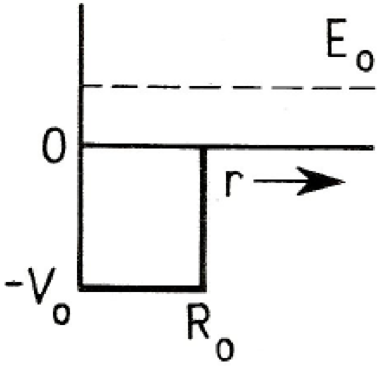
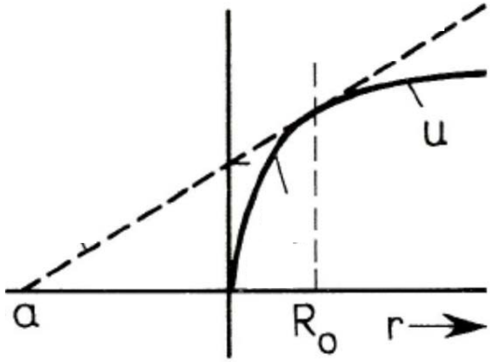
3.2

Visualisation of scattering length

attractive potential:

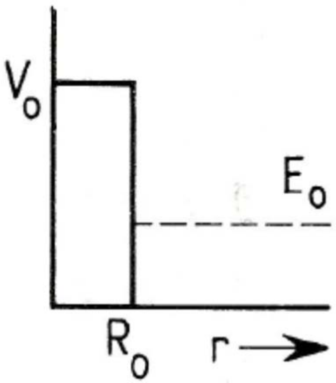


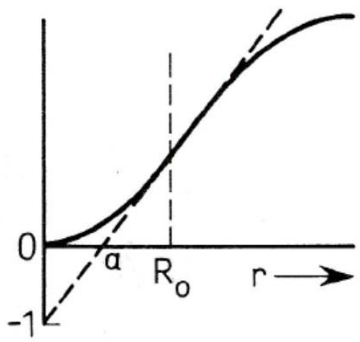


wavefunction pulled into potential region,

a<0

repulsive potential:





wavefunction expelled from potential region,

$0<a<R_0$

3.2

n-p cross section

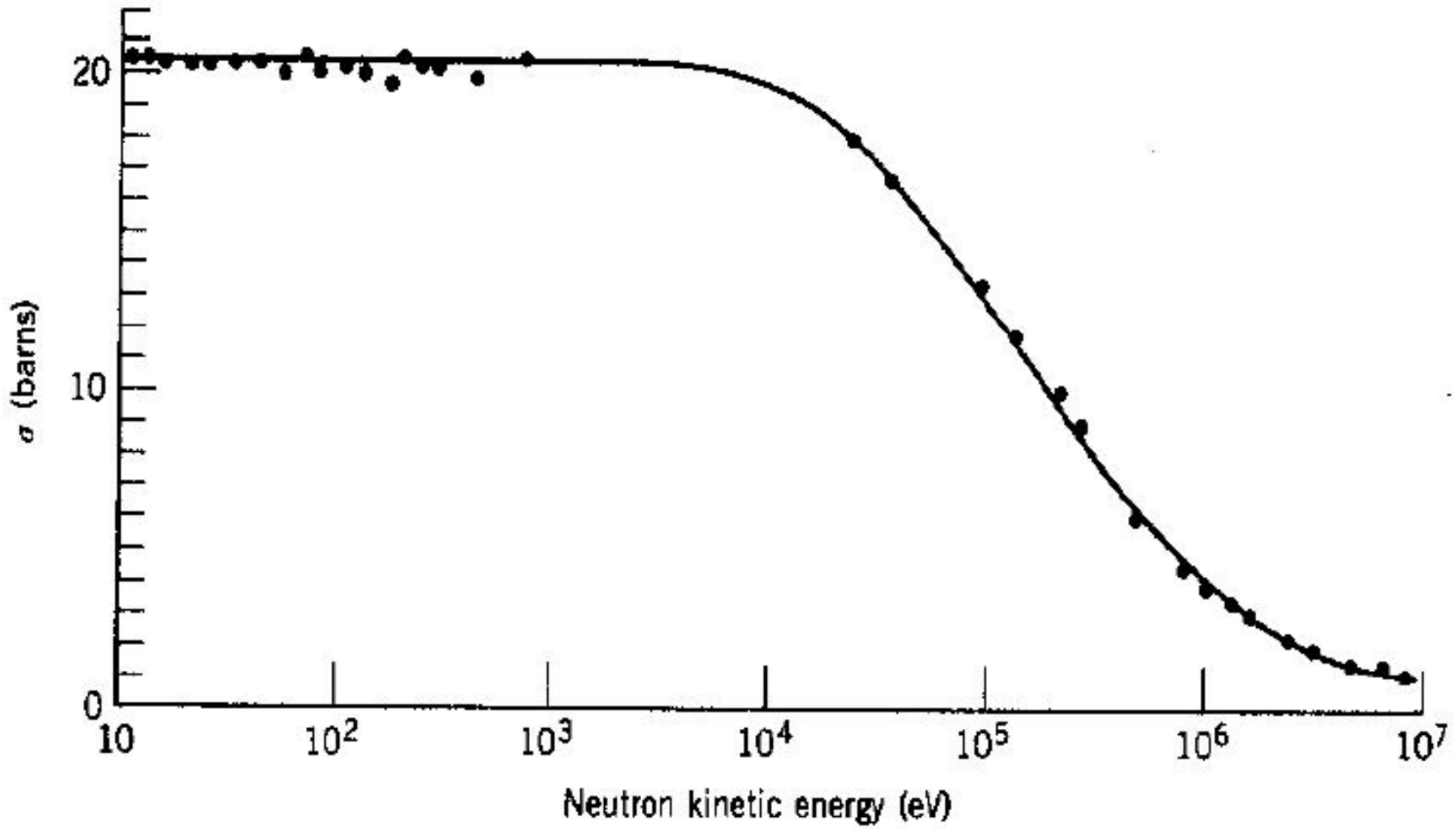
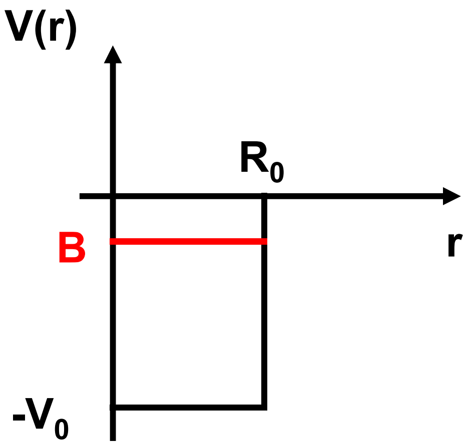


