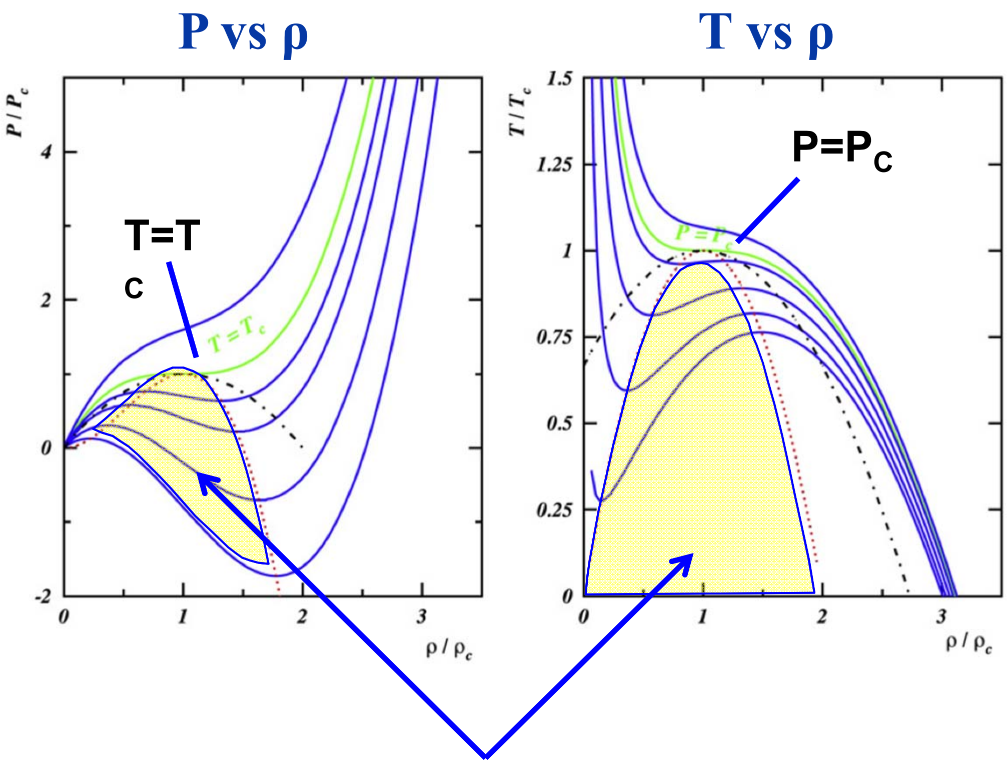
Liquid gas phase transition of nuclear matter


P vs ρ

T vs ρ

T=T_C

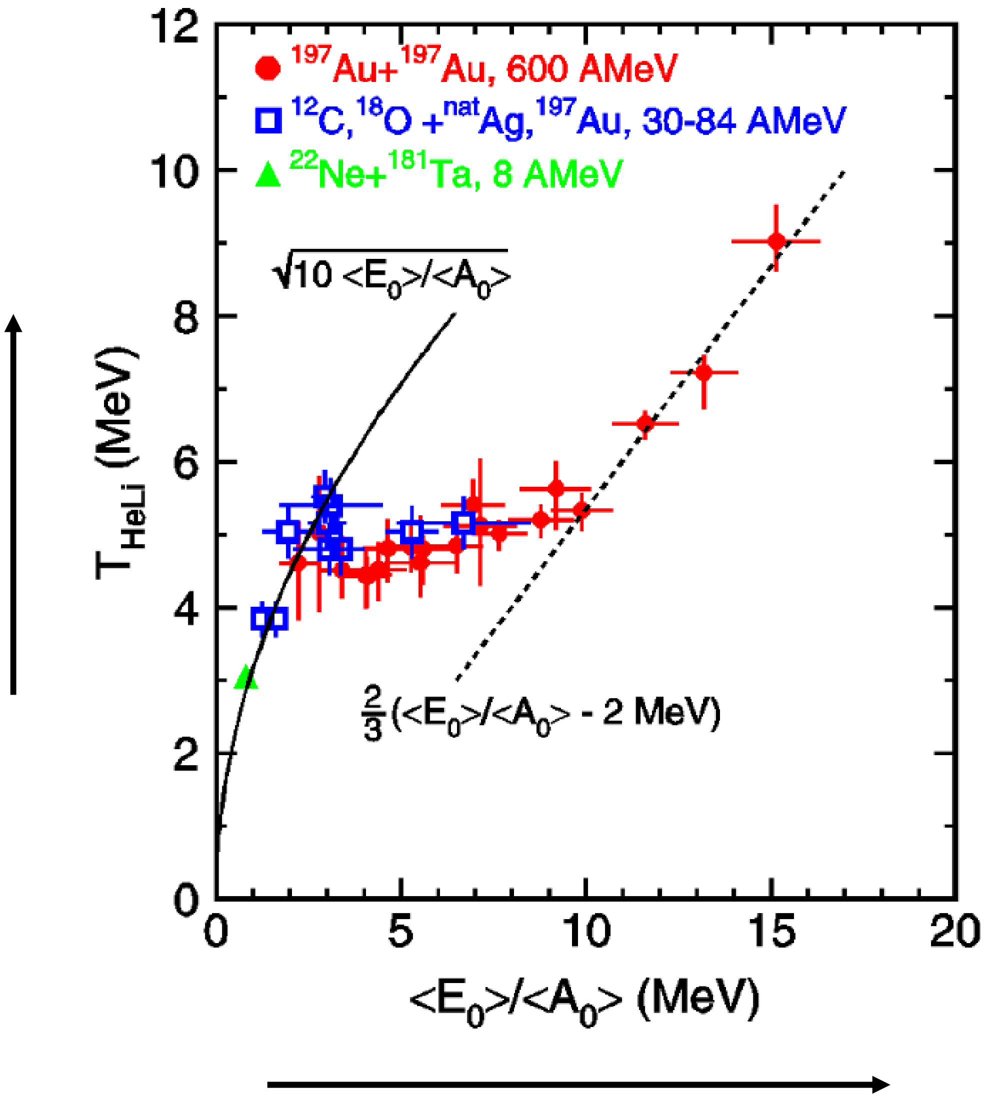
P=P_C

Borderie, Rivet, Prog.Part.Nucl.Phys. 61 (2008)

Spinodal instability region (K<0):
Liquid-gas phase transitions
at $\rho<\rho_0$ and T<15 MeV?

