2.5

Magnetic moments of nuclei

Measurable verification: magnetic moment of nuclei (nuclear resonance method, Rabi)

Nucleons:

$$\vec{\mu}_N = g_N \mu_N \vec{s} \quad \text{mit } g_N = \begin{cases} 5.58 & \text{Proton} \\ -3.82 & \text{Neutron} \end{cases} \quad \text{und } \mu_N = \frac{e\hbar}{2m_p c} = 3.152 \cdot 10^{-14} \frac{MeV}{T}$$

Nuclei: spin orbit coupling

$$\vec{\mu}_j = \vec{\mu}_l + \vec{\mu}_s = g_j \mu_N \vec{j}$$

$$g_j \vec{j} = g_l \vec{l} + g_s \vec{s} \quad \text{mit } g_l = \begin{cases} 1 & \text{Proton} \\ 0 & \text{Neutron} \end{cases} \quad \text{und } g_s = g_N$$

$$\Downarrow$$

$$g_j \vec{j} \cdot \vec{j} = g_l \vec{l} \cdot \vec{j} + g_s \vec{s} \cdot \vec{j}$$

$$\vec{l} \cdot \vec{j} = \frac{1}{2}\left[j(j+1) + l(l+1) - s(s+1) \right] \quad \text{aus } \vec{s}^2 = \left(\vec{j} - \vec{l}\right)^2 = \vec{j}^2 - 2\vec{l} \cdot \vec{j} + \vec{l}^2$$

$$\vec{s} \cdot \vec{j} = \frac{1}{2}\left[j(j+1) + s(s+1) - l(l+1) \right] \quad \text{aus } \vec{l}^2 = \left(\vec{j} - \vec{s}\right)^2 = \vec{j}^2 - 2\vec{s} \cdot \vec{j} + \vec{s}^2$$

$$g_j = g_l \frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

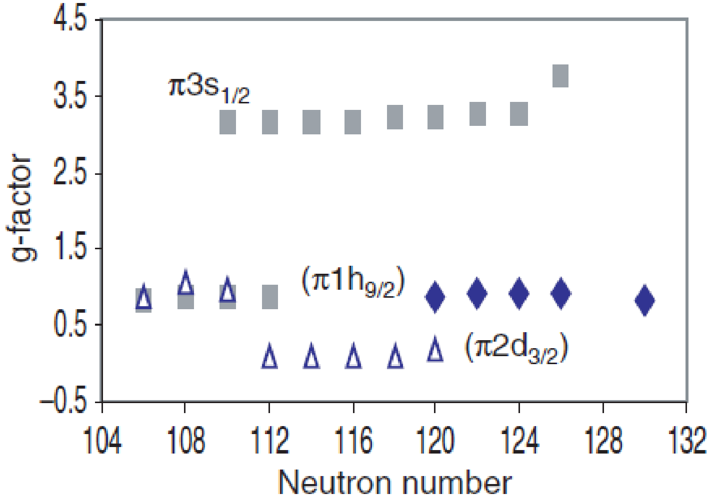
$$\Downarrow \; j = l \pm \tfrac{1}{2}$$