Figure 4.6 The neutron–proton scattering cross section at low energy. Data taken from a review by R. K. Adair, *Rev. Mod. Phys.* **22**, 249 (1950), with additional recent results from T. L. Houk, *Phys. Rev. C* **3**, 1886 (1970).

Bound states



Ansatz for 2 particle bound state:
μ – reduced mass

$$r < R_0 : k_1^2 = 2\mu(V_0 - B) \longrightarrow u_1(r) = A_1 \sin k_1 r$$

$$r > R_0 : k_2^2 = -2\mu B \longrightarrow u_2(r) = A_2 e^{-\frac{r}{R}}, R = \sqrt{2\mu B}$$

Applying continuity conditions:

$$\cot\left[2\mu(V_0 - B)R_0^2\right]^{\frac{1}{2}} = -\left(\frac{B}{V_0 - B}\right)^{\frac{1}{2}}$$

$$\Downarrow$$

Range depth relation for n-p system

$$V_0 = \frac{100\,\text{MeV}}{(R_0(\text{fm}))^2}$$

Typical numbers for deuteron: **R_0 ~ 1.4fm, B=2.225 MeV => V_0=50 MeV**

Size of bound state corresponds to scattering length **a = R_0 + R (a>R_0)**

Total s- wave cross section

$$\sigma_{tot} = 4\pi a^2 \approx 4b$$

n – p – cross section

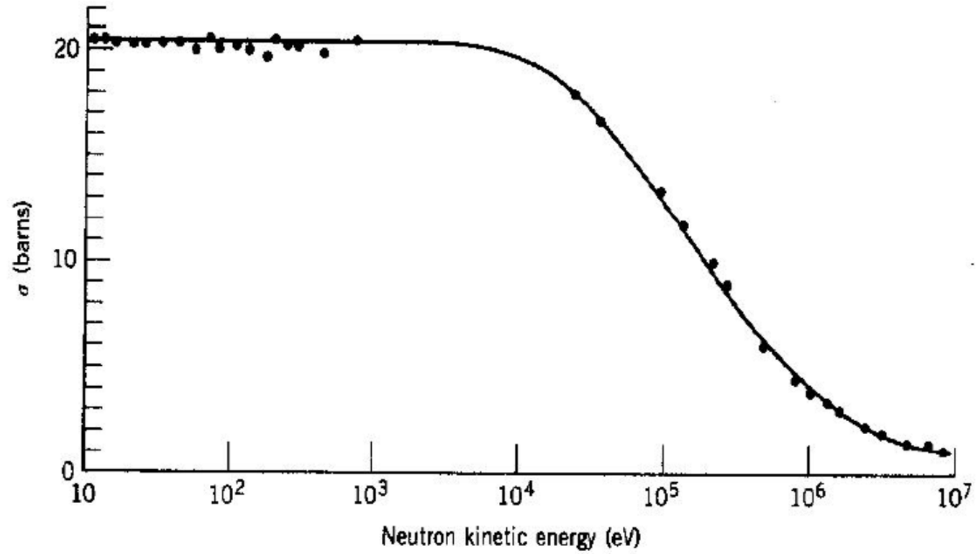


Figure 4.6 The neutron–proton scattering cross section at low energy. Data taken from a review by R. K. Adair, *Rev. Mod. Phys.* **22**, 249 (1950), with additional recent results from T. L. Houk, *Phys. Rev. C* **3**, 1886 (1970).

