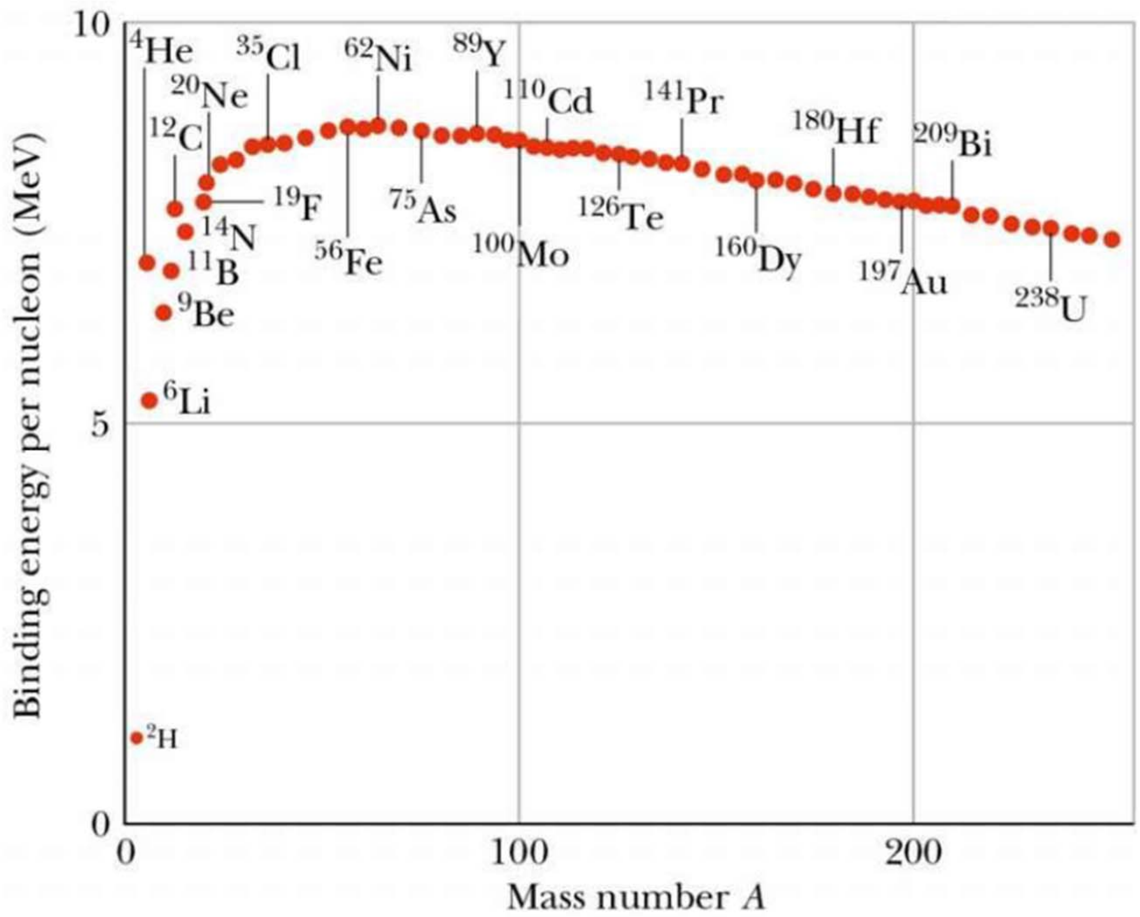
What we discussed last time

BINDING ENERGIES

2.5

Binding energy of stable nuclei



Binding energy ≈ 8 MeV/N (~ 1% of the atom's mass)

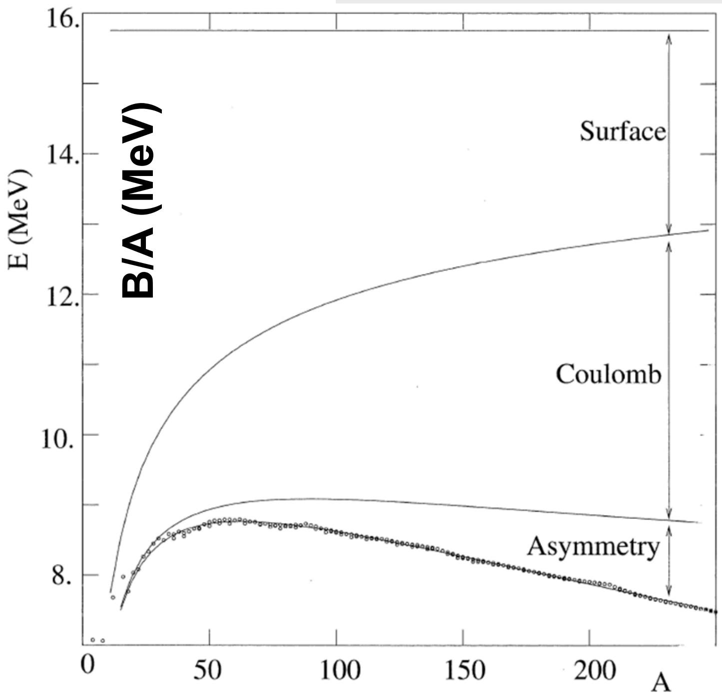
Liquid drop model

Bethe – Weizsäcker mass formula (1935):

Mass of nucleus: $$M(Z,\mathrm{N}=\mathrm{A}-\mathrm{Z}) = Nm_n + Zm_p + Zm_e - a_V A + a_S A^{\frac{2}{3}} + a_C \frac{Z^2}{A^{\frac{1}{3}}} + a_A \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{\frac{1}{2}}}$$

Binding energy: $$B(\mathrm{Z},\mathrm{N}) = Zm_H + Nm_n - M(Z,N)$$

$$= \underbrace{a_V A}_{\text{volume}} - \underbrace{a_S A^{\frac{2}{3}}}_{\text{surface}} - \underbrace{a_C \frac{Z^2}{A^{\frac{1}{3}}}}_{\text{Coulomb}} - \underbrace{a_A \frac{(N-Z)^2}{4A}}_{\text{asymmetry}} - \underbrace{\frac{\delta}{A^{\frac{1}{2}}}}_{\text{pairing}}$$