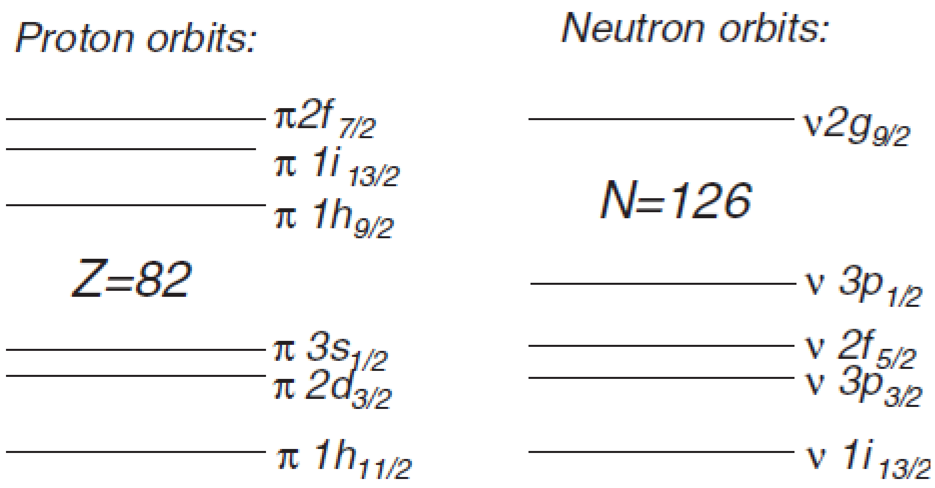
$$g_j = g_l \pm \frac{g_s - g_l}{2l+1}$$

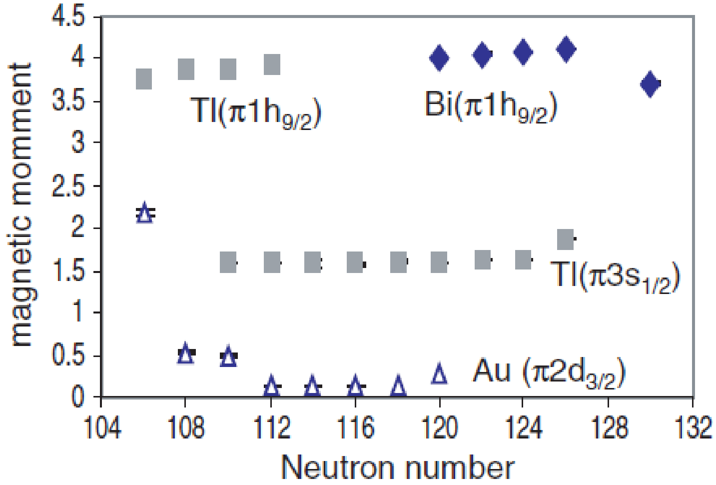
2.5

Magnetic moments and g-factors

http://mri-q.com/uploads/3/4/5/7/34572113/1938_rabi_physrev.53.318.pdf







g-factors of heavy nuclei insensitive to deformations

magnetic moments sensitive to core polarization and details of the configurations

https://fys.kuleuven.be/iks/nm/files/publications/rpp-gerda-2003-30405.pdf

3.

Nucleon – Nucleon Interaction

Characteristics

- attractive: bound states
- short range: ~ 1 fm
- spin dependent
- non-central: Tensor force
- Isospin symmetry
- Hard core
- Spin-orbit force
- Parity conserving

Access to NN forces

- simple system NN
- scattering

3.1. The deuteron

3. HADRON-HADRON INTERACTION

3.1

Properties of the deuteron

Important features of the NN – interaction are visible in the properties of the deuteron.

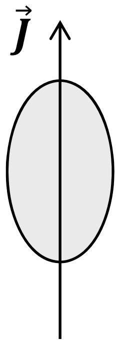
1) Spin of the deuteron: J = 1+

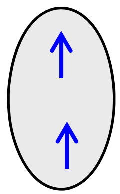
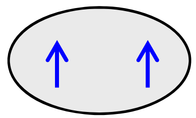
2) Magnetic moment: μ_d=0.8574 μ_N with nuclear magnetic moment $\mu_N = \frac{e}{2m_p}$

proton: μ_p= 2.7928 μ_N
neutron: μ_n=-1.9130 μ_N

=> spins of neutron and protons are almost aligned,
deuteron ground state is mostly L = 0 with a small admixture of L=2.
=> NN – interaction has a spin – spin and a spin – orbit contribution.

3) Deuteron has a quadrupol moment: Q_d = 0.282 efm^2.