needs some heating and low densities

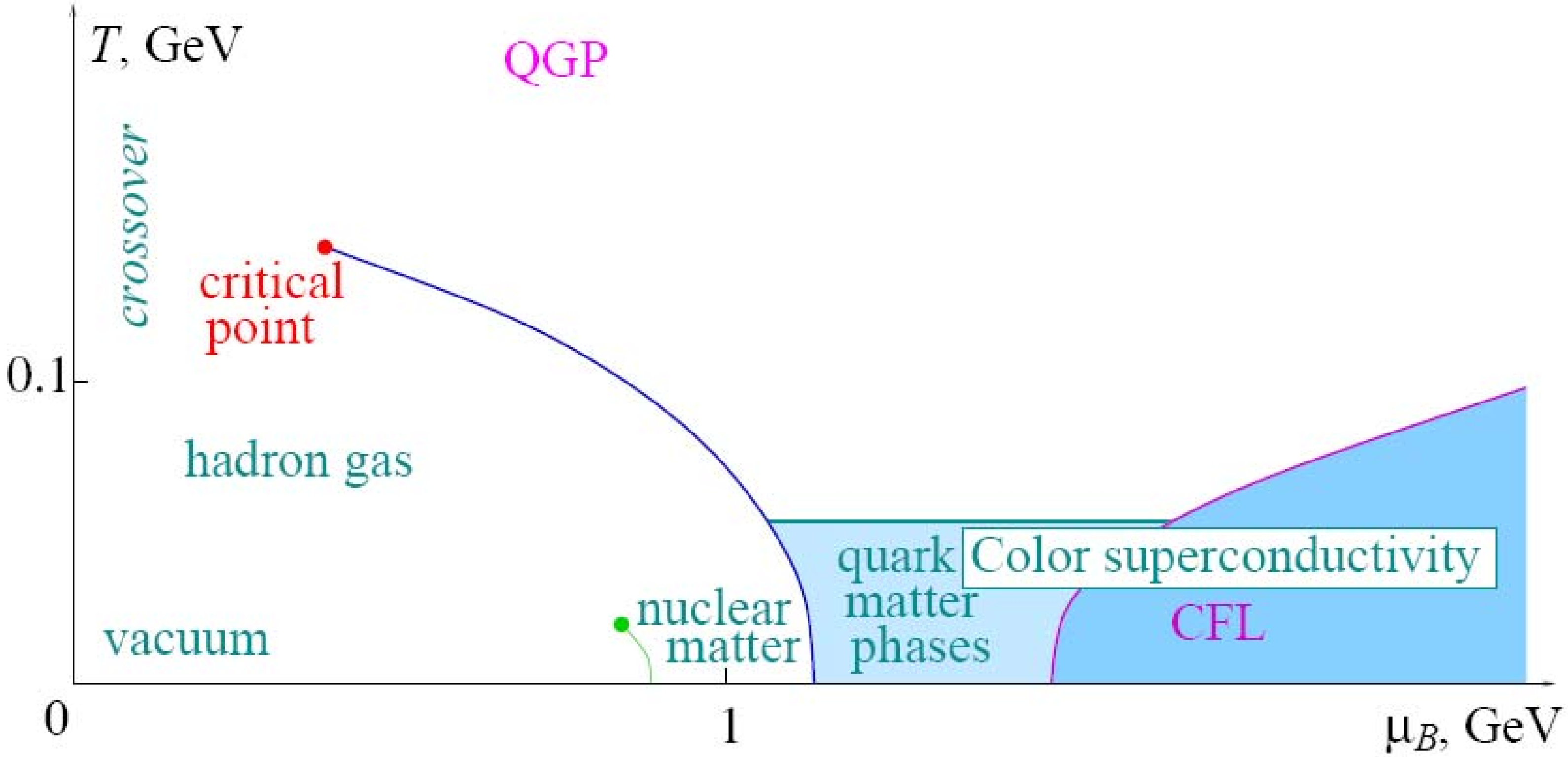
Signals of the phase transition



caloric curve
multi-fragmentation
fluctuations
critical exponents

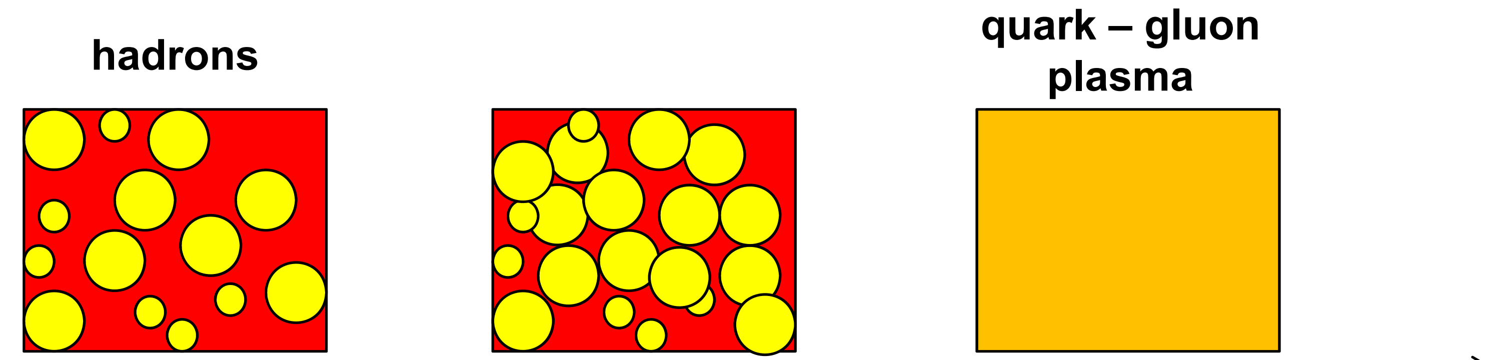
QCD phase diagram

Stephanov (2008)



Phase transitions in QCD matter

Quark gluon plasma



hadrons

quark – gluon
plasma

pressure and heat

lower symmetry

higher symmetry

4.3

“Energy density” from Thermal model for particle production

P. Braun-Munzinger et al., arXiv:nucl-th/0304013

Chemical equilibrium concept.

Density of particle state i:

$$n_i(\mu,T)=\frac{N_i}{V}=-\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu}=\frac{g_i}{2\pi^2}\int\frac{p^2dp}{e^{\frac{E_i-\mu_i}{T}}\pm 1}$$

$$\mu_i=\mu_B B_i+\mu_S S_i+\mu_{I_3} I_{3,i}$$

“+” for fermions, “-” for bosons
g_i – spin degeneracy factor

Chemical potentials μ_i are constrained by conservation of quantum numbers:

baryon number: $V\sum_i n_i B_i = Z+N \rightarrow V$

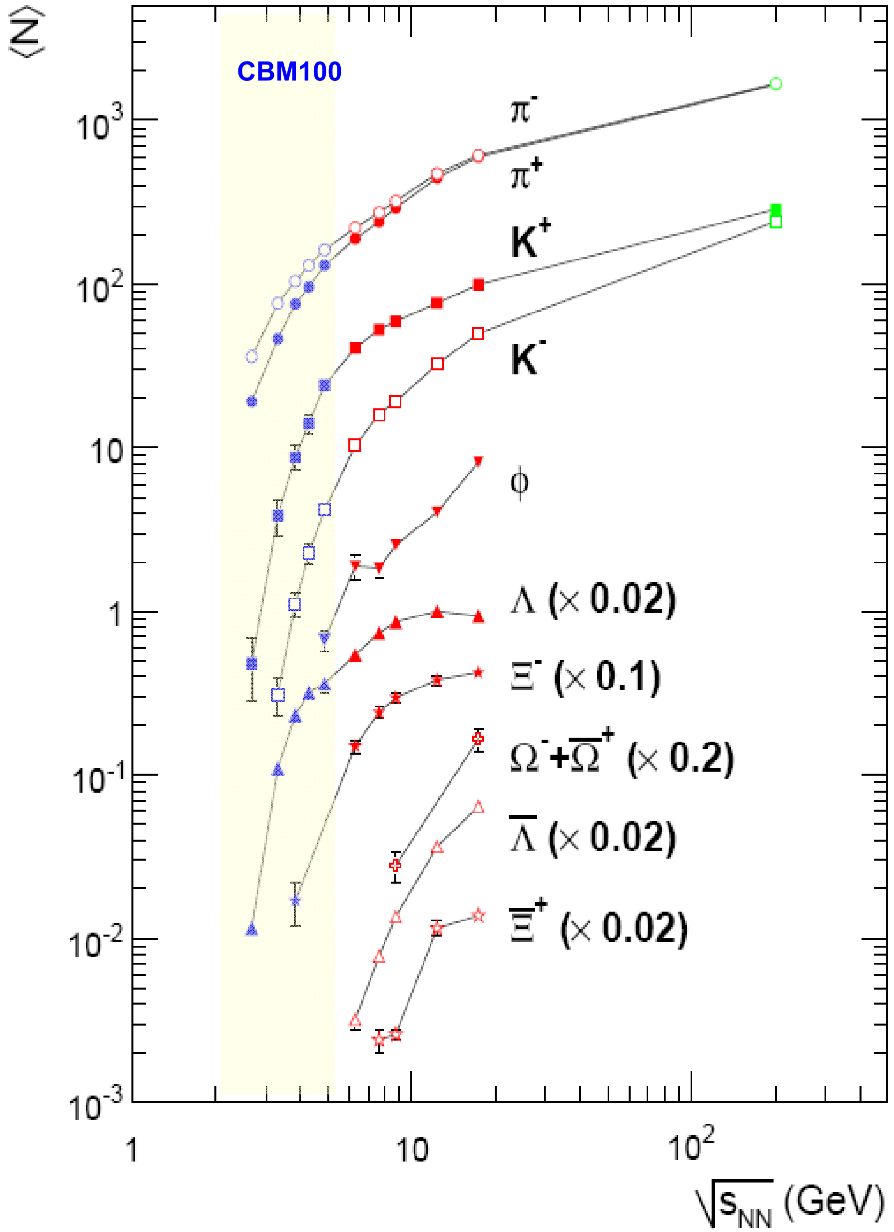
strangeness: $V\sum_i n_i S_i = 0 \rightarrow \mu_S$

charge: $V\sum_i n_i I_{3,i}=\frac{Z-N}{2}\rightarrow \mu_{I_{3,i}}$

⇒

3 equations,
5 unknows
↓
2 free parameter

Measured particle yields



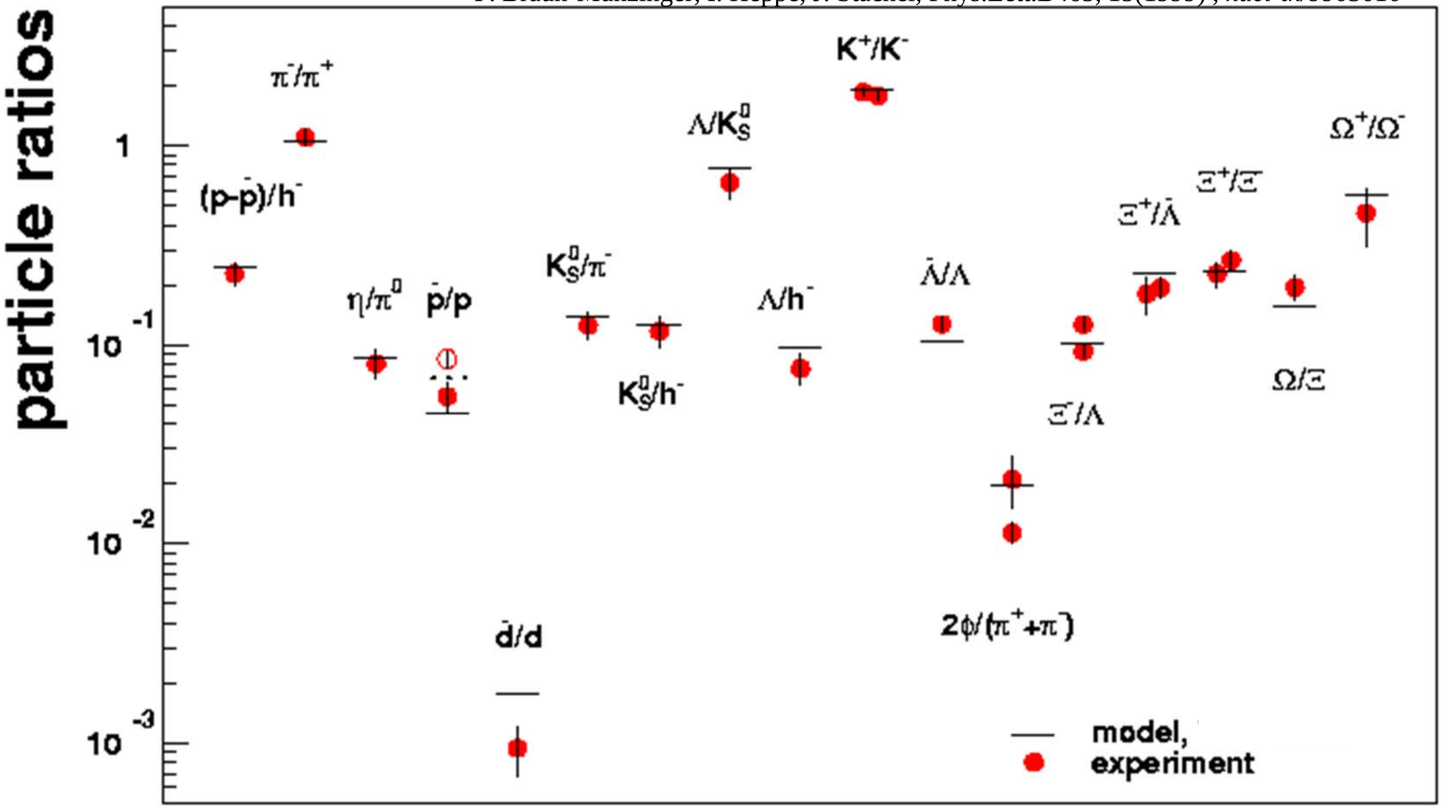
C. Blume, J. Phys. G 31 (2005) 57

4.3

Chemical equilibrium

Example: SPS data, E_{beam}=158 AGeV, Pb+Pb

P. Braun-Munzinger, I. Heppe, J. Stachel, Phys.Lett.B465, 15(1999) , *nucl-th/9903010*



Model parameter:

$$T = 168 \pm 2.4 \text{ MeV}$$

$$\mu_B = 266 \pm 5 \text{ MeV}$$

$$\mu_S = 71.1 \text{ MeV}$$

$$\mu_{I_3} = -5. \text{ MeV}$$

Note: volume is not needed for description of particle ratios.

