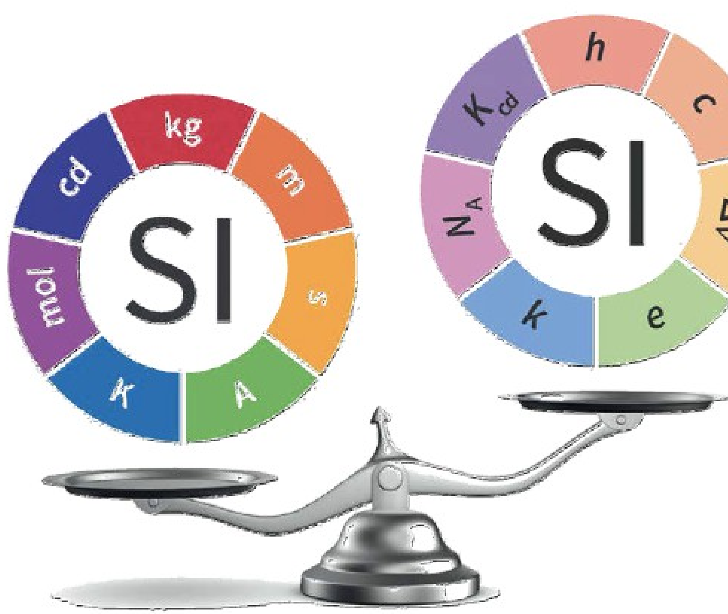
**the new international system of units (SI)**
**a (also historic) introduction**
**pbm, Friday, Nov. 15, 2019**

**Master/Bachelor seminar**
**WS 2019/2020**

Replacing the Ur-Kilogram with High-Precision: The new system of units and the precise determination of the defining constants.



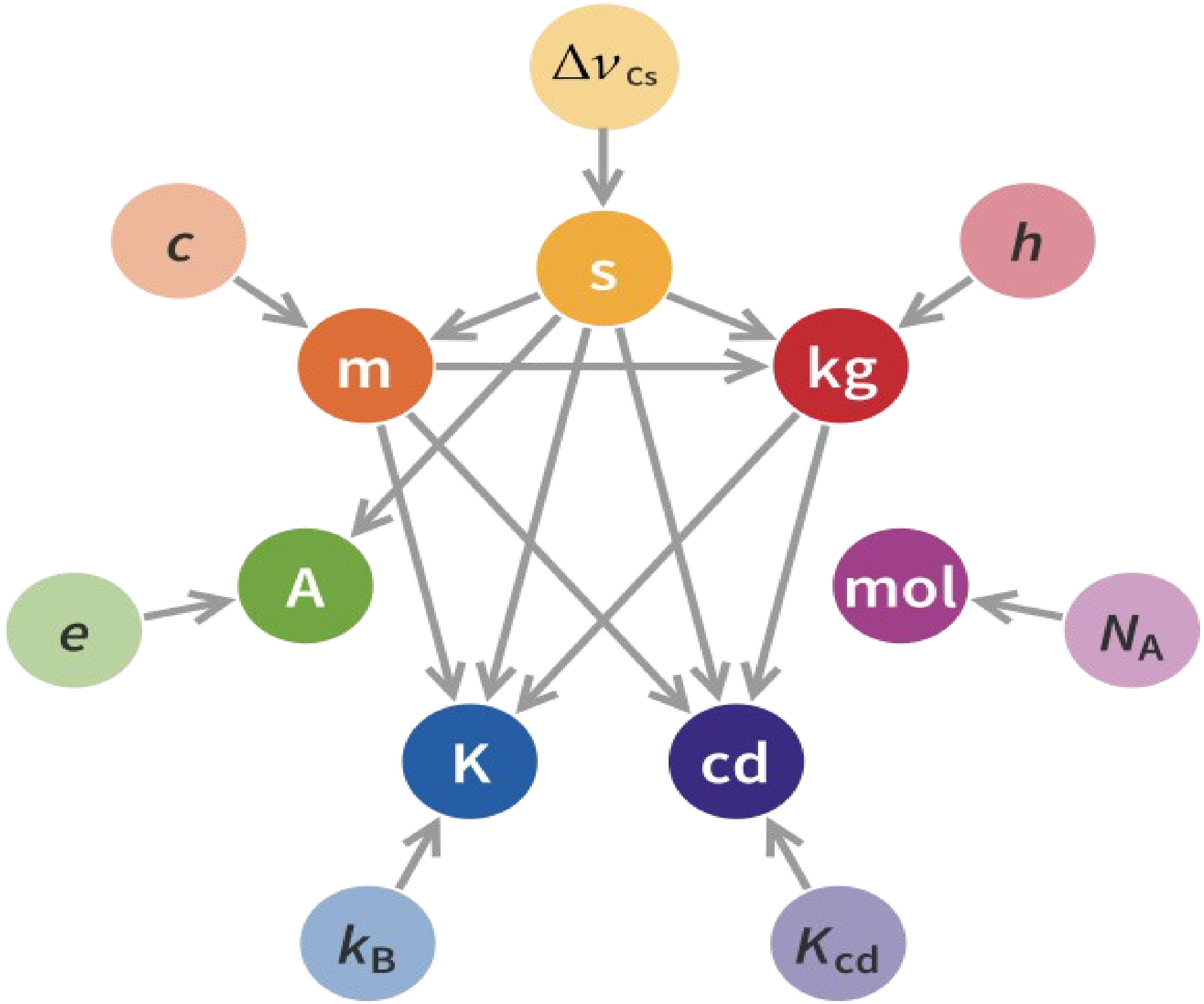
References:

Liebisch et al, Annalen der Physik 2019 and references there

Ernst Goebel and Uwe Siegner, The new international system of units, Wiley-VCH, 2019

Wikipedia entry on the international SI system

# New SI



# historical comments

the metric system originated during the French revolution. As member of the Academie Francaise, Antoine de Lavoisier, scientist, founder of modern chemistry, proposed 1790 to set-up a decimal system based on the kilogram and meter.
The basic units were taken from the natural world: the unit of length, the meter, was based on the dimensions of the Earth, and the unit of mass, the kilogram, was based on the mass of water having a volume of one liter or a 1000 cubic centimeters.

The system was set-up in 1799. Lavoisier died on the guillotine in 1794.

Maxwell in the middle of the $19^{th}$ century defined the base units mass, length and time, from which all other units should be derived. Giovanni Giorgi in 1901 proved that a coherent system that incorporated electromagnetism had to have an electromagnetic unit as a fourth base unit.

In 1960, the CGPM launched the International System of Units (in French the Système international d'unités or SI) which had six "base units": the meter, kilogram, second, ampere, kelvin and candela. A seventh base unit, the mole was added later. During this period, the meter was redefined in terms of the speed of light, and the second was redefined in terms of the microwave frequency of a cesium atomic clock.

**On November 16, 2018, in Versailles, France, a group of 60 countries  vote unanimously for the new international system that underpins global science and trade.**

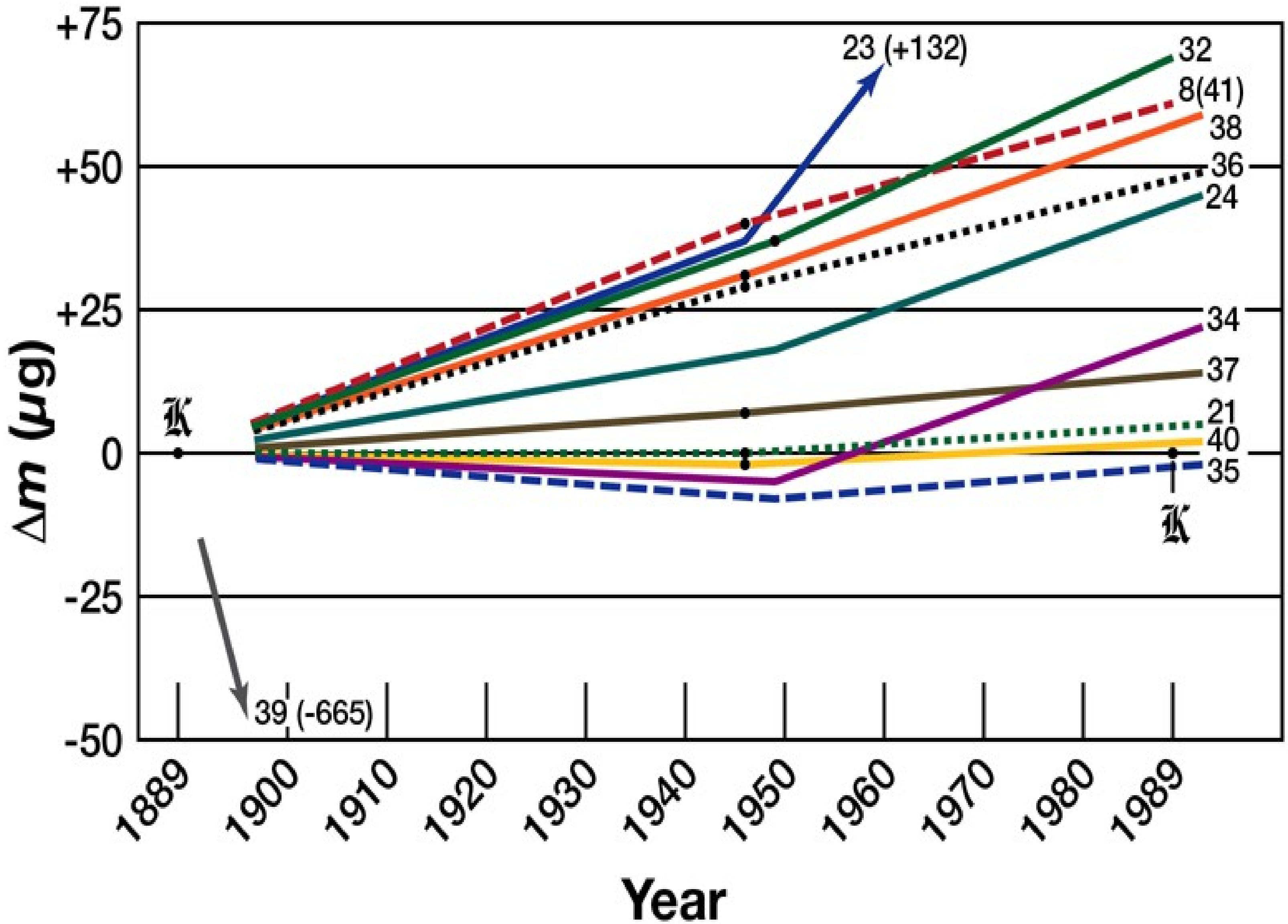
**The new Si system was put in place on May 20, 2019.**