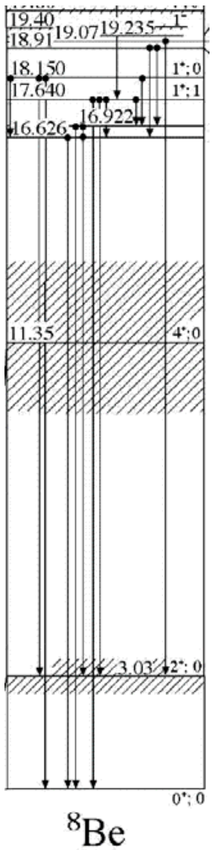
Measure cross section larger than the one anticipated from deuteron binding energy.

Explanation: Spin – dependent NN – force

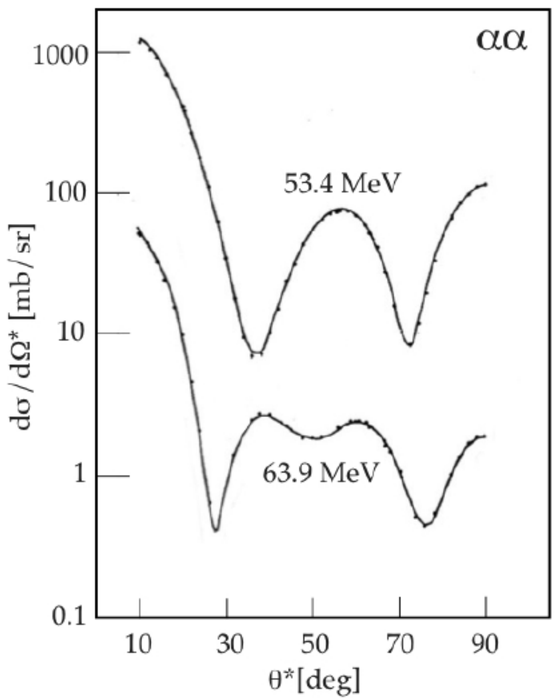
$$\sigma_{\text{scattering}} = \frac{3}{4}\sigma_{\text{triplett}} + \frac{1}{4}\sigma_{\text{singlett}}$$

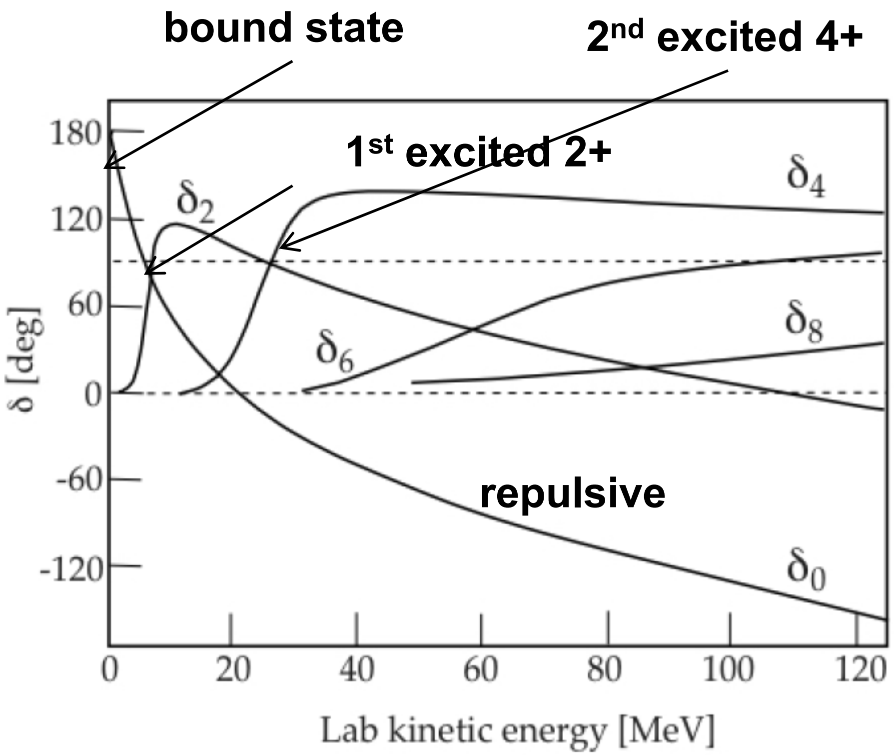
Deuteron: $J^P = 1^+$, **inferred singlett scattering cross section:** $\sigma_{\text{singlett}} = 68b$

