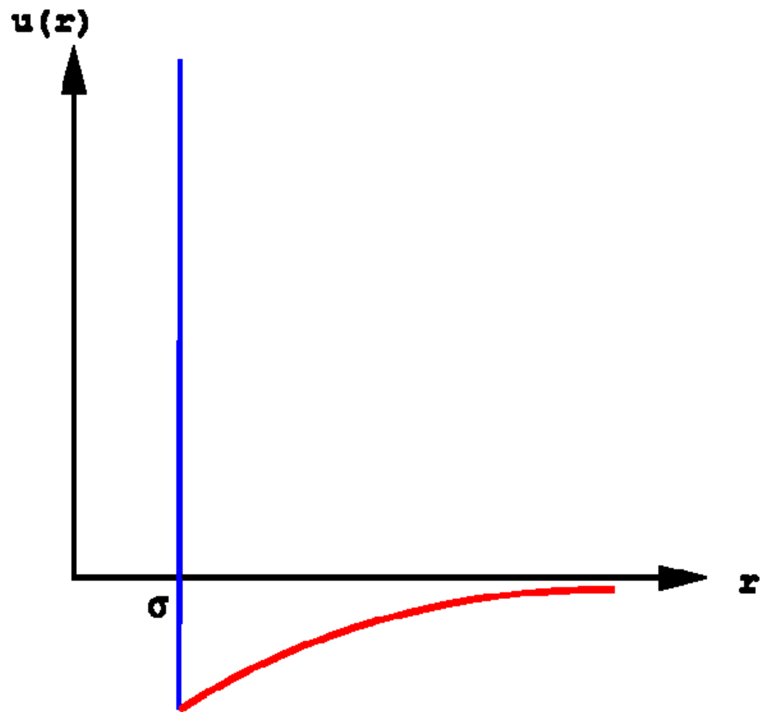
From the potential to the equation of state



From a potential to an equation of state

In absence of interactions the canonical partition function Z of an ideal gas consisting $N = nN_A$ identical particles with mass m runs as follows:

$$Z = \frac{z^N}{N!} \;\; with \;\; Z = \frac{1}{N!}\left[\int\int e^{-\beta\left(\frac{p^2}{2m}+U_e\right)} \frac{d^3r d^3p}{h^3}\right]^N$$

with $\beta = 1/kT$
integration over momenta yields

$$Z = \frac{1}{N!}\left[\left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}} \int \mathrm{e}^{-\beta U_e} d^3r\right]^N$$

Each particle moves independently in an average potential generated by the other particles. The interaction between a pair of particles, which are hard spheres, is taken to be:

u$(r) = \infty$ for $r < d$
u$(r) = -\epsilon\left(\frac{d}{r}\right)^6$ for $r \geq d$,
r is the distance between the centers of the spheres and d is the distance where the hard spheres touch each other.
The integrand vanishes in the regions $V < V_x$ where U_e is infinity. Hence, the integration volume is reduced to $V - V_x$. Otherwise, U(r) only moderately changes with r, and U_e is placed by an effective potential $< U_e >$.

From potential to equation of state

Thus the previous equation becomes

$$Z = \frac{1}{N!}\left[\left(\frac{2\pi m}{h^2\beta}\right)^{\frac{3}{2}}(V - V_x)e^{-\beta<U_e>}\right]^N .$$

Now we have to estimate the values of the parameters $< U_e >$ and V_x. The total mean potential energy of the molecules is $N < U_e >$. But, because these are $\left(\frac{1}{2}\right)N(N-1) \cong \left(\frac{1}{2}\right)N^2$ pairs of molecules in the gas, it follows that the total mean potential energy is also $\frac{1}{2}N^2 < u >$, where u is the potential energy of interaction between one given pair of molecules. Equating the different expressions fo the total mean potential energy of the gas, we obtain $N < U_e > = \frac{1}{2}N^2 < u >$, or

$$< U_e > = \frac{1}{2}N < u >$$

With the above definition of the potential $u(r)$ the molecules are weakly-attracting spheres of radius $\frac{1}{2}R_0$. Under the assumption that another molecule is equally likely to be anywhere in the container. Hence

$$< u > = \frac{1}{V}\int_0^\infty u(R)4\pi R^2 dR = -\frac{4\pi u_0}{V}\int_{R_0}^\infty \left(\frac{R}{R_0}\right)^6 R^2 dR = \frac{4\pi u_0}{V}R_0(1/3)$$

with that

$$< U_e > = \frac{1}{2}N < u > = \frac{2\pi}{3}R_0^3\frac{1}{3}u_0\frac{N}{V}$$

From the potential to an equation of state

The distance of closest approach between molecules is R_0. This, in each encounter between a pair of molecules, there is a volume excluded to one molecule by the presence of the other one. Because of $\frac{1}{2}N^2$ pairs of molecules, the total excluded volume is $\frac{1}{2}N^2\frac{4}{3}\pi R_0^3$. This volume must be equal to NV_x . Thus it follows

$$V_x = \frac{2\pi}{3}R_0^3 = 4\left[\frac{4\pi}{3}\left(\frac{R_0}{2}\right)^3\right]$$

This is four times the volume of the hard sphere molecule.

The equation of state is than given by

$$< p > = \frac{1}{\beta}\frac{\partial \ln Z}{\partial V} = \frac{1}{\beta}\frac{\partial}{\partial V}[N \ln(V - V_x) - N\beta < U_e >] \, .$$

one obtains

$$< p > = \frac{kTN}{V - b'N} - \frac{a'N^2}{V^2}$$

with $b' = \frac{2\pi}{3}R_0^3$ and $a' = \frac{2\pi}{3}R_0^3 u_0 \frac{1}{3}$

Rearranging the terms give

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

where $a = N_A^2 a'$ and $b = N_A b'$ and v is the molar volume.