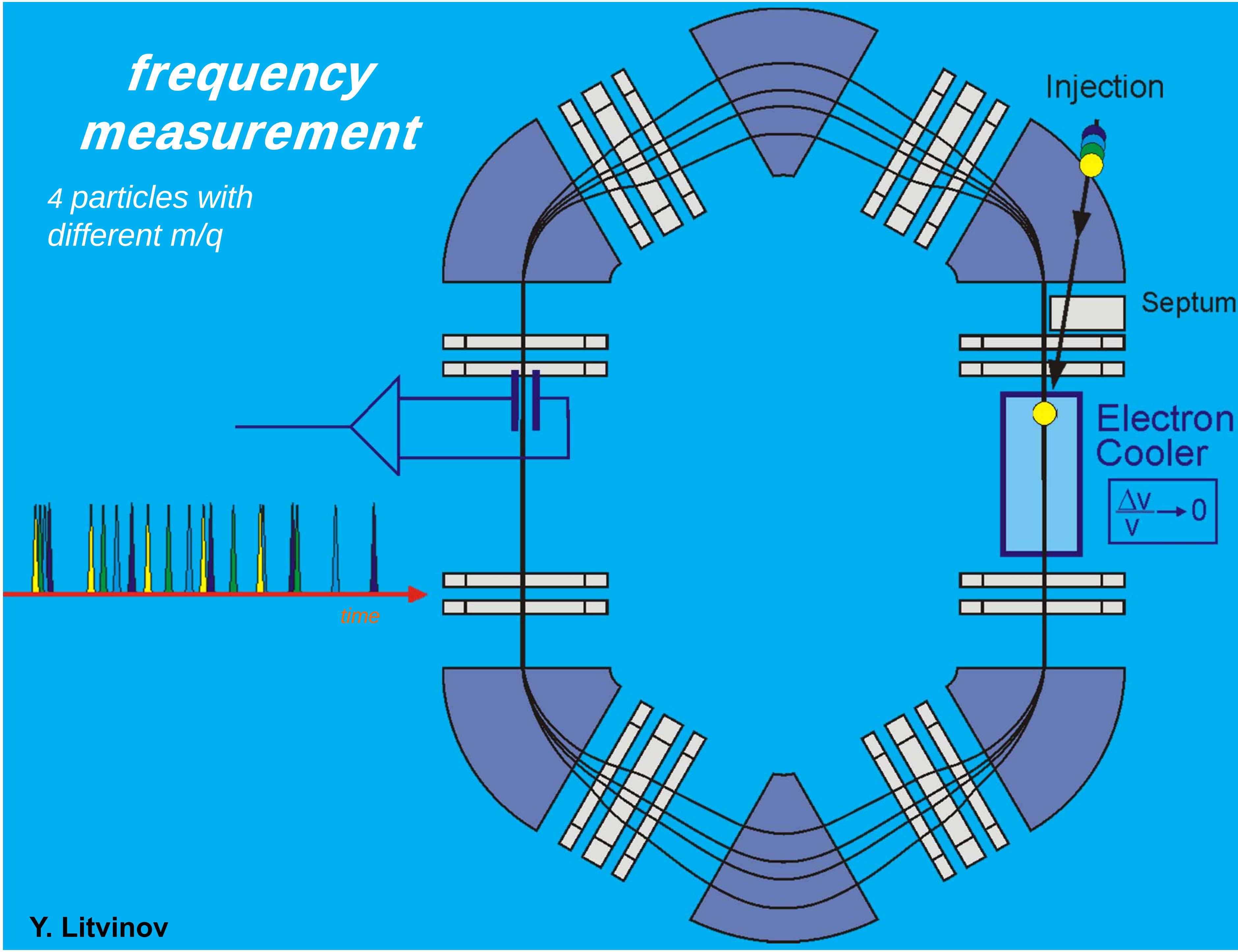
A – mass number
Z – proton number
N – neutron number

Parameter (from fit to data):
a_V=15.67 MeV
a_S=17.23 MeV
a_C=0.714 MeV
a_A=93.15 MeV

δ **=+11.2 MeV for (even,even)**
0 for (even, odd) , (odd,even) nuclei
-11.2 MeV for (odd,odd)