Example of partial wave analysis



αα scattering





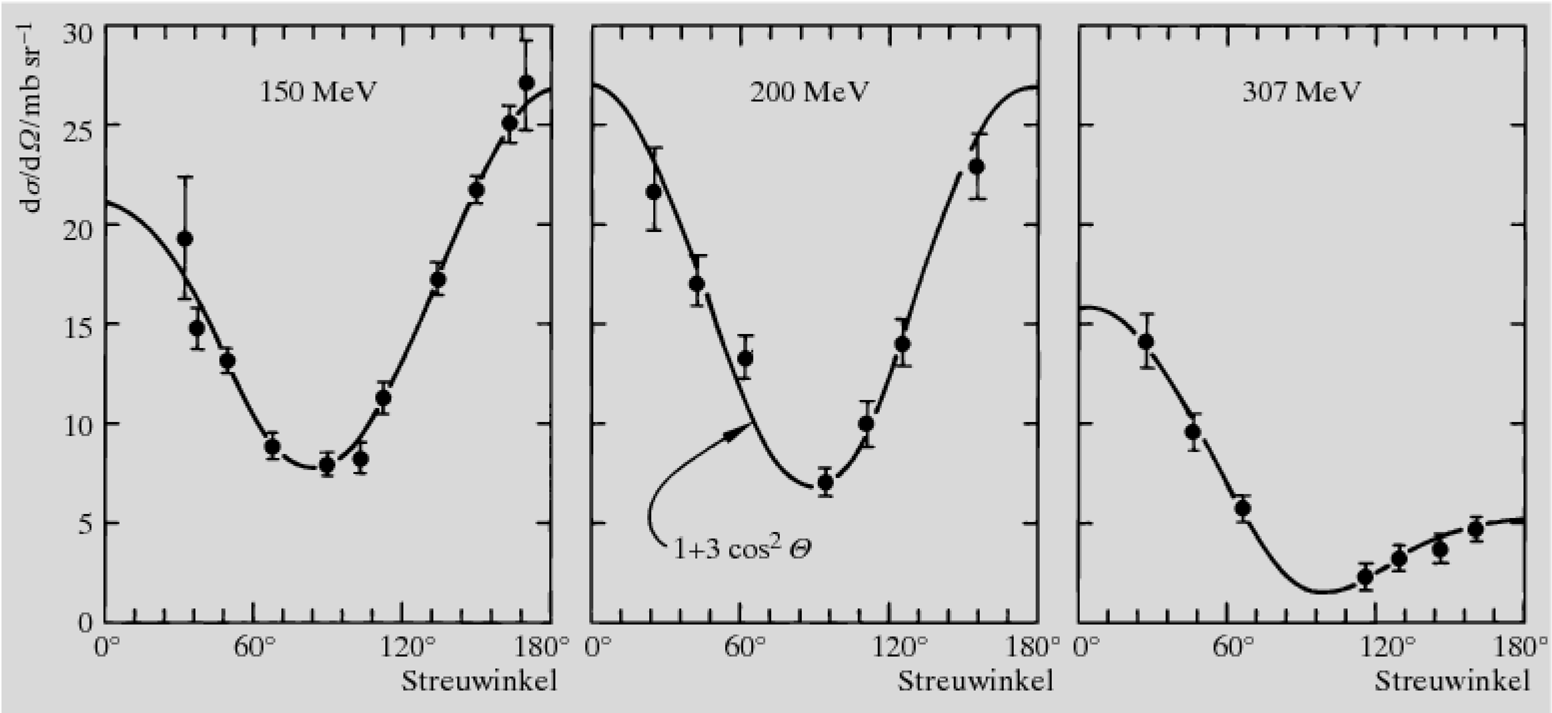
Fitting cross sections to extract scattering amplitudes $f(\theta)$

$$\frac{d\sigma}{d\Omega} = |\,f(\theta)|^2 = \frac{1}{k^2}\sum_l \sum_{l'} (2l+1)(2l'+1)e^{i(\delta_l-\delta_{l'})} sin\delta_l \, sin\delta_{l'} P_l(cos\theta) P_{l'}(cos\theta)$$

