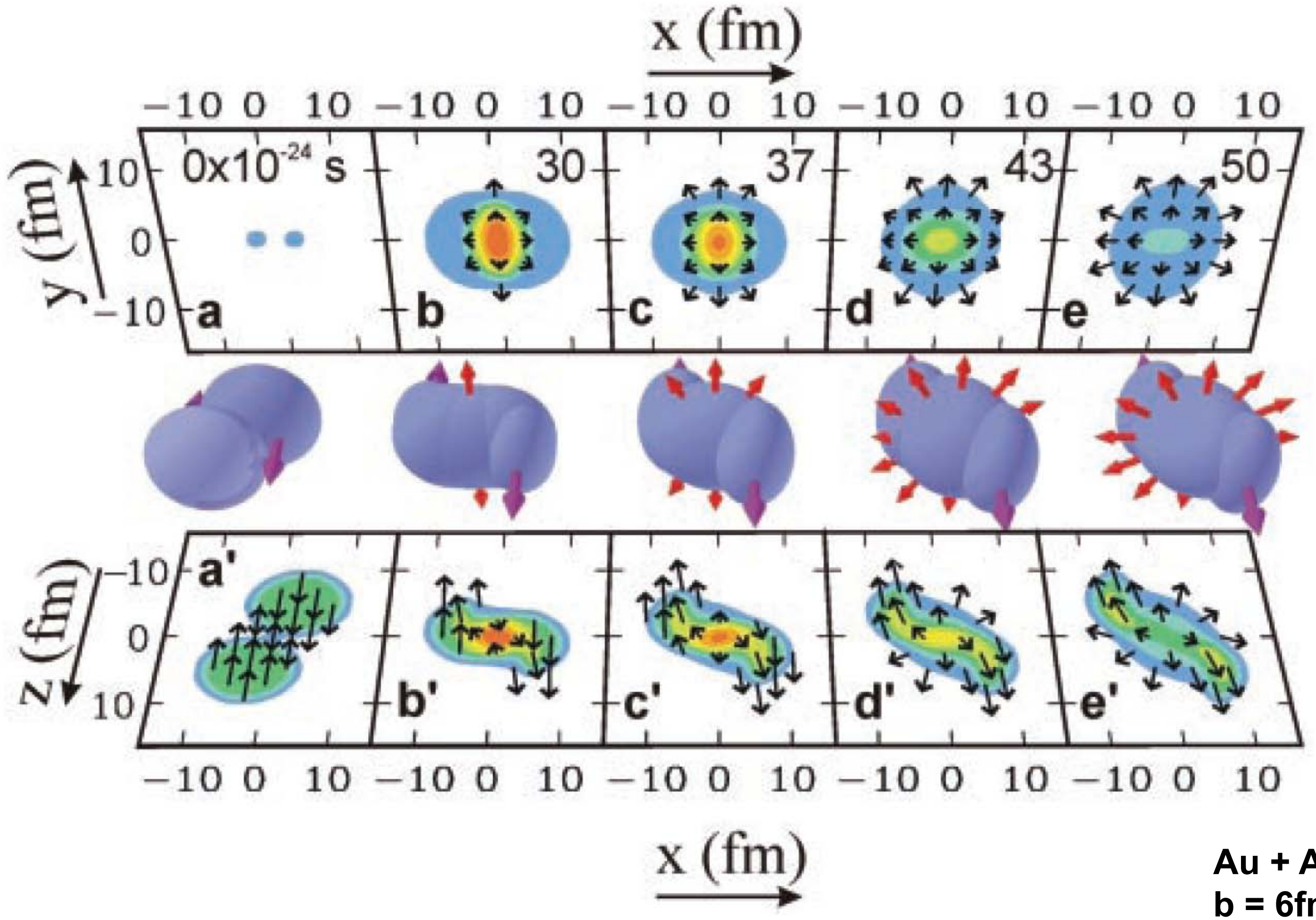
5 DYNAMICAL MODELS

Heavy-ion collisions

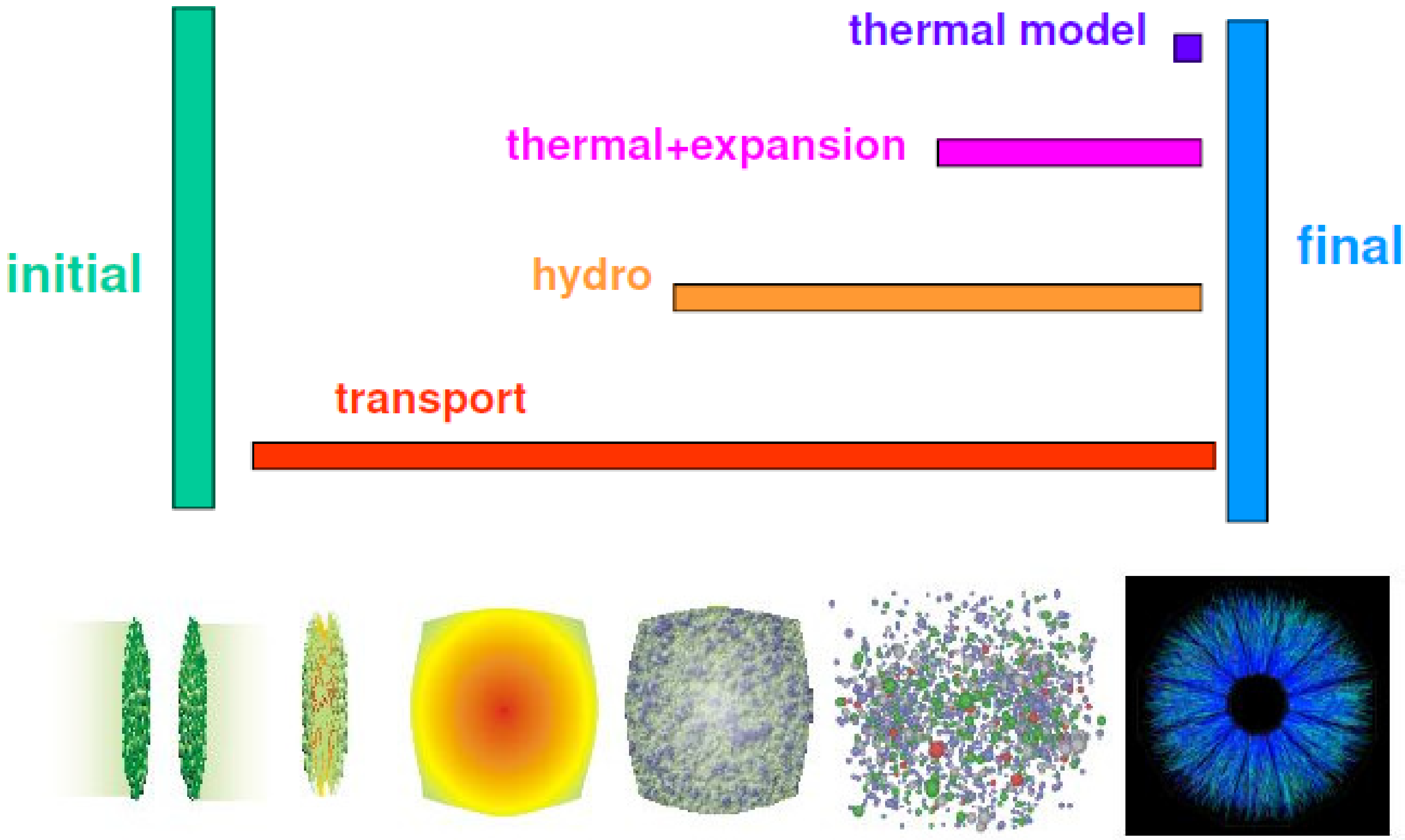


Au + Au
b = 6fm

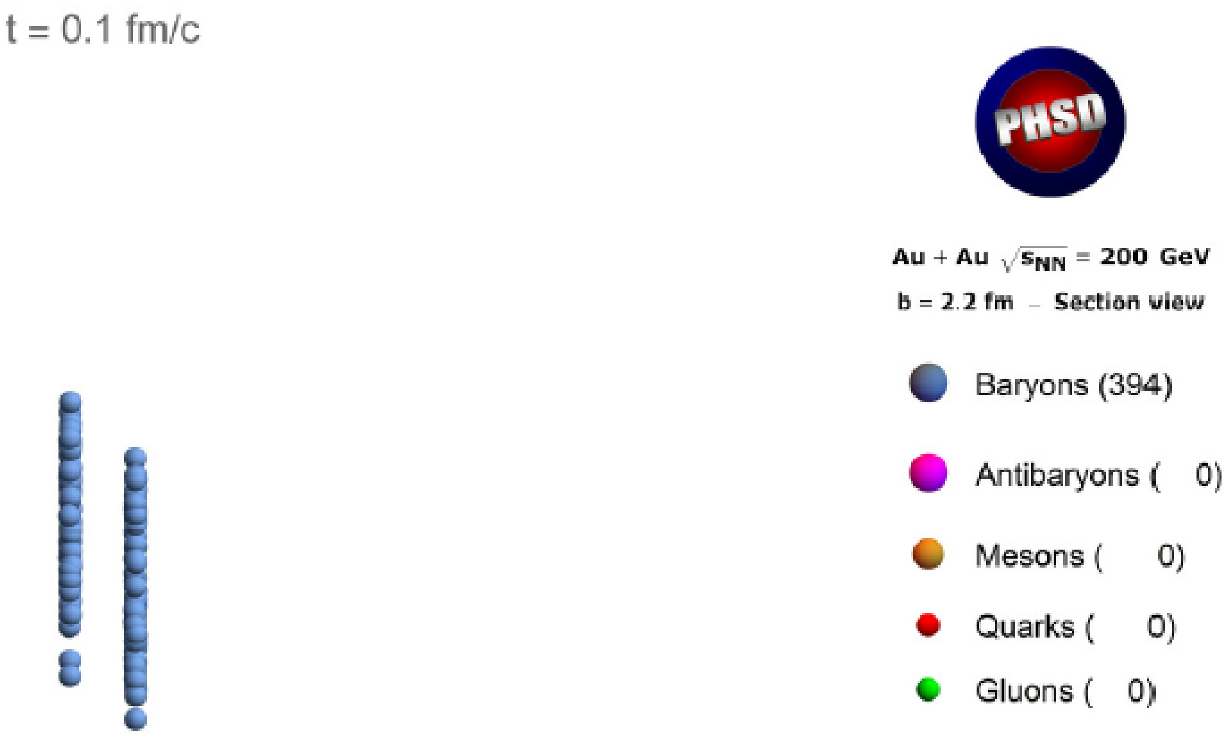
P. Danielewicz et al.
Science 298, 1592 (2002)