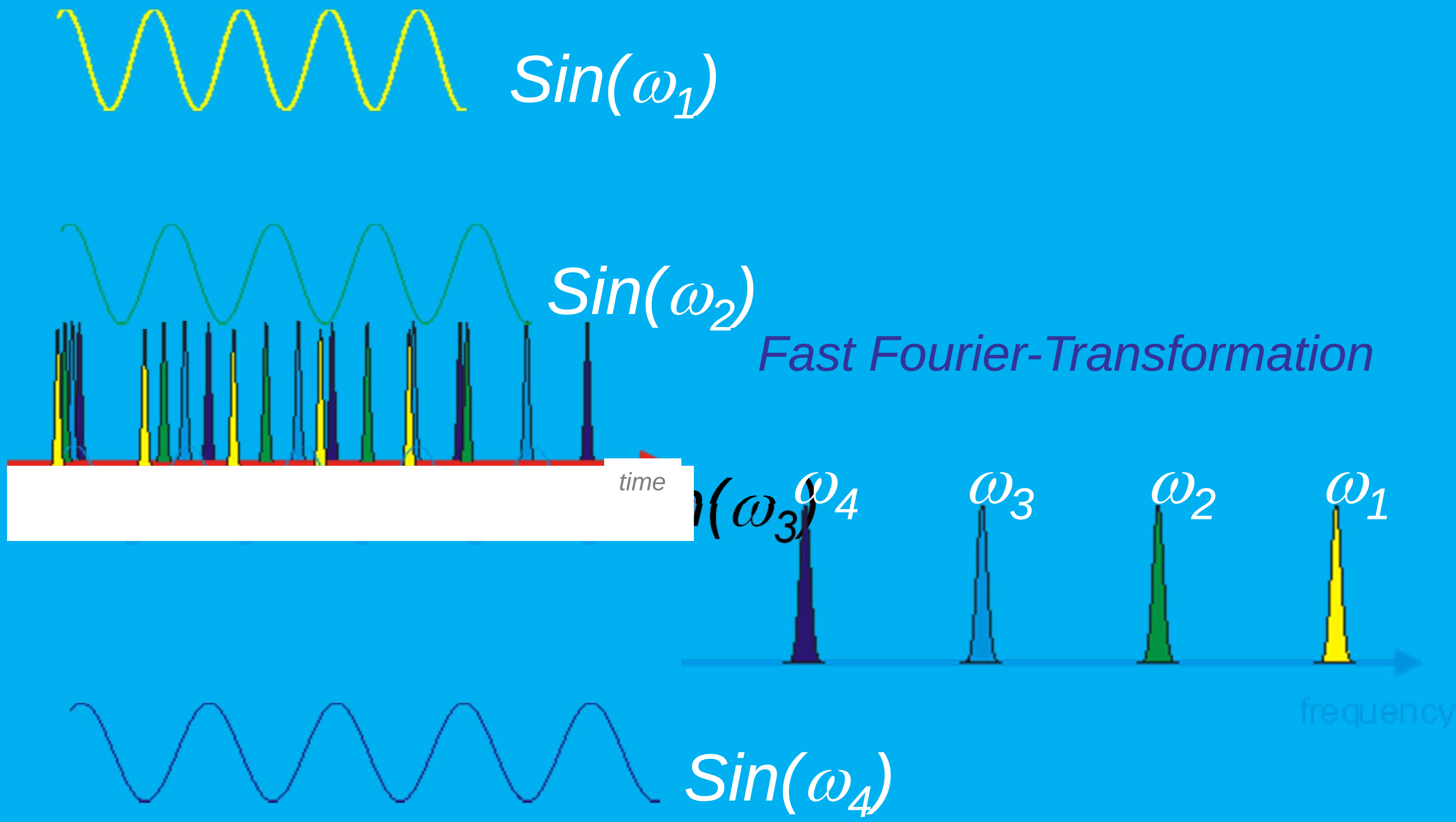
frequency measurement

4 particles with different m/q



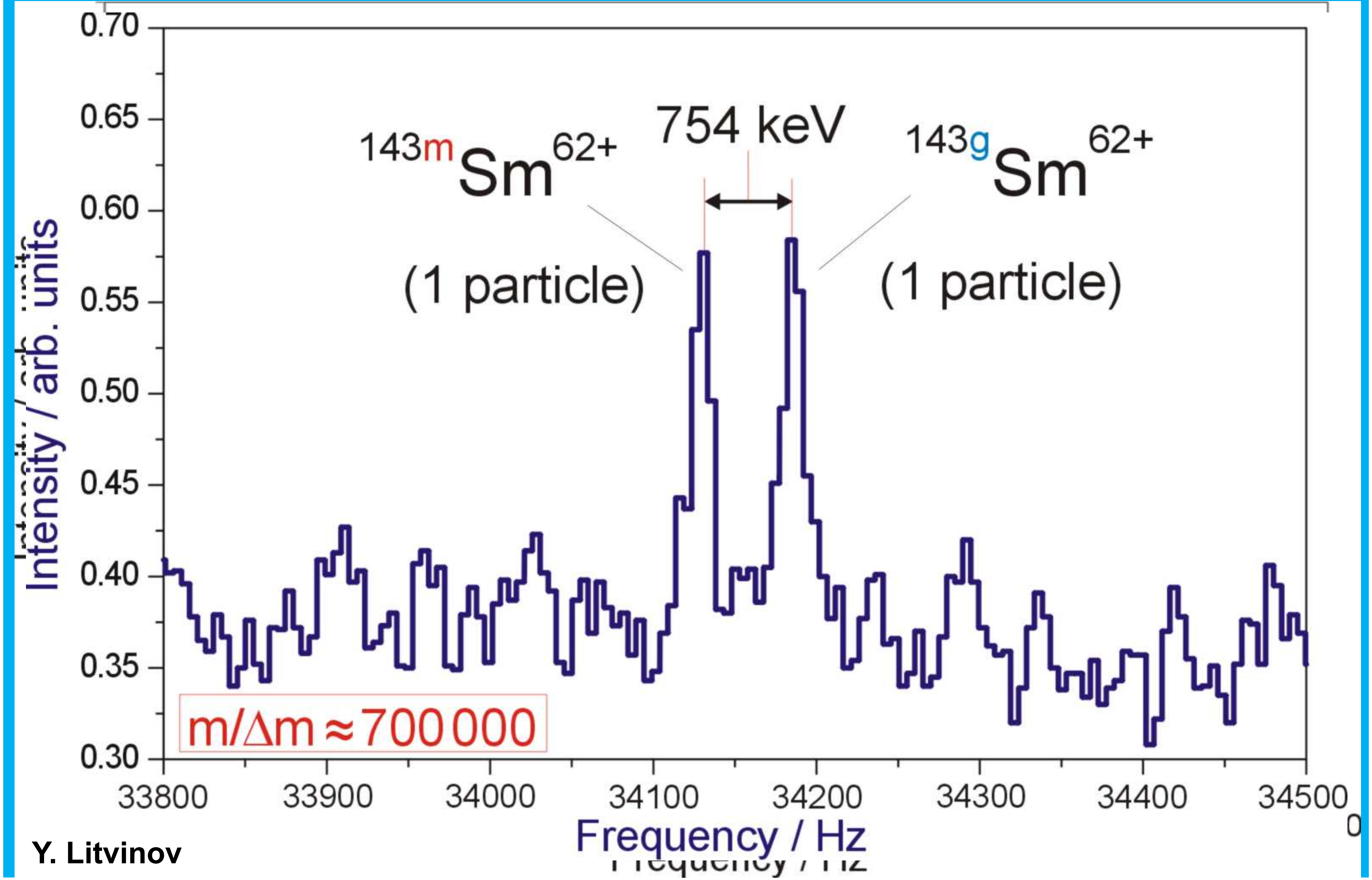
Y. Litvinov

Frequency measurement



Y. Litvinov

Measured mass spectrum



Y. Litvinov

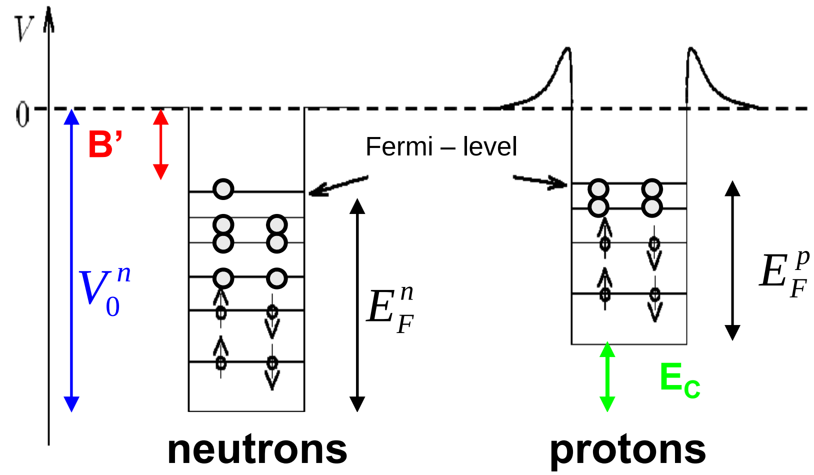
2.5

Fermigas model of nuclei

Phase space single particle state density:

(2s+1) – spin degeneracy

$$dn = \frac{4\pi p^2 dp}{(2\pi\hbar)^3} V(2s+1)$$



Total number of nucleons N in Volume V defines the maximum momentum p_F.

$$N_{p,n} = \int_0^{p_F} dn = \frac{p_F^3}{3\pi^2\hbar^3} V$$

$$p_F^3 = 3\pi^2\hbar^3 \rho_{p,n}$$

$$p_F = \hbar\left(3\pi^2\right)^{\frac{1}{3}} \rho_{p,n}^{\frac{1}{3}}$$

separately for neutrons and protons

Typical numbers for N=Z nuclei (equal number of neutrons and protons):

$$p_F \approx 250 \text{ MeV/c}$$

$$\mathrm{E_F} \approx 33 \text{ MeV}$$

2.5

Asymmetry energy in Fermi gas model

Consider mean kinetic energy of a nucleon

$$\langle E_{kin} \rangle = \frac{\int_o^{p_F} E_{kin}\, p^2 dp}{\int_o^{p_F} p^2 dp} = \frac{3}{5}\frac{p_F^2}{2m_{p,n}} \approx 20 MeV$$

Contribution to total kinetic energy of the nucleus:

$$E_{kin}(N,Z) = N\langle E_{kin}^n \rangle + Z\langle E_{kin}^p \rangle = \frac{3}{10 m_{p,n}}\left(N p_{F,n}^2 + Z p_{F,n}^2\right)$$

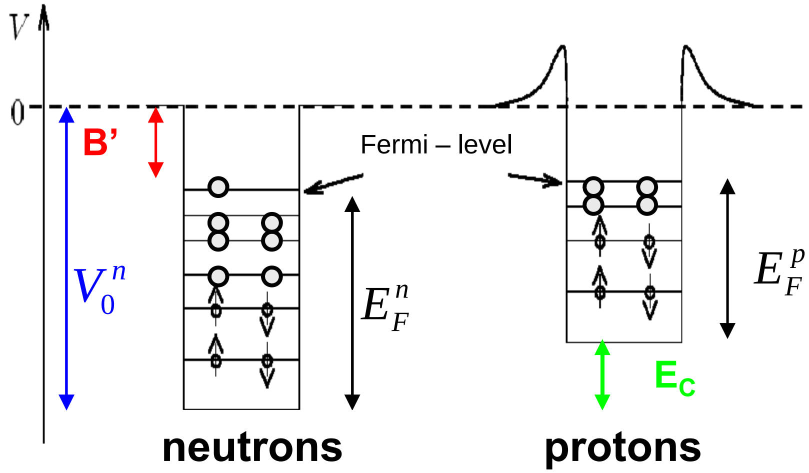
$$= \frac{3}{10 m_{p,n}} \frac{\hbar^2}{r_0^2}\left(\frac{9}{4}\pi\right)^{\frac{2}{3}} \frac{N^{\frac{5}{3}} + Z^{\frac{5}{3}}}{A^{\frac{2}{3}}}$$

using:

$$Z = \frac{p_{F,p}^3}{3\pi^2\hbar^3}V, \qquad N = \frac{p_{F,n}^3}{3\pi^2\hbar^3}V$$

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_0^3 A$$

$$\Rightarrow p_{F,p} = \frac{\hbar}{r_0}\left(\frac{9}{4}\pi\frac{Z}{A}\right)^{\frac{1}{3}}, \qquad p_{F,n} = \frac{\hbar}{r_0}\left(\frac{9}{4}\pi\frac{N}{A}\right)^{\frac{1}{3}}$$



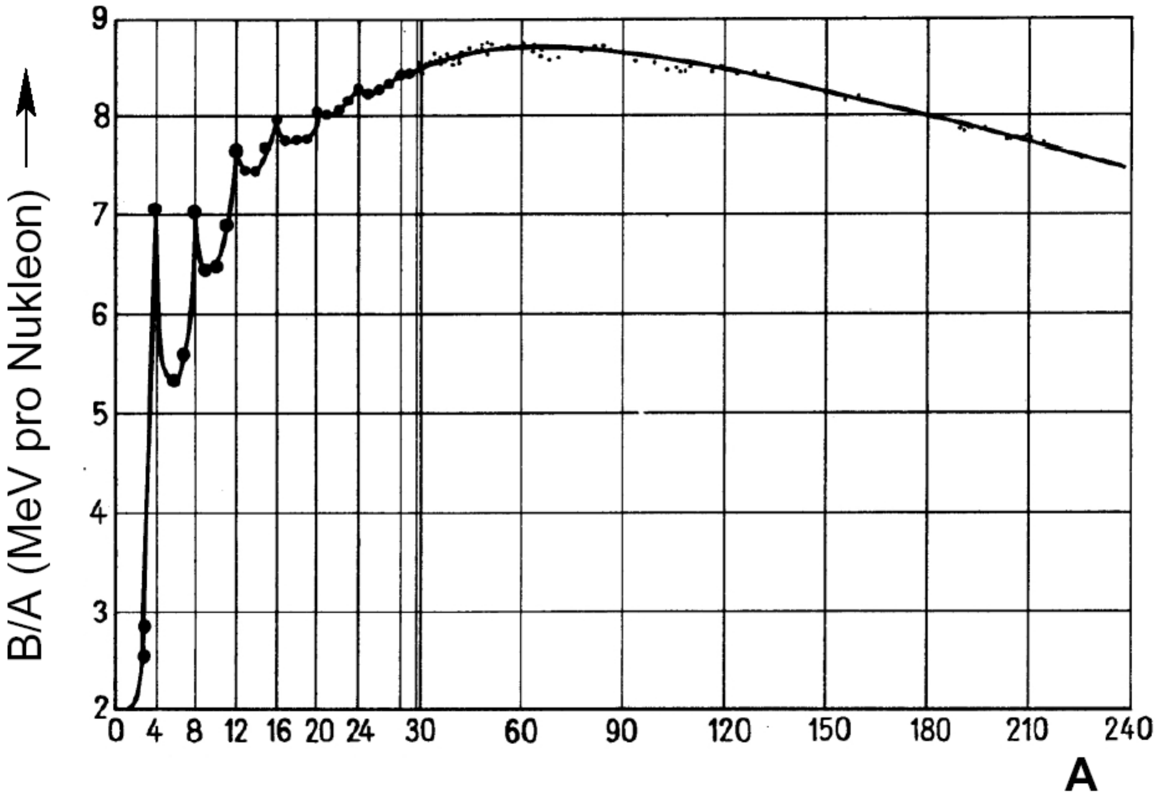
Expansion wrt. N-Z:

$$E_{kin}(N,Z) = \frac{3}{10 m_{p,n}} \frac{\hbar^2}{r_0^2}\left(\frac{9}{8}\pi\right)^{\frac{2}{3}}\left(A + \frac{5}{9}\frac{(N-Z)^2}{A} + \ldots\right)$$

… as in liquid drop model …

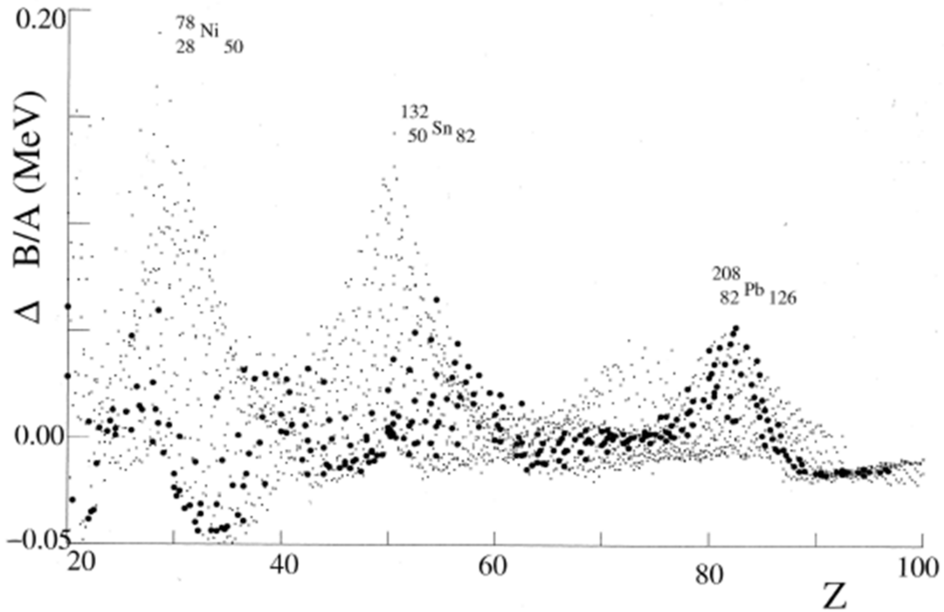
2.5

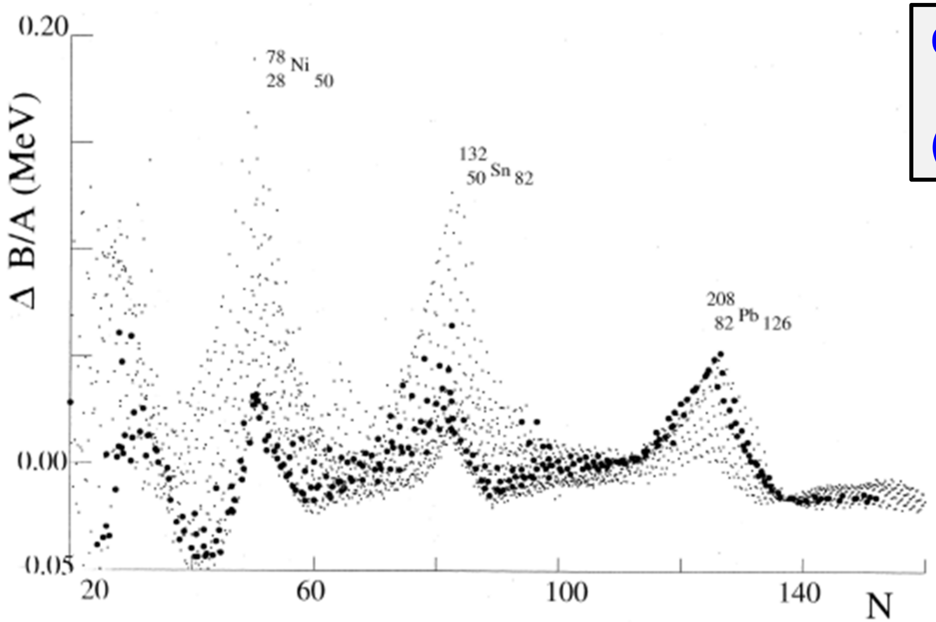
Binding energies of nuclei



Structures are indicative of shell effects.