and deduce phase shifts

Partial wave analysis for elastic scattering

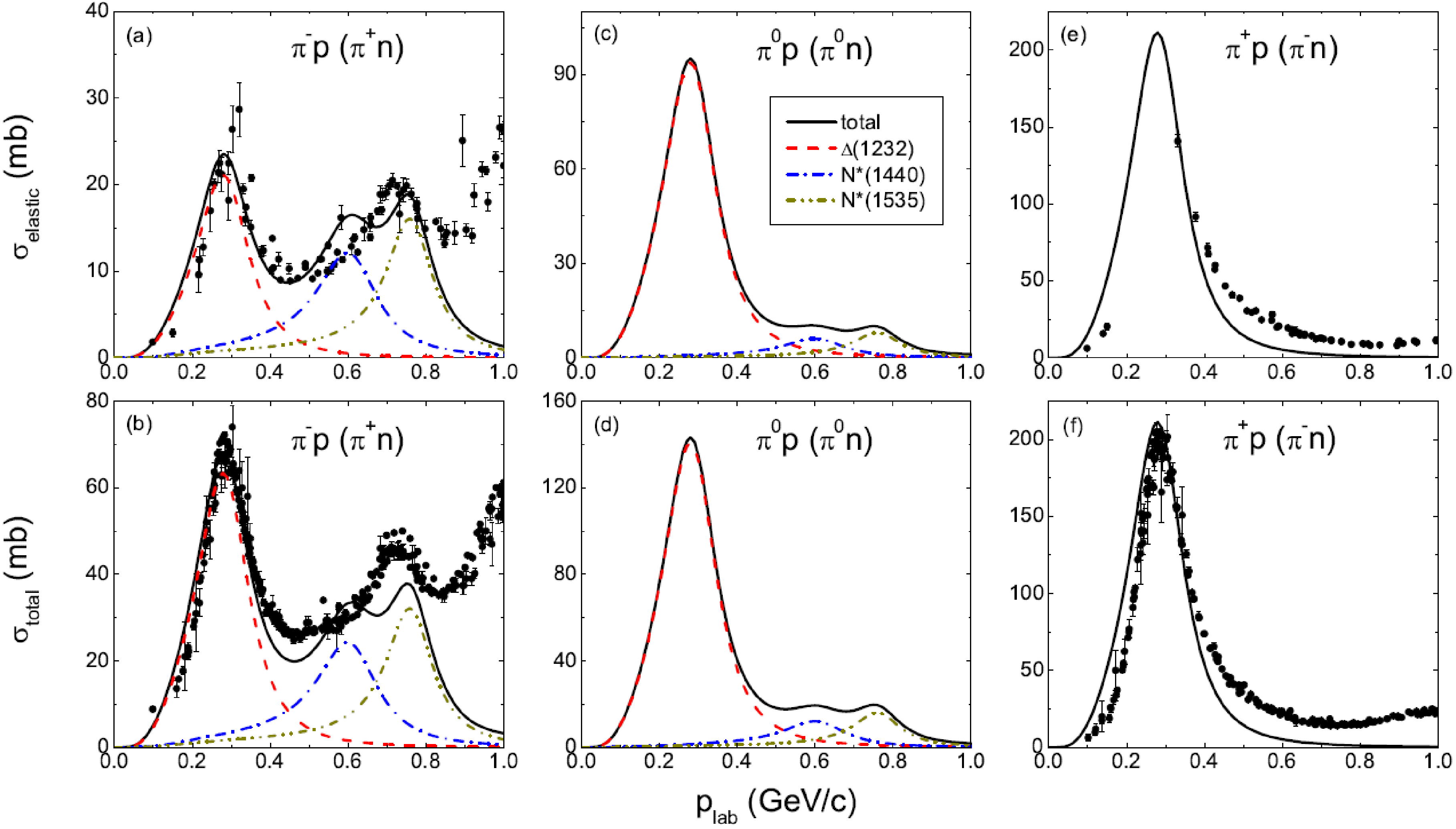
π+p scattering at different pion energies