**already done for length, where until 1960, the SI standard of length was disseminated using platinum-iridium meter bars, then replaced by speed of light and Cs frequency, i.e. fundamental constants**

## what about the mass standard?

NIST's platinum-iridium kilogram K92 (front) with stainless-steel kilogram masses in the background.
Credit: J.L. Lee/NIST

# the need for a new definition of the kilogram




try to become independent of artefacts, otherwise precision is limited to about $5 \times 10^{-8}$

The reason for the observed mass changes is not well understood

# the new kilogram standard

**the revised SI redefines the kilogram using a fixed value for the Planck constant and using the definition of the meter and second, which are already based on constants.**

**can this be? is the Planck constant, which rules quantum mechanics, somehow related to gravity?**

# an aside: Max Planck's 1900 paper on thermal radiation and quantization, Ver. DPG 2 (1900) 237

in this paper, Planck introduced his famous constant h with dimensions J s or $m^2$ kg $s^{-1}$

in the same paper, Planck speculated that a unit of mass could be introduced since, using c, h, and the gravitational constant G the following 3 relations are obtained:

$$\text{Planck mass, } M_{pl} = \left(\frac{\hbar c}{G}\right)^{1/2} = 2.2\times10^{-5}\, gm$$

$$\text{Planck length, } L_{pl} = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6\times10^{-33}\, cm$$

$$\text{Planck time, } t_{pl} = \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5\times10^{-44}\,\text{sec}$$

together with the charge of the electron, these could define the 4 fundamental constants of the unit system

however, these units turned out to be impractical

## another aside: Josephson constant and von Klitzing constant

Given the success of measurement standards based on the Josephson and the quantum Hall effect, the choice of the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ for volt and ohm, respectively, could be considered as defining constants. However, they were not chosen because the definition of the base units ampere and kilogram would be less obvious. Dissemination of electrical units still fully benefit from these two quantum effects, because with the fixed numerical values of $e$ and $h$, those of $K_J$ and $R_K$ are fixed automatically.

Liebisch et al., 2019 Annalen der Physik

**now: all the defining constants**

**note: as defining constants they are independent of each other, have units and fixed numerical values**

**the current numerical values are based on the best values determined by measurements as of today**

**should a value change in the future by improved measurements, this new value can be inserted into the system without any further changes**

# the seven base units and defining constants

| Defining constant | Symbol | Exact numerical value | Unit |
|---|---|---|---|
| Hyperfine splitting of Cs | $\Delta \upsilon_{Cs}$ | 9 192 631 770 | $Hz = s^{-1}$ |
| Speed of light in vacuum | $c$ | 299 792 458 | $m\ s^{-1}$ |
| Planck constant | $h$ | $6.62607015 \times 10^{-34}$ | $J\ s = kg\ m^2\ s^{-1}$ |
| Elementary charge | $e$ | $1.602176634 \times 10^{-19}$ | $C = A\ s$ |
| Boltzmann constant | $k$ | $1.380649 \times 10^{-23}$ | $J/K = kg\ m^2\ s^{-2}\ K^{-1}$ |
| Avogadro constant | $N_A$ | $6.02214076 \times 10^{23}$ | $mol^{-1}$ |
| Luminous efficacy | $K_{cd}$ | 683 | $lm/W = cd\ sr\ kg^{-1}\ m^{-2}\ s^3$ |