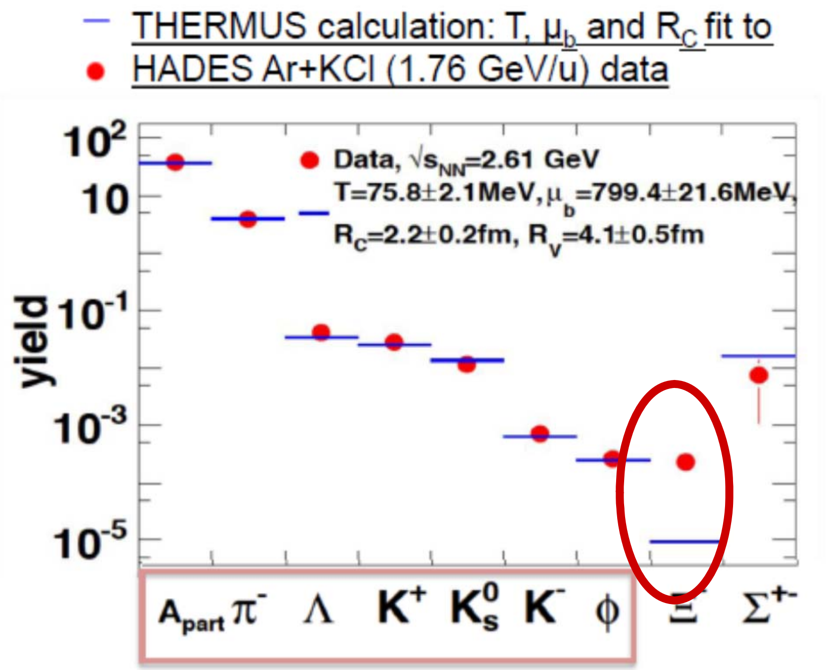
HADES: Sub-threshold Ξ^- production

Ar+KCl reactions at 1.76A GeV

- Ξ^- yield by appr. factor 25 higher than thermal yield
- strangeness exchange reactions like

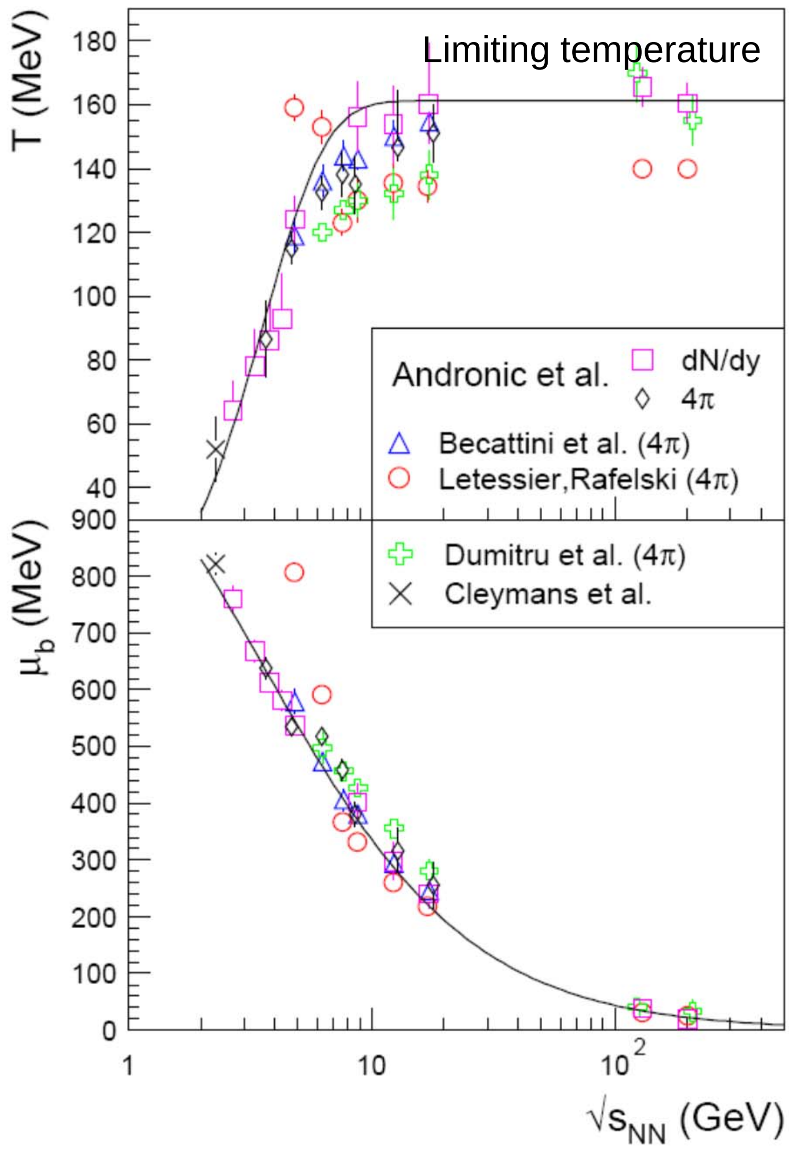
$\overline{K}Y \rightarrow \pi\Xi$ (Y=Λ,Σ) ?

[HADES: PRL103, 132301, (2009)]



THERMUS fit: J.Cleymans, J.Phys.G31(2005)S1069
HADES: Eur. Phys. J. A 47:21, 2011.

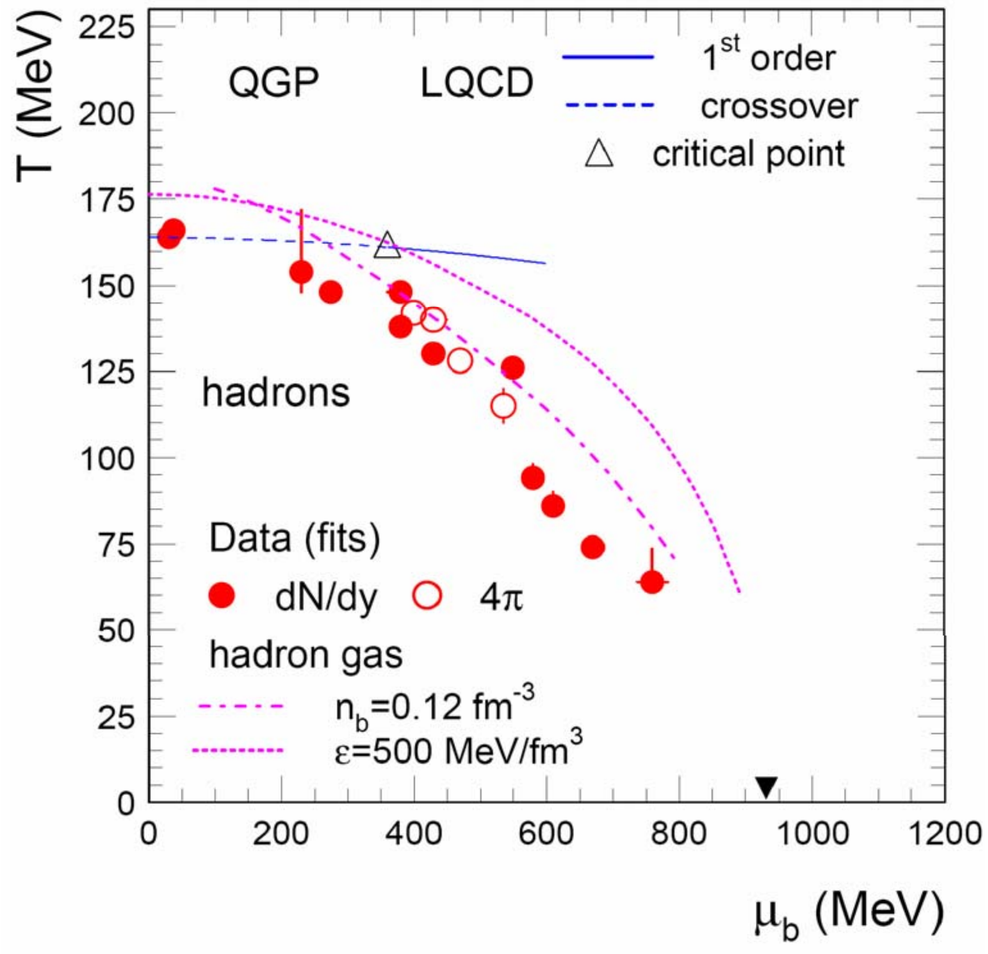
Excitation function of particle production



http://www.gsi.de/forschung/fair_experiments/CBM/PhysicsBook.html

Phase diagram with freeze-out data

A. Andronic et al., Phys. Lett. B 673 (2009).



Equilibration times in hadronic matter

Naïve estimate:

3 collisions needed for equilibration (result from kinetic theory)

hadronic cross section: σ=40 mb = 4 fm^2

strangeness production cross section: σ=400 μb = 4 · 10^{-2} fm^2

mean free path

$$\lambda = \frac{1}{n\sigma} = \frac{1}{0.17\, fm^{-3} \cdot 4\, fm^2} = 1.5\, fm$$

time between collisions

$$\tau = \lambda / c = 1.5\, fm / c$$

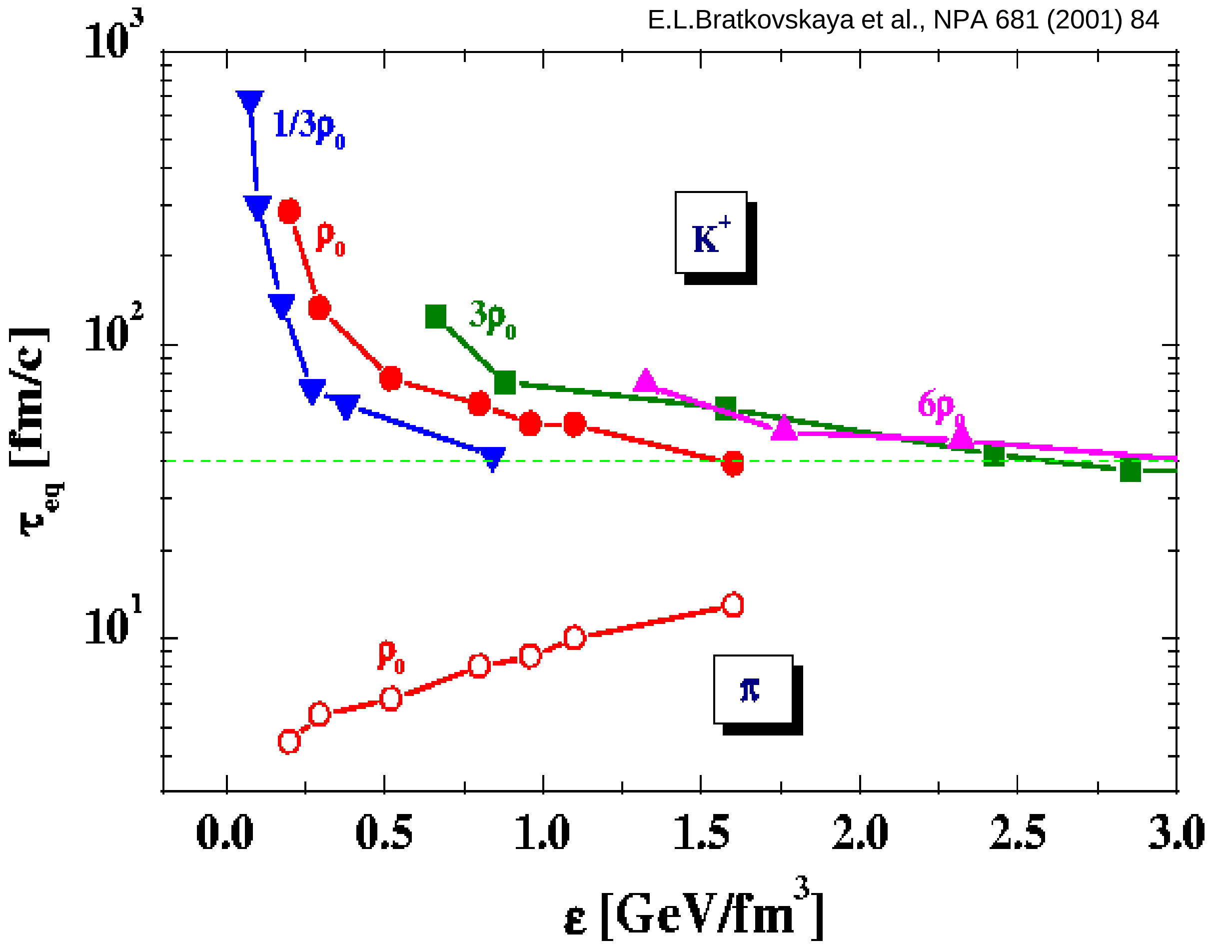
minimal equilibration time

$$\tau_{eq}^{pion} = 4.5\, fm / c$$

$$\tau_{eq}^{strangeness} = 450\, fm / c$$

4.5

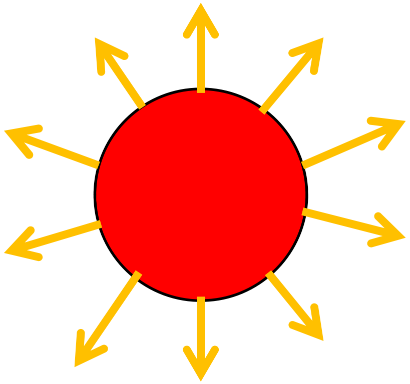
Chemical equilibration in transport models



E.L.Bratkovskaya et al., NPA 681 (2001) 84

4.3

Thermal Source



Limiting case: Fireball nucleons in thermodynamic equilibrium

Momentum space distribution: Isotropic emission in CMS
(e.g. in rest frame of spectator nucleus)

Invariant spectrum of particles radiated by a thermal source:

$$E\frac{d^3N}{dp^3} = \frac{d^3N}{m_T\ dm_T\ \mathrm{dy}\ d\phi}$$

$$\propto \frac{\mathrm{E}}{\mathrm{e}^{(E-\mu)/T} \pm 1} \xrightarrow{(E-\mu)>>T} Ee^{-(E-\mu)/T}$$

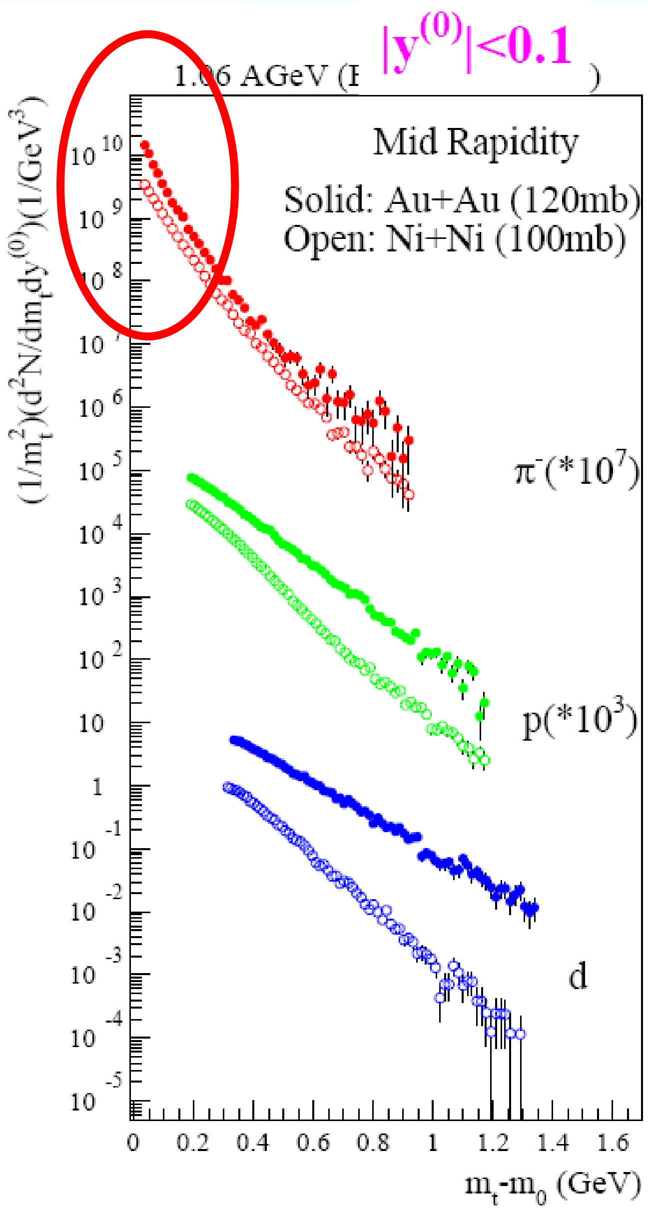
where:	$m_T = (m^2+p_T^2)^{½}$	transverse mass (Note: requires knowledge of mass)
	μ	chemical potential
	T	temperature of source

At mid-rapidity
$E = m_T \cosh y = m_T$ and hence:

$$\frac{dN}{m_T\ dm_T} \propto m_T\ e^{-m_T/T}$$

5.2.2

Transverse Mass Spectra

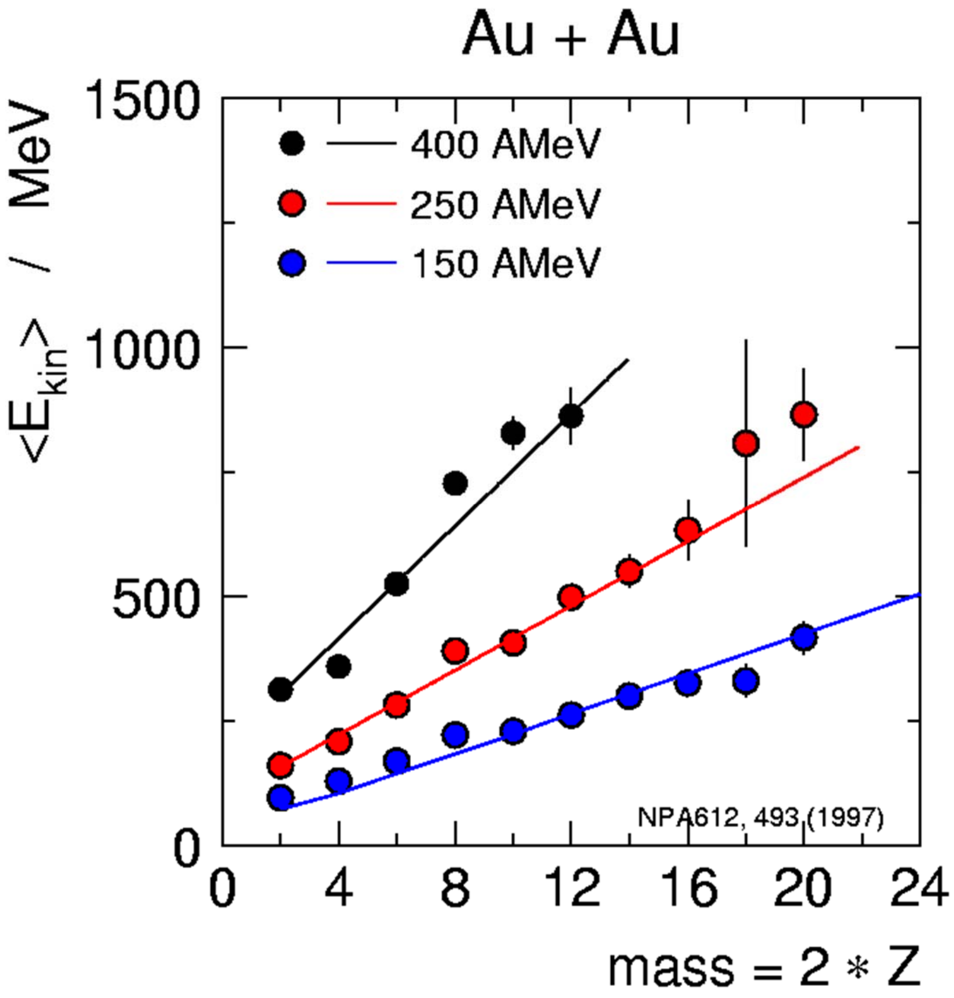


Temperature can be determined by slope constant of transverse mass spectra.

