standard Model, neutrinos masses are zero because of the missing RH ν-singlets. The observation of non-vanishing neutrino masses indicates thus physics beyond the Standard Model.

Remark: Nowadays massive neutrinos are often treated as “part of the SM” assuming the existence of RH neutrinos: the additional new particle is not modifying the gauge structure for the theory.

The very small neutrino masses as well as the existence of a sterile particle is not motivated.

4.2 Majorana mass terms

As indicated above, mass terms with $\bar{\psi}^c\psi$ and $\bar{\psi}\psi^c$ also satisfy Lorentz invariance. Moreover $(\psi_L)^C$ is a RH chiral state – thus no need to introduce an additional RH neutrino component. Using LH particle and the anti-particle spinor (RH), the mass term would have the following form:

$$\mathcal{L}^M_{Mass} = -\frac{1}{2}\sum_{i,j}\left\{\bar{\nu}^i_L M^{ij}_M \left(\nu^j_L\right)^C + h.c.\right\}$$

i, j are the flavor indices: e, μ, τ

With the matrix M_M (3x3 complex matrix, in general non-diagonal) the mass term can be rewritten in the following matrix form:

$$\mathcal{L}^M_{Mass} = -\frac{1}{2}\bar{\nu}_L \mathbf{M}_M \left(\nu_L\right)^C + h.c. \quad \text{with} \quad \nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

We can diagonalize the matrix $\mathbf{M_M}$ with an unitary transformation **U:**

$$\mathcal{L}^M_{Mass} = -\frac{1}{2}\bar{\nu}^M \mathbf{m}\, \nu^M \quad \text{with} \quad \nu^M = U^\dagger \nu_L + \left(U^\dagger \nu_L\right)^C = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (*)$$

$$\mathbf{m} = diag\left(m_1, m_3, m_3\right)$$

ν_i is the field of the neutrino with mass m_i. From Eq. (*) follows:

$$\left(\nu^M\right)^C = \nu^M$$

Thus the fields of the neutrinos with definite mass satisfy the Majorana condition:

$$\nu_i^C = \nu_i$$

For neutrino field satisfying the Majorana condition: neutrino = antineutrino

The field ν^M is the sum of a LH and RH component:

$$\nu^M = \nu_L^M + \nu_R^M$$

Comparing with $\nu^M = U^\dagger \nu_L + \left(U^\dagger \nu_L\right)^C$ one finds

$$\nu_L^M = U^\dagger \nu_L \quad \text{and} \quad \nu_R^M = \left(U^\dagger \nu_L\right)^C$$

i.e. the LH and RH component of the Majorana field are connected by

$$\nu_R^M = \left(\nu_L^M\right)^C \quad \text{and consequently} \quad \nu_{i,R} = \left(\nu_{i,L}\right)^C$$

Which means also the fields $\nu_{1,2,3}$ satisfy the condition

$$\nu_i = \nu_{i,L} + \left(\nu_{i,L}\right)^C$$

While for Dirac fermion LH and RH components are independent, for Majorana fermions they are connected: $\nu_{i,R} = \left(\nu_{i,L}\right)^C$

Under a global phase transformation the two components transform as:

$$\nu'_{i,L} = e^{i\Lambda}\nu_{i,L} \quad \text{and} \quad \left(\nu'_{i,L}\right)^C = e^{-i\Lambda}\left(\nu_{i,L}\right)^C$$

Mass terms $m_i \bar{\nu}_{i,L}\left(\nu_{i,L}\right)^C$ therefore violates lepton number conservation.

It should be stressed that in case of the introduced Majorana mass term only active left-handed neutrino fields $\nu_{l,L}$ (RH anti-neutrinos) enter the total Lagrangian: weak interaction cannot distinguish if neutrinos are Dirac or Majorana fermions.

Mass terms $m_i \bar{\nu}_{i,L}\left(\nu_{i,L}\right)^C$ cannot be generated in a gauge invariant way within the SM:

$$\nu_L \left\{ \begin{array}{l} I_3 = +\frac{1}{2} \\ Y = -1 \end{array} \right. \qquad \left(\bar{\nu}_L\right)^C \nu_L \left\{ \begin{array}{l} I_3 = +1 \\ Y = -2 \end{array} \right.$$

To generate such a mass term via a Higgs-coupling a Higgs triplet with I=1, Y=2 is necessary $\rightarrow$ does not exist in SM.

Neutrino mass term (Dirac or Majorana) requires physics beyond SM:
ν_R or Higgs-triplet or new mass generation.

4.3: Dirac and Majorana mass terms and seesaw model

For simplicity we discuss here only the case of one neutrino generation. For 3 generations a diagonalization of the mass matrices is required – only a technical complication.