Magic numbers





Difference of measured binding energies to liquid drop approximation
(J.-L. Basdevant, J. Rich, M. Spiro, Springer 2005)

Full dots: β – stable nuclei
Small dots: instable isotopes

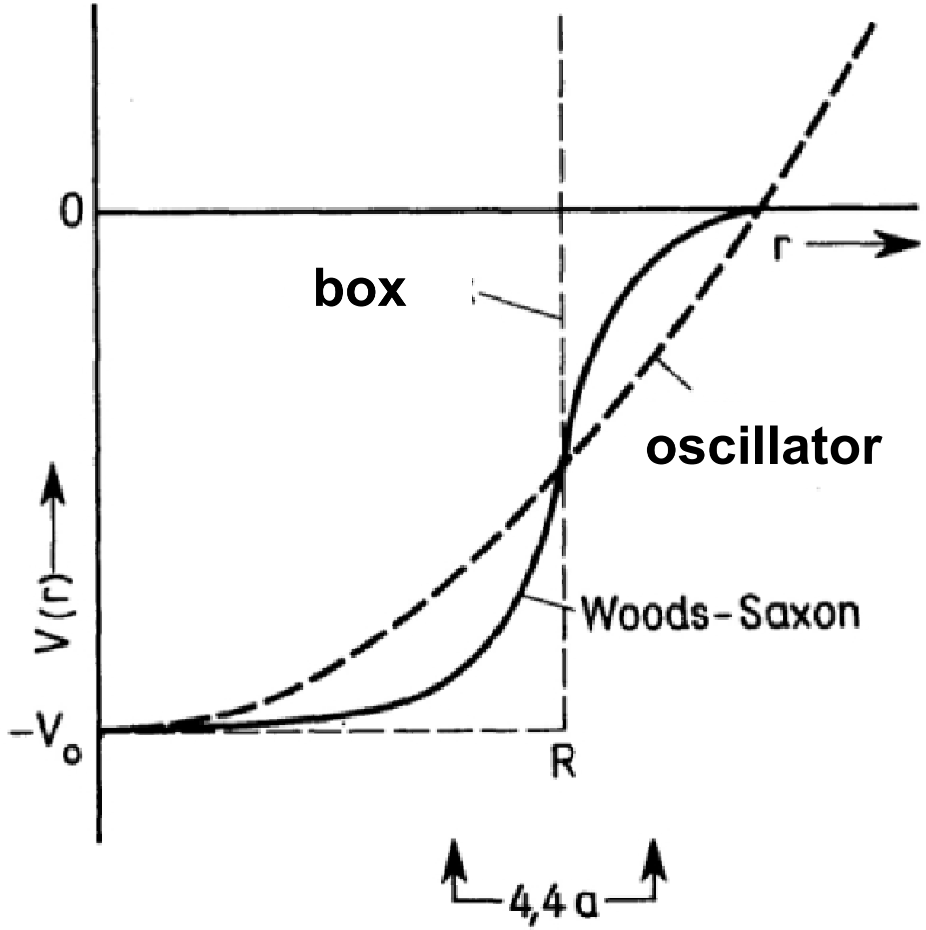
Observed shell closures at 2,8,20,28,50,82,126,…
(independently for protons and neutrons)

Double magic nuclei

$^{4}_{2}He, ^{16}_{8}O, ^{40}_{20}Ca, ^{48}_{20}Ca, ^{208}_{82}Pb; ^{100}_{50}Sn$

2.5

Nuclear potential



For large nuclei, neither a box nor a harmonic oscillator fits the potential.

Empirically from electron scattering: Woods – Saxon – Potential

$$V(r) = \frac{-V_0}{1 + e^{\frac{r-R}{a}}}$$

$$\text{with } R = 1.07\ A^{\frac{1}{3}}, \quad a = 0.54\text{fm}$$

Due to the reduced symmetry

$\Rightarrow$ N – degeneracy of shells is broken.

2.5

Spin-Orbit-interaction

Historic remark: contribution by Heidelberg professors Haxel and Jensen
Conjecture Haxel, Suess 1948
Calculation Göppert-Mayer, Jensen 1949 (Nobelprize 1963)

Nuclear potential has strong contribution from spin – orbit coupling:

$$V(r) = V_{central} + V_{ls}(r) \cdot \left\langle \vec{l} \cdot \vec{s} \right\rangle$$

Level ordering depends on total angular momentum j:

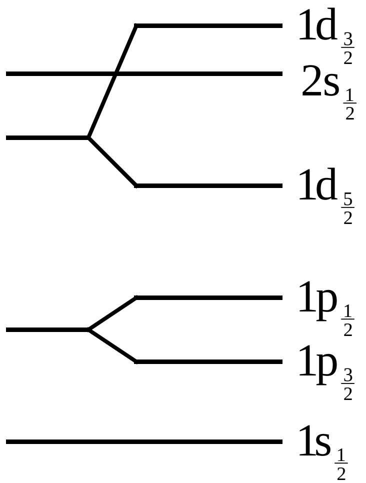
$$\vec{j} = \vec{l} + \vec{s}$$

$$j = l \pm \tfrac{1}{2}$$

$$\left\langle \vec{j}^2 \right\rangle = \left\langle \left(\vec{l} + \vec{s} \right)^2 \right\rangle = \left\langle \vec{l}^2 \right\rangle + 2\left\langle \vec{l} \cdot \vec{s} \right\rangle + \left\langle \vec{s}^2 \right\rangle$$

$$\left\langle \vec{l} \cdot \vec{s} \right\rangle = \frac{1}{2}\left(j(j+1) - l(l+1) - s(s+1) \right)$$

$$= \begin{cases} l/2 & \text{for } j = l + \tfrac{1}{2} \\ -(l+1)/2 & \text{for } j = l - \tfrac{1}{2} \end{cases}$$



Spin-orbit splitting

$$\Delta E_{ls} = \frac{2l+1}{2} \left\langle V_{ls}(r) \right\rangle$$

Splitting grows with rising l, determines shell structure for large l !

Level with j=l+1/2 is energetically favored.
(opposite to atomic (electronic) shell levels (fine structure))