5.1

Models for heavy ion collisions

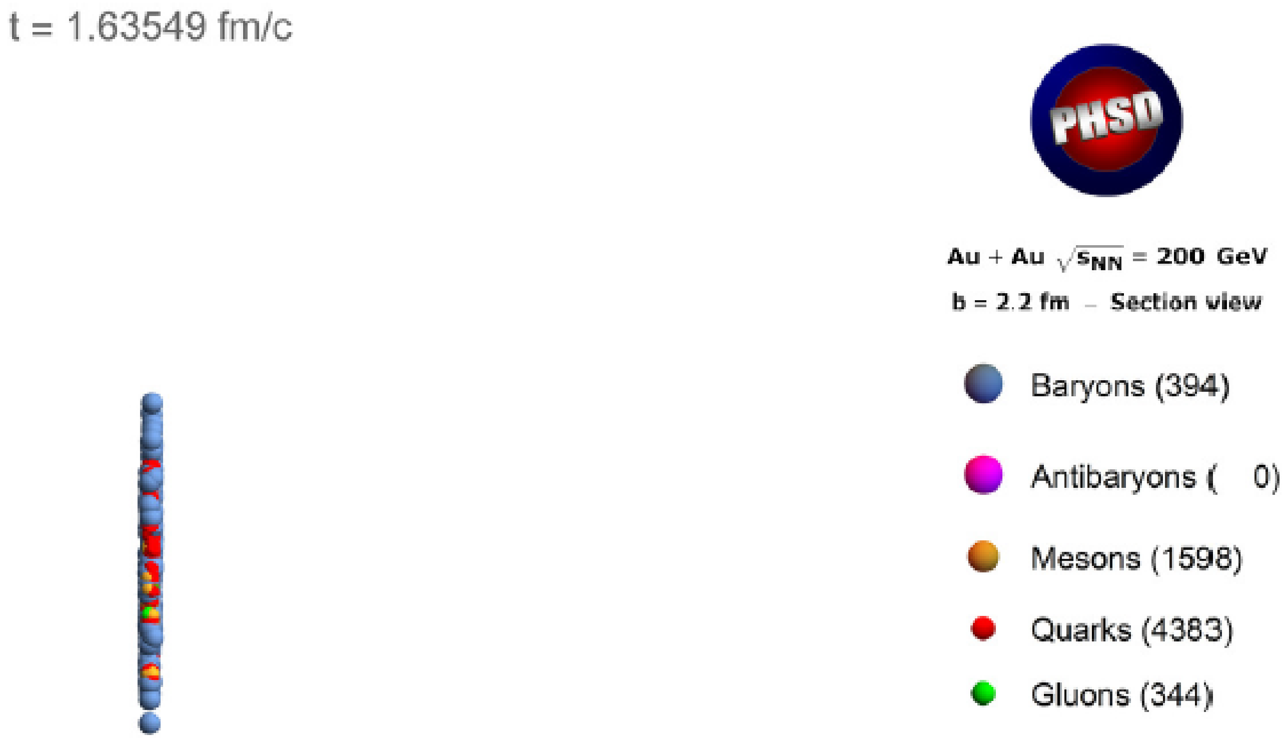


Au+Au at 200 A GeV, b=2.2 fm



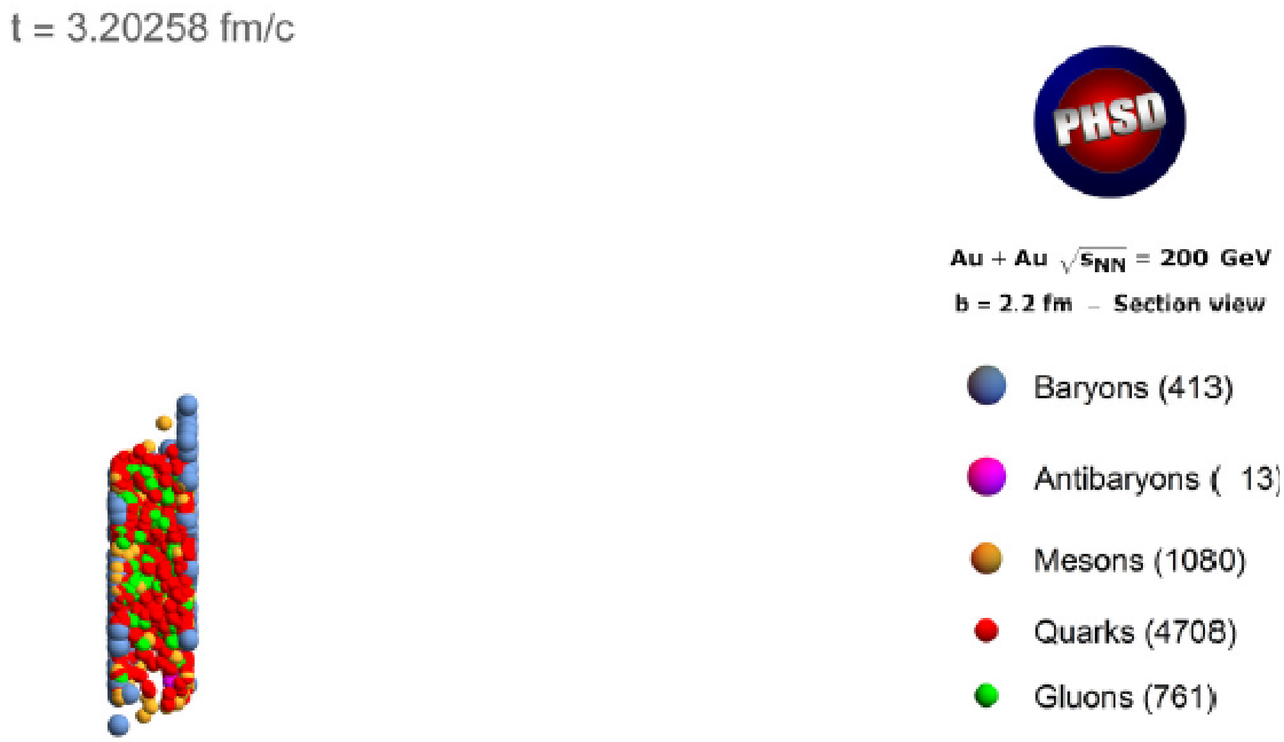
P.Moreau

Au+Au at 200 A GeV, b=2.2 fm



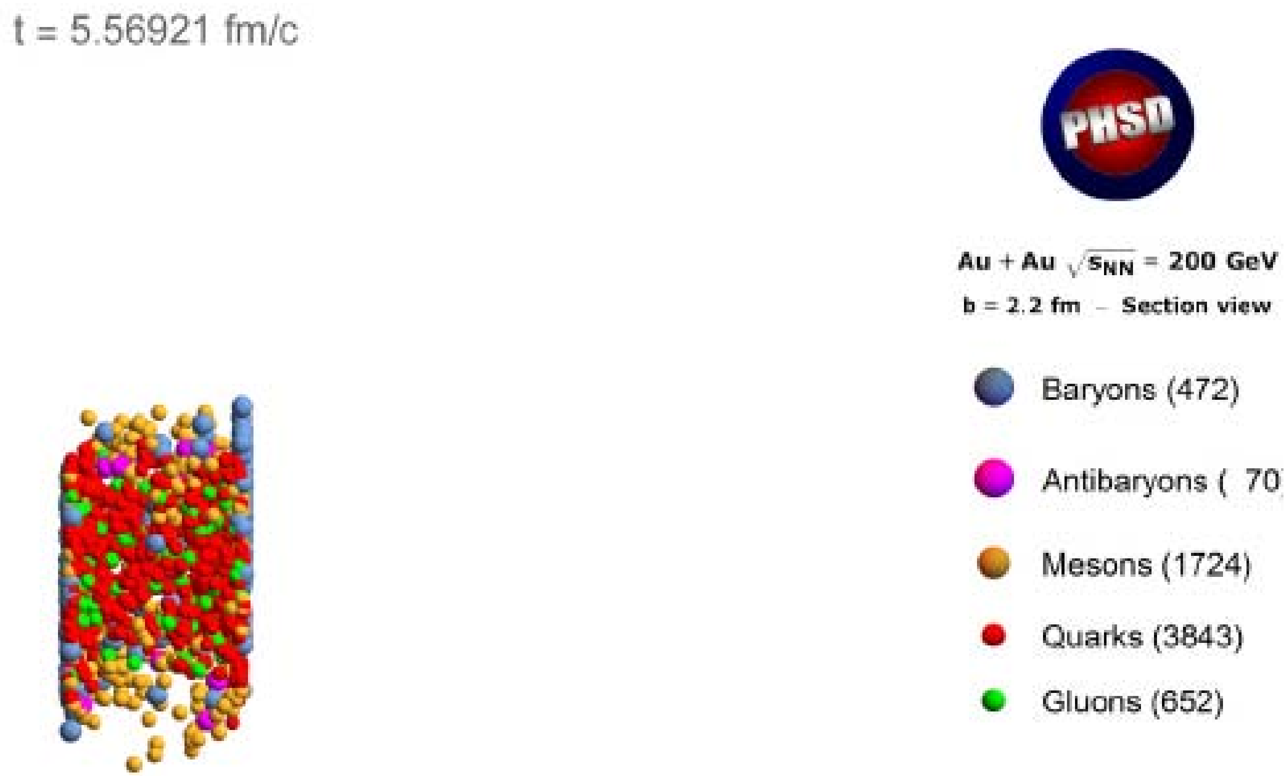
P.Moreau

Au+Au at 200 A GeV, b=2.2 fm



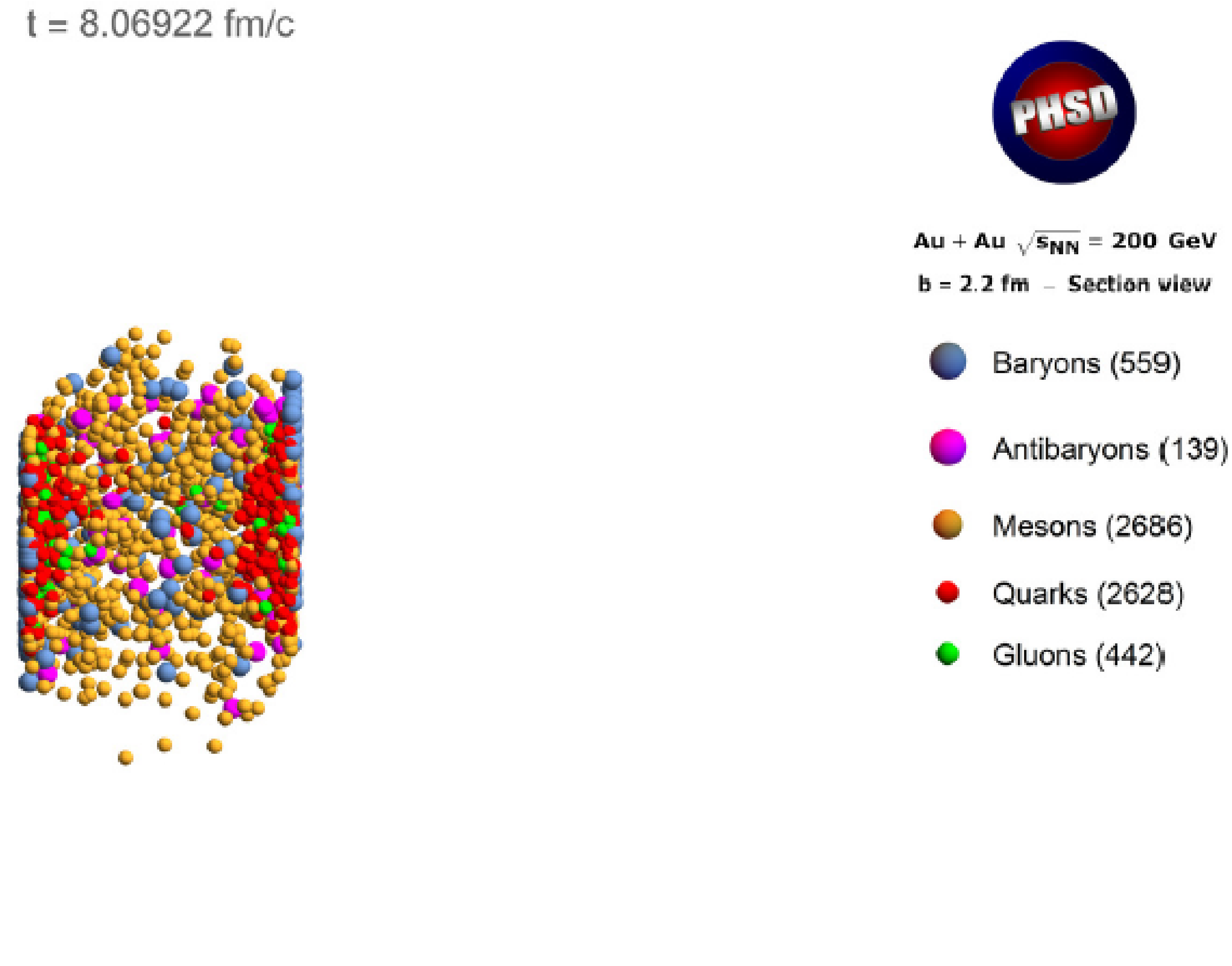
P.Moreau

Au+Au at 200 A GeV, b=2.2 fm

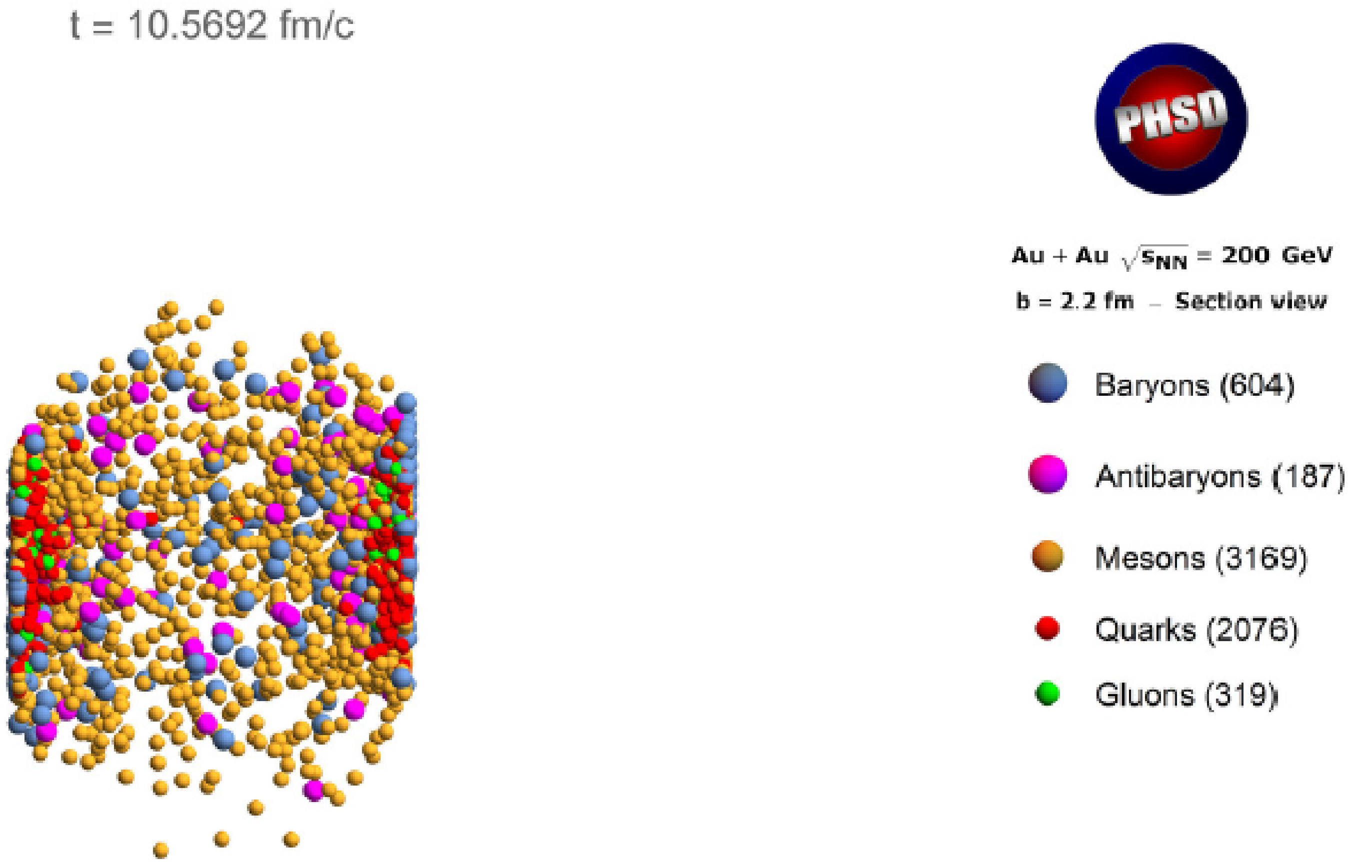


P.Moreau

Au+Au at 200 A GeV, b=2.2 fm

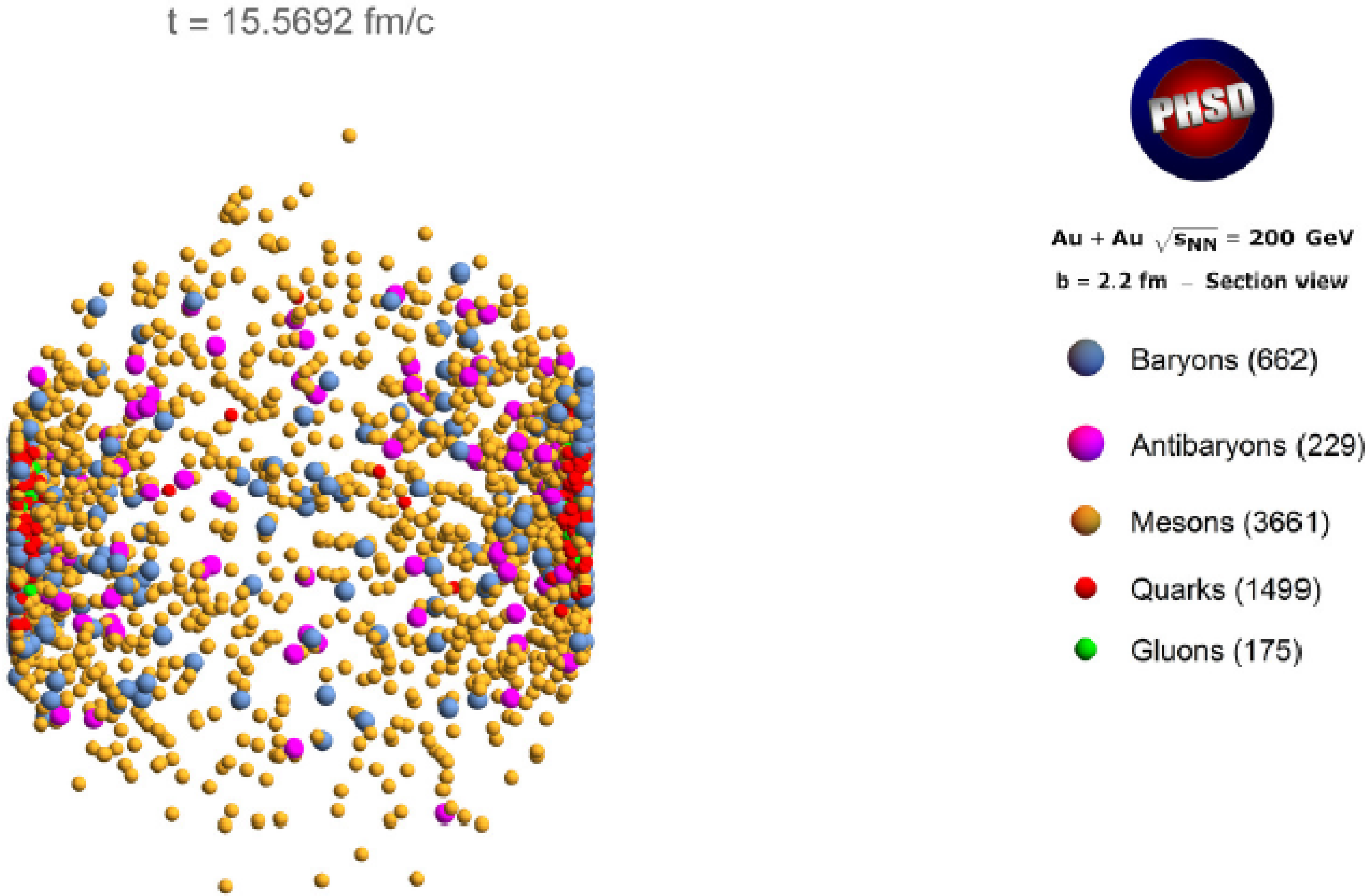


Au+Au at 200 A GeV, b=2.2 fm



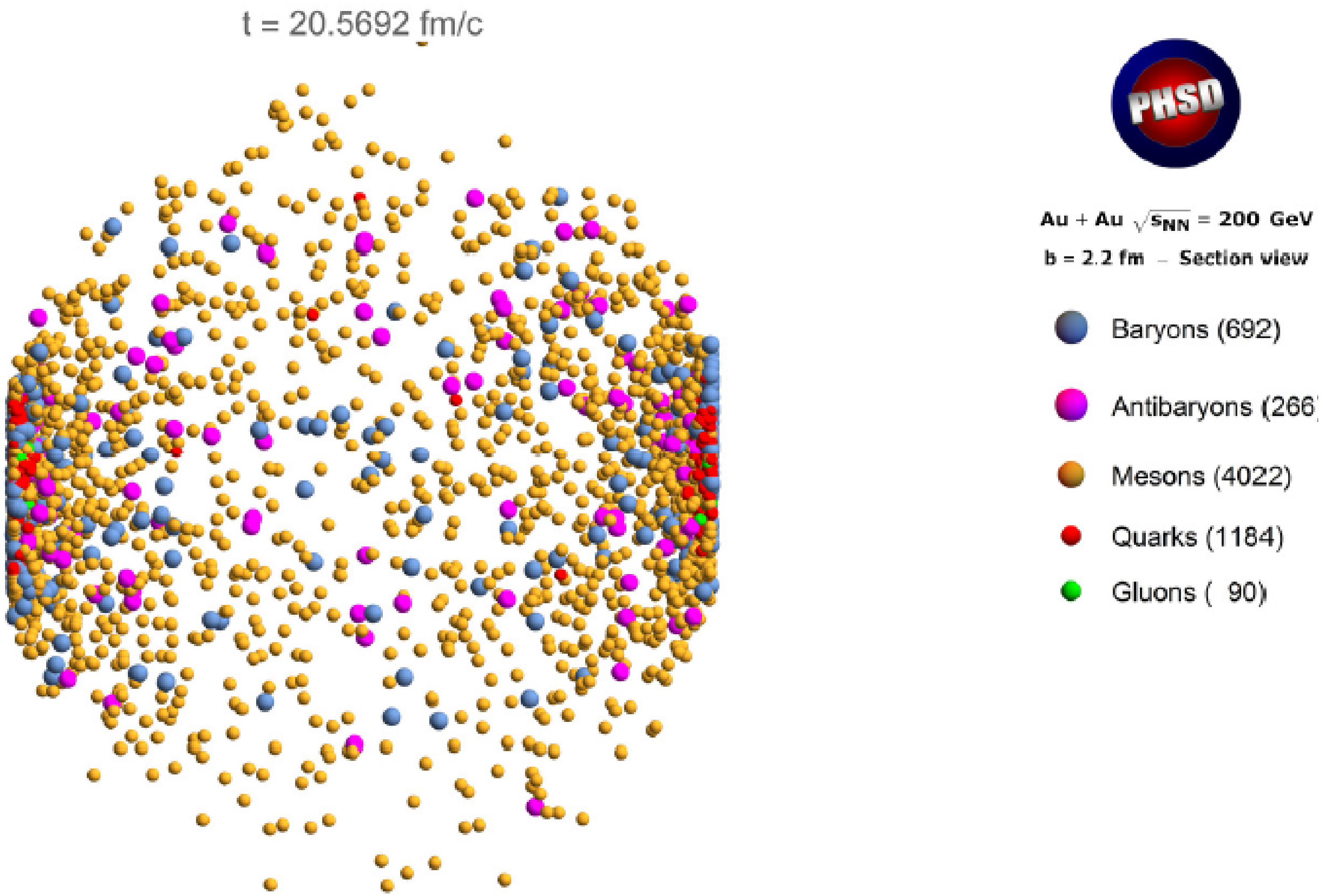
P.Moreau

Au+Au at 200 A GeV, b=2.2 fm



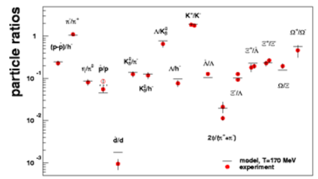
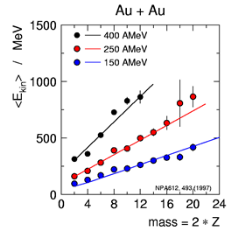
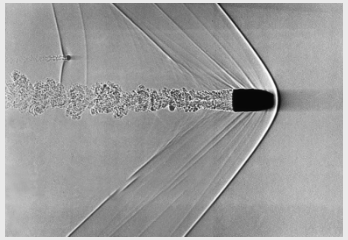
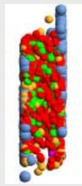
P.Moreau

Au+Au at 200 A GeV, b=2.2 fm



P.Moreau

Classification of models

Thermal/ statistical models	System described by (grand) canonical ensemble of non-interacting articles (fermions and bosons) • No dynamics ➢ Particle yields predicted, but no flow	
Thermal models + radial or longitudinal flow	Thermally expanding fireball (Boltzman distribution for particle spectra) with additional explosive pressure yielding rapid longitudinal or radial expansion ➢ Particle spectra predicted (Boltzman distribution + flow pattern)	
Hydro-dynamics	Treat nuclear matter as viscous liquid: assume local thermal and chemical equilibrium. Variables describing the system are density ρ, particle number N, four-velocity $\vec{u}$, temperature T, specific heat capacity c_S, pressure P, internal energy E, specific energy, enthalpy h, ... all thermodynamic variables describing a macro-system • Simplified dynamics	
Transport	Non-equilibrium microscopic transport models based on many body theory: hadron-hadron interactions, parton-parton interactions; including potentials, cross sections, life times of particles, in-medium characteristics of particles etc. • Full dynamics; many particles	

Dynamical models

5.2

Hydrodynamic model

Central equations of motion:

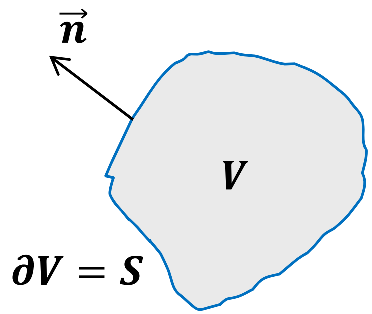
Conservation of mass:

$$\vec{n} \perp \partial V$$

$$\frac{\partial}{\partial t}\int_V \rho dV = -\int_{\partial V} \rho\vec{u}\cdot\vec{n}dS$$

mass change

mass flow through surface



Gauss theorem:

$$\frac{\partial}{\partial t}\int_V \rho dV = -\int_V \nabla\cdot(\rho\vec{u})dV$$

Continuity equation:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

Momentum conservation:

$$\frac{\partial}{\partial t}(\rho\vec{u} + \nabla\cdot(\rho\vec{u}\,\vec{u}) + \nabla P = 0$$

Force on surface

Reminder:

$\rho\vec{u}\,\vec{u} + \Im P =$ **Stress tensor ($\Im$ is the unit tensor)**

Hydrodynamic model

Central equations of motion:

Energy conservation:

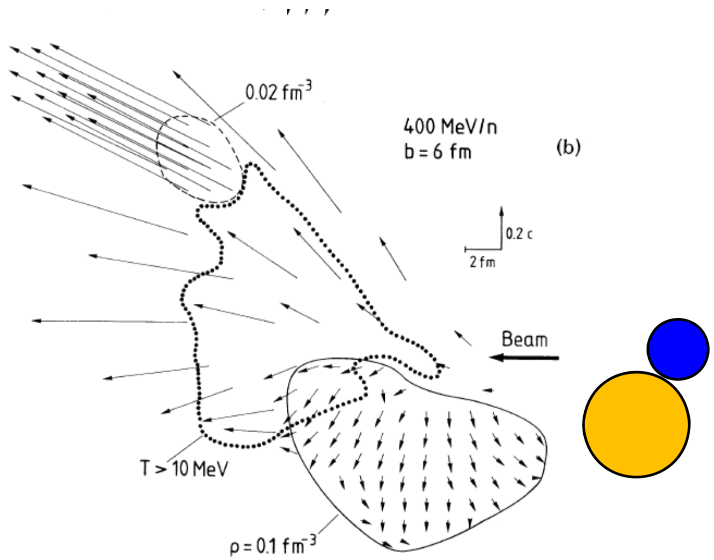
Total thermal energy + kinetic energy = $\int \rho\left(\epsilon + \frac{u^2}{2}\right) dV$

$$\frac{\partial}{\partial t}(\rho\epsilon_{tot}) + \nabla\cdot(\rho\,\epsilon_{tot} + P)\vec{u} = 0$$

Fluid is accelerated by pressure gradients
→ Equation of state is ingredient to the model

Ingredients:

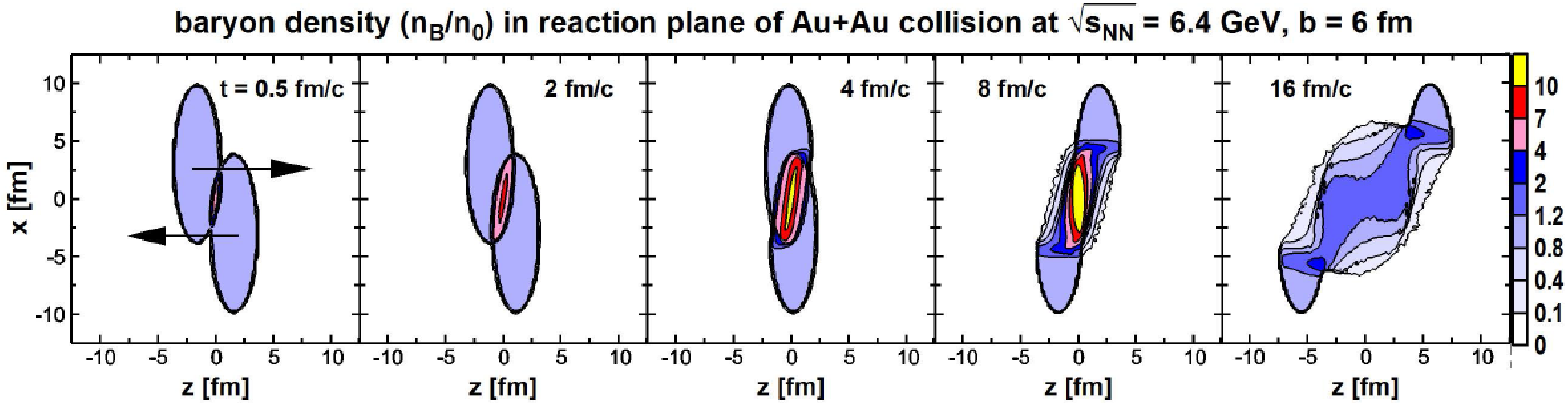
- **thermalisation time** τ_0
- **shape of the initial energy density profile**
- **maximum entropy**
- **baryon density/entropy (constant during acceleration)**
- **initial radial flow**
- **freeze-out parameter** τ_f → **"particlization"**



Assumptions:

Hydrodynamics assumes local equilibrium

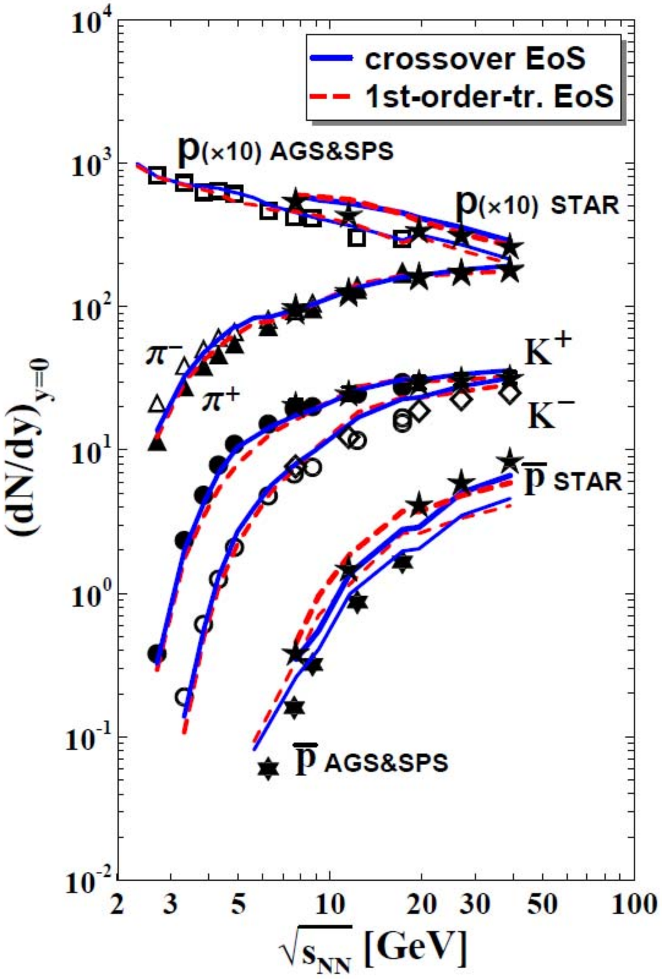
Hydrodynamic model

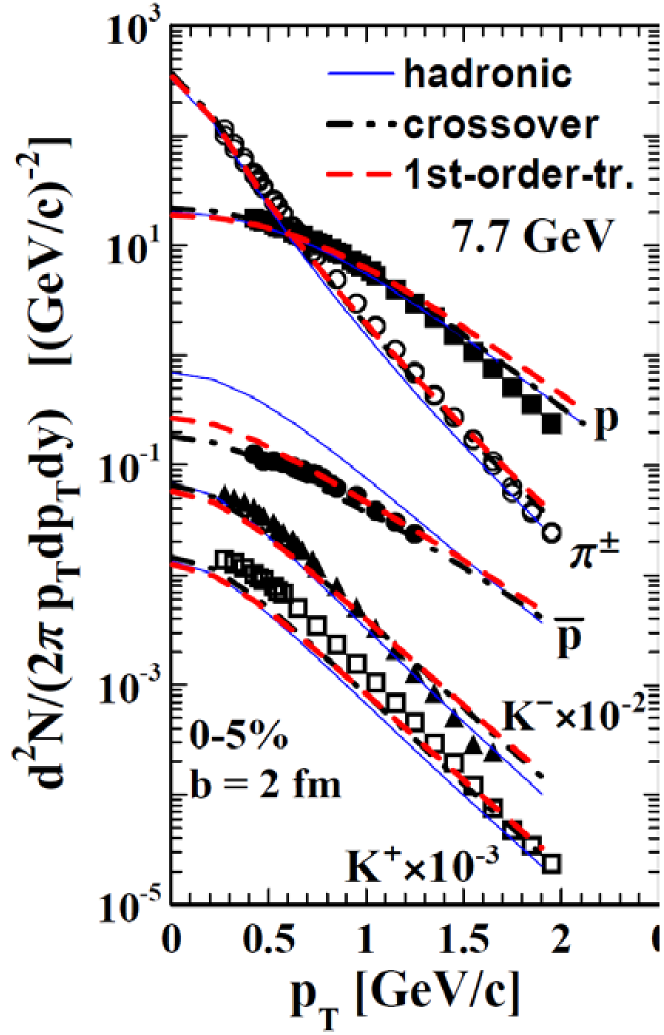


Ivanov et al. 1801.01764

Predictions of hydrodynamics

3 fluid-hydrodynamics + hadronization according to grand canonical distribution functions: Ivanov et al. 1801.01764





The Boltzmann – Uehlig – Uhlenbeck approach

Assumption:

- deterministic trajectories
- 2 body collisions
- no correlations between collisions

Ingredients:

- EOS via a mean field
- many cross sections for particle interactions

Central equation of motion:

BUU equations can be deduced from Schrödinger Equations for n-particles.