The most general Lorentz invariant mass term has a Dirac and Majorana contributions for LH and RH neutrinos:

$$\mathcal{L}^{D+M} = -\frac{1}{2} m_L \bar{\nu}_L (\nu_L)^C - m_D \bar{\nu}_L \nu_R - \frac{1}{2} m_R \overline{(\nu_R)^C} \nu_R + h.c.$$

m_L and m_R are LH and RH Majorana masses, m_D is the Dirac mass.

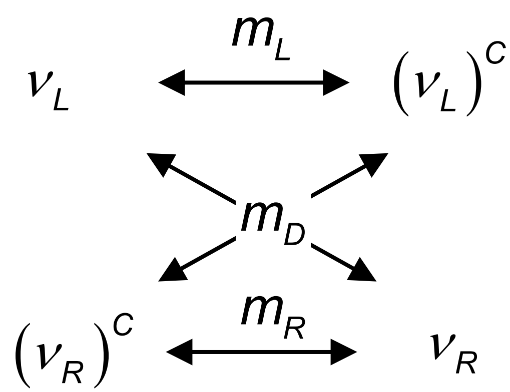
Introducing the neutrino vector n_L the mass term can be written in matrix form:

$$\mathcal{L}^{D+M} = -\frac{1}{2} \bar{n}_L \mathbf{M}^{D+M} (n_L)^C + h.c. \quad \text{with} \quad n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^C \end{pmatrix} \text{ and } \mathbf{M}^{D+M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

$$(n_L)^C = \begin{pmatrix} (\nu_L)^C \\ \nu_R \end{pmatrix} = \begin{pmatrix} \nu_R^C \\ \nu_R \end{pmatrix}$$

Mass matrix couples the chiral states in the following way:

$$\mathbf{M}^{D+M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$



The chiral fields ν_L and $(\nu_R)^C = \nu^C_L$ are not the mass eigenstates - these are found by diagonalizing the matrix $\mathbf{M}^{D+M}$ using the orthogonal matrix $\mathbf{O}$:

$$\mathbf{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad \mathbf{M}^{D+M} = \mathbf{O}\mathbf{M}'\mathbf{O}^T \qquad \mathbf{M}' = diag(m_1', m_2')$$

with $\tan\theta = \dfrac{2m_D}{m_R - m_L}$ and $m'_{1,2} = \dfrac{1}{2}(m_R + m_L) \mp \dfrac{1}{2}\sqrt{(m_R - m_L)^2 + 4m_D^2}$

As $m'_{1,2}$ can be positive and negative one rewrites $m'_{1,2}$

$$m_i' = \eta_i m_i \text{ with } \eta_i = \pm 1 \text{ and } m_i > 0$$

Taking this into account one can express the diagonalization of $\mathbf{M}^{D+M}$ as

$$(*) \qquad \mathbf{M}^{D+M} = \mathbf{O}\eta\mathbf{M}\mathbf{O}^T = U\mathbf{M}U^T$$

with $\mathbf{M} = diag(m_1, m_2)$ and $U = \sqrt{\eta}O =$ unitary matrix

For the neutrino mass eigenstates one finds from $(*)$:

$$(**) \qquad \nu^M = U^\dagger n_L + \left(U^\dagger n_L\right)^C = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Definition of the Majorana neutrino (see p. 59)

$$\nu^M = U^\dagger \nu_L + \left(U^\dagger \nu_L\right)^C$$

and thus $$\mathcal{L}^{D+M} = -\frac{1}{2}\bar{\nu}^M \mathbf{M}\nu^M = -\frac{1}{2}\sum_{i=1}^{2} m_i \bar{\nu}_i \nu_i$$

Evidently $(\nu_i)^C = \nu_i \rightarrow$ mass eigenstates are Majorana neutrinos.

Using $(**)$ one obtains the following mixing equation:

$$\nu_L = \sqrt{\eta_1}\cos\theta\nu_{1,L} + \sqrt{\eta_2}\sin\theta\nu_{2,L}$$
$$\left(\nu_R\right)^C = -\sqrt{\eta_1}\sin\theta\nu_{1,L} + \sqrt{\eta_2}\cos\theta\nu_{2,L}$$

The parameter η_l determines the CP parity of the Majorana neutrino ν_i.

Seesaw model: (simples case for one neutrino family)

The seesaw mechanism was proposed at the end of the 1970s and is based on the Dirac and Majorana mass terms. It is a natural and viable way to generate neutrino masses.

The three parameters m_L, $m_{R,}$ and m_D characterize the LH and RH Majorana mass terms and the Dirac mass term. The mass eigenstates characterized by m_1 and m_2 are Majorana states (see above).