Slope constants different for different particle species.

Collective flow: radial expansion

SIS – energy



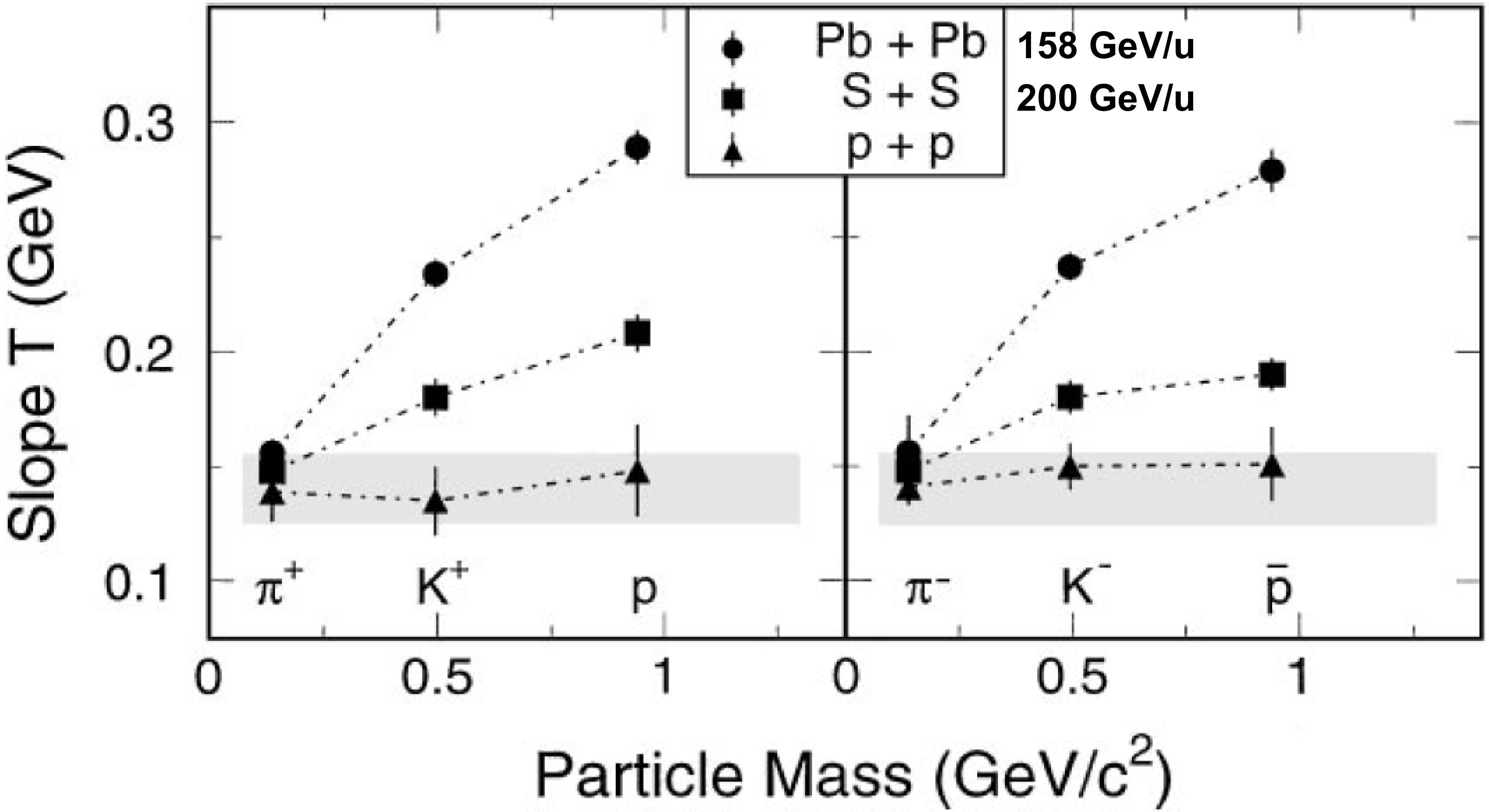
$<E_{kin}>=E_{thermal}+M_{*}<e_{coll}(\beta)>$

First observation of radial expansion
S.C. Jeong et al. (FOPI),
Collective Motion in Selected Central Collisions of Au + Au at 150*A* MeV
Phys. Rev. Lett. 72 (1994) 3468

5.2.2

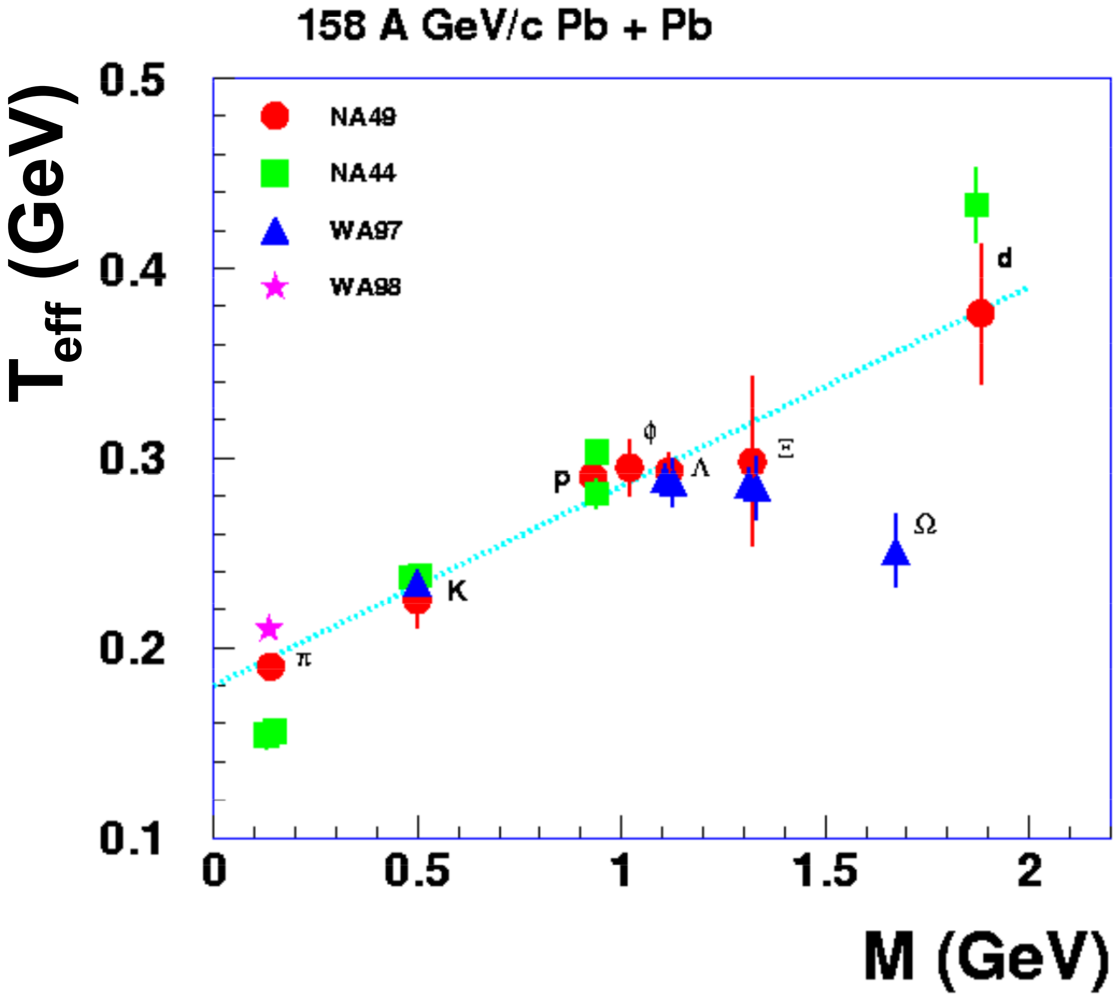
Slope parameter of transverse mass spectra