Single particle phase space density $f = f(\vec{r}, \vec{p}, t)$ "moving" in a mean field U

Vlasov equation:

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\nabla_r - \nabla_r U \nabla_p\right) f(\vec{r}, \vec{p}, t) = 0$$

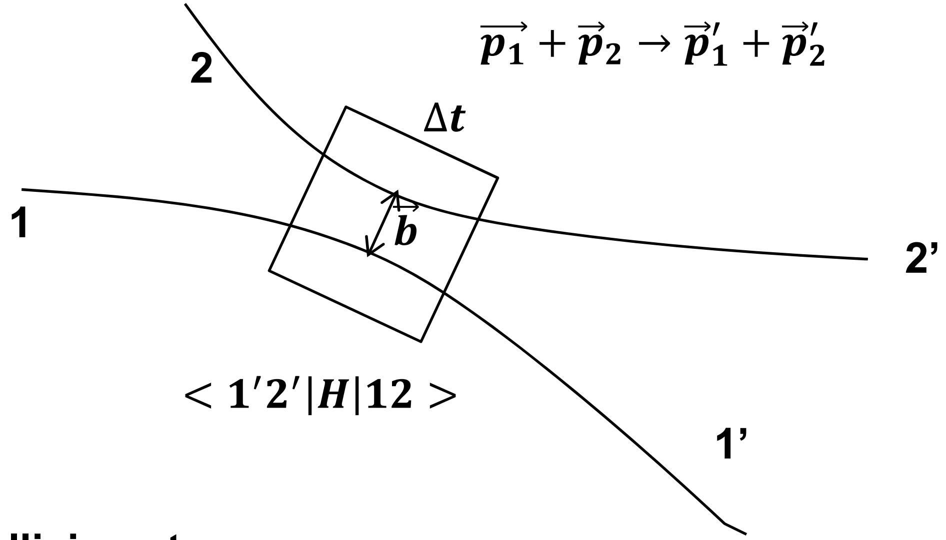
Boltzmann-Uehlig-Uhlenbeck Equation

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\nabla_r - \nabla_r U \nabla_p\right) f(\vec{r}, \vec{p}, t) = \left(\frac{\partial f}{\partial t}\right)_{coll} = I$$

5.3

The Boltzmann – Uehlig – Uhlenbeck approach

Add collisions:



$(\vec{r}_1, \vec{p}_1), (\vec{r}_2, \vec{p}_2) \rightarrow (\vec{r}_1`, \vec{p}_1'), (\vec{r}_2', \vec{p}_2')$ has to obey Fermi – statistics: NO two particles can occupy the same phase space cell.

If phase space around $(\vec{r}_1`, \vec{p}_1'), (\vec{r}_2', \vec{p}_2')$ is empty, collision may happen.

Collisions term:

$$I_{coll} = \frac{4}{(2\pi)^3} \int d^3p_2 d^3p_1 \int d\Omega \, |\mathrm{v}_{12}| \, \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \cdot \frac{d\sigma}{d\Omega}(1 + 2 \rightarrow 1' + 2') \cdot P$$

Probability including Pauli blocking of Fermions:

$$P = f_1' f_2' (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_1')(1 - f_2')$$

gain term
1'+2' → 1+2

loss term
1+2→1'+2'

Pauli blocking factors

5.3

The Boltzmann – Uehlig – Uhlenbeck approach

Numerical realization:

Vlasov part:

Approximate f by test particles: $f(\vec{r},\vec{p},t) = \frac{1}{N}\sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i(t))\delta(\vec{p} - \vec{p}_i(t))$

N = Number of test particles

Trajectories $\vec{r}_i, \vec{p}_i$ result from solution of classical equation of motion

$$\frac{\partial H}{\partial \vec{r}_i} = -\dot{\vec{p}}_i \qquad \frac{\partial H}{\partial \vec{p}_i} = -\dot{\vec{r}}_i$$

Collision term solved by Monte Carlo:

- **interaction takes place $\pi b^2 < \sigma$**
- **final state selected by Monte Carlo according to cross section and angular distribution**
- **final state accepted by Monte Carlo (obeying Pauli principe)**

5.3

Quantum Molecular Dynamics approach

<u>Assumption</u>:

Particles moving in a potential of other particles and are described by Gaussian wave packets

$$f_i(\vec{r},\vec{p},t) = \frac{1}{(\pi\hbar)^3} e^{\frac{-2\left(\vec{r}-\vec{r}_i(t)\right)^2}{L}} e^{-(\vec{p}-\vec{p}_i)^2\left(\frac{L}{2\hbar^2}\right)}$$

The Hamiltonian contains 2 and 3 body interactions

$$U = V^{loc} + V^{Yuk} + V^{Coul} + V \ldots. \rightarrow \text{EOS}$$

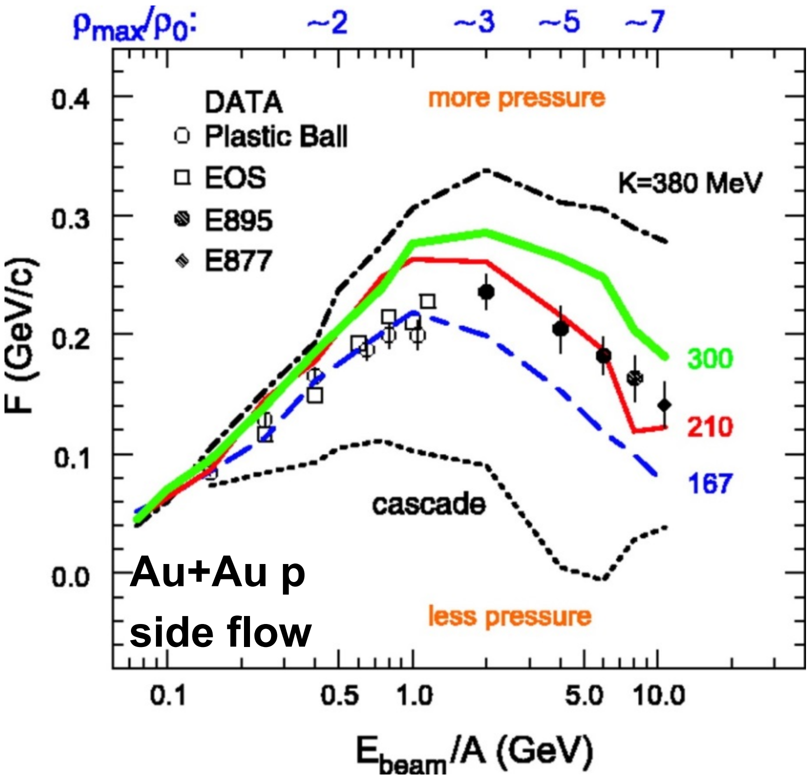
Wigner density = $\sum_i f_i(\vec{r},\vec{p},t)e^{i\vec{p}\,\vec{r}}$ **obeys BUU equations BUT on N-body level**

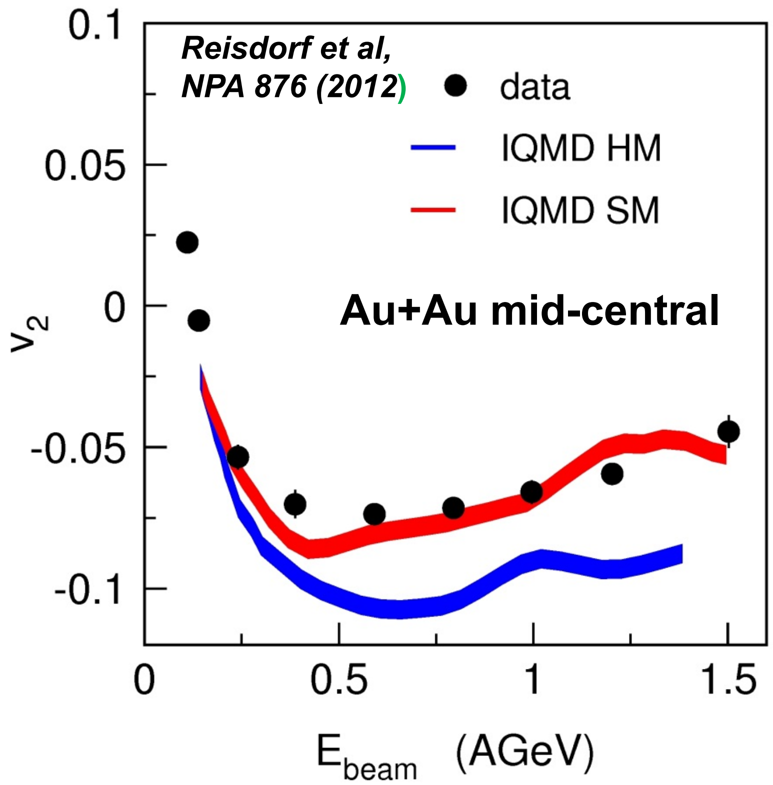
This approach includes

- **fluctuations**
- **correlations between particles**

5.4

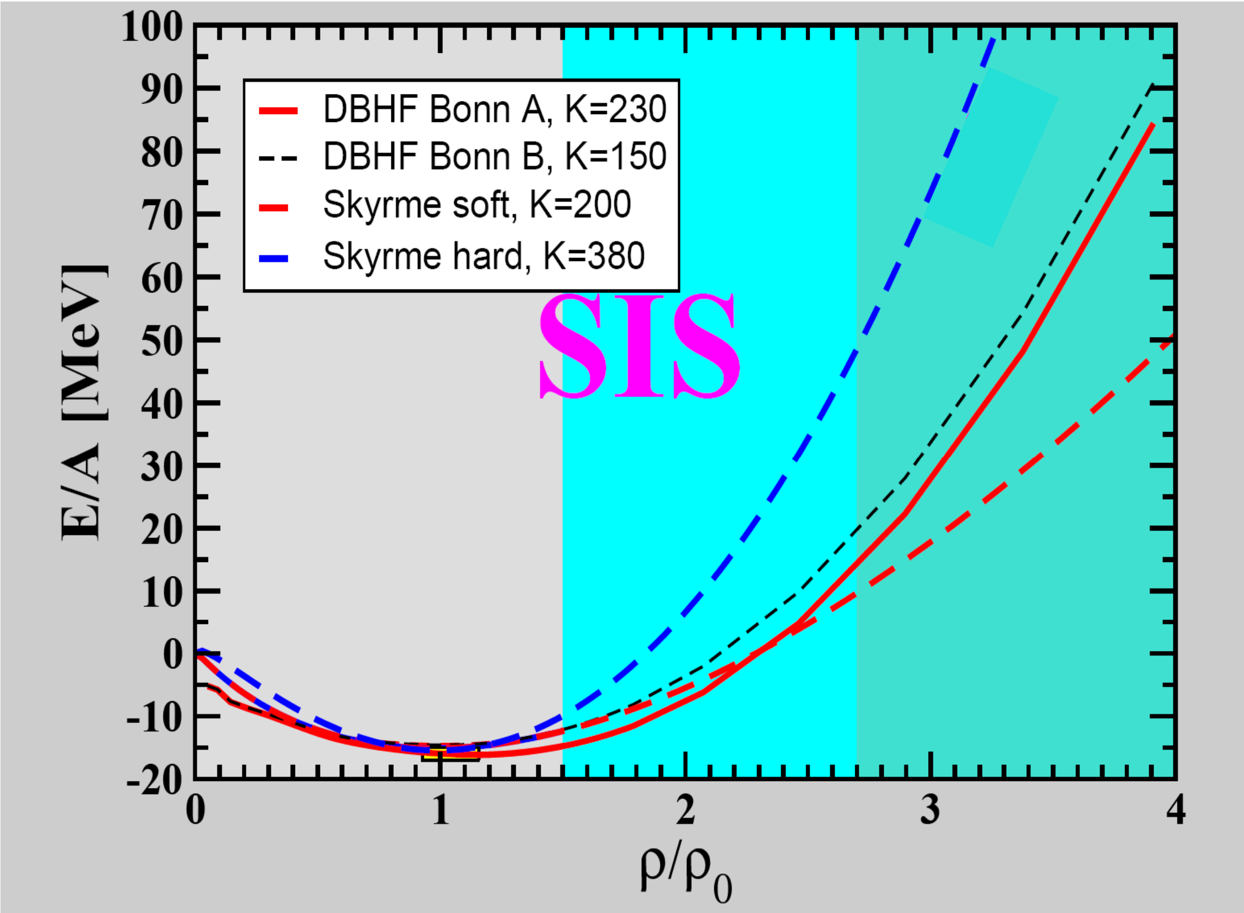
Application of transport models





Nuclear Matter Equation-of-State

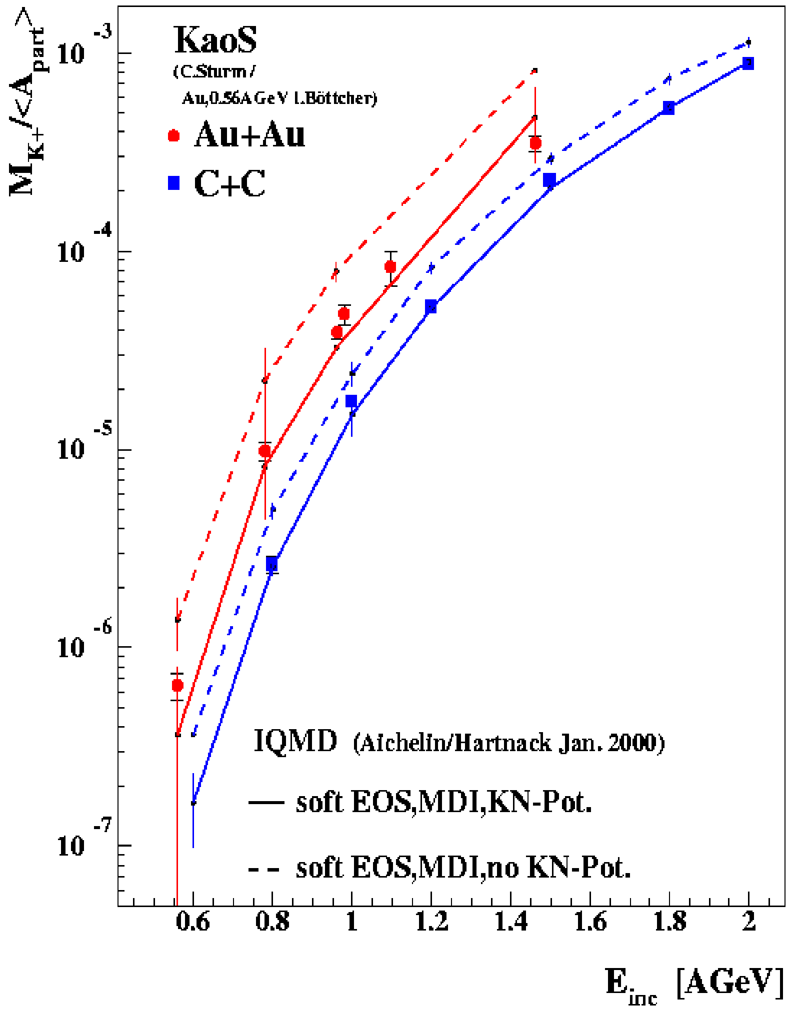
C. Fuchs, Prog. Part. Nucl. Phys. 56 (2006) 1