For nuclear matter

Calculate the equivalent to the partition function: energy density functional because we are dealing with a quantum mechanical system

$$E = <\Psi|\mathrm{T}+\mathrm{V}|\Psi>$$

T = kinetic Energy
V = potential Energy
Ψ = Slater determinant

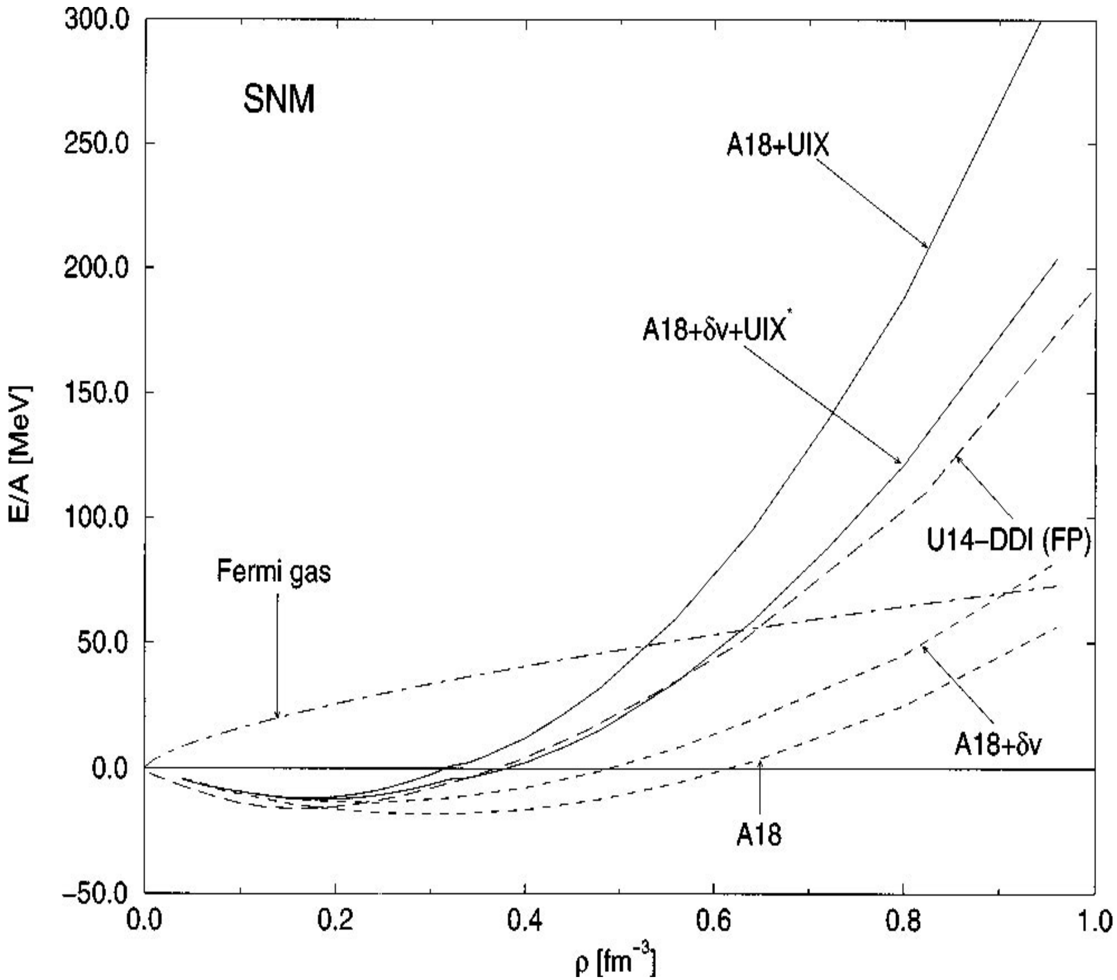
$$\Psi(1,2,3\ldots \mathrm{N}) = \mathrm{A_N}\psi_1(1)\psi_2(2)\ldots\psi_N(N)$$

where A_N is the anti-symmetrization operator, which exchanges pairs of particles.

This can be re-written as

$$A_N[\phi_1(1)\phi_2(2)\ldots\phi_N(N)] = \frac{1}{\sqrt{N!}}\begin{vmatrix} \phi_1(1) & \phi_2(1) & \ldots & \phi_N(1) \\ \phi_1(2) & \phi_2(2) & \ldots & \phi_N(2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(N) & \phi_2(N) & \ldots & \phi_N(N) \end{vmatrix} = |\phi_1\phi_2\ldots\phi_N|$$