I. Bearden et al.,(NA44), PRL78,2030 (1997)



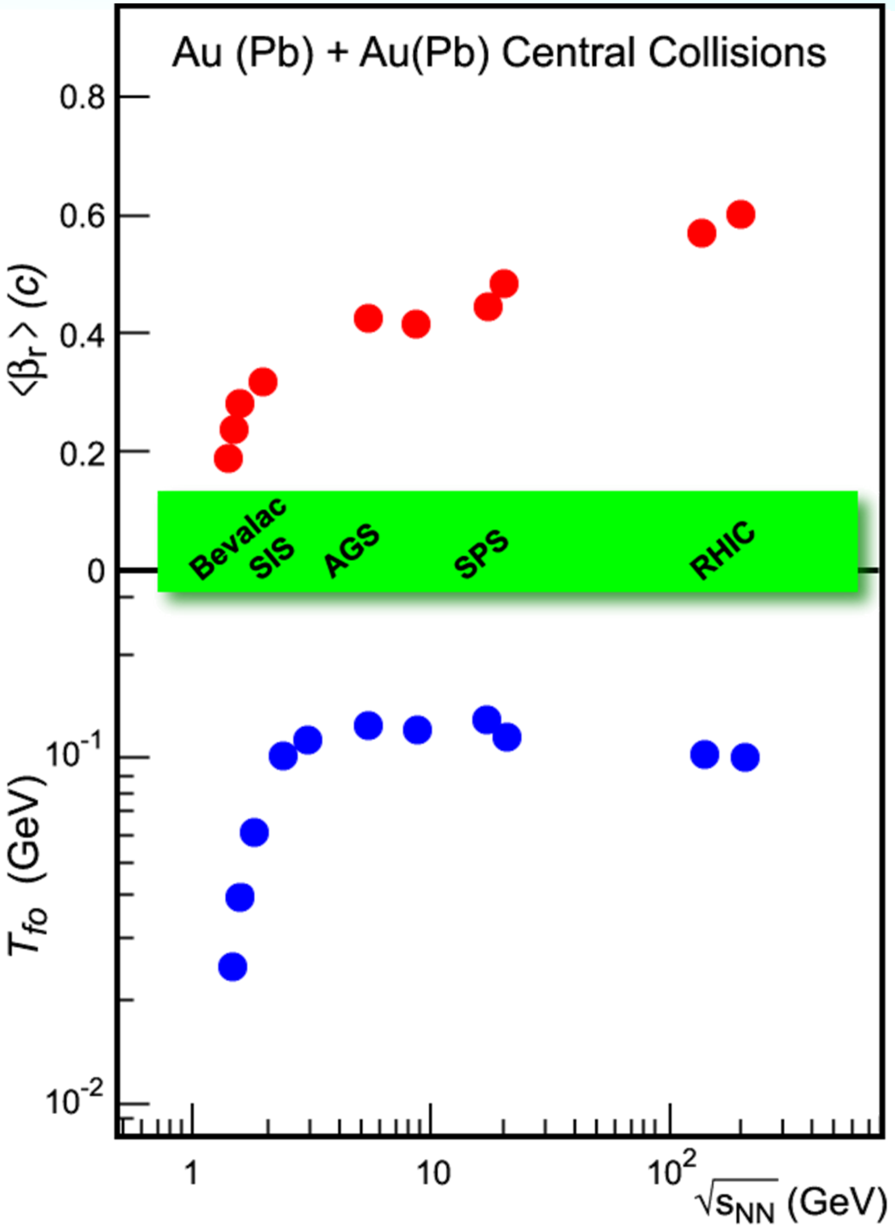
5.2.2

Transverse expansion



Slopes are proportional to mass except for multiple strange particles.

Excitation function for transverse flow



Blastwave: Hydrodynamically inspired description of spectra

Schnedermann, Sollfrank & Heinz,PRC48 (1993) 2462

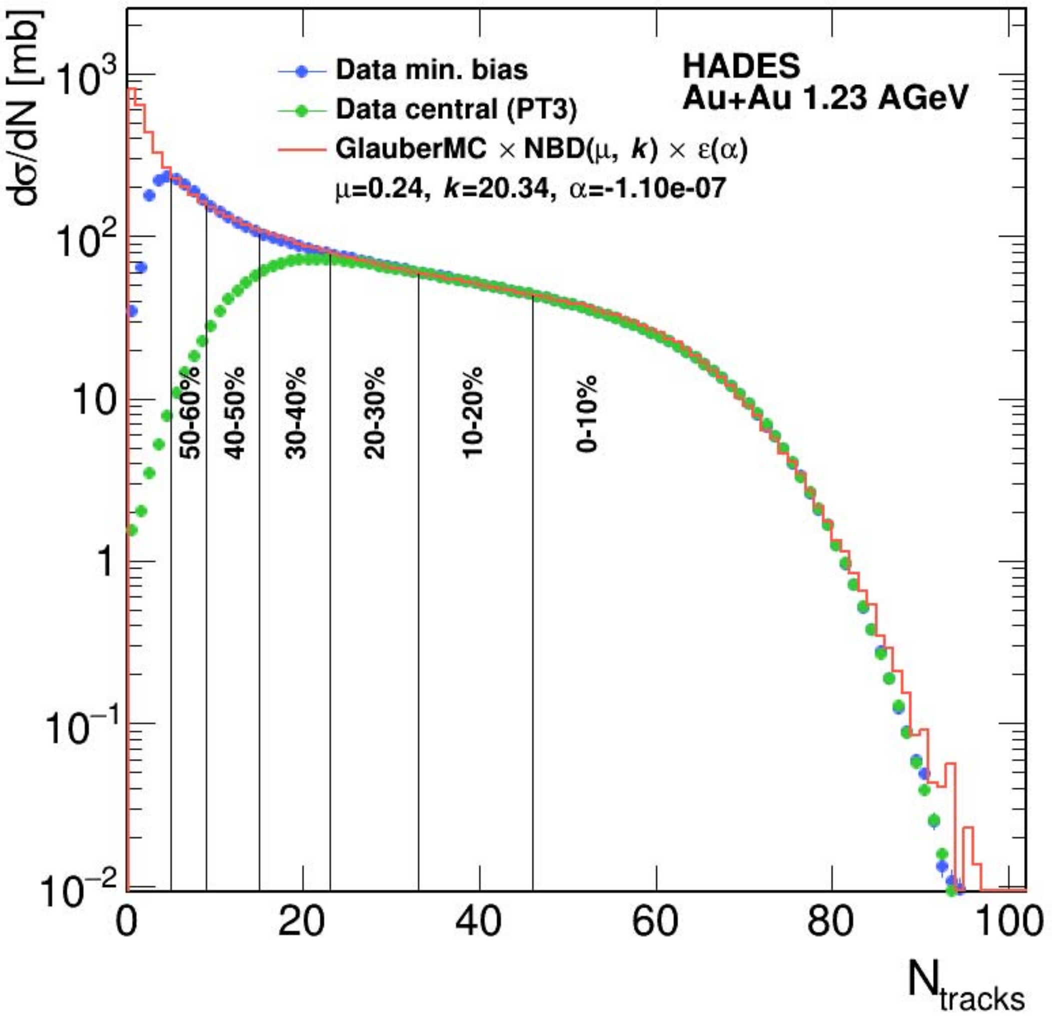
$$\frac{dN}{m_T\, dm_T} \propto \int_0^R r\, dr\; m_T\, I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T \cosh\rho}{T}\right)$$

with

$$\beta_r(r) = \beta_s\left(\frac{r}{R}\right)^n$$ **Transverse velocity distribution**

$$\rho = \tanh^{-1}\beta_r$$ **Boost angle (boost rapidity)**

Determination of impact parameter



Glauber model

Also: wounded nucleon model

Bialas, Bleszynski, and Czyz (see Nucl. Phys. B111(1976)461)
Kari Eskola, Nucl. Phys. B 323(1989)37

http://www-linux.gsi.de/~misko/overlap/

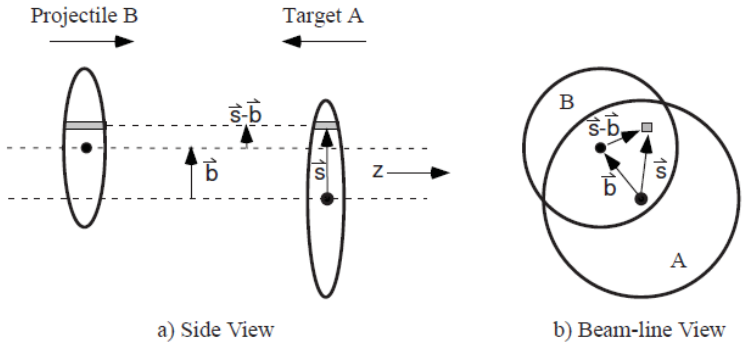
R. Glauber: http://www.kfki.hu/~qm2005/PROC05/Glauber/Glauber_qm05.pdf

Woods-Saxon density profile

$$n_A(r) = \frac{n_0}{1+\exp\left(\frac{r-R}{d}\right)} \quad \text{with } n_0 = 0.17\, fm^{-3}, d = 0.54\, fm$$

$$R = (1.12 A^{1/3} - 0.86 A^{-1/3})\, fm$$

$$\int d^3 r\, n_A(r) = 4\pi \int_0^\infty r^2 n_A(r) dr = A$$



a) Side View b) Beam-line View

- Projectile B is colliding with A at relativistic speed
- Impact parameter b, flux tube of nucleons at s relative to nucleus center

Thickness function

Probability per unit transverse area of nucleon in flux tube

$$T_A(b) = \int_{-\infty}^{\infty} dz\, n_A\left(\sqrt{b^2+z^2}\right)$$

$$\int d^2 b\; T_A(b) = A$$

n_A= prob. of location per unit volume

4.4

Glauber model

- Product of T_A, T_B can be used to define nuclear thickness function:

$$T_{AB}(\boldsymbol{b}) = \int T_A(\boldsymbol{s}) T_B(\boldsymbol{s}-\boldsymbol{b}) d^2 s$$

- effective overlap of nuclei A and B
- $T(b)\sigma_{inel}^{NN}$ = probability interaction σ_{inel}^{NN}, elastic cross section have little energy loss
- Probability of n interactions than given by binomial distribution

$$P(n,b) = \binom{AB}{n} \left[T_{AB}(b)\sigma_{inel}^{NN}\right]^n \left[1 - T_{AB}(b)\sigma_{inel}^{NN}\right]^{AB-n}$$