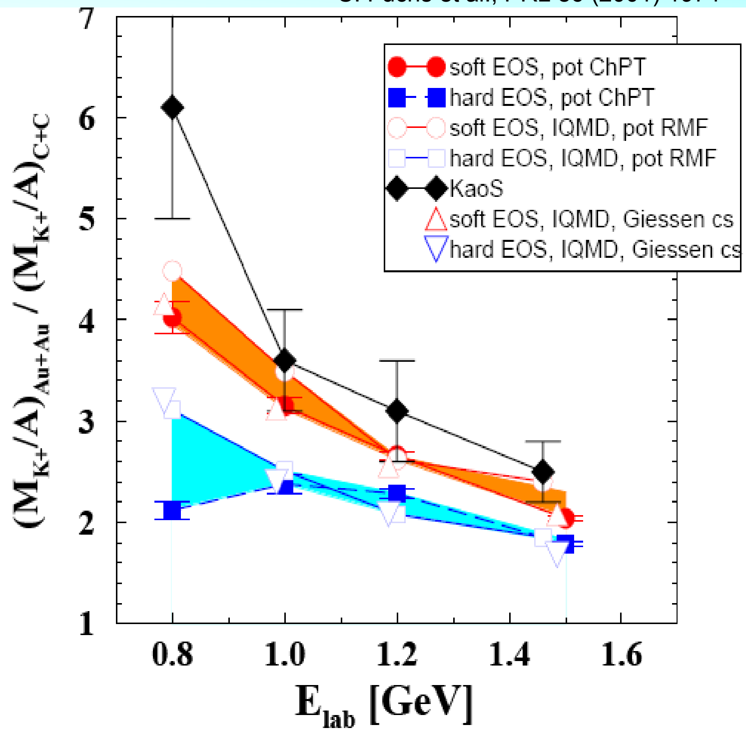
5.4

Subthreshold Kaon Yields

C.Sturm et al. (KaoS), PRL 86 (2001) 39



C. Fuchs et al., PRL 86 (2001) 1974



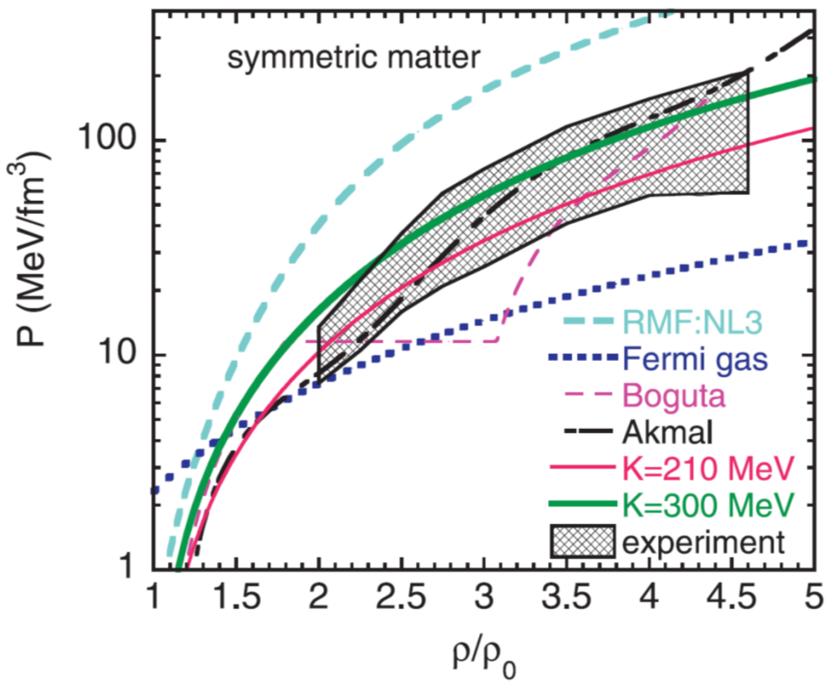
Ratio of yields stable against variation of K^+ production cross section

Strong sensitivity to EOS -> soft EOS (K=200)

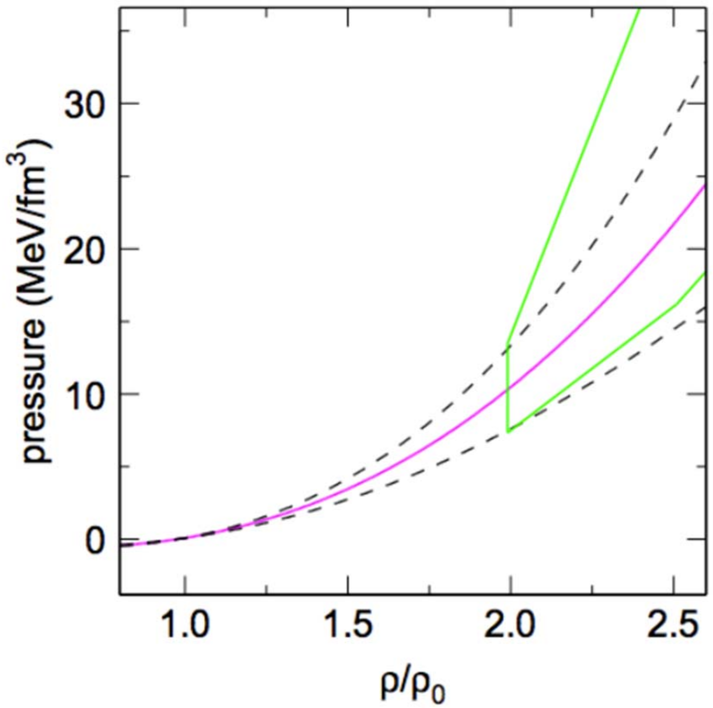
K^+ yield is described only when KN potential is used.

EOS from HI – collisions

(Danielewicz 2002)

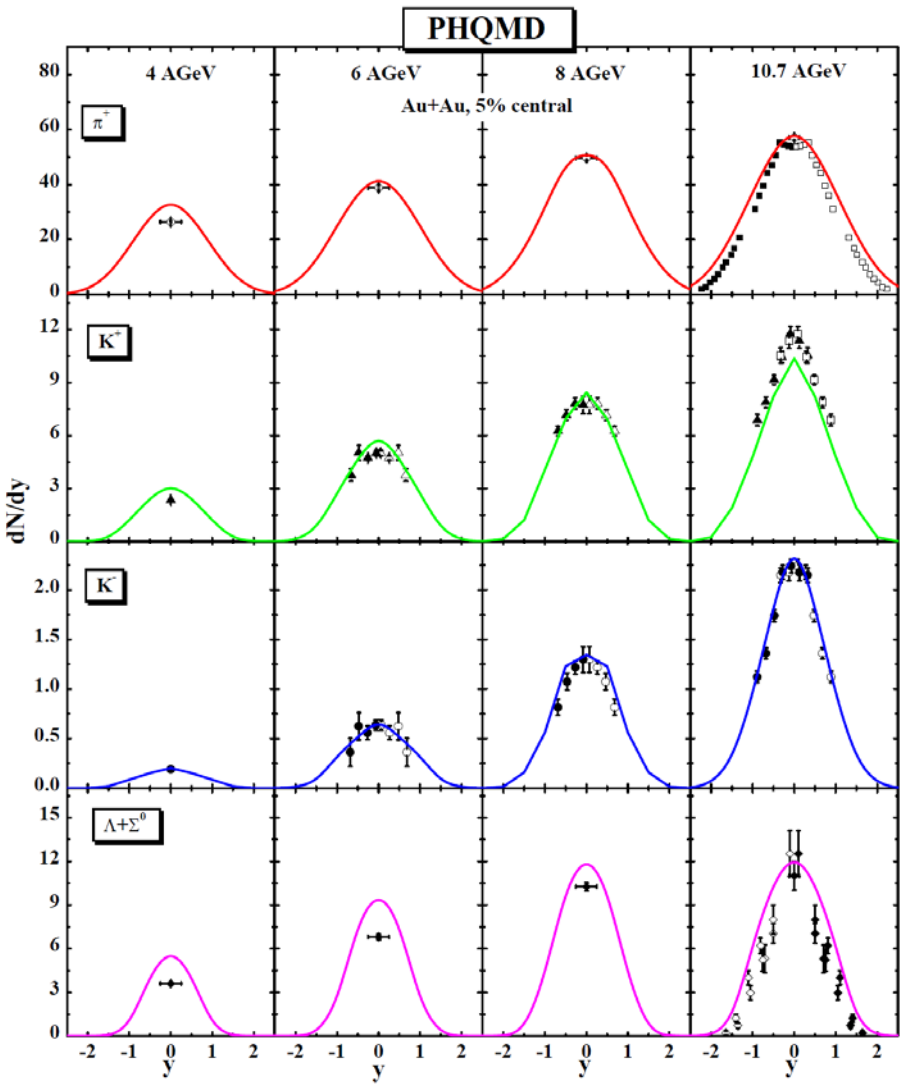


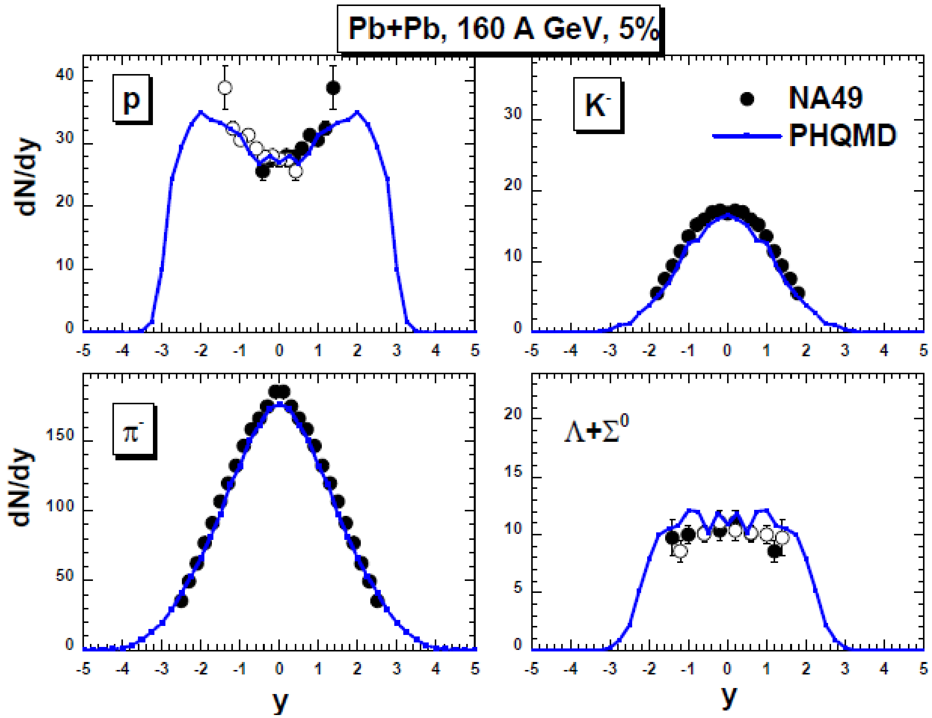
A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, Ch. Hartnack, Nucl.Phys. A945 (2016) 112--133



5.4

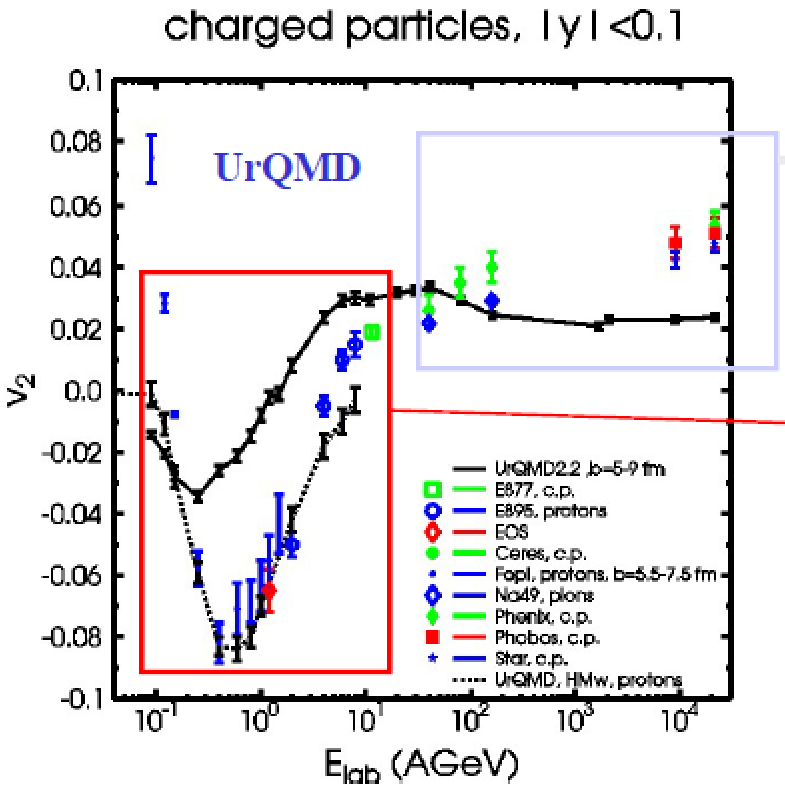
Observables at higher energies





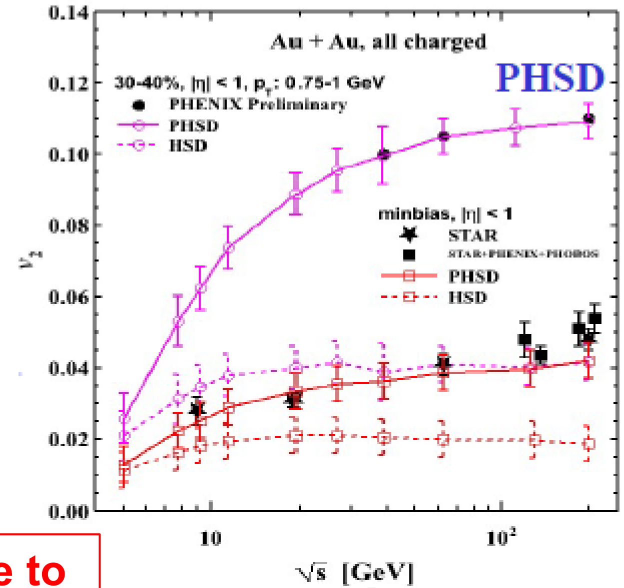
Hadron-string dynamics

Above a critical energy density $\epsilon_{crit} > 0.5$ GeV/fm^3 the program uses partons (quarks and gluons) as constituents.