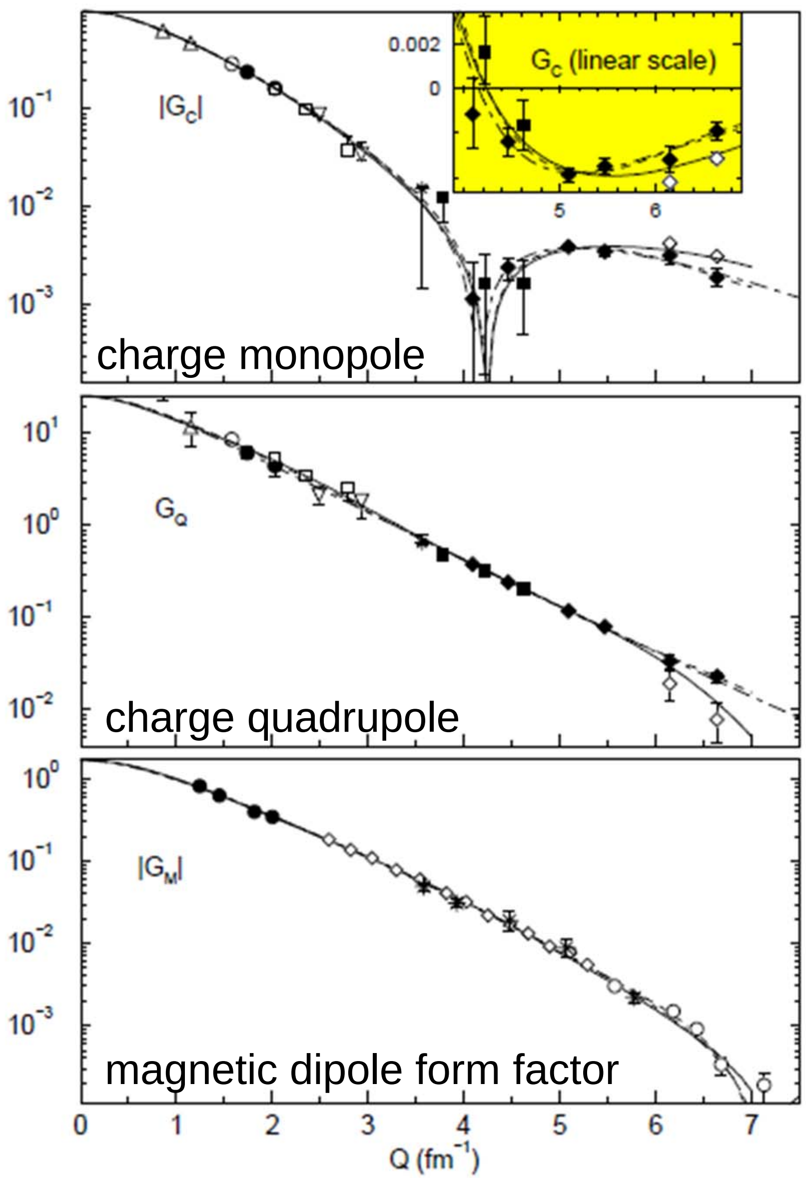


=> the NN – interaction favors  **over** 

This is called a 'tensor force'.

3.1

Deuteron form factor from electron scattering



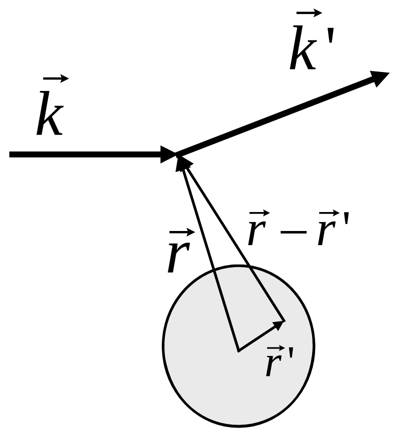
Review: https://arxiv.org/pdf/nucl-th/0102049.pdf