2.5

Shell model levels

P.E. Hodgson, 'Introductory Nuclear Physics', Oxford Science Publications

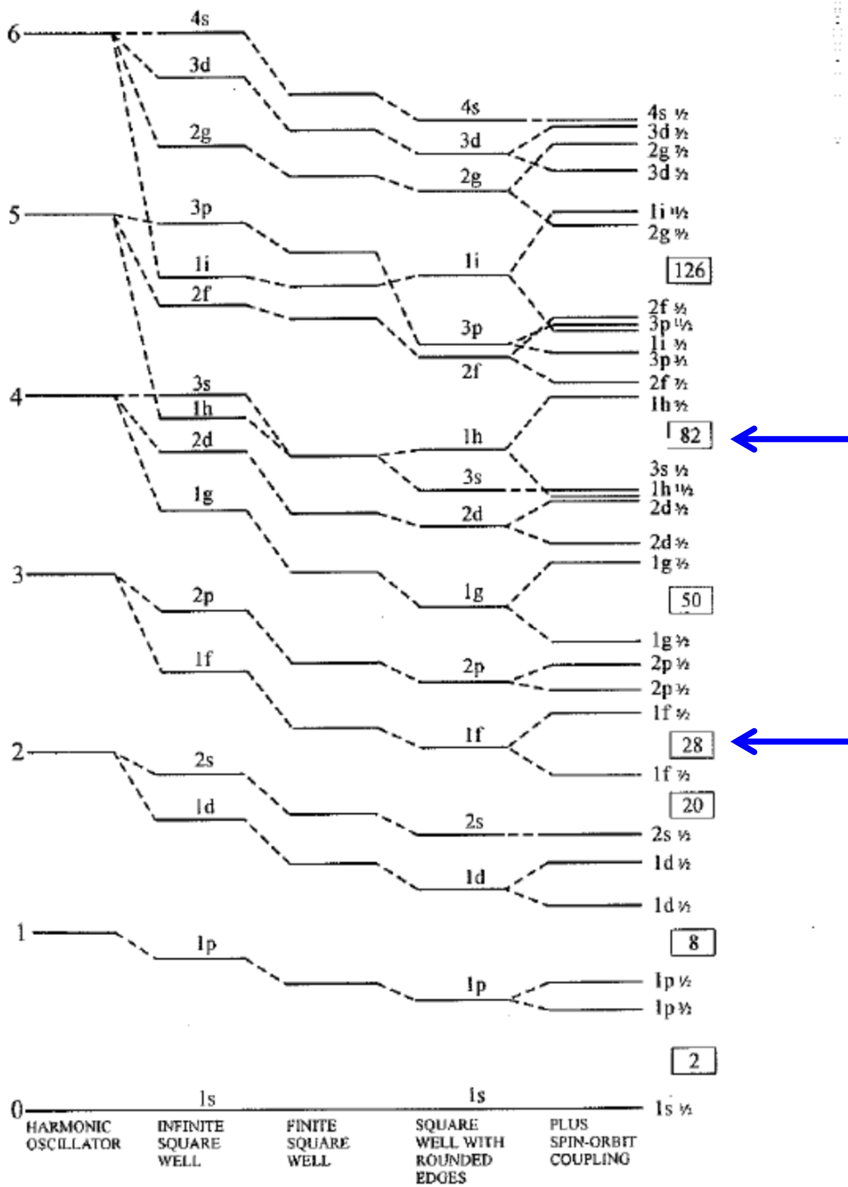


Fig. 17.6 Order of energy levels according to the independent-particle model with various assumptions for the shape of the nuclear potential (Feld 1953).

Magic number 82 arises from level splitting of 1h – shell.

Magic number 28 arises from level splitting of 1f – shell.

2.5

Shell model levels

P.E. Hodgson, 'Introductory Nuclear Physics', Oxford Science Publications

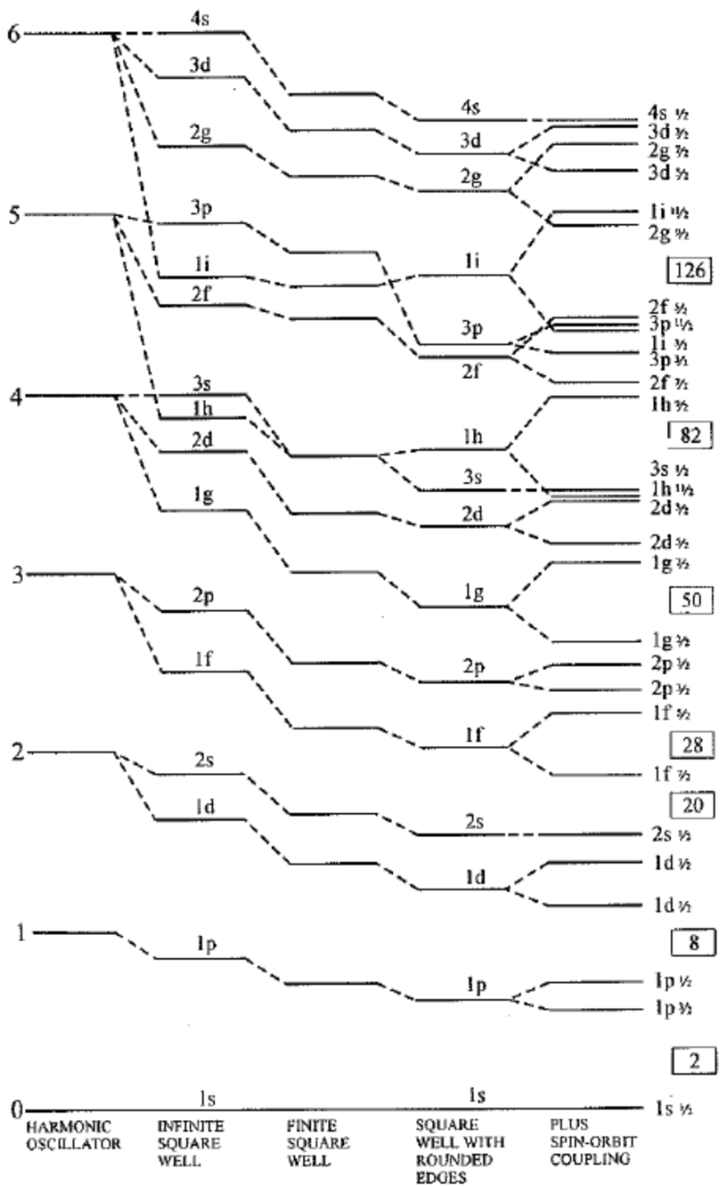
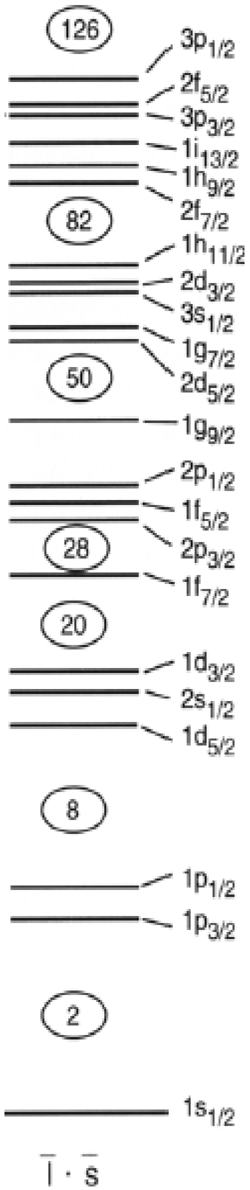


Fig. 17.6 Order of energy levels according to the independent-particle model with various assumptions for the shape of the nuclear potential (Feld 1953).