QGP

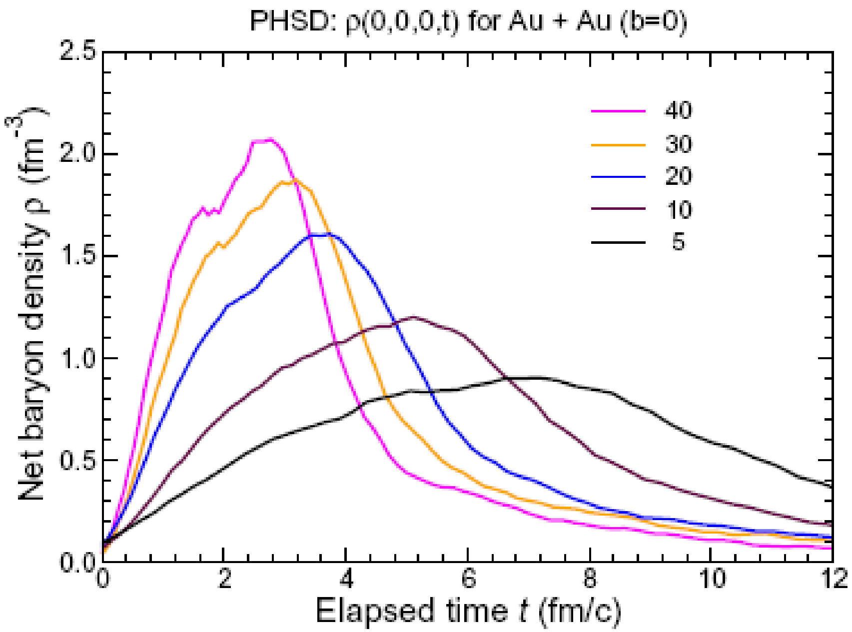
sensitive to equation of state of nuclear matter

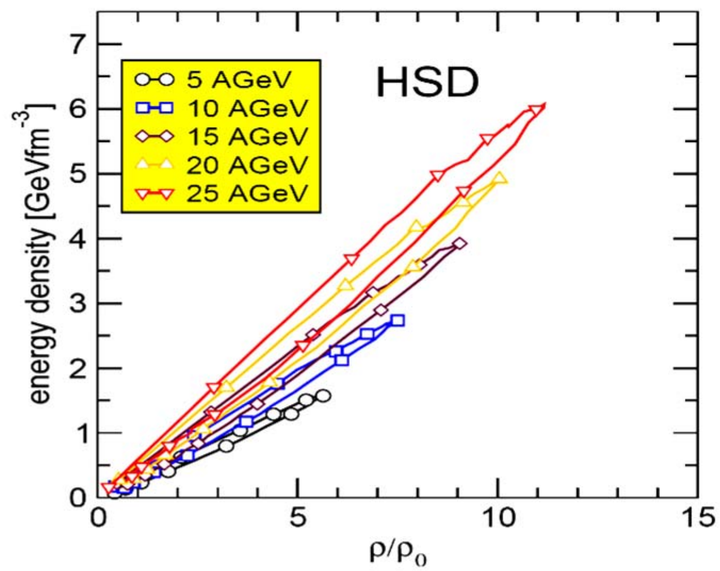


sensitive to the QCD phases: HSD hadron gas, pHSD gas out of partons above critical density

IMPORTANT: results are model dependent! Needs cross checks with other models and different experimental data

Time evolution of collisions



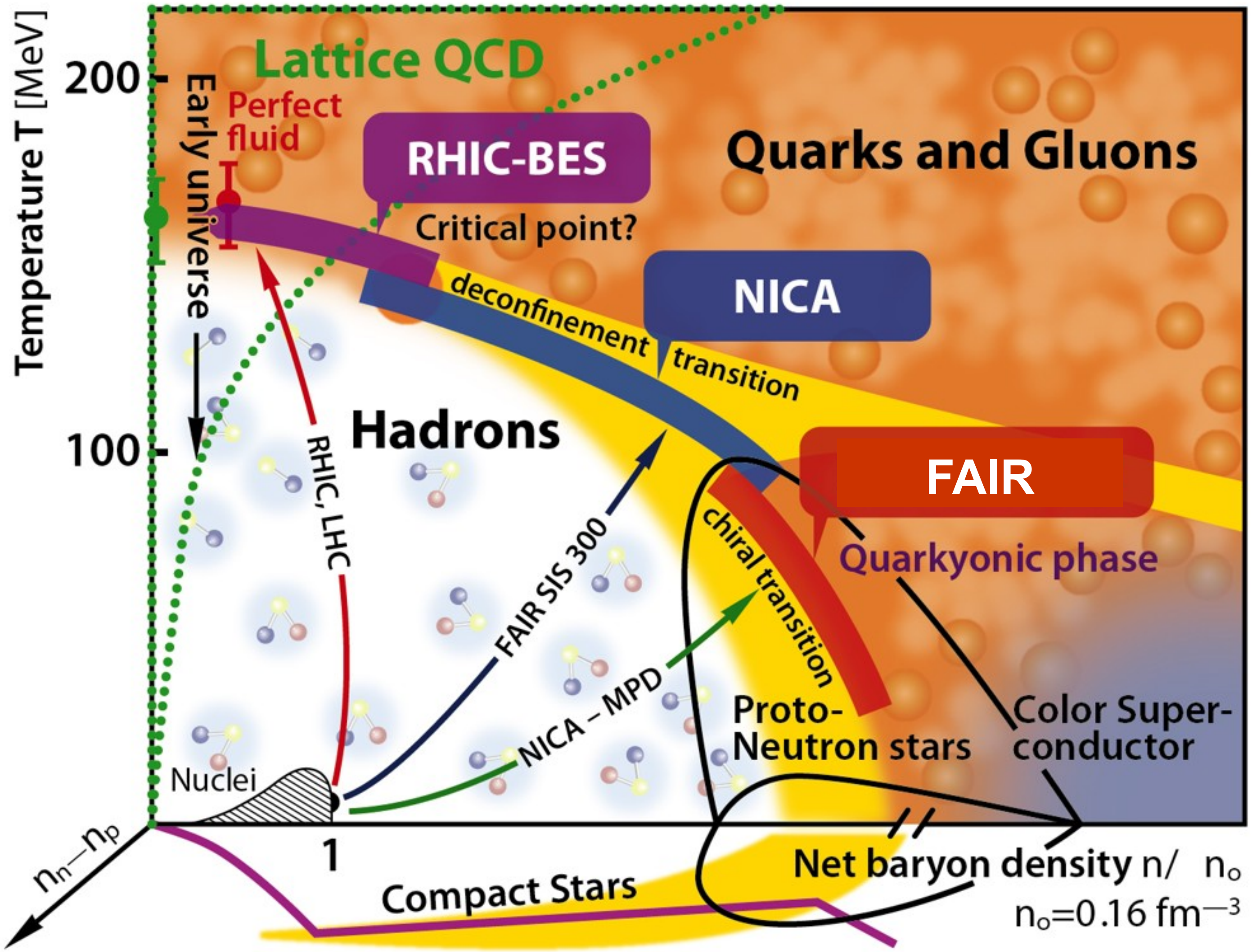


huge energy baryon densities reached at FAIR energies ($\epsilon > \epsilon_{crit}$) E= 5AGeV.

6

CHIRAL SYMMETRY

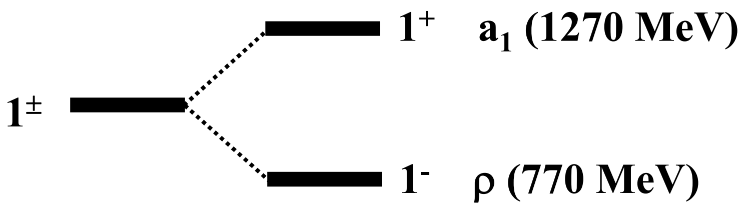
Phase transition in QCD matter



6.

Consequences of Spontaneous Chiral Symmetry Breaking

1) All hadrons have well defined parity, chiral J^P doublets not observed.



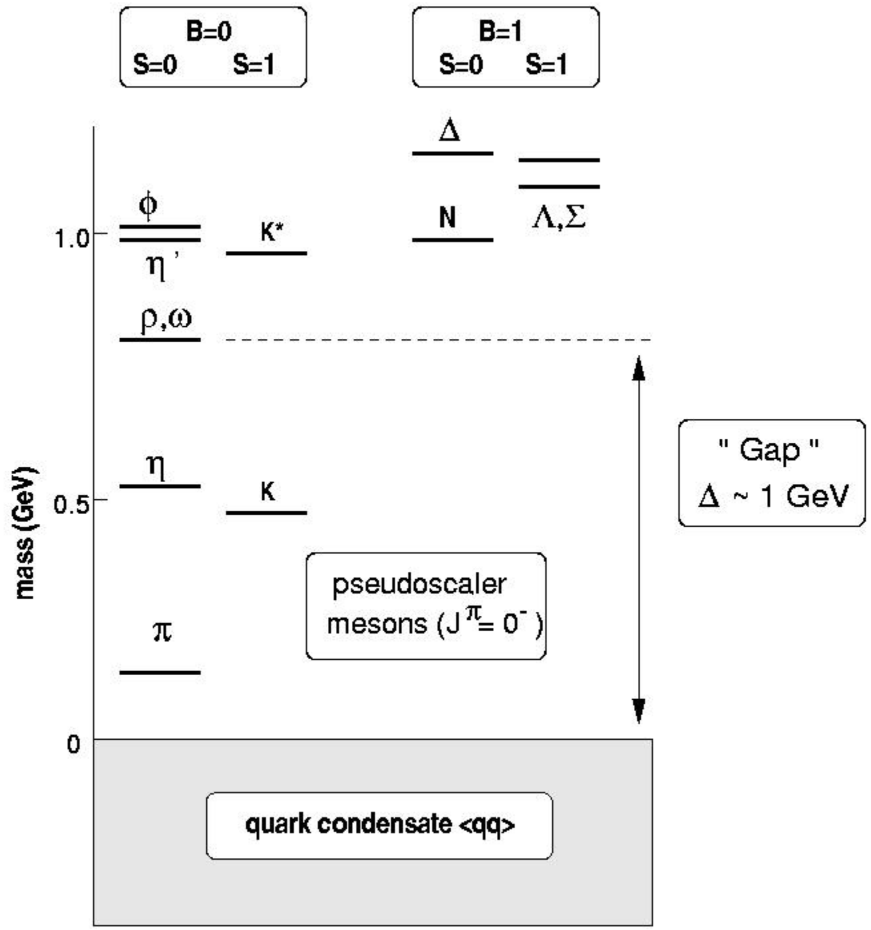
2) Chiral symmetry spontaneously broken, vacuum is filled with $q\bar{q}$ – condensate.

3) Goldstone theorem: Any spontaneously broken continous symmetry generates a massless boson (→ Goldstone bosons).

4) Characteristic mass scale of hadrons

1 GeV mass gap to quark condensate

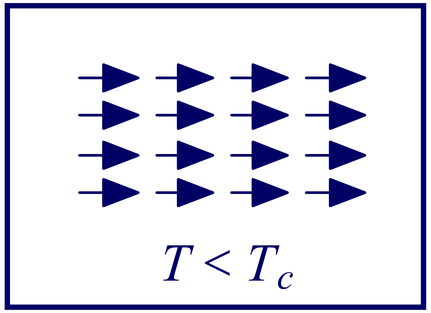
except pseudoscalar mesons that are the Pseudo - Goldstone bosons: π, η, and K



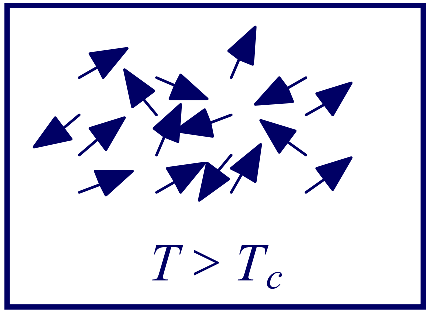
6.

(Hidden) Symmetry in ferromagnetism

- **Example of a hidden symmetry restored at high temperature**
  - **Ferromagnetism - the spin-spin interaction is rotationally invariant.**



Below the Curie temperature the underlying rotational symmetry is hidden.



Above the Curie temperature the rotational symmetry is restored.

  - **In the sense that any direction is possible the symmetry *is* still present at $T<T_c$.**

  - **Curie – Weiss – Law: Phase transition at T_c**

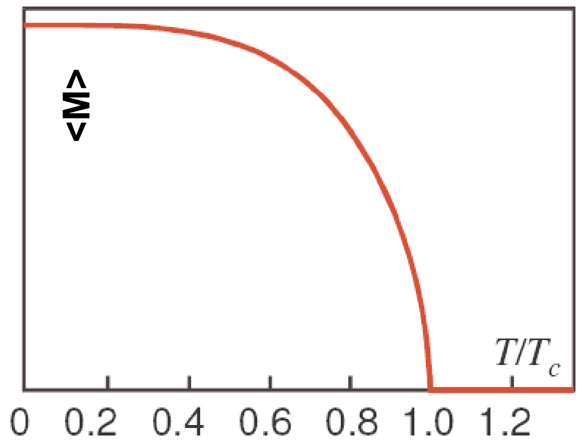
magnetic susceptibility

$$\chi = \frac{C}{T - T_c}$$

with magnetisation M

$$B = B_{ext} + \mu_0 M = (1 + \chi) B_{ext}$$

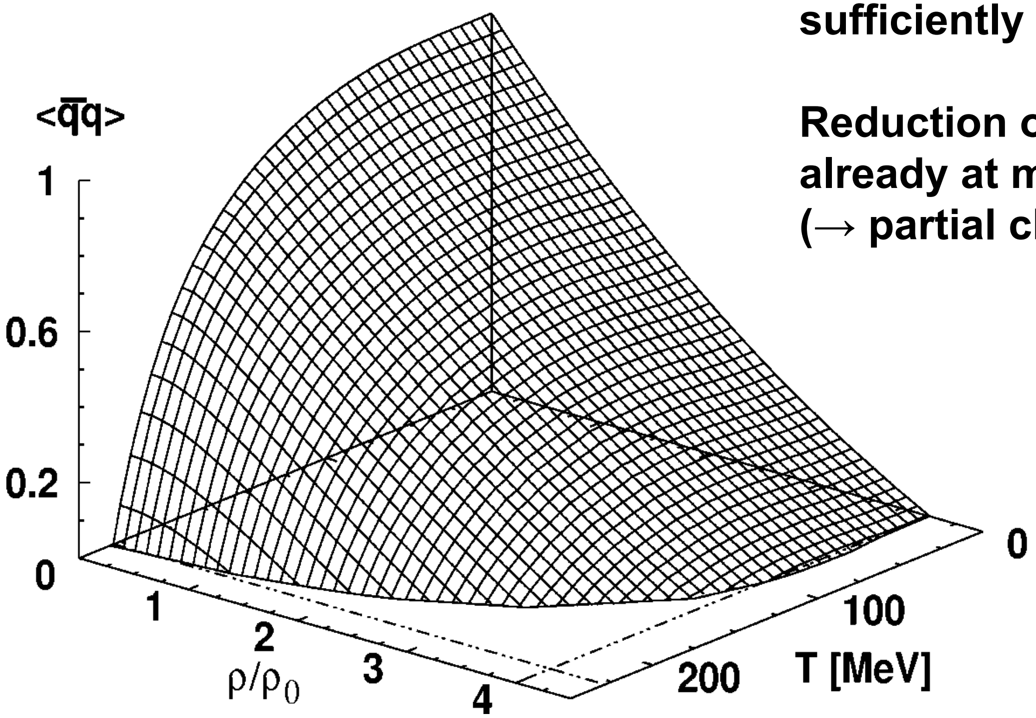
$$\chi = \frac{\mu_0 M}{B_{ext}}$$



6.

Chiral symmetry restoration of QCD

Chiral Condensate



W.Weise, Prog. Theor. Phys. Suppl. 149 (2003) 1

initially: S.Klimt et al.., PLB 249, 386 (1990)

Chiral symmetry should be restored at sufficiently high temperatures and baryon densities.