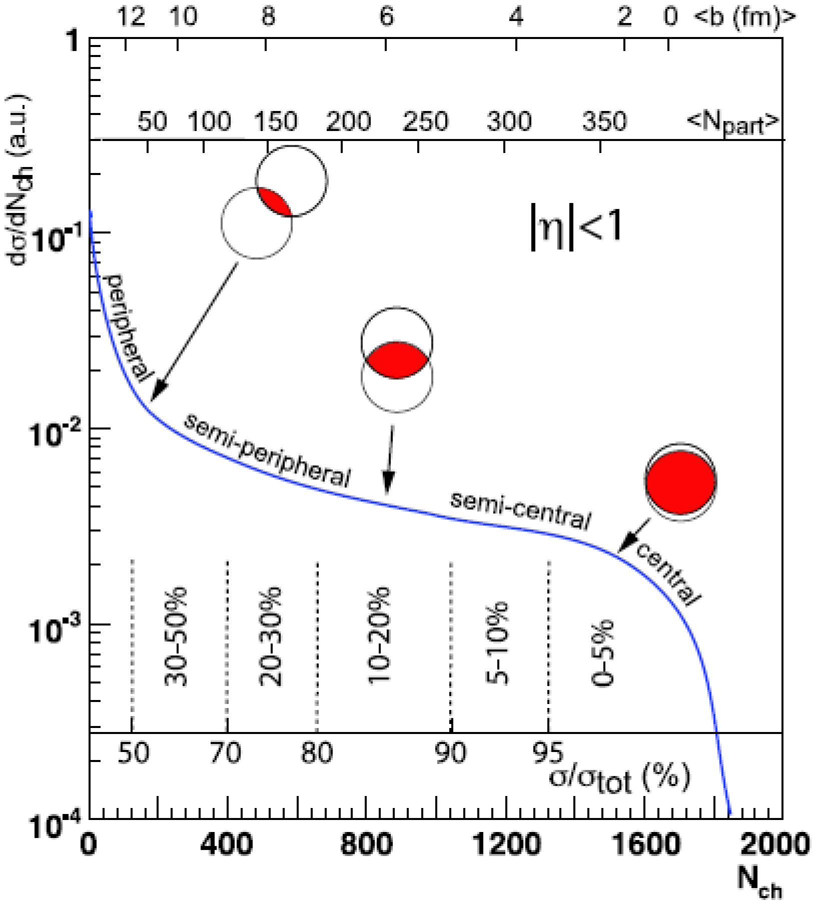
- Can be used to calculate $N_{coll}, N_{part}, \sigma_{AA}$

$$\boxed{\sigma_{inel}^{A+B} = \int_0^\infty 2\pi b db \left\{1 - \left[1 - T_{AB}(b)\sigma_{inel}^{NN}\right]^{AB}\right\}}, \boxed{N_{coll}(b) = \sum_{n=1}^{AB} nP(n,b) = AB\, T_{AB}(b)\sigma^{NN}{}_{inel}}$$

$$\boxed{\begin{array}{c} N_{part(b)} = A\int T_A\,(s)\left\{1 - \left[1 - T_B(s-b)\sigma_{\hat{\imath}nel}^{NN}\right]^B\right\} d^2 s + \\ \mathrm{B}\int T_B\,(s)\left\{1 - \left[1 - T_A(s-b)\sigma_{\hat{\imath}nel}^{NN}\right]^A\right\} d^2 s \end{array}}$$

Glauber model - results

Jeremy Wilkinson
ALICE

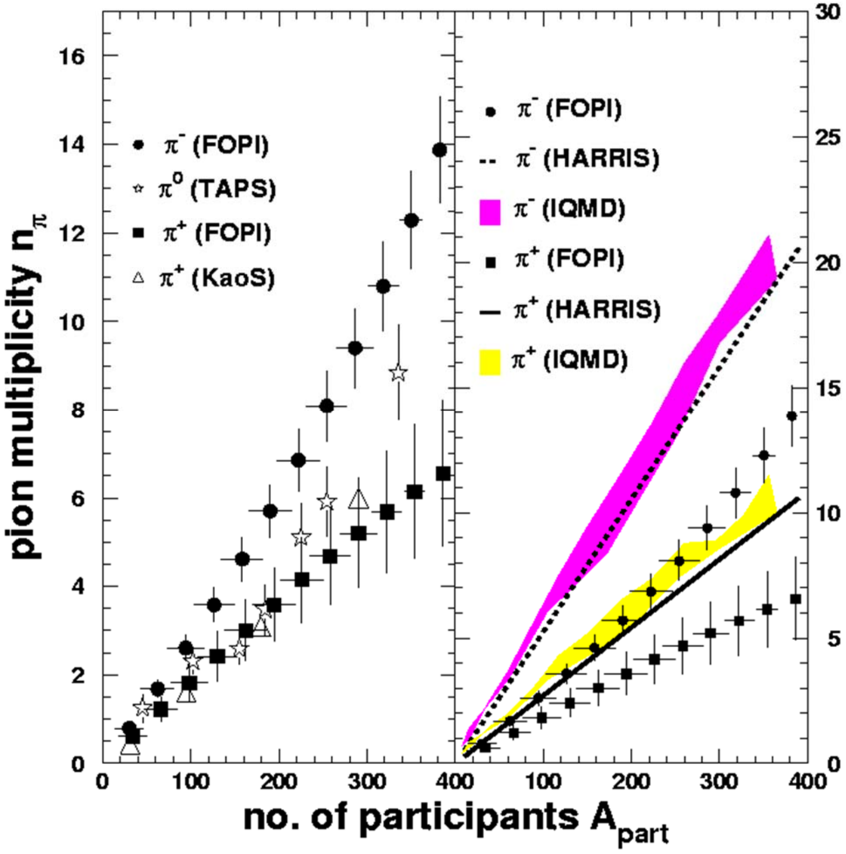


5.2.1

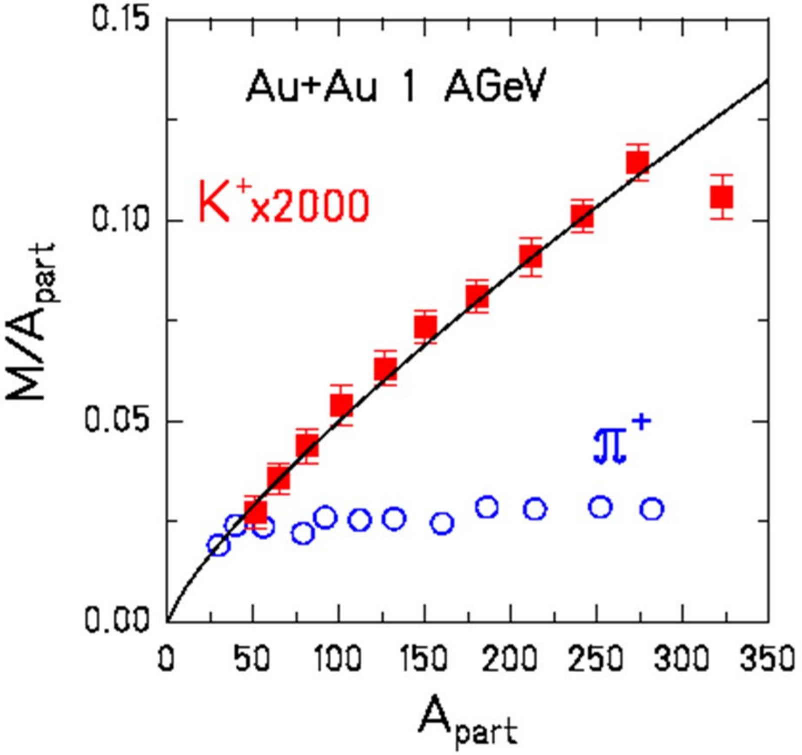
Pion / Kaon Production @ SIS18

Au + Au 1AGeV

D.Pelte et al., (FOPI), Z.Phys.A357, 215 (1997)
(abs. scale revised: W. Reisdorf et al. (FOPI), NPA 781, 459 (2007)

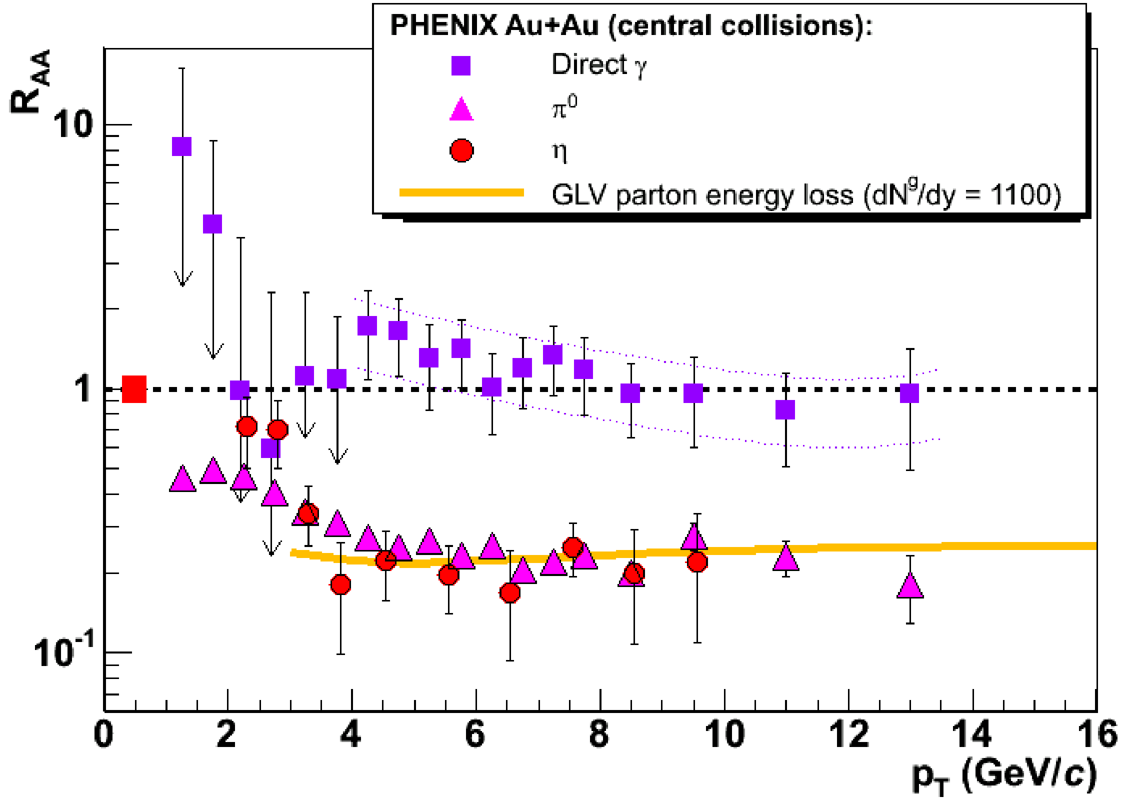


P.Senger, H.Ströbele., (KaoS),
J.Phys. G25 (1999) R59

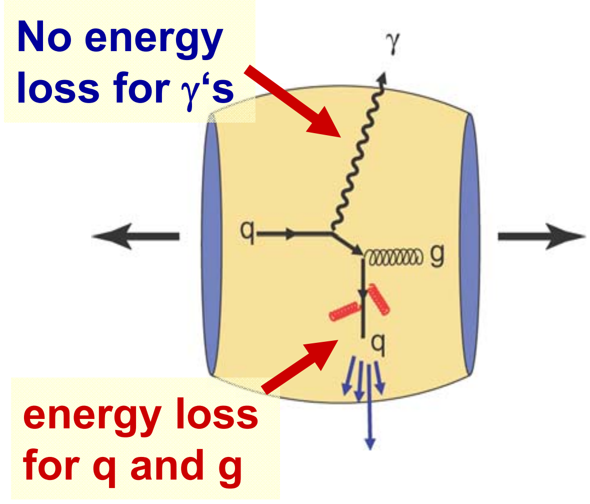


Pion production scales linear with A_{part}, bulk property!