$$r_{ch} = -6dG_C / dQ^2 |_{Q^2=0}$$

Reminder: Form factor

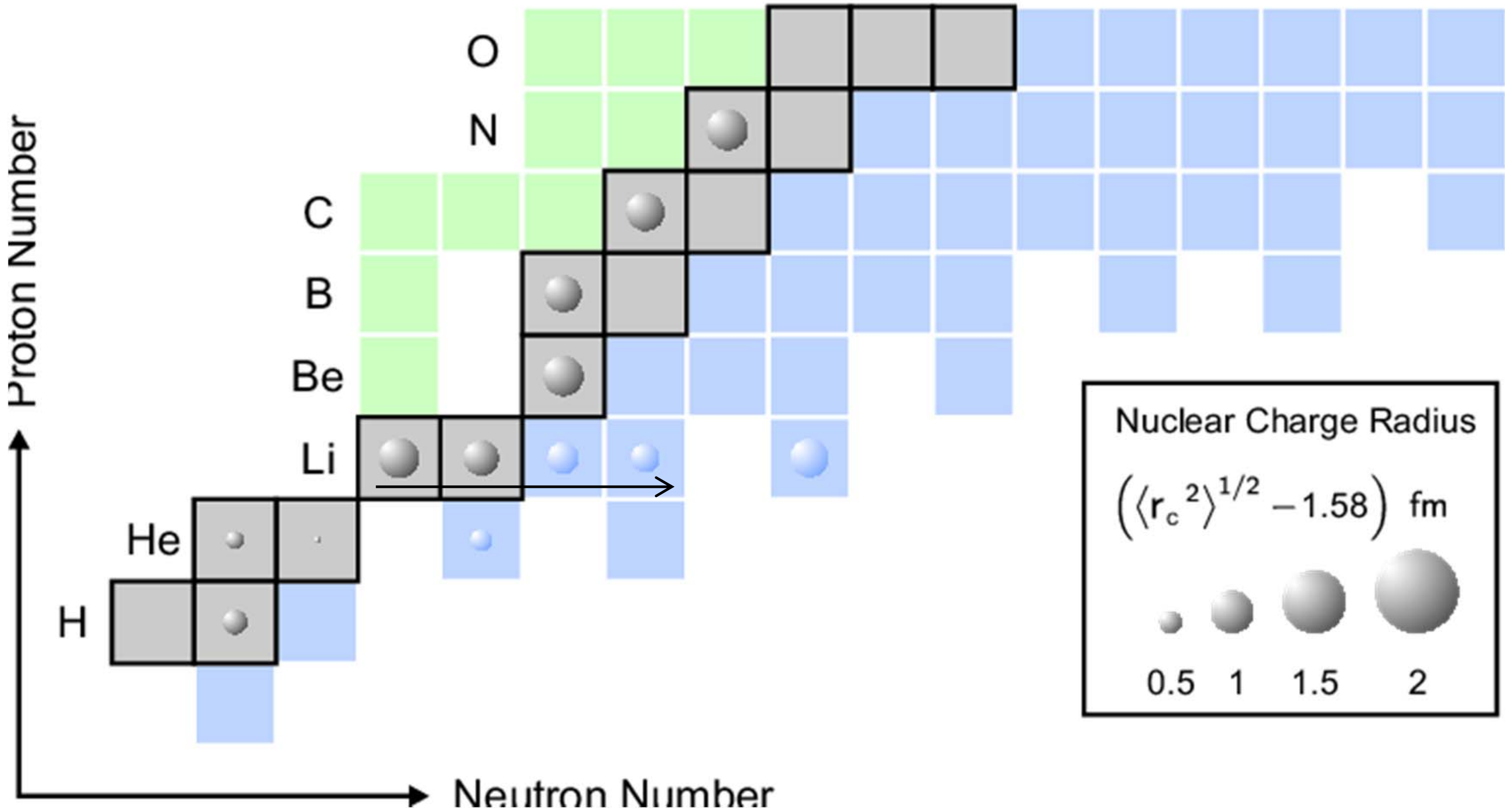
$$F(\vec{q}) = \frac{1}{Z}\int d^3r\rho(\vec{r})e^{i\vec{q}\cdot\vec{r}}$$