Reduction of vacuum value should be visible already at moderate densities (→ partial chiral symmetry restoration)

Symmetry breaking pattern of Chiral Symmetry of QCD

Gell-Mann-Oaks-Renner Relation:

$$m_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle + O(m_u^2)$$

$$m_K^2 f_K^2 = -\frac{1}{2}(m_u + m_s)\langle \bar{u}u + \bar{s}s \rangle + O(m_s^2)$$

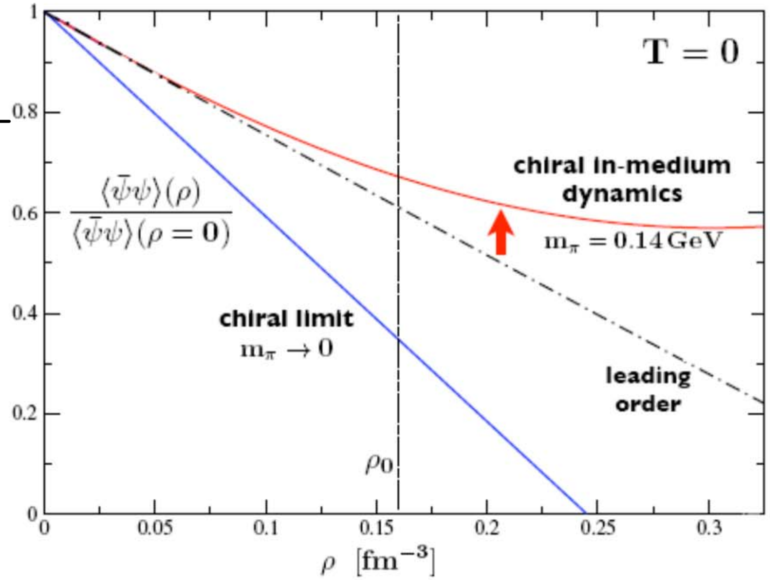
spontaneous symmetry breaking

explicit symmetry breaking

6.

Chiral in-medium dynamics

$$\frac{\langle q\bar{q}\rangle_{T,\rho}}{\langle q\bar{q}\rangle_0}$$

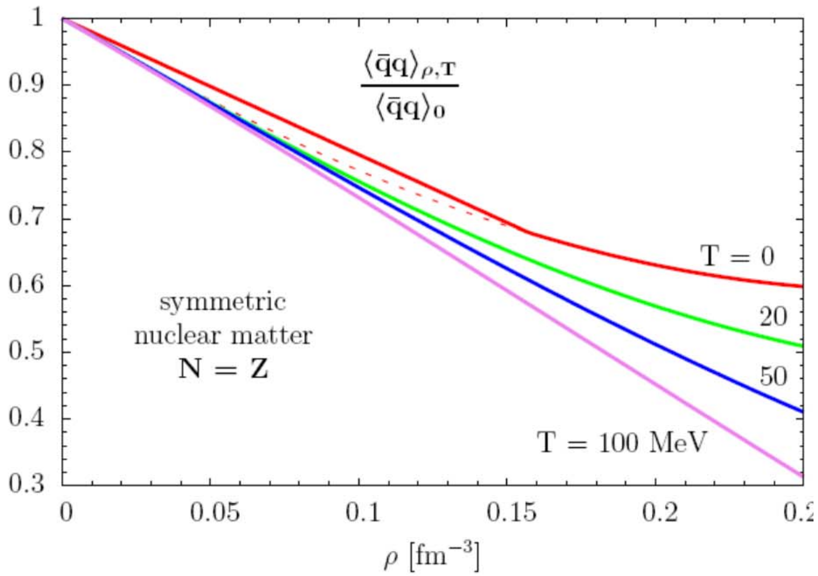


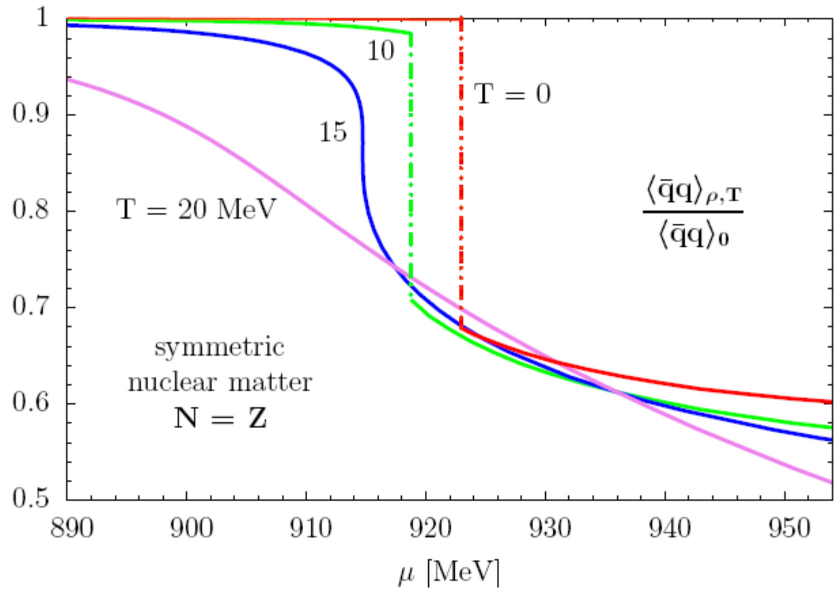
W. Weise, Prog.Part.Nucl.Phys. 67 (2012) 299

Calculations within
„chiral effective field theory“

↓

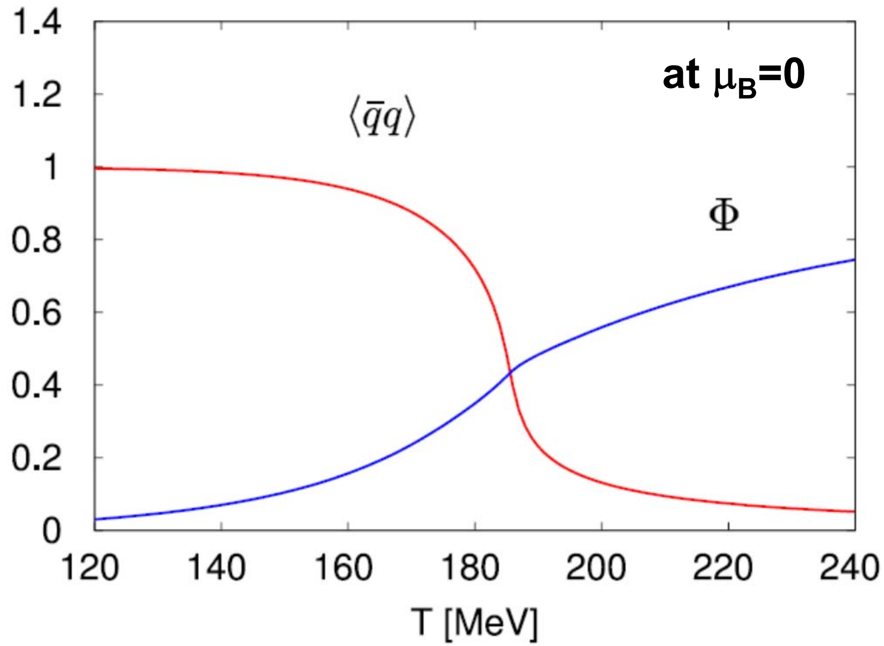
Nuclear matter (baryons) has to be taken into account.





6.

Order parameters in QCD



Order parameters:

chiral symmetry: Quark condensate $\langle\bar{q}q\rangle$

deconfinement: Polyakov loop $\Phi \sim e^{-\beta F_q}$

with β=1/T, F_q = free energy of free quark

Chiral and Polyakov order parameters show transition at the same temperature.

6.0

Thermal Electromagnetic Emission Rates

Electromagnetic - correlation function:

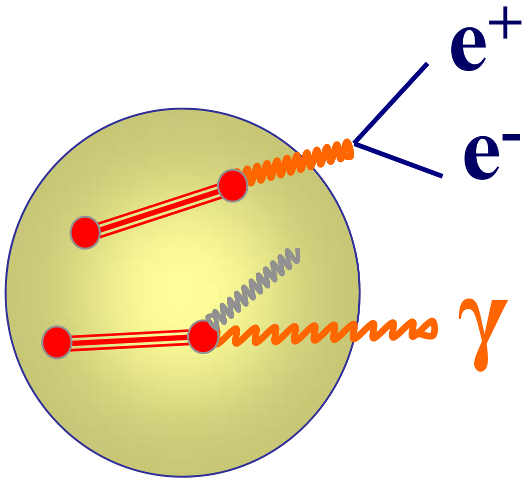
See CBM physics book, ch. 2.2

$$\Pi_{\mathrm{em}}(q) = -i\int d^4x\, e^{iqx}\,\langle j_{\mathrm{em}}(x)\, j_{\mathrm{em}}(0)\rangle_T$$

L.D. McLerran, T. Toimela, Phys. Rev. D 31, 545 (1985)

Average has to be taken from statistical ensemble.

Connected to thermal emission rates by electromagnetic spectral function Im Π_{em}.



$$\frac{dR_{ee}}{d^4q} = \frac{-\alpha^2}{\pi^3 M^2}\, f^B(T)\; Im\;\Pi_{\mathrm{em}}(M,q)$$

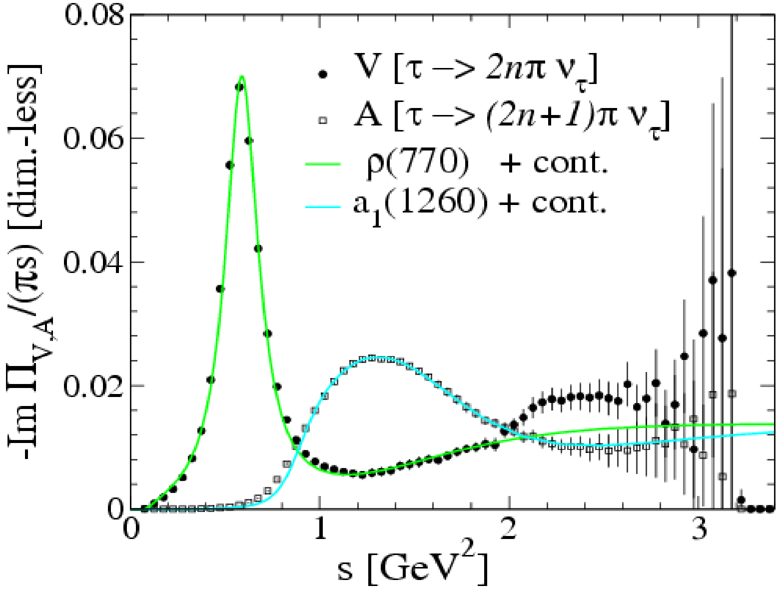
$$q_0\frac{dR_\gamma}{d^3q} = \frac{-\alpha}{\pi^2}\, f^B(T)\; Im\;\Pi_{\mathrm{em}}(q_0=q)$$

electromagnetic spectral function

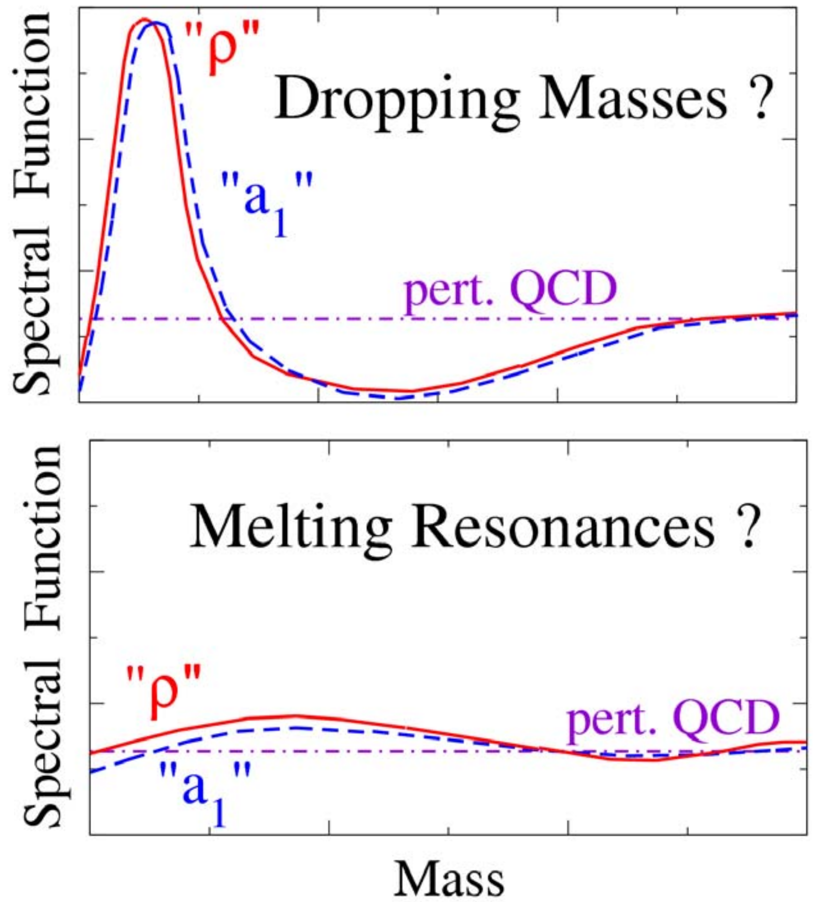
6.

In-medium spectral function

Vacuum



Chiral Restoration



R. Rapp et al.

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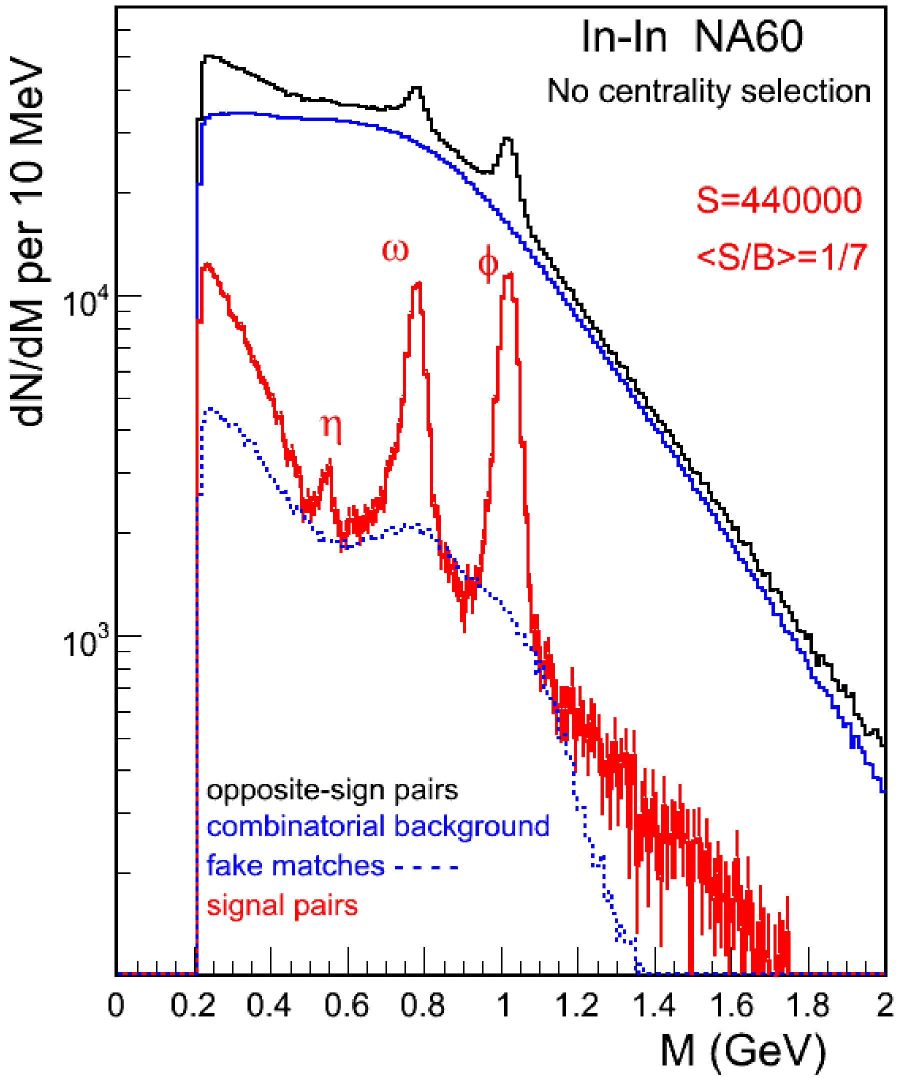
Light vector mesons by Myons