Assumptions:

1. There is no LH Majorana mass term
2. Dirac mass term generated by a SM higgs-coupling $\rightarrow$ m_D is of the order of a lepton or quark mass.
3. RH Majorana mass term $\neq 0$ for neutrino N_R, breaks lepton number conservation: we assume that this happens at a mass scale M_R much larger than the electroweak scale:

$$m_R \equiv M_R >> M_W, M_Z >> m_D$$

One obtains for the mass eigenvalues (see above): See p. 62

$$m_1 \approx \frac{m_D^2}{m_R} = \frac{m_D^2}{M_R} << m_D \qquad m_2 \approx M_R >> m_D$$

$$m'_{1,2} = \frac{1}{2}(m_R + m_L) \mp \frac{1}{2}\sqrt{(m_R - m_L)^2 + 4m_D^2}$$

With the mixing angle $\theta \approx \frac{m_D}{M_R} << 1$ and w/ $\eta_1 = -1$ and $\eta_2 = +1$:

⟹ mixing relations:

$$\nu_L = i\nu_{1,L} + \frac{m_D}{M_R}\nu_{2,L}$$

$$(N_R)^C = -i\frac{m_D}{M_R}\nu_{1,L} + \nu_{2,L}$$

For the physical states one obtains (up to phases):

$\nu_1 \approx \nu_L$ LH neutrino w/ low mass → active

$\nu_2 \approx N_R$ RH neutrino w/ high mass → sterile

<u>Estimation of scale M_R :</u>

$$m_D \le m_t \approx 170\,\text{GeV}$$

$$m_1 \approx \sqrt{\Delta m^2}\Big|_{\substack{\text{heaviest}\\ \text{Neutrino}}} \approx 5\cdot 10^{-2}\,\text{eV}$$

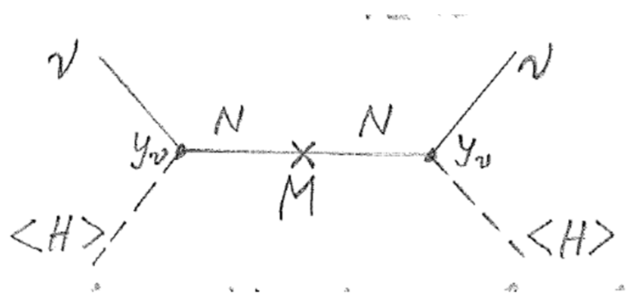
$$\Longrightarrow \quad M_R \approx \frac{m_D^2}{m_1} \approx 10^{15}\,\text{GeV}$$

Thus, the seesaw mechanism explains why the neutrinos which we observe are so light. The energy scale M_R is far beyond the energies colliders can reach.

The Majorana mass term can be generated through BSM extensions of the Standard Model:

Interaction of "lepton-Higgs pairs" with a heavy Majorana singlet fermion N_R. Due to the "decoupling" of the heavy neutrino the virtual exchange of N_R leads to an effective dimension five operator (gauge invariant) with only SM fields:

(only dim-5 operator which breaks lepton number at tree-level)



$$\mathcal{L}_{eff} \sim \frac{c}{\Lambda_{NP}} \left(\bar{L}_L \tilde{\phi}\right)\left(\tilde{\phi}^T L_L^C\right) + h.c.$$

$$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \tilde{\phi} = \sigma_2 \phi = \sigma_2 \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

After symmetry breaking

$$\mathcal{L}_{eff} \sim \frac{c}{\Lambda_{NP}} \frac{v^2}{2} \nu_L \nu_L^C + h.c.$$

In the limit $M \to \infty$, where N_R decouples, neutrinos are effectively massless.

$$\frac{c}{\Lambda_{NP}} \frac{v^2}{2} = \frac{y_\nu^2 v^2}{2M} \equiv m_\nu$$

We notice that a theory where New Physics is composed of heavy sterile neutrinos, provides an specific example of a theory which at low energy contains three light mass eigenstates with an effective dim-5 interaction with Λ_{NP}=M. In this case the New Physics scale is the characteristic mass scale of the heavy sterile neutrinos

If seesaw is realized in nature:

- Neutrinos are Majorana particles
- Neutrino masses are much smaller than lepton and quark masses
- Sterile heavy Majorana particle - seesaw partner – must exist.

Question:

If neutrinos are Majorana particles $\nu = \bar{\nu}$ why does the reaction

$$\bar{\nu} + n \rightarrow e^- + p$$

not exist? $\rightarrow$ only the LH component of $\nu = \bar{\nu}$ can interact in the weak charged current reaction: strong helicity suppression (see below).

Neutrino mixing for Majorana neutrinos:

The weak eigenstates ν_α which by default are the states produced in the weak CC interaction of a charged lepton l_α (flavor eigenstates) are the linear combinations of the mass eigenstates ν_i determined by the PMNS mixing matrix U:

$$\left|\nu_\alpha\right\rangle = \sum_{i=1}^{3} U^*_{\alpha i}\left|\nu_i\right\rangle$$

While for Dirac neutrinos the PMNS mixing matrix is given by three mixing angles and one phase δ.

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For Majorana neutrinos there are two additional Majorana phases which can not be absorbed in the redefinition of the neutrinos states:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(different conventions, here PDG convention)