Examples for EOS of symmetric nuclear matter

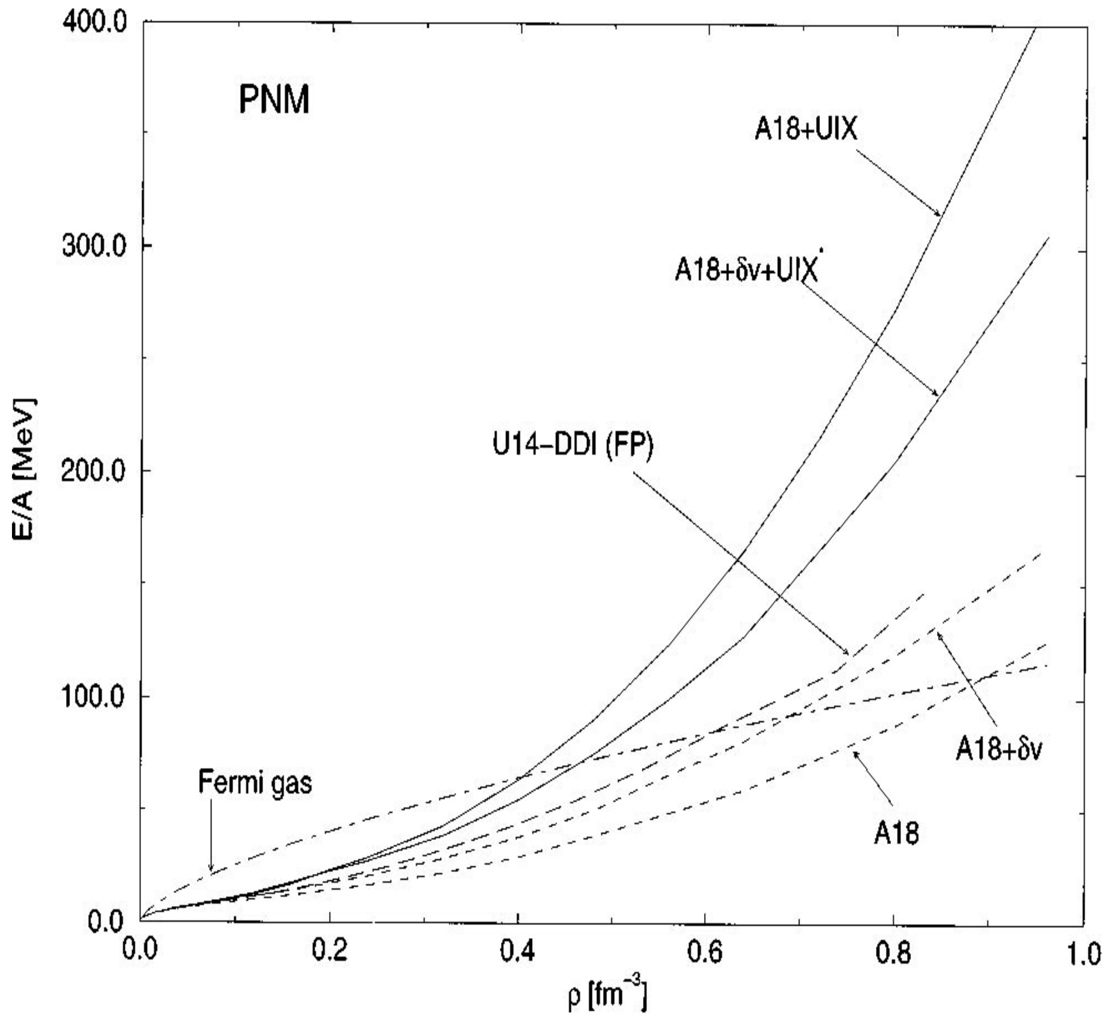


Variational method

A18: Argonne potential
UIX: including three body forces
δv: relativistic corrections

Akmal et al.
PRC 58 (1998) 1804

EOS for pure neutron matter

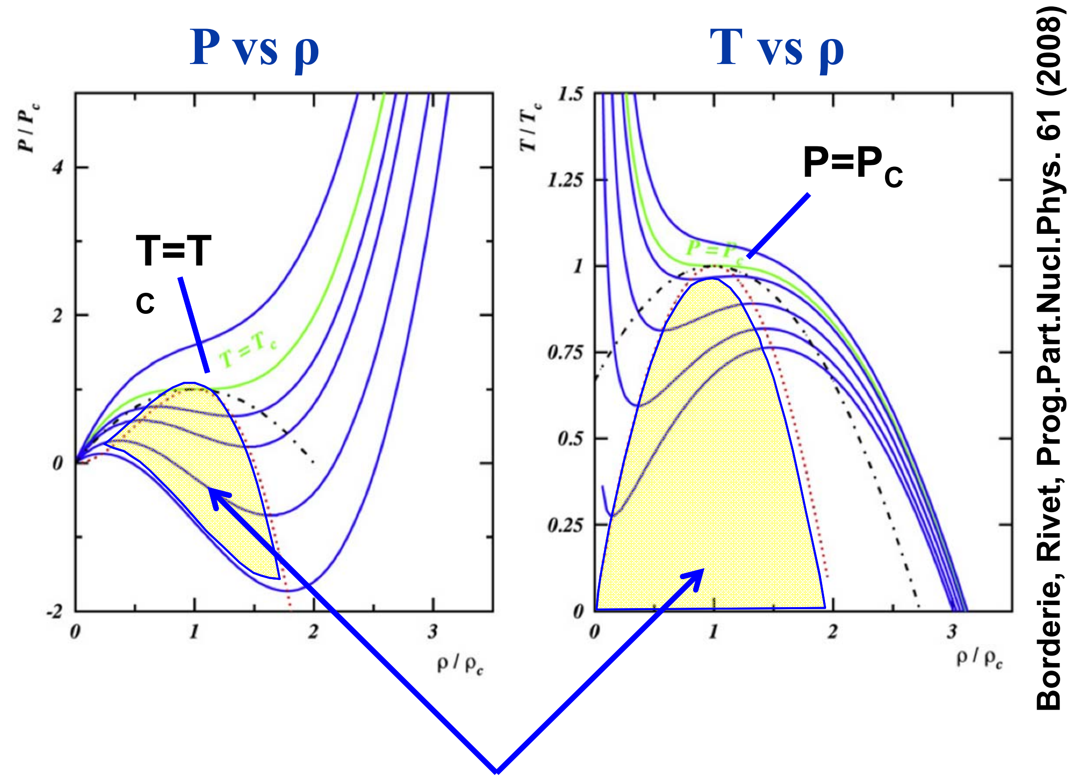


Variational method

A18: Argonne potential
UIX: including three body forces
δv: relativistic corrections

Akmal et al.
PRC 58 (1998) 1804

Equation of state as a function of temperature



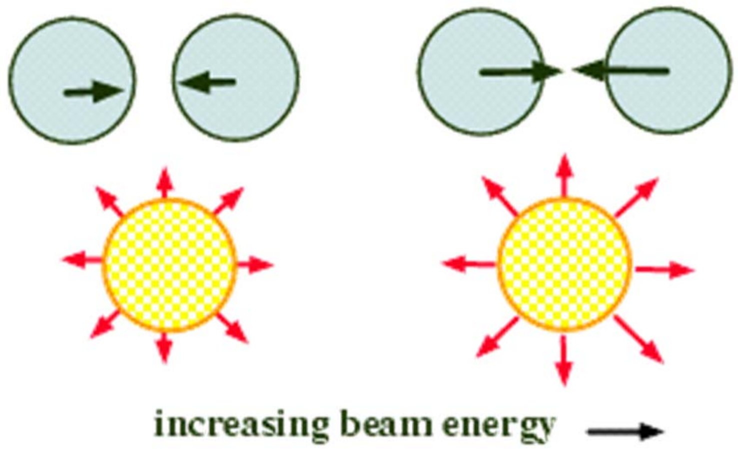
Spinodal instability region (K<0):
Liquid-gas phase transitions
at $\rho<\rho_0$ and T<15 MeV?

needs some heating and low densities

4.2

Methods to heat nuclei

a) Heating of participant:



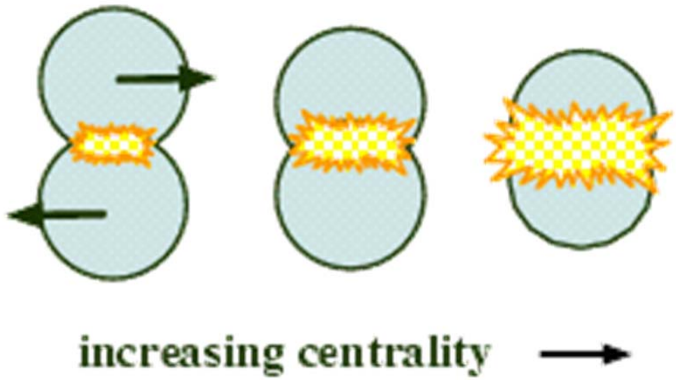
Typical Incident Energy ≈ 30AMeV

Flow

$\sigma_{centralcollision} \approx 0$

Overlap Participant & Spectators

b) Heating of spectator(s):