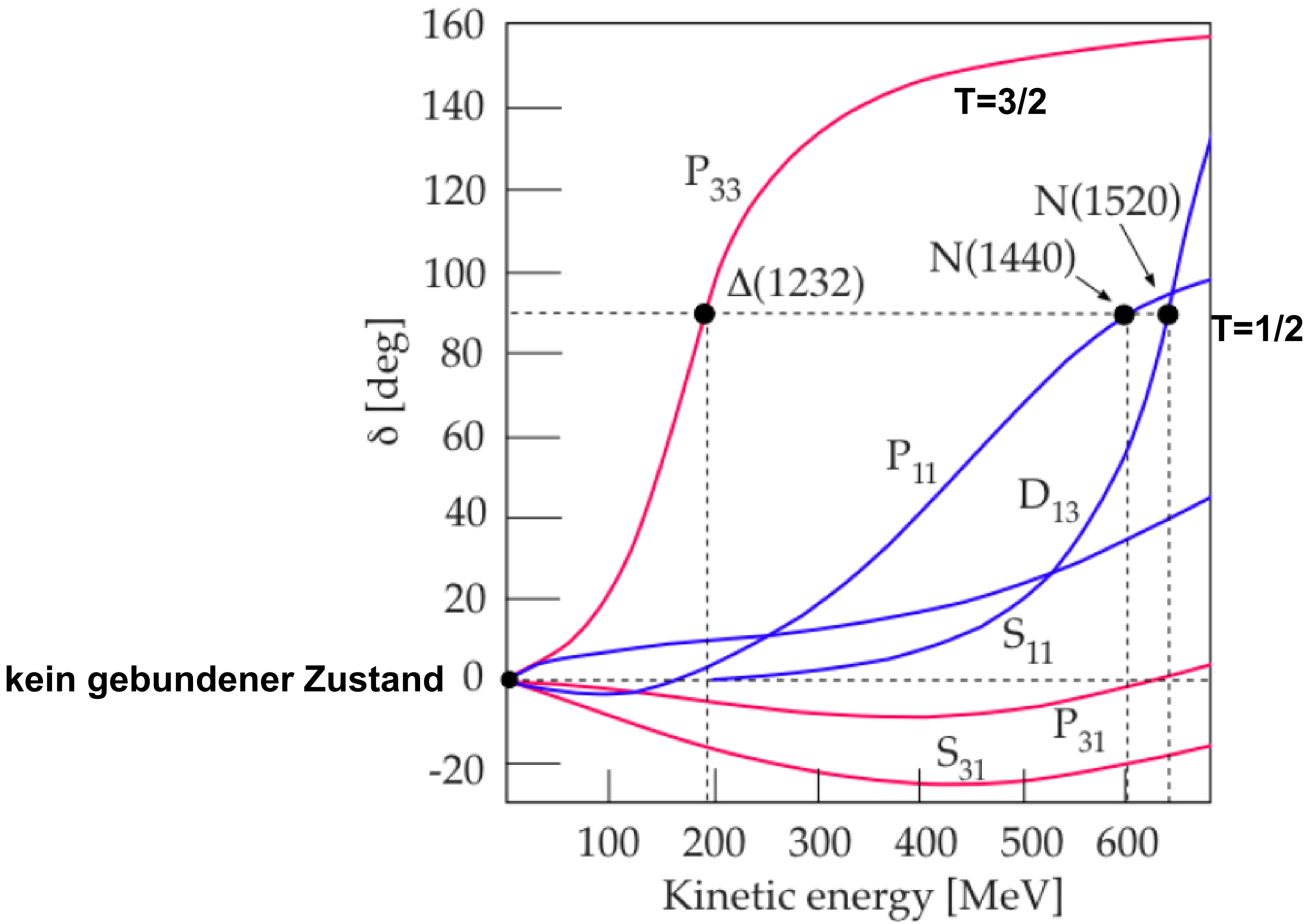
At 200 MeV p-wave scattering

https://link.springer.com/chapter/10.1007/978-3-642-41753-5_2

Elastic and total cross section of pion nucleon scattering



Phase shifts of pion-nucleon scattering



Energy dependence of np - phase shift

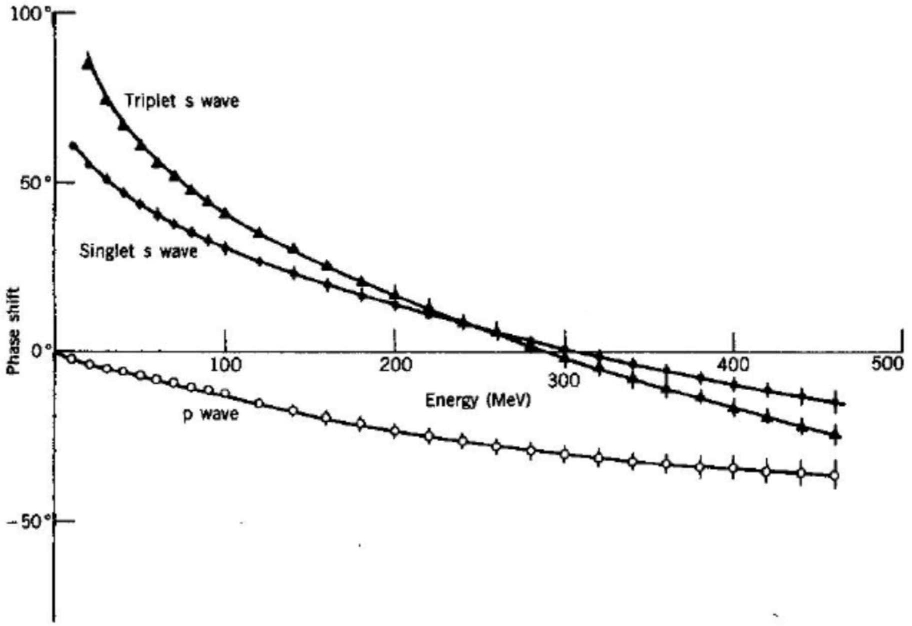
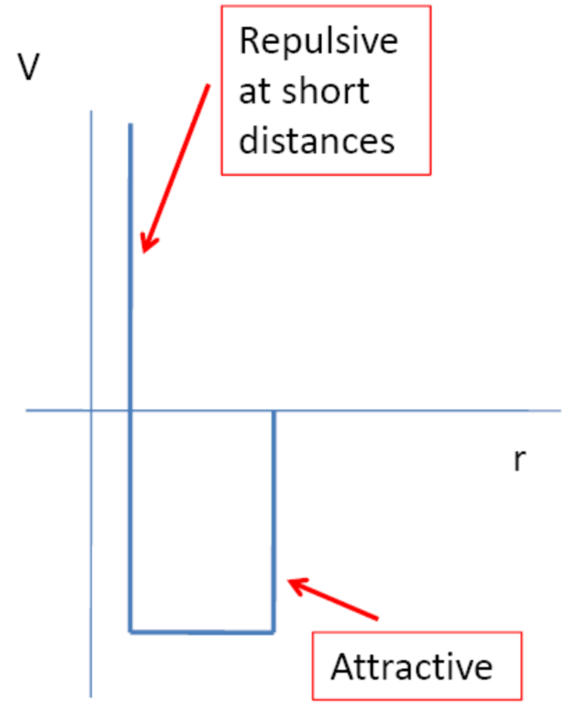


Figure 4.12 The phase shifts from neutron–proton scattering at medium energies. The change in the s-wave phase shift from positive to negative at about 300 MeV shows that at these energies the incident nucleon is probing a repulsive core in the nucleon–nucleon interaction. ▲, $^3S_1$; ●, $^1S_0$; O, $^1P_1$. Data from M. MacGregor et al., *Phys. Rev.* **182**, 1714 (1969).