Nuclear modification factor R_{AA}



$$R_{AA} = \frac{\left.\frac{d\sigma}{dp_t}\right|_{A+A}}{N_{coll} \cdot \left.\frac{d\sigma}{dp_t}\right|_{p+p}}$$

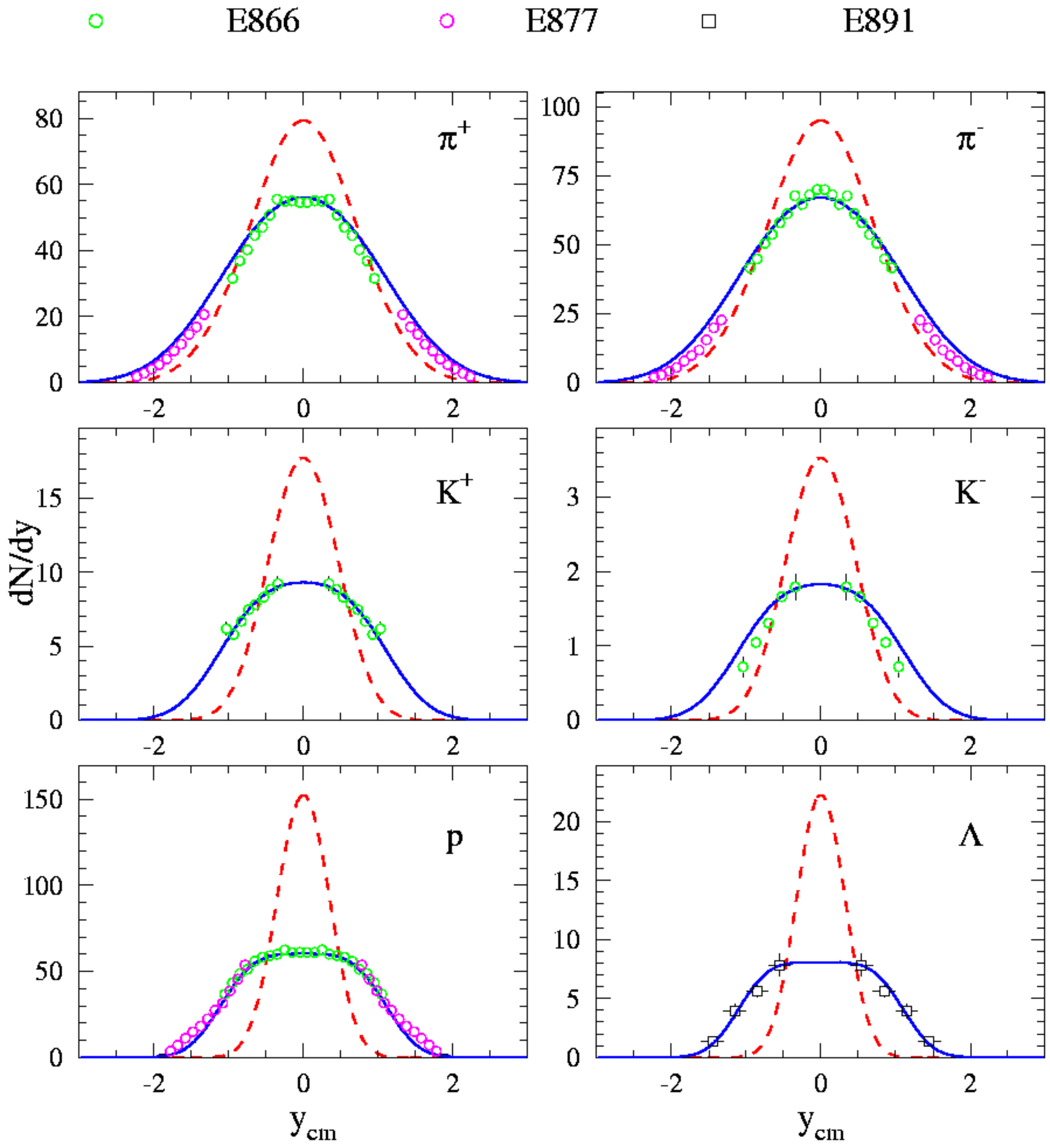


No energy loss for γ's

energy loss for q and g

5.2.2

Stopping



AGS: Au + Au @ 10.7 AGeV

Rapidity density distributions
Incompatible with
isotropic thermal source

↓

Longitudinal expansion.

N.Herrmann,
J.P. Wessels,
T.Wienold,
Ann.Rev.Nucl.Part.
Sci.49,581 (1999)

5.2.2

Thermal width of rapidity distribution

Width of isotropic thermal source can be calculated analytically:

$$\frac{dN_{isotropic}}{dy} \propto m^2 T\left(1+2\chi+2\chi^2\right)\exp(-1/\chi),$$

$$\chi = \frac{T}{m\cosh(y)}$$

T can be (has to be) extracted from slopes of thermal spectra at midrapidity.

Measured distributions are at variance with isotropic thermal emission picture.

Possible scenario: longitudinally expanding source(s) with source velocities β_l

$$\langle \beta_l \rangle = \tanh(\langle y' \rangle)$$

$$\frac{dN}{dy} = \int_{-y'_{\max}}^{y'_{\max}} dy' \frac{dN_{iso}(y-y')}{dy'}$$

5.2.2

Stopping

Average rapidity loss:

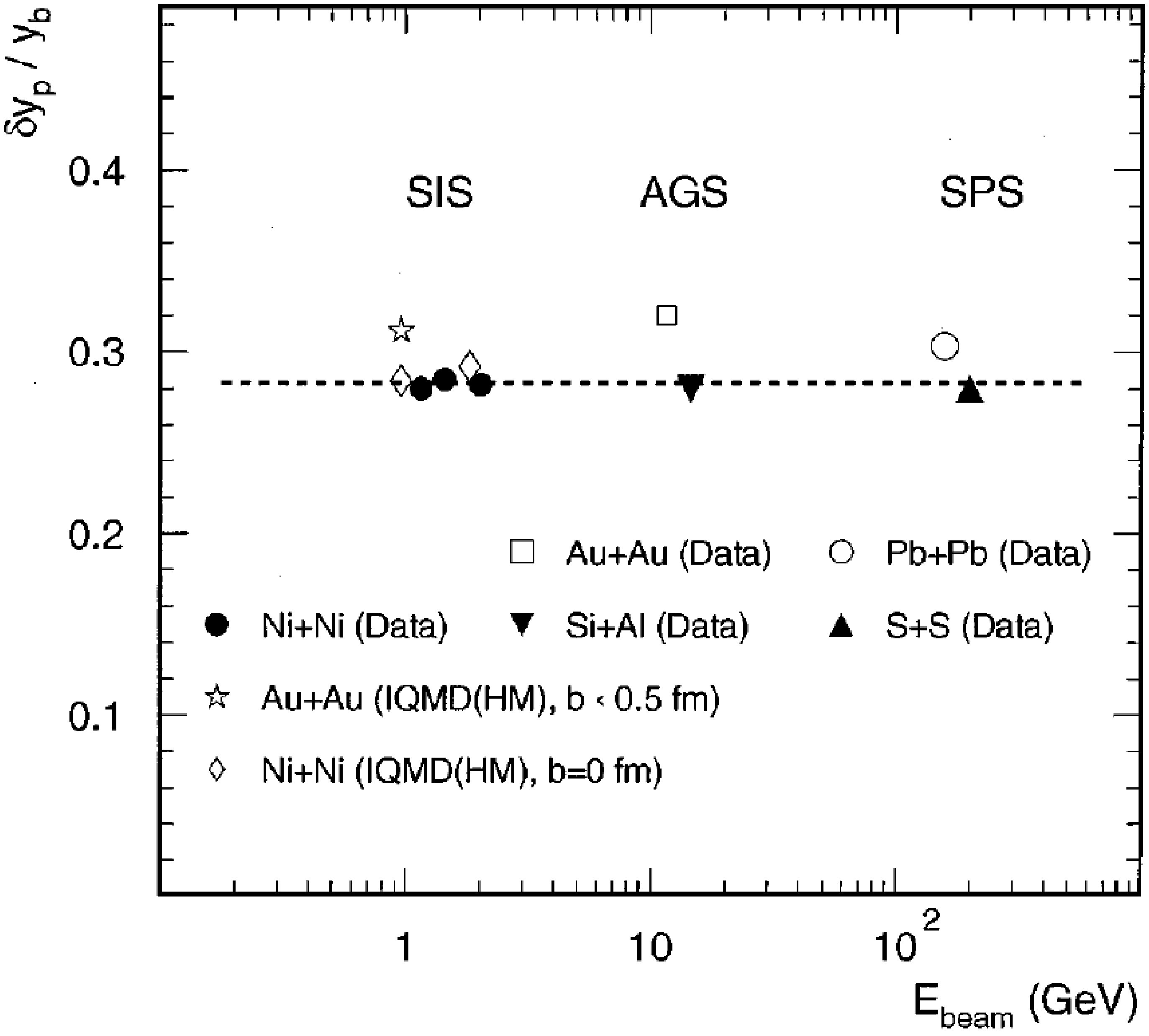
$$\left\langle \delta y_p \right\rangle = y_p - \left\langle y_b \right\rangle$$

$\langle y_p \rangle$ - average net baryon rapidity after the collision

$$\left\langle \delta y_p \right\rangle = \frac{\int_{-\infty}^{0} \left| y_p - y_{t(b)} \right| \left(dN_p / dy \right) dy}{\int_{-\infty}^{\infty} \left(dN_p / dy \right) dy}$$

5.2.2

Excitation function of stopping



N.Herrmann,
J.P. Wessels,
T.Wienold,
Ann.Rev.Nucl.Part.
Sci.49,581 (1999)

5.2.2

Brahms – rapidity loss