Fouriertransform of charge distribution



$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \cos^2\frac{\theta}{2} \cdot \left|F\left(\vec{q}^{\,2}\right)\right|^2$$

Charge radii of light nuclei

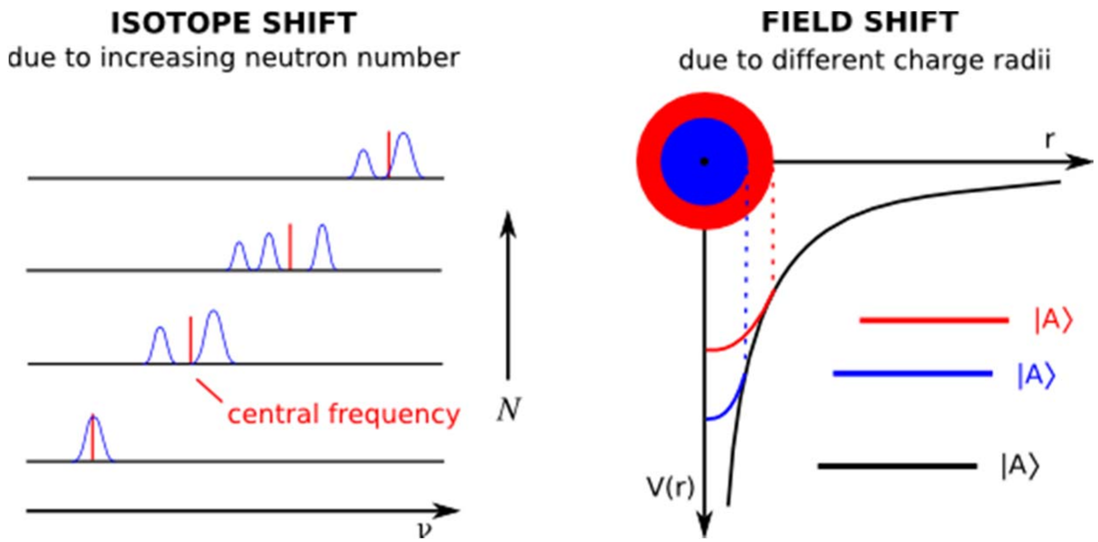


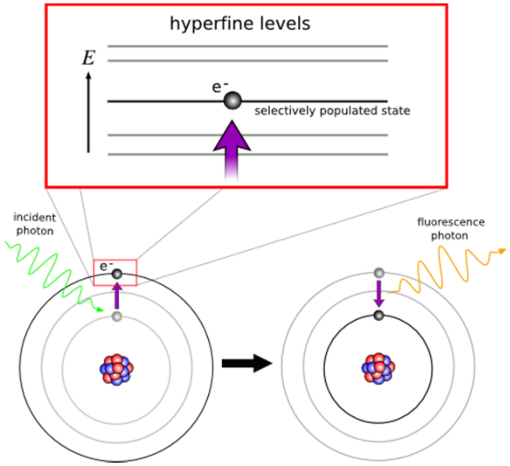
radii of Li isotopes decrease with N, but 11Li is larger again

how charge radii of unstable isotopes are measured?
isotope shifts: hyper fine splitting (coupling of electron spins to nuclear spins sensitive to the charge radius)

Charge radii from atomic physics

http://www.triumf.ca/theory-0





Hyperfine structure:

nucleus spin I couples with electron spin J

$$\vec{F} = \vec{I} + \vec{J}$$

m_f=|I-J|...J+I, each multiplet state will have a different strength of magnetic interaction.