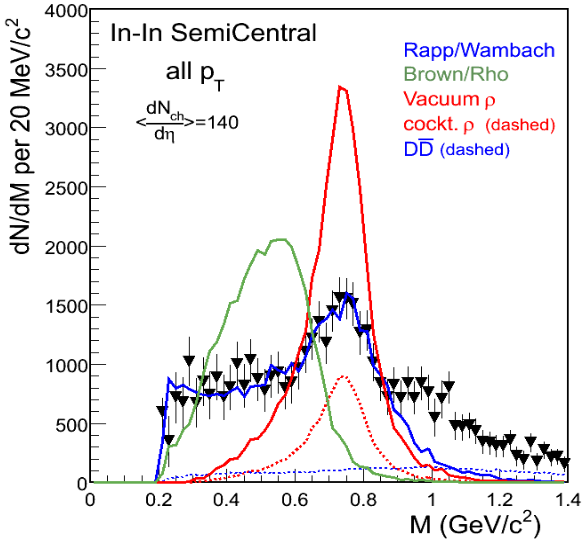
- **In-In collisions at 158 AGeV**
  - **5 weeks in Oct.-Nov. 2003**
  - **~ 4 · 10^{12} ions delivered**
  - **~ 230 million dimuon triggers**

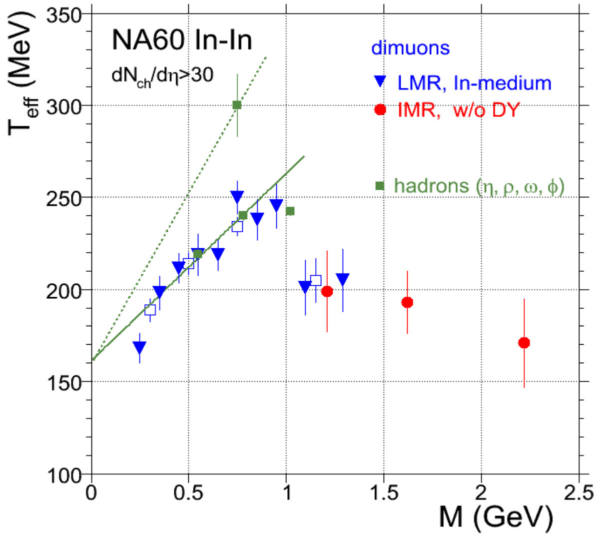
- **Data analysis for dimuons**
  - **Select events with only one reconstructed vertex in target region (avoid re-interactions)**
  - **Match muon tracks from Muon Spectrometer with charged tracks from Vertex Tracker (candidates selected using weighted distance squared → matching χ^2)**
  - **Subtract Background**



Phase transition observables (?)

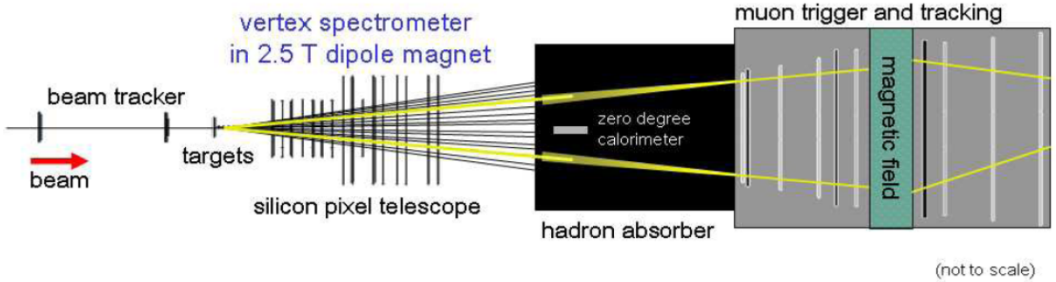
R. Arnaldi et al. (NA60), PRL 100 (2008) 022302





NA60

In + In collisions at 158 AGeV (SPS)



Clean measurement of ρ – meson spectral function.

Slope parameter of transverse momentum spectra in agreement with hadrons up to M ~ 1 GeV

integral yield sensitive to coexistence time

Spectra above 1 GeV are conjectured to originate from partonic source

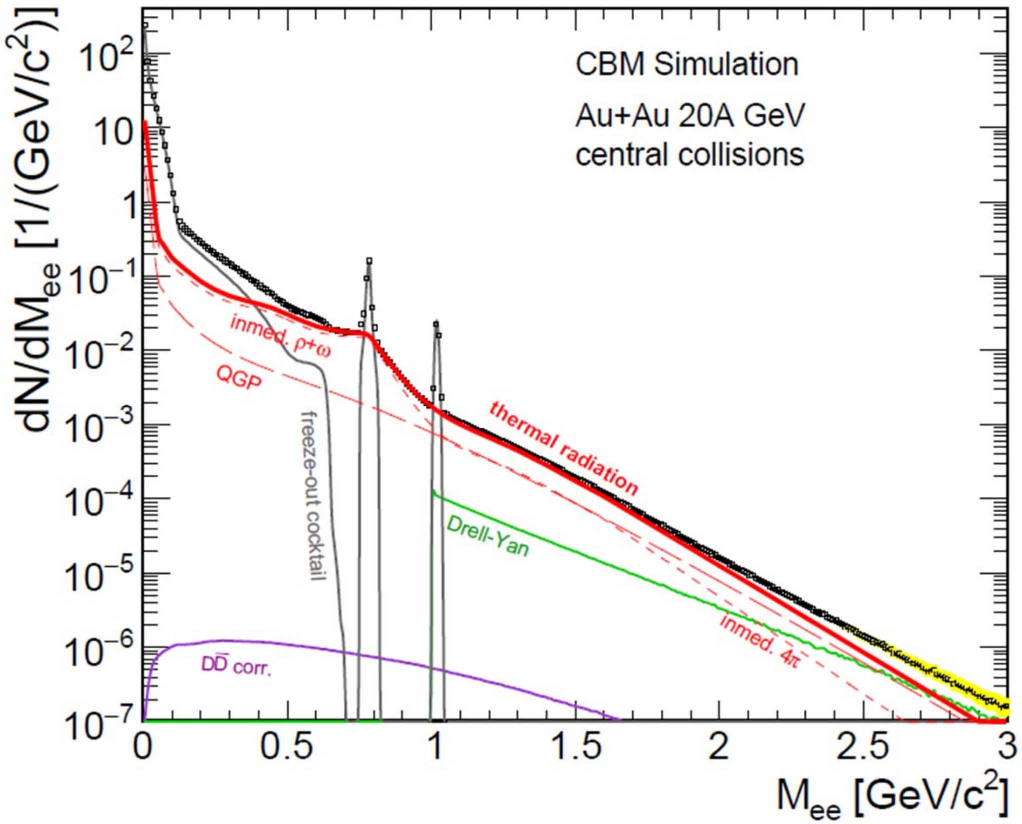
plateau as function of √s might signal latent heat

6.

Dileptons at CBM

S. Chattopadhyay et al.(CBM), arXiv:1607.01487 [nucl-ex]

(R. Rapp, H. v. Hees, priv. comm.)



- Background sources strongly reduced with respect to SPS
- Dilepton measurement can provide

  Temperature of fireball
  Lifetime of fireball
  Chiral symmetry restoration

- Large statistics needed to achieve sufficiently small errors !

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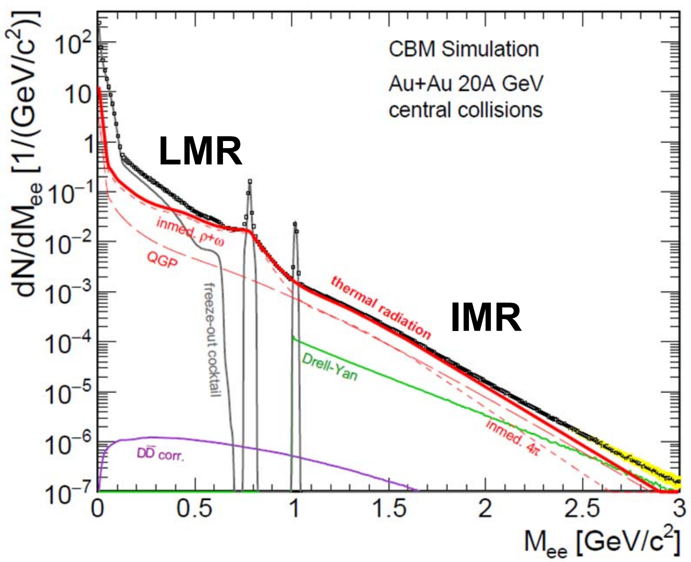
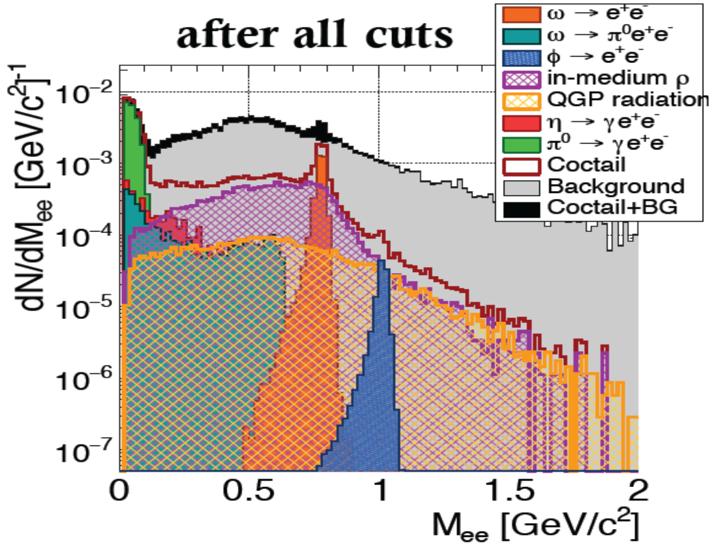
Dileptons as probes for dense matter

[R. Rapp, H. v.Hees, PLB 753 (2016) 586]





- 1M Au+Au (b=0 fm), 8 AGeV
- IMR: S/B > 1/100
- Statistical accuracy of 10% requires ~1 week of beamtime

LMR: ρ – chiral symmetry restoration
fireball space – time extension

IMR: access to fireball temperature
ρ-a_1 chiral mixing

Measurement program:
e.g. excitation function of IMR - slope

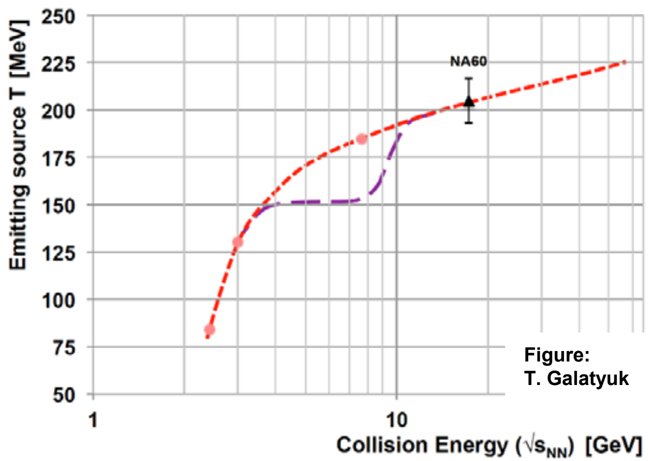


Figure:
T. Galatyuk

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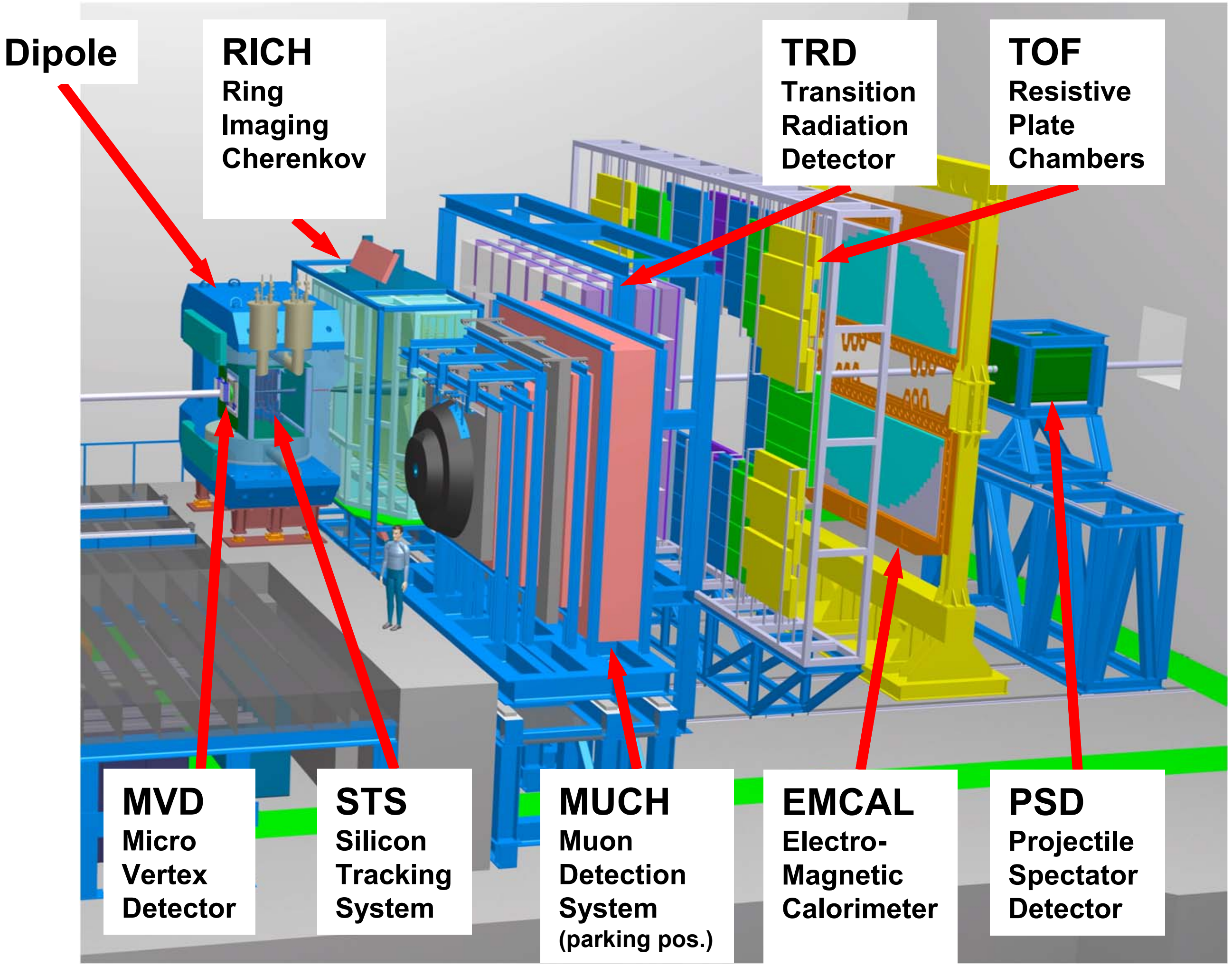
CBM Experimental Setup



- **Tracking acceptance:** **$2^o < \theta_{lab} < 25^o$**
- **Free streaming DAQ**

  R_{int} = 10 MHz (Au+Au)

  except:
  R_{int} (MVD)=0.1 MHz
- **Software based event selection**