Typical Incident Energy ≈ 1 AGeV

(*almost*) no flow

Source well localized in rapidity

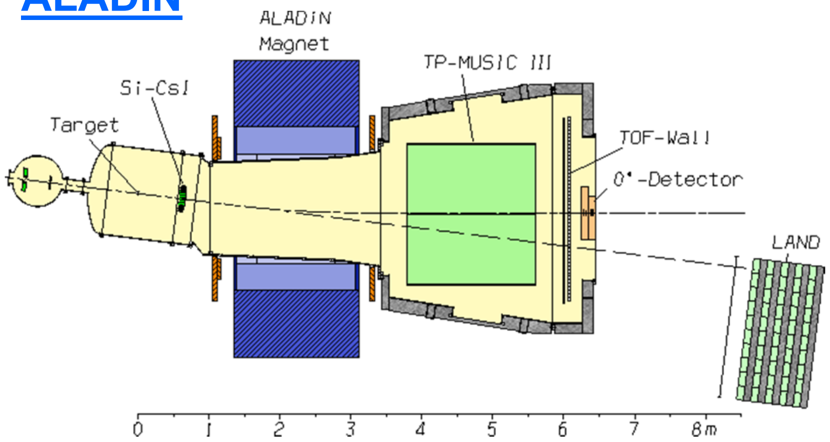
Equilibrated System

Easy 4π coverage for fragments

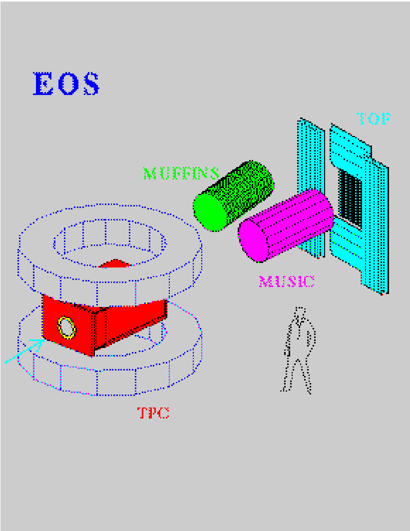
Detectors for LG phase transition

Spectrometers

ALADIN

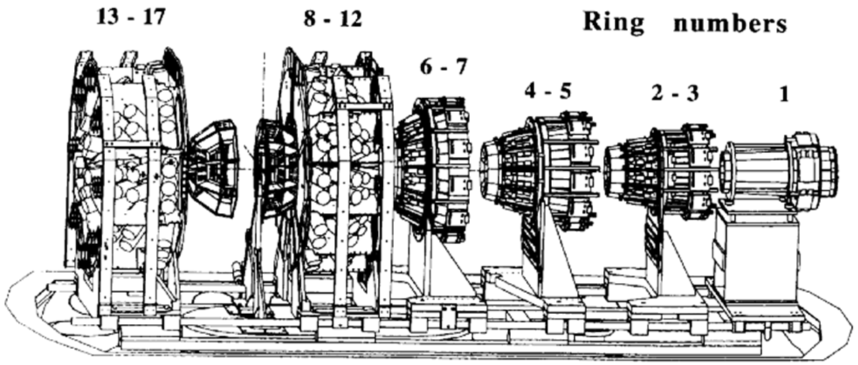


EOS

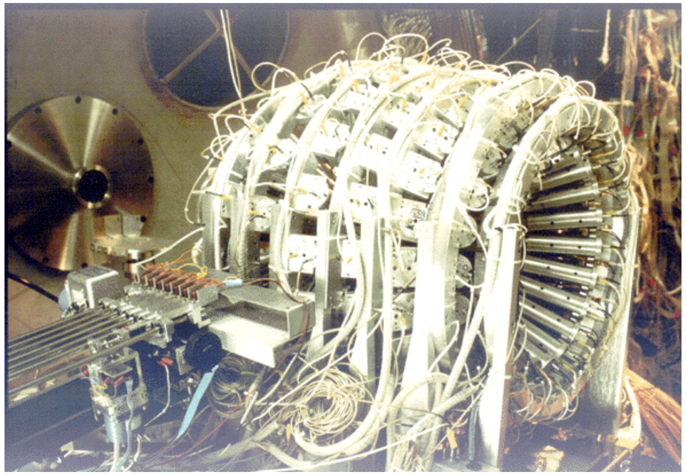


4π detectors

INDRA



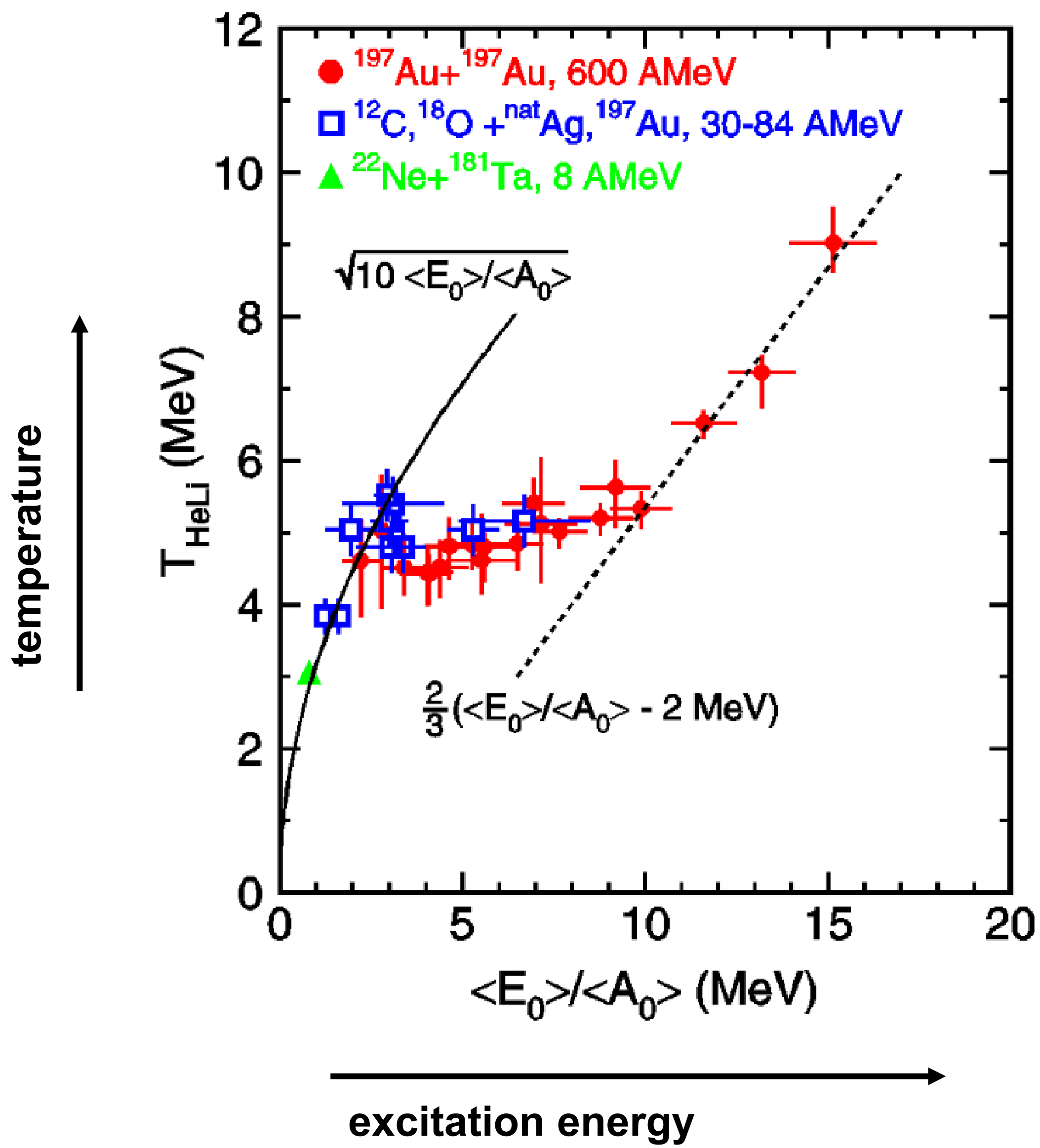
MINIBALL



4.2

Caloric curve of nuclei

J. Pochodzalla et al. (ALADIN), PRL 75(1995) 1040



Recent review:
B. Borderie, M.F. Rivet, "Nuclear multifragmentation and phase transition for hot nuclei", Progress in Particle and Nuclear Physics 61 (2008) 551-601

4.3

Statistical thermal model

P. Braun-Munzinger et al., arXiv:nucl-th/0304013

Chemical thermal model:

assume a common 'surface' at which all particles decouple (inelastic collisions stop)

Grand canonical formulation (i.e. energy and particle exchange with heat bath)

Partition function:

$$Z^{GC}(T,V,\mu_Q) = Tr\left[e^{-\beta\left(H - \sum_i \mu_{Q_i} Q_i\right)}\right]$$