Predictions of shell model:

1. The spin of nuclei is given by the total angular momentum of the nucleons.
2. Even number of nucleons combine to angular momentum 0. The spin of even – even nuclei is 0, closed shells have spin 0.
3. Nuclei with only one valence nucleon (~hole) outside of a closed shell have the spin – parity of the total angular momentum of the valence nucleon (hole).
4. In odd-even nuclei the nucleons in the odd shell couple to J=j (very seldom to J=j-1)
5. In odd-odd nuclei nucleons prefer triplett states
   a) $j_1=l_1 + ½, j_2=l_2 – ½ \Rightarrow J=|j_1-j_2|$ (strong Nordheim rule)
   b) $j_1=l_1 + ½, j_2=l_2 + ½$
      $j_1=l_1 - ½, j_2=l_2 – ½ \Rightarrow J=j_1+ j_2$ (weak Nordheim rule)

Shell model – predictions



Complete shell + 1

$$^{41}_{20}Ca_{21} \qquad J^P = \frac{7}{2}^-$$

$$^{91}_{40}Zr_{51} \qquad J^P = \frac{5}{2}^+$$

Complete shell - 1

$$^{39}_{20}Ca_{19} \qquad J^P = \frac{3}{2}^+$$

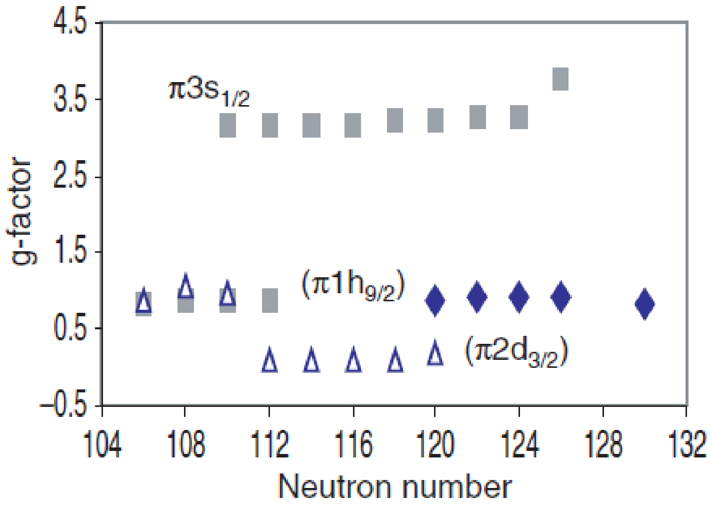
Complete orbital + 1

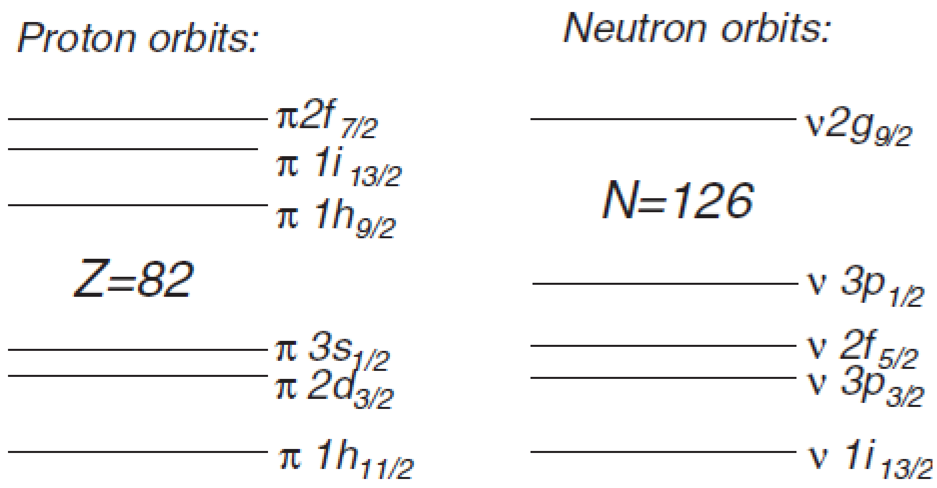
$$^{35}_{15}P_{20} \qquad J^P = \frac{1}{2}^+$$

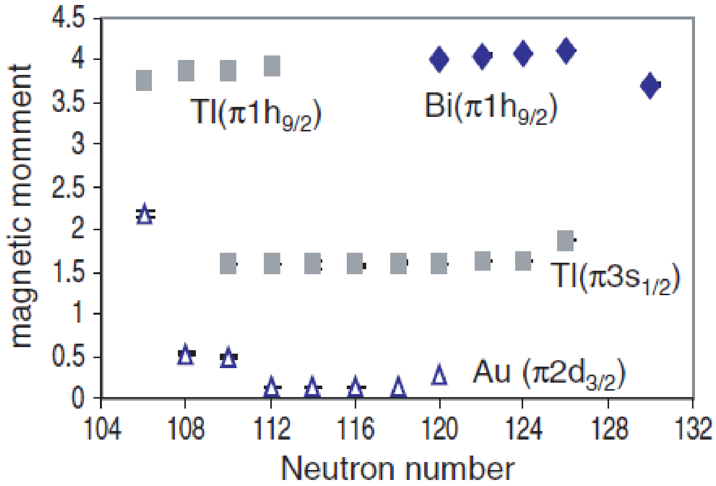
2.5

Magnetic moments and g-factors

http://mri-q.com/uploads/3/4/5/7/34572113/1938_rabi_physrev.53.318.pdf







g-factors of heavy nuclei insensitive to deformations

magnetic moments sensitive to core polarization and details of the configurations

https://fys.kuleuven.be/iks/nm/files/publications/rpp-gerda-2003-30405.pdf