The speed of light c is exactly 299792458 metres per second ($\mathrm{m{\cdot}s^{-1}}$);

The ground state hyperfine structure transition frequency of the caesium-133 atom ΔvCs is exactly 9192631770 hertz (Hz);

The luminous efficacy $K_{cd}$ of monochromatic radiation of frequency $540\times10^{12}$ Hz (540 THz) – a frequency of green-colored light at approximately the peak sensitivity of the human eye – is exactly 683 lumens per watt ($\mathrm{lm{\cdot}W^{-1}}$).

The Planck constant h is exactly $6.62607015\times10^{-34}$ joule-second (Jקs).

The elementary charge e is exactly $1.602176634\times10^{-19}$ coulomb (C).

The Boltzmann constant k is exactly $1.380649\times10^{-23}$ joule per kelvin (JקK−1).

The Avogadro constant NA is exactly $6.02214076\times10^{23}$ reciprocal mole (mol−1).

ΔvCs = Δv(133Cs)hfs = 9192631770 s$^{-1}$

c = 299792458 mקs$^{-1}$

h = 6.62607015×10$^{-34}$ kgקm2קs−1

e = 1.602176634×10$^{-19}$ Aקs

k = 1.380649×10$^{-23}$ kgקm2קK−1קs−2

$N_A$ = 6.02214076×10$^{23}$ mol−1

$K_{cd}$ = 683 cdקsrקs$^3$קkg$^{-1}$קm$^{-2}$

## definitions of the base units

### the second

2019 definition: The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta \nu_{Cs}$, the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom to be 9192631770 when expressed in the unit Hz, which is equal to $s^{-1}$.

The second may be expressed directly in terms of the defining constants:

$$1\ s = 9192631770/\Delta \nu_{Cs}.$$

## definitions of the base units

### the second

2019 definition: The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom to be 9192631770 when expressed in the unit Hz, which is equal to $s^{-1}$.

The second may be expressed directly in terms of the defining constants:

$$1\ s = 9192631770/\Delta\nu_{Cs}.$$

## definitions of the base units

### the meter

2019 definition: the meter, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit $m\,s^{-1}$, where the second is defined in terms of the caesium frequency ΔvCs.

The meter may be expressed directly in terms of the defining constants:

$$1\ m = 9192631770/299792458 c/\Delta v Cs$$

n.b. if a new measurement in the future would get a new (improved) value for c, then the meter will change accordingly, the defining constants stay unchanged

## definitions of the base units

### the meter

2019 definition: the meter, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit $m \cdot s^{-1}$, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.

The meter may be expressed directly in terms of the defining constants:

$$1 \text{ m} = \frac{9192631770}{299792458} \frac{c}{\Delta\nu_{Cs}}$$

## definitions of the base units

### the kilogram

2019 definition: the kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.62607015\times10^{-34}$ when expressed in the unit $J\cdot s$, which is equal to $kg\cdot m^2\cdot s^{-1}$, where the meter and the second are defined in terms of c and ΔvCs.

the kilogram may be expressed directly in terms of the defining constants:

$$1\ kg = (299792458)^2 /\{(6.62607015\times10-34)(9192631770)\}\ h\Delta v_{Cs}/c^2.$$

note: this definition via the Planck constant assumes that the value of h can be determined to an accuracy of a few parts in $10^8$ by at least 3 independent experiments
this was achieved in:

Kibble balance, Avogadro experiment

## definitions of the base units

### the ampere

2019 definition: the ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602176634\times10^{-19}$ when expressed in the unit C, which is equal to Aקs, where the second is defined in terms of ΔvCs.

The ampere may be expressed directly in terms of the defining constants as:

1 A = eΔvCs/{(1.602176634×10−19)(9192631770)}.

# definitions of the base units

## the kelvin

2019 definition: the kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be $1.380649\times10^{-23}$ when expressed in the unit Jק$K^{-1}$, which is equal to kgק$m^{2}$ק$s^{-2}$ק$K^{-1}$, where the kilogram, meter and second are defined in terms of h, c and ΔvCs.