Q_i = conserved quantum numbers (baryon number, strangeness, isospin, charm,…)

β= 1/T, T= Temperature

H = Hamiltonian of non-interacting hadron gas

Grand canonical potential J:

$$J(T,V,\mu_Q) = -T \ln Z^{GC}(T,V,\mu_Q)$$

$$F(T,V,N) = J(T,V,\mu_Q) + \sum_i \mu_{Q_i} N_i$$

Decomposition into individual hadronic species:

$$\ln Z^{GC}(T,V,\mu) = \sum_i \ln Z_i^{GC}(T,V,\mu)$$

Density of particle species i:

$$n_i(\mu,T) = \frac{N_i}{V} = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu_i}$$

4.3

Thermal model for particle production

P. Braun-Munzinger et al., arXiv:nucl-th/0304013

Chemical equilibrium concept.

Density of particle state i:

$$n_i(\mu,T)=\frac{N_i}{V}=-\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu}=\frac{g_i}{2\pi^2}\int\frac{p^2dp}{e^{\frac{E_i-\mu_i}{T}}\pm 1}$$

$$\mu_i=\mu_B B_i+\mu_S S_i+\mu_{I_3} I_{3,i}$$

"+" for fermions, "-" for bosons
g_i – spin degeneracy factor

Chemical potentials μ_i are constrained by conservation of quantum numbers:

baryon number: $V\sum_i n_i B_i = Z+N \rightarrow V$

strangeness: $V\sum_i n_i S_i = 0 \rightarrow \mu_S$

charge: $V\sum_i n_i I_{3,i} = \frac{Z-N}{2} \rightarrow \mu_{I_{3,i}}$

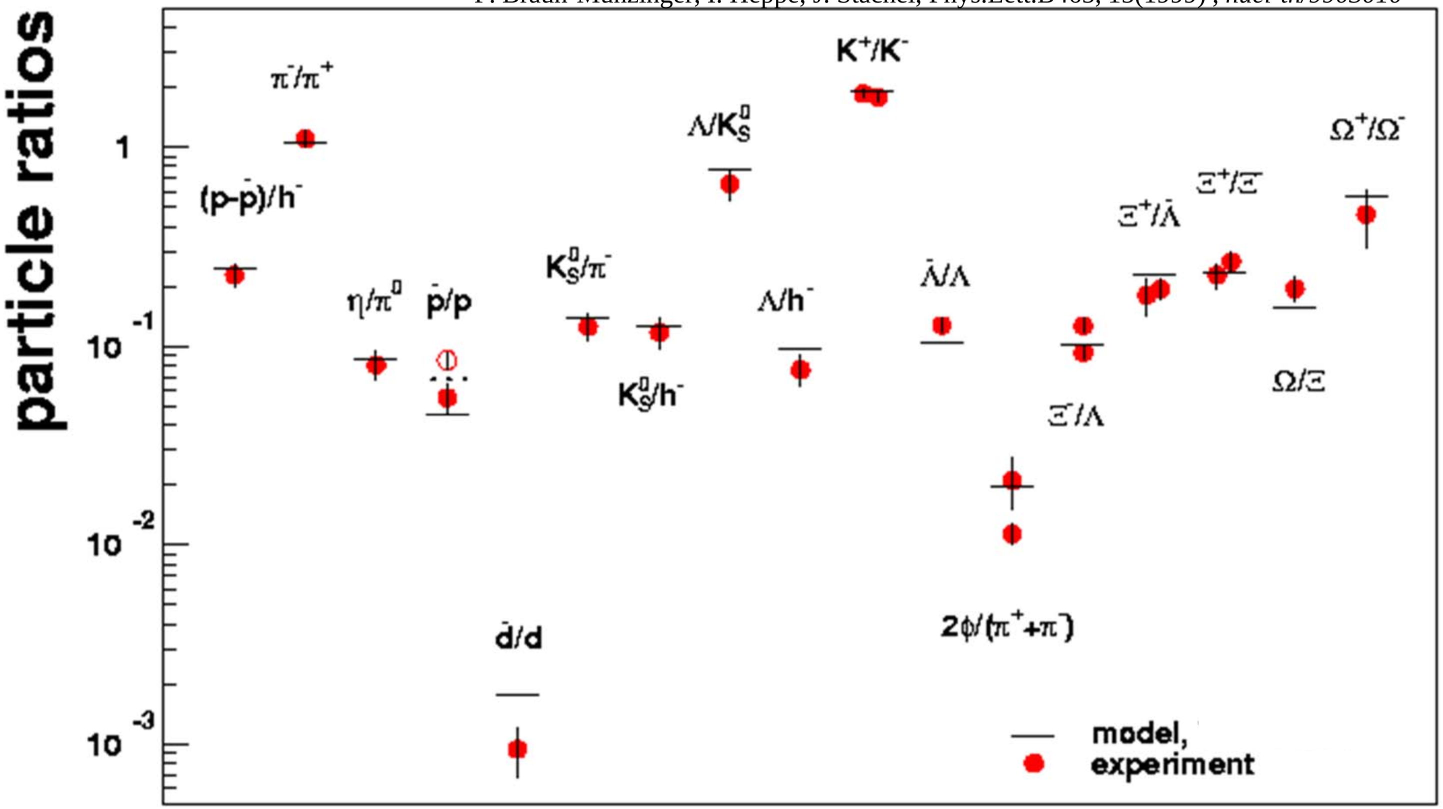
⇒

3 equations,
5 unknows
↓
2 free parameter

Chemical equilibrium

Example: SPS data, E_{beam}=158 AGeV, Pb+Pb

P. Braun-Munzinger, I. Heppe, J. Stachel, Phys.Lett.B465, 15(1999) , *nucl-th/9903010*



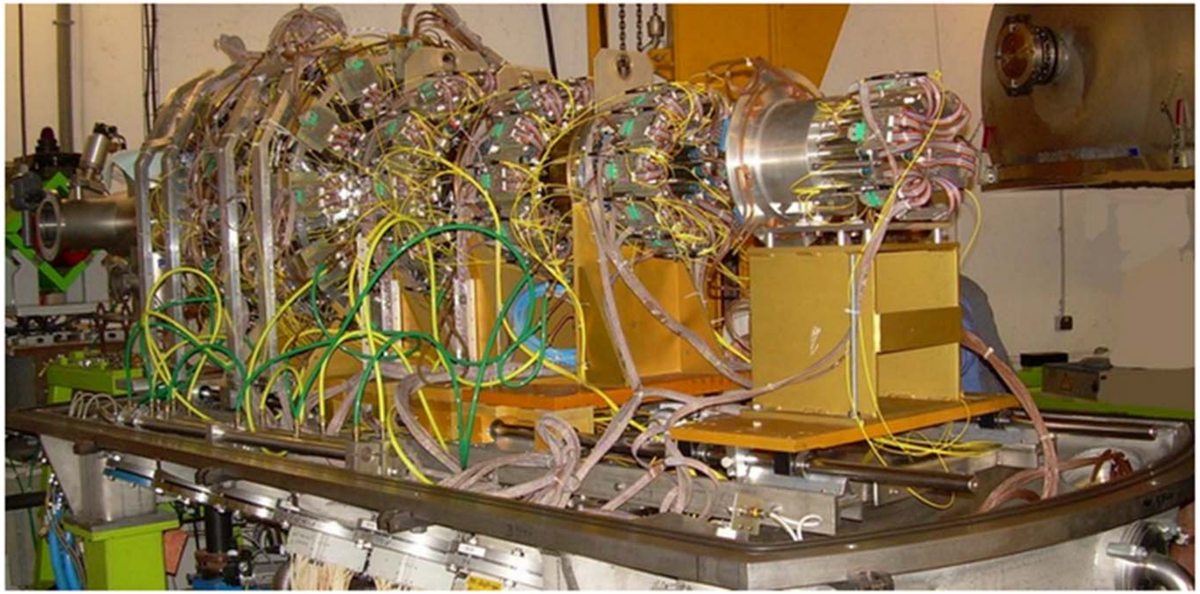
Model parameter:

$$T = 168 \pm 2.4 \text{ MeV}$$
$$\mu_B = 266 \pm 5 \text{ MeV}$$
$$\mu_S = 71.1 \text{ MeV}$$
$$\mu_{I_3} = -5. \text{ MeV}$$

Note: volume is not needed for description of particle ratios.

Experimental Setup

INDRA (Identification des Noyaux et Détection à Résolution Accrue)
http://pro.ganil-spiral2.eu/laboratory/detectors/indra