For two different isotopes one observes a shift in the central frequency:

$$\delta\nu^{AA'} = \nu^{A} - \nu^{A'}$$

This shift comprises out of

$$\delta\nu^{AA'} = \delta\nu_{NMS} + \delta\nu_{FS} + \ldots$$

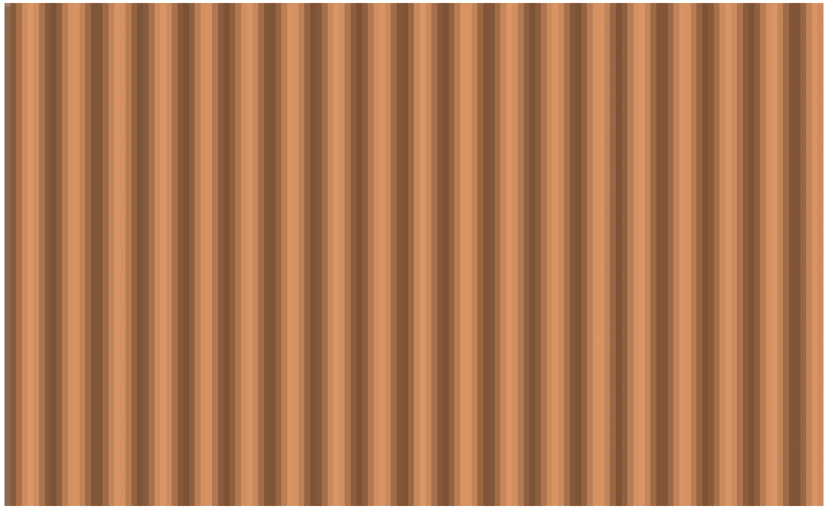
effective masses (→ $\delta\nu_{NMS}$) charge radius (→ $\delta\nu_{FS}$)