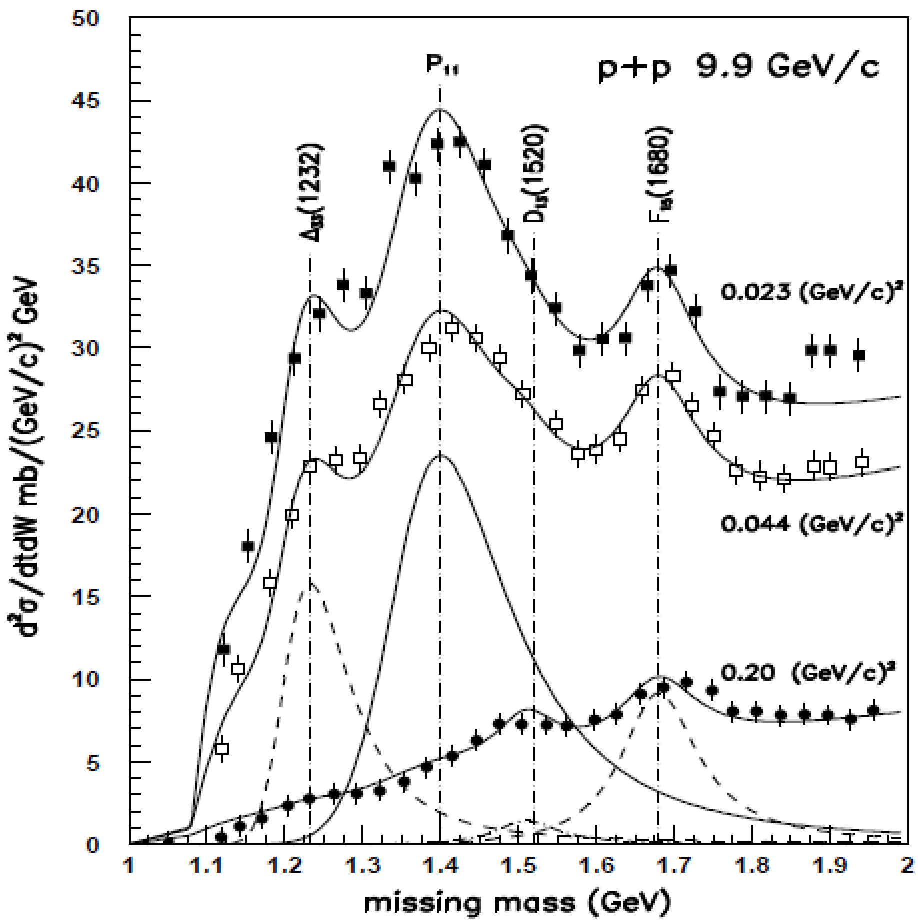
Change of sign at

E_{kin} ~ 300 MeV

Interpretation:



New measurements on P



3.2

NN – potential

Further properties of NN – interaction:

Deuteron has electric quadrupole moment -> non – central force (Tensor potential)

Deuteron spin -> D – wave contribution to wave function -> Spin – orbit potential

General form for fixed isospin:

$\vec{s}_1, \vec{s}_2$ – spin

$\vec{r}$ – relative distance

$\vec{p}$ – relative momentum

$\vec{L}$ – relative orbital momentum

$$
\begin{aligned}
V_{NN} = & V_0(r) \\
& + V_{ss}(r)\vec{s}_1 \cdot \vec{s}_2 \\
& + V_T(r)\left(3\vec{s}_1 \cdot \vec{r} \;\; \vec{s}_2 \cdot \vec{r} / r^2 - \vec{s}_1\vec{s}_2\right) \\
& + V_{LS}(r)(\vec{s}_1 + \vec{s}_2) \cdot \vec{L} \\
& + V_{Ls}(r)\left(\vec{s}_1 \cdot \vec{L} \;\; \vec{s}_2 \cdot \vec{L}\right) \\
& + V_{ps}(r)\left(\vec{s}_1 \cdot \vec{p} \;\; \vec{s}_2 \cdot \vec{p}\right) / m^2
\end{aligned}
$$

Terms constrained by requiring invariance under translations, rotations and symmetry under particle exchange.

Radial dependence not calculable from first principles.