Measurement of excitation energy

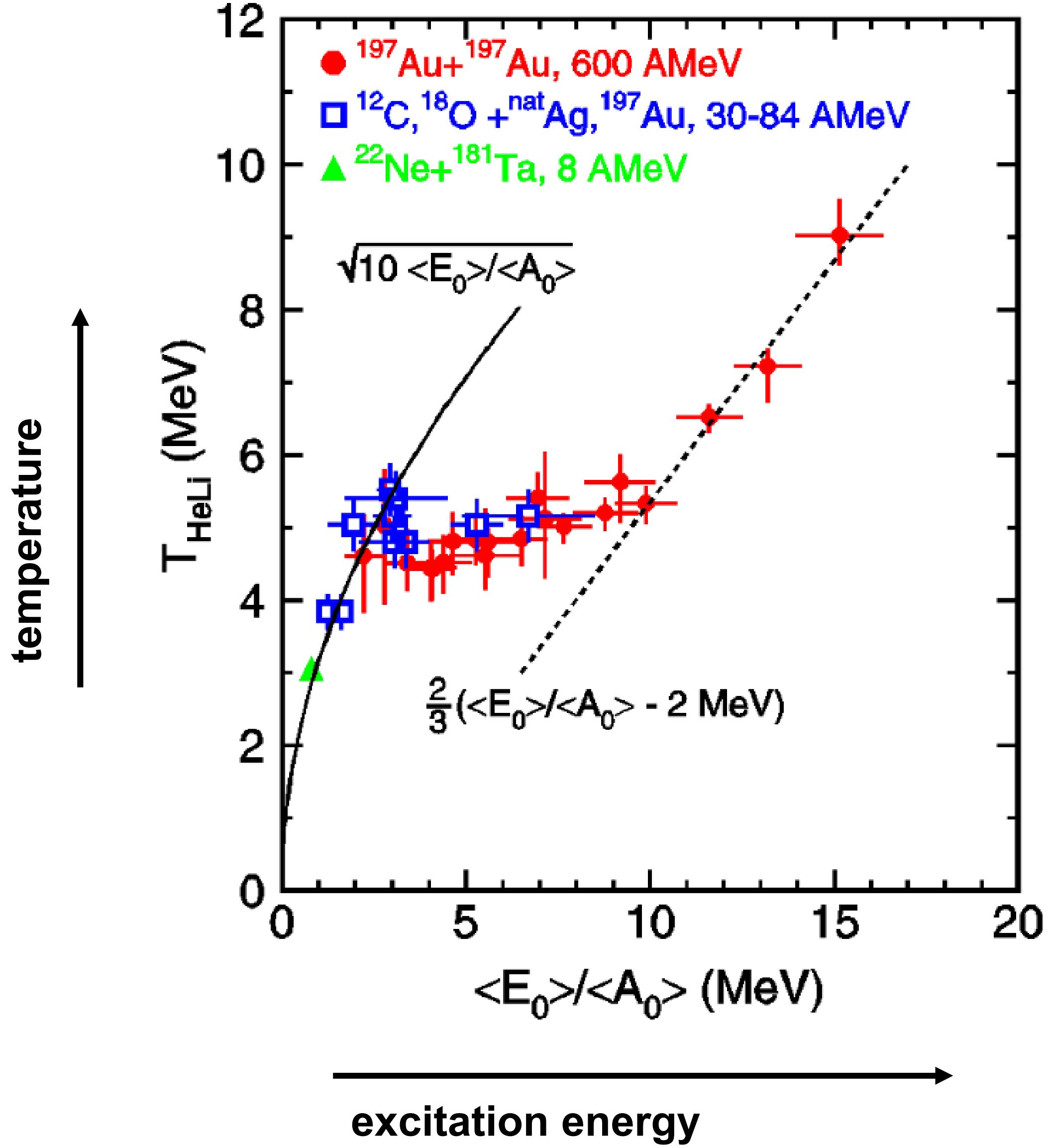
$$\langle E_0 \rangle = \underbrace{\left(\left\langle \sum_i m_i \right\rangle + \left\langle \sum_i K_i \right\rangle \right)}_{\text{measured}} - \underbrace{\left(\langle m_0 \rangle + \langle K_0 \rangle \right)}_{\text{beam}}$$

$$A_0 = \sum_i A_i$$

4.4

Caloric curve of nuclei

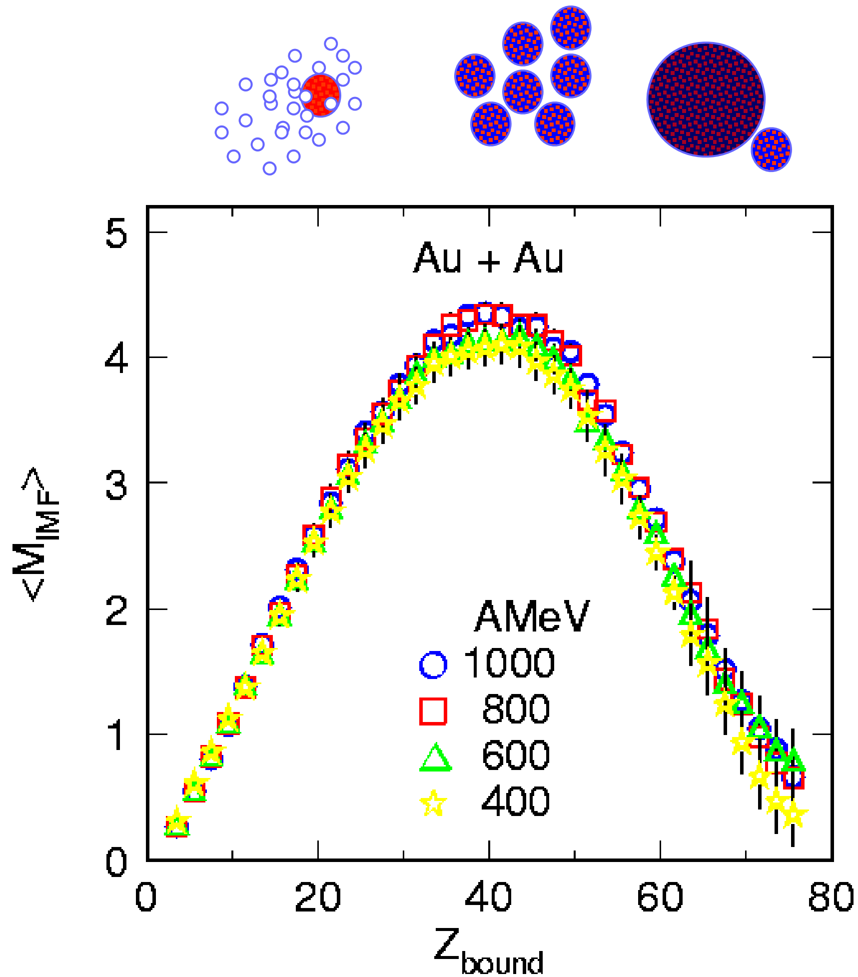
J. Pochodzalla et al. (ALADIN), PRL 75(1995) 1040