The kelvin may be expressed directly in terms of the defining constants as:

$$1\ K = 1.380649\times10^{-23}/\{(6.62607015\times10^{-34})(9192631770)\}\ h\Delta vCs/k.$$

## definitions of the base units

### the mole

2019 definition: The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, $N_A$, when expressed in the unit $mol^{-1}$ and is called the Avogadro number. The amount of substance, symbol $n$, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

The mole may be expressed directly in terms of the defining constants as:

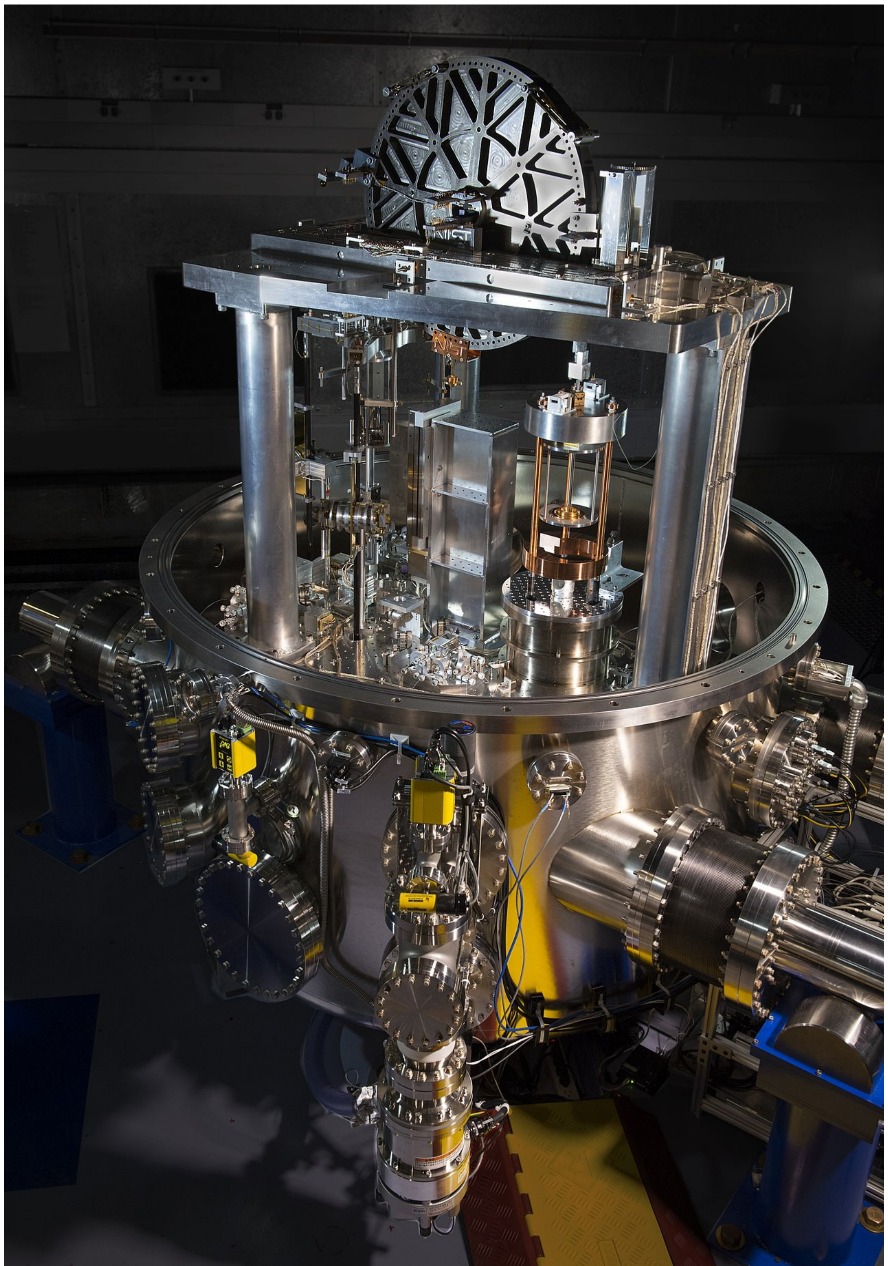
$$1 \text{ mol} = 6.02214076 \times 10^{23}/N_A.$$

## definitions of the base units

### the candela

2019 definition: the candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}$ Hz, $K_{cd}$, to be 683 when expressed in the unit lmקW$^{-1}$, which is equal to cdקsrקW$^{-1}$, or cdקsrקkg$^{-1}$קm$^{-2}$קs$^{3}$, where the kilogram, meter and second are defined in terms of h, c and ΔvCs.

examples: the Kibble balance for the Planck constant
and the Si sphere for the Avogadro constant



# the practical realisation of SI units

apart from the more general issues and 'philosophy' as how to setup such a system, measuring the value of the  defining constants is the main meat of the seminar and talks ahead