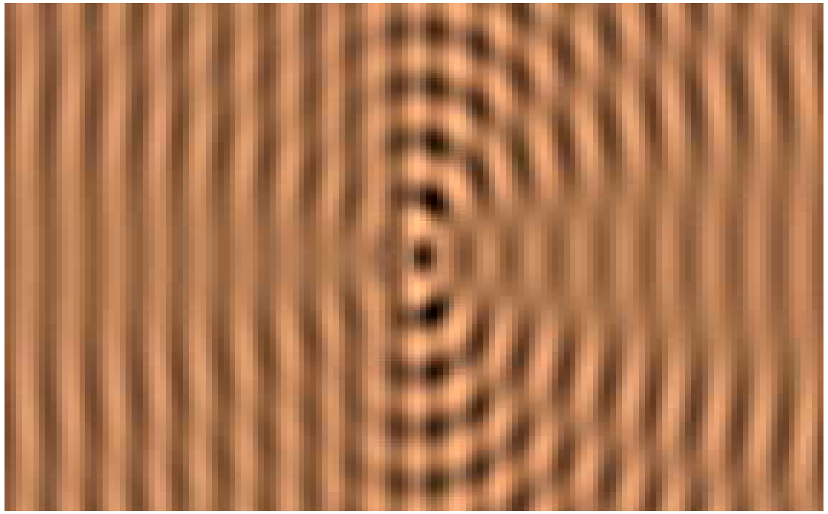
3.2. Scattering formalism

3. HADRON-HADRON INTERACTION

3.2

Consider plane wave on a localized target

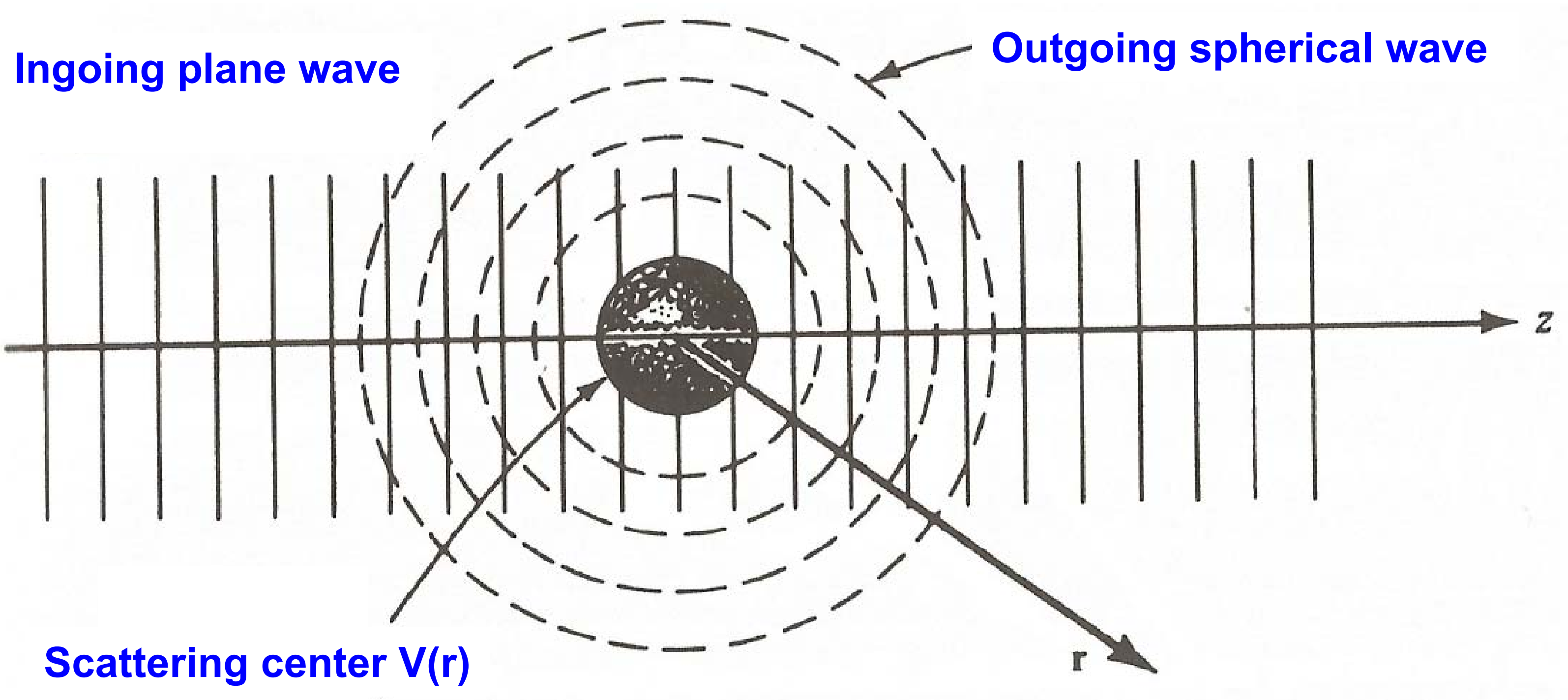




outside localized region superposition of incident and scattered waves

Scattering formalism

Non – relativistic scattering on a potential



Time independent Schrödinger equation (SE):

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

$$\left(\nabla^2 + k^2\right)\psi = 2mV\psi$$

Ansatz for solution:

$$\psi_{tot} = \psi_{in} + \psi_{scat}$$

$$= N\left[e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right]$$

Differential Cross Section

With ‘classical’ definition of currents:

$$j_{in} = v_{in}\psi^*_{in}\psi_{in} = v_{in}P_{in} = v_{in}N^2$$

$$j_{out}dA = v_{out}\psi^*_{out}\psi_{out}dA = v_{out}\left|Nf(\theta)\frac{e^{ikr}}{r}\right|^2 dA = v_{out}N^2|f(\theta)|^2\frac{dA}{r^2}$$

$$= v_{out}N^2|f(\theta)|^2 d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{j_{out}dA}{j_{in}} = |f(\theta)|^2$$

