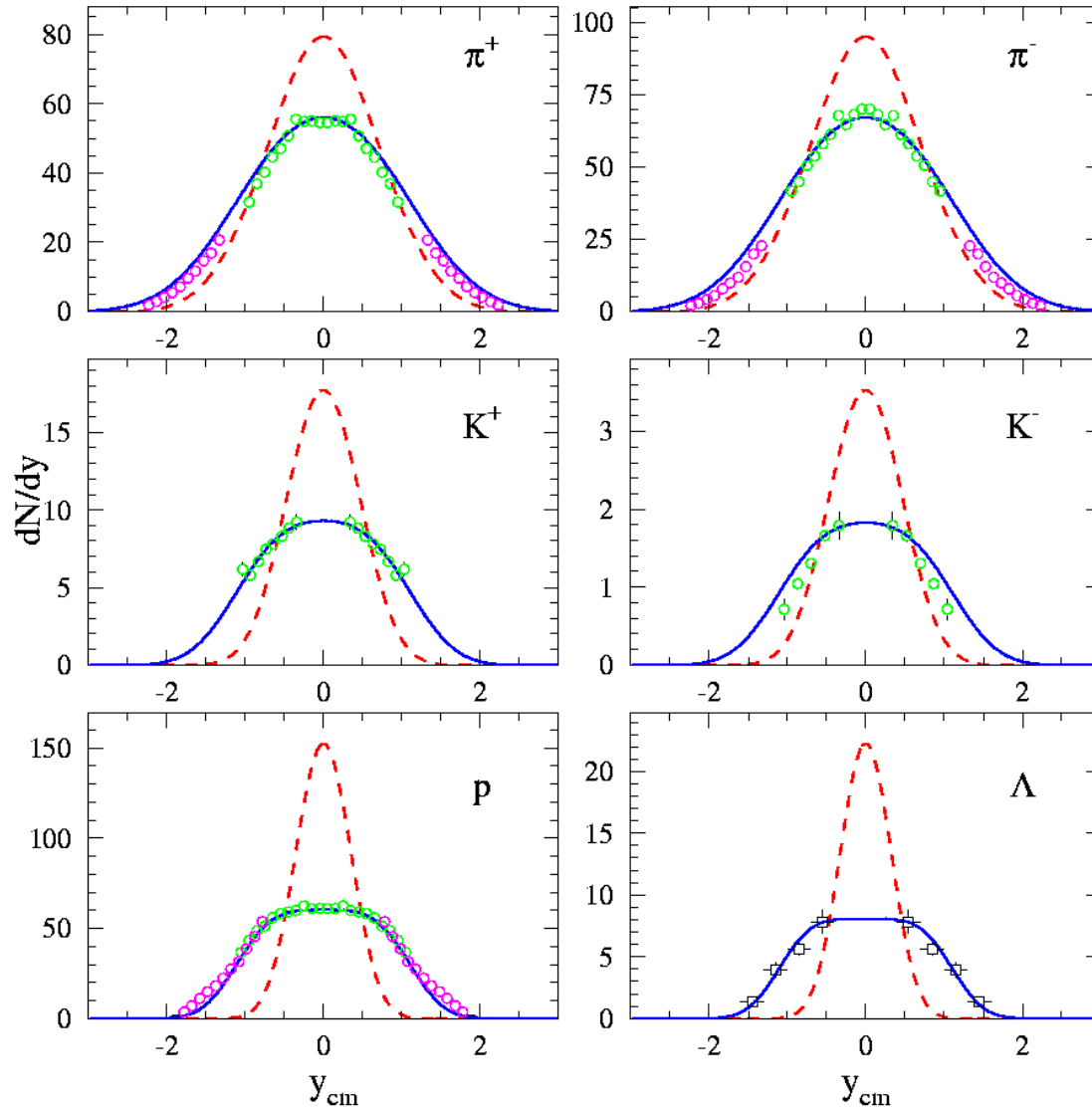


# 5.4

# Stopping

○ E866    ○ E877    □ E891



AGS: Au + Au @ 10.7 AGeV

Rapidity density distributions  
Incompatible with  
isotropic thermal source



Longitudinal expansion.

N.Herrmann,  
J.P. Wessels,  
T.Wienold,  
Ann.Rev.Nucl.Part.  
Sci.49,581 (1999)

## Thermal width of rapidity distribution

**Width of isotropic thermal source** can be calculated analytically:

$$\frac{dN_{isotropic}}{dy} \propto m^2 T (1 + 2\chi + 2\chi^2) \exp(-1/\chi),$$

$$\chi = \frac{T}{m \cosh(y)}$$

T can be (has to be) extracted from slopes of thermal spectra at midrapidity.