NN – potential

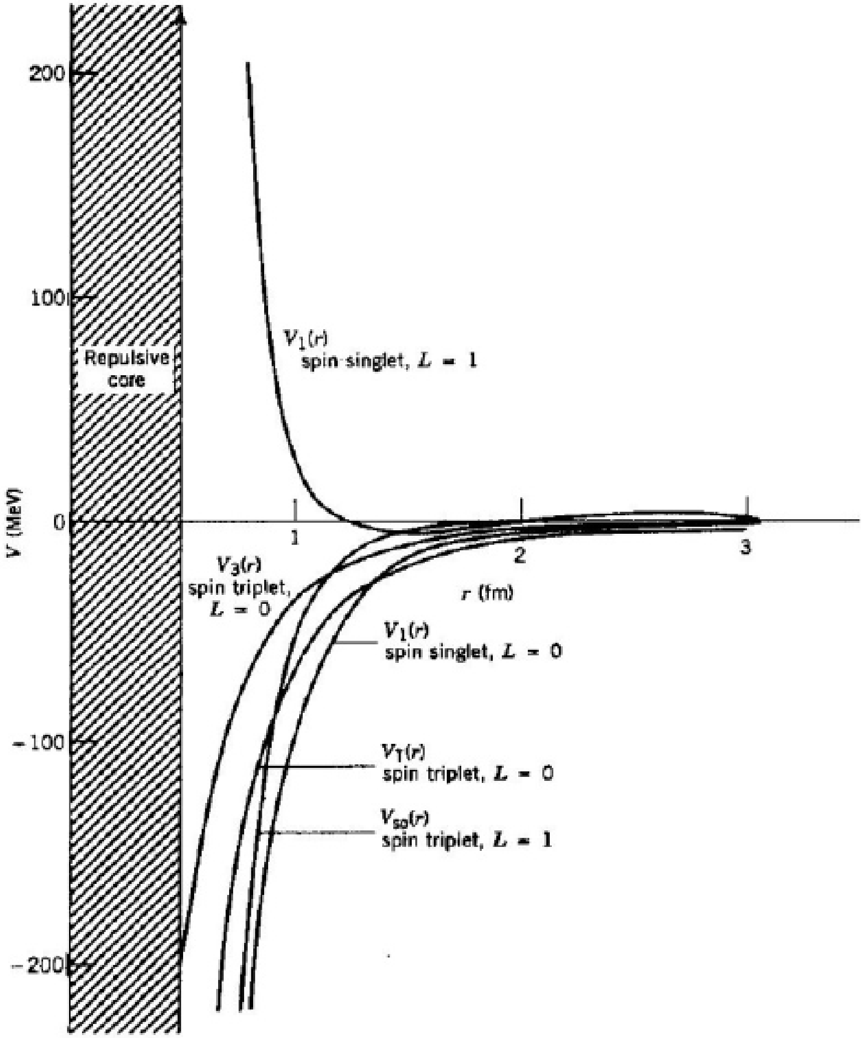
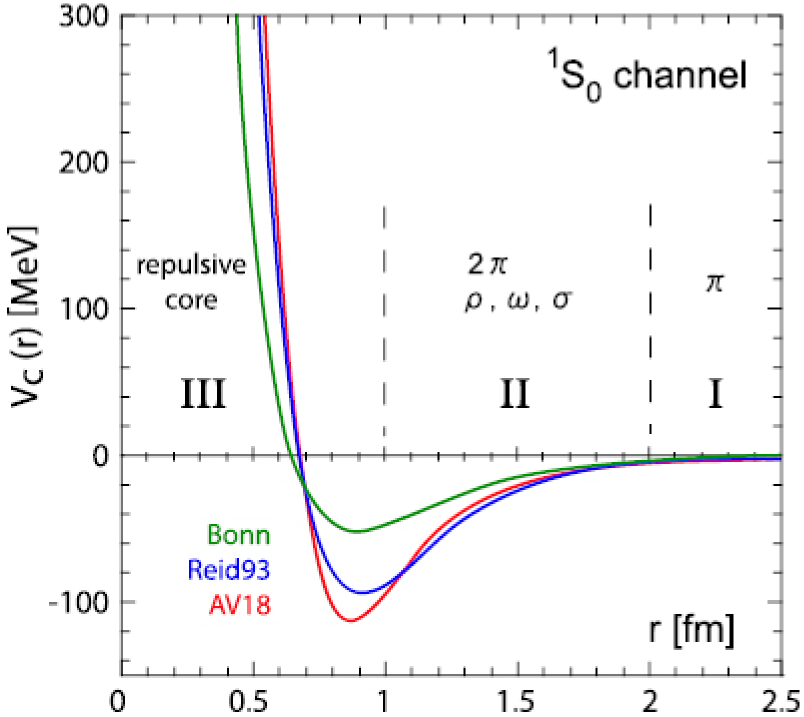


Figure 4.16 Some representative nucleon–nucleon potentials. Those shown include the attractive singlet and triplet terms that contribute to s-wave scattering, the repulsive term that gives one type of p-wave ($L = 1$) scattering, and the attractive tensor and spin-orbit terms. All potentials have a repulsive core at $r = 0.49$ fm. These curves are based on an early set of functional forms proposed by T. Hamada and I. D. Johnston, *Nucl. Phys.* **34**, 382 (1962); other relatively similar forms are in current use.

NN potential

Different potentials based on Boson-exchange picture
short range interactions mediated by heavier mesons with different characteristics
pions: pseudo scalar
rhos: vector
sigmas: scalar



AV18: R.B. Wiringa, V.G.J. Stoks, and R. Schiavilla. An accurate nucleon-nucleon potential with charge-independence breaking. Phys .Rev. C, 51:38–51, 1995.arXiv:nucl-th/9408016v1

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doi:10.1088/1749-4699/1/1/015009

One boson exchange potential – exchange particles

Type of meson	Physical meson	Interaction terms
Scalar	σ-meson	1, L-S
Pseudo scalar	π,η,η‘	Tensor S_{12}
Vector	ρ,Φ,ω	all

sigma – meson not seen experimentally, 2 pion exchange
The isovector mesons π, ρ carry an additional factor of isospin dependence
The masses and coupling constants are fitted to experimental data!