Note: Concept holds for QM – currents as well

$$\vec{j} = \frac{\hbar}{2mi}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

Partial wave decomposition

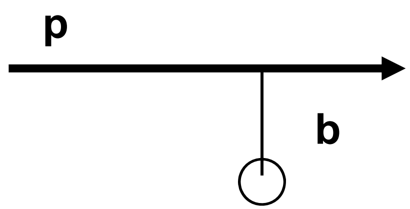
V=V(r)

↓

Implement conservation of angular momentum in description of scattering process

Incoming particle:

$$p = k = \frac{1}{\bar{\lambda}}$$

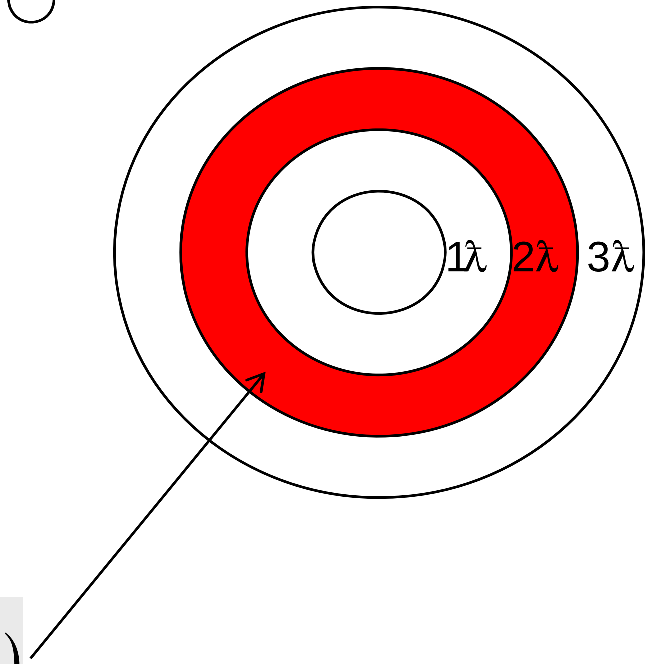


Orbital angular momentum:

express impact parameter in multiples of de-Broglie wavelength

$$b = l\bar{\lambda}$$

$$L = pb = \frac{1}{\bar{\lambda}} l\bar{\lambda} = l$$



Classical contribution of orbital angular momentum
l to total cross section

$$\sigma_l = (2l+1)\pi\bar{\lambda} = \frac{\pi}{k^2}(2l+1)$$

3.2

Partial wave decomposition in QM

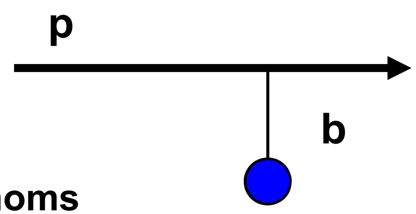
For spherically symmetric potentials V=V(r), SE is separable into radial and angular part.

$$\psi(\vec{r}) = R(r)Y_{l,m}(\theta,\varphi)$$

Incoming plane wave:

$$\vec{L} \perp \vec{p} \perp \vec{b} \Rightarrow m = 0$$

$$Y_{L,0} = \sqrt{\frac{2l+1}{4\pi}} P_L(\cos\theta)$$ **Legendre polynoms**



Solution of radial SE:

$$u(r) = rR(r)$$

$$\frac{d^2u}{dr^2} + \left(k^2 - \frac{l(l+1)}{r^2} - 2mV(r)\right)u = 0$$

Free particle:

$$V(r) = 0$$

$$u(r) = rj_l(kr)$$

Spherical Bessel functions

Plane wave:

$$e^{ikz} = \sum_{l=0}^{\infty} A_l(r)P_l(\cos\theta) = \sum_{l=0}^{\infty}(2l+1)i^l j_l(kr)P_l(\cos\theta)$$