**Measured distributions are at variance with isotropic thermal emission picture.**

**Possible scenario:** longitudinally expanding source(s) with source velocities  $\beta_l$

$$\langle \beta_l \rangle = \tanh(\langle y' \rangle)$$

$$\frac{dN}{dy} = \int_{-y'_{\max}}^{y'_{\max}} dy' \frac{dN_{iso}(y - y')}{dy'}$$

# Stopping

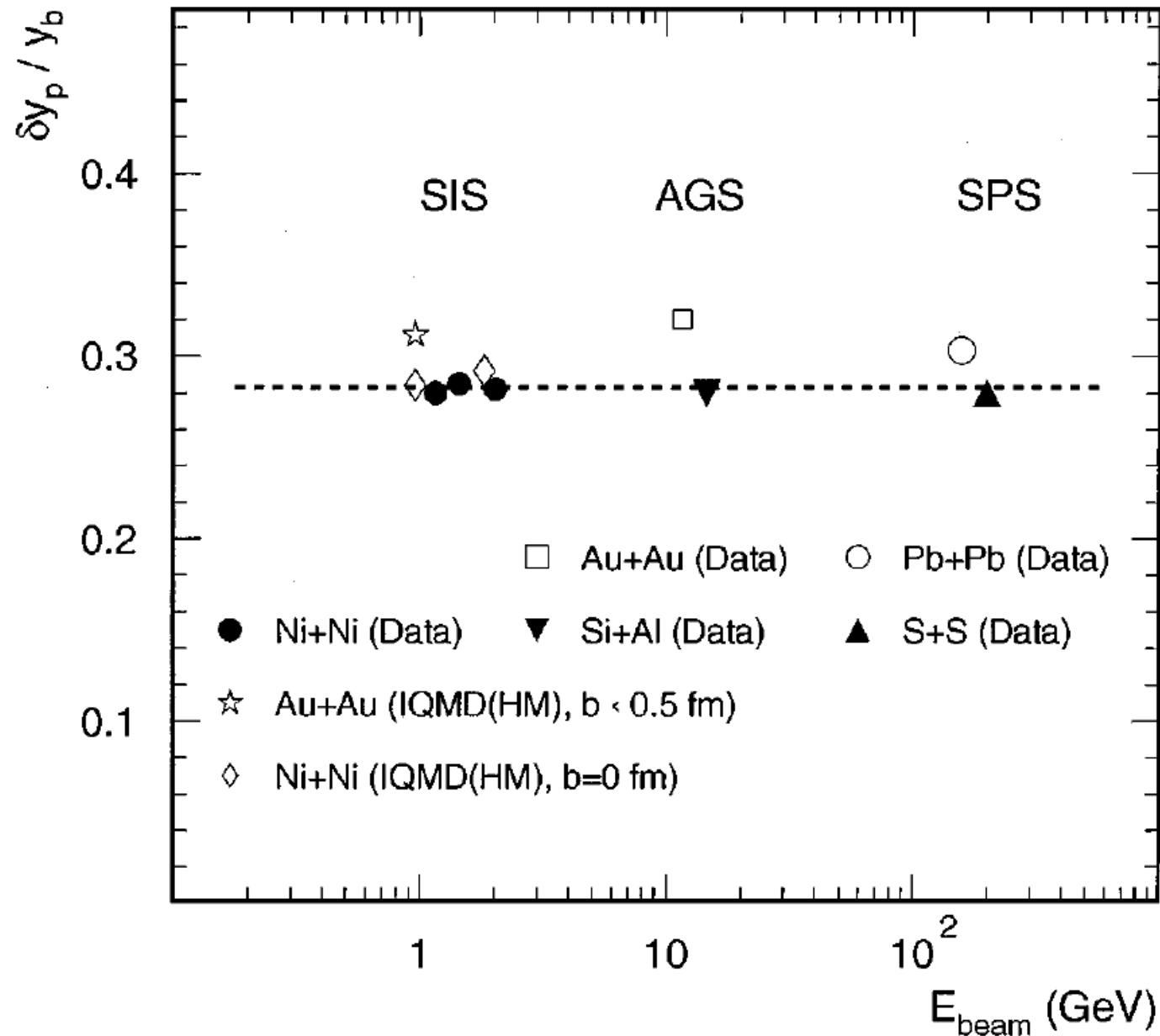
Average rapidity loss:

$$\langle \delta y_p \rangle = y_p - \langle y_b \rangle$$

$\langle y_p \rangle$  - average net baryon rapidity after the collision