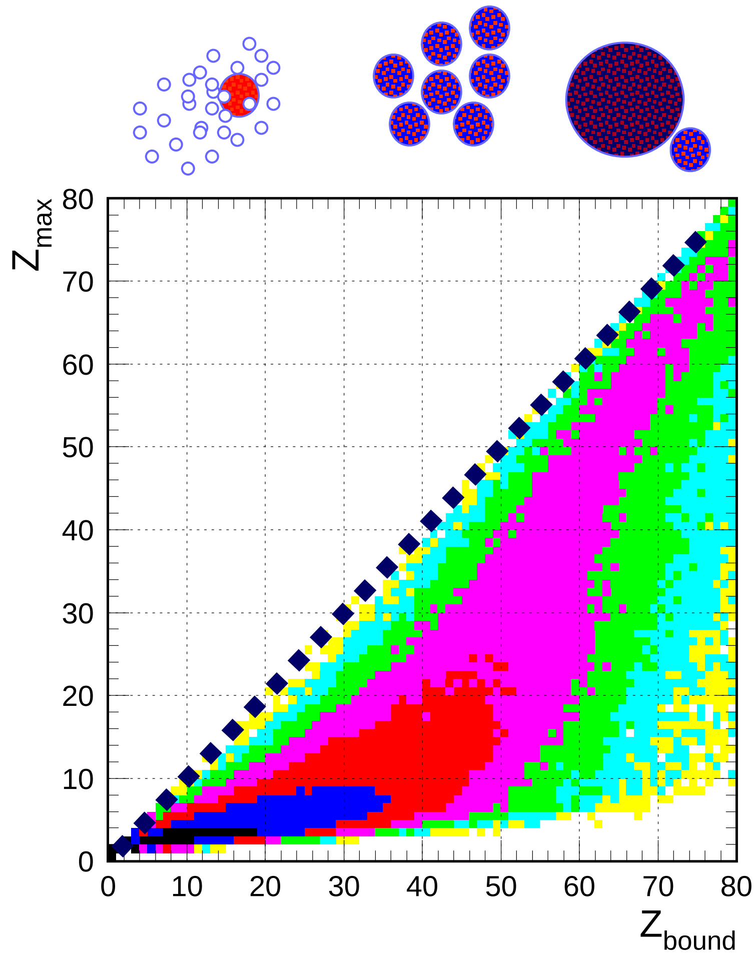


Recent review:

B. Borderie, M.F. Rivet, "Nuclear multifragmentation and phase transition for hot nuclei", Progress in Particle and Nuclear Physics 61 (2008) 551-601

Multifragmentation - Bimodality





4.4

Critical exponents

Close to a critical point the thermodynamic behaviour of physical systems is universal and depends within a universality class only on

$$t = \frac{T - T_C}{T_C}$$

Thermodynamic quantities show a power law behaviour:

Heat capacity: $C \sim |t|^{-\alpha}$

Order parameter: $M \sim |t|^{\beta}$

Susceptibility: $\chi \sim |t|^{-\gamma}$

Equation – of – state: $M \sim |H|^{\frac{1}{\delta}}$

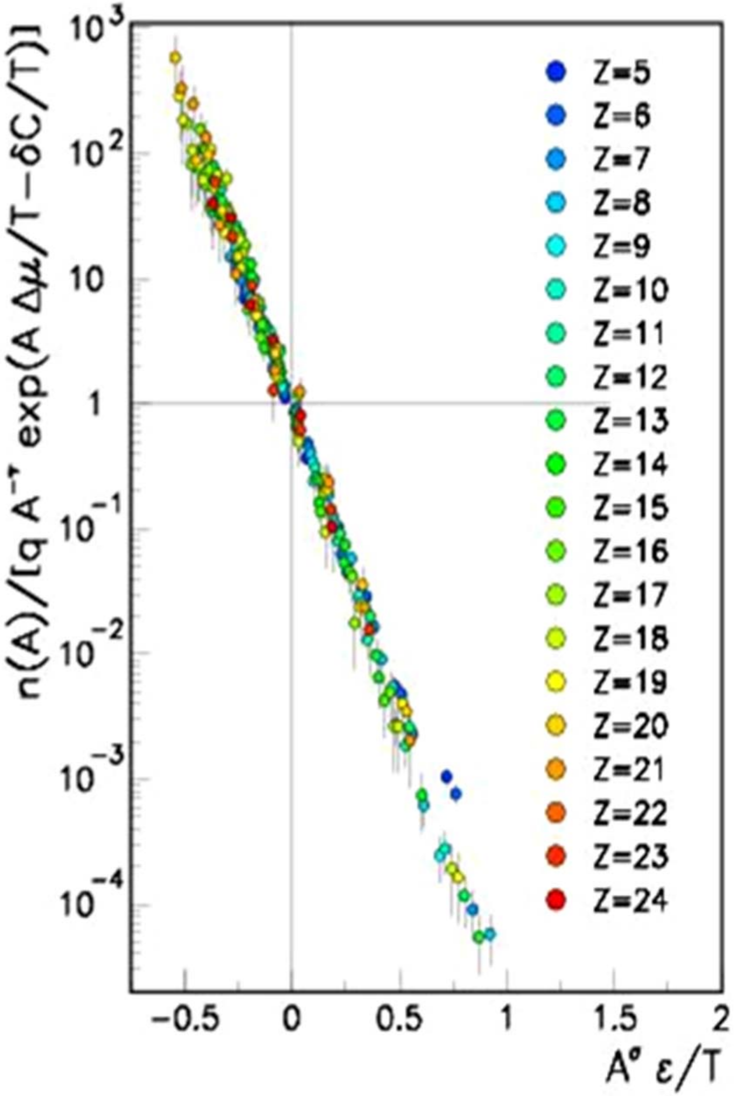
Correlation length: $\xi \sim |t|^{-\nu}$

Griffiths universality hypothesis
R.B. Griffiths, PRL 24, 1479 (1970):
Critical exponents are universal and depend only on:
1) **Dimension of the system**
2) **Range of the interaction**
3) **Spin dimensionality**

Note: Only 2 of the critical exponents are independent.

Nuclear liquid - gas phase transition

M. D'Agostino et al., NPA 724, 455 (2003)



Fragment distribution

(Fisher droplet model 1967)

$$n_A = q_o A^{-\tau} \exp\left(\frac{-c_o \varepsilon A^{\sigma}}{T}\right)$$

$$\varepsilon = \frac{T_c - T}{T_c}$$

Universal critical exponents in the vicinity of the critical point:

	Au	Liquid-Gas
τ	2.1±0.1	2.196 ±0.024
σ	0.66±0.02	0.647 ±0.006

4.4

Current status Nuclear LG – phase transition

B. Borderie, M.F. Rivet, Progress in Particle and Nuclear Physics 61 (2008) 551-601, Table 1

Excitation energies (MeV/nucleon) where LG - phase transition signatures occur:

variable	QP A_s ~36	QP Z_s ~68	monosources Z_s ~82
E_{crit}^{Fisher}	5-6	4.2	3.8-4.5
Δ scaling	5-6	-	6.2
max $A_s\sigma_k^2/T^2$	4-6.5	4-5	$\leq$4
max $\sigma^2_{Z_{max}}/\langle Z_{max}\rangle$	5-6	-	-
$c < 0$	-	[2.5:5.5]	[$-$:6.5]
max σ_{Z_{max}/Z_s}	-	4-5	$\leq$ 5
max $\langle Z_{max2}\rangle$	6	5	4.5-6
E_{Zipf}: $\lambda = 1$	5.6	8.5	7.5
$S_p = 0.5$	5.2	8.5 and above	3.2 - 6
change slope S_p	5.6	4	-
$E_{tr}^{bimodality}$	5.6	[4.75:5.25]	7.8
spinodal	-	[5:8]	[5:9]
threshold ε_{rad}	-	~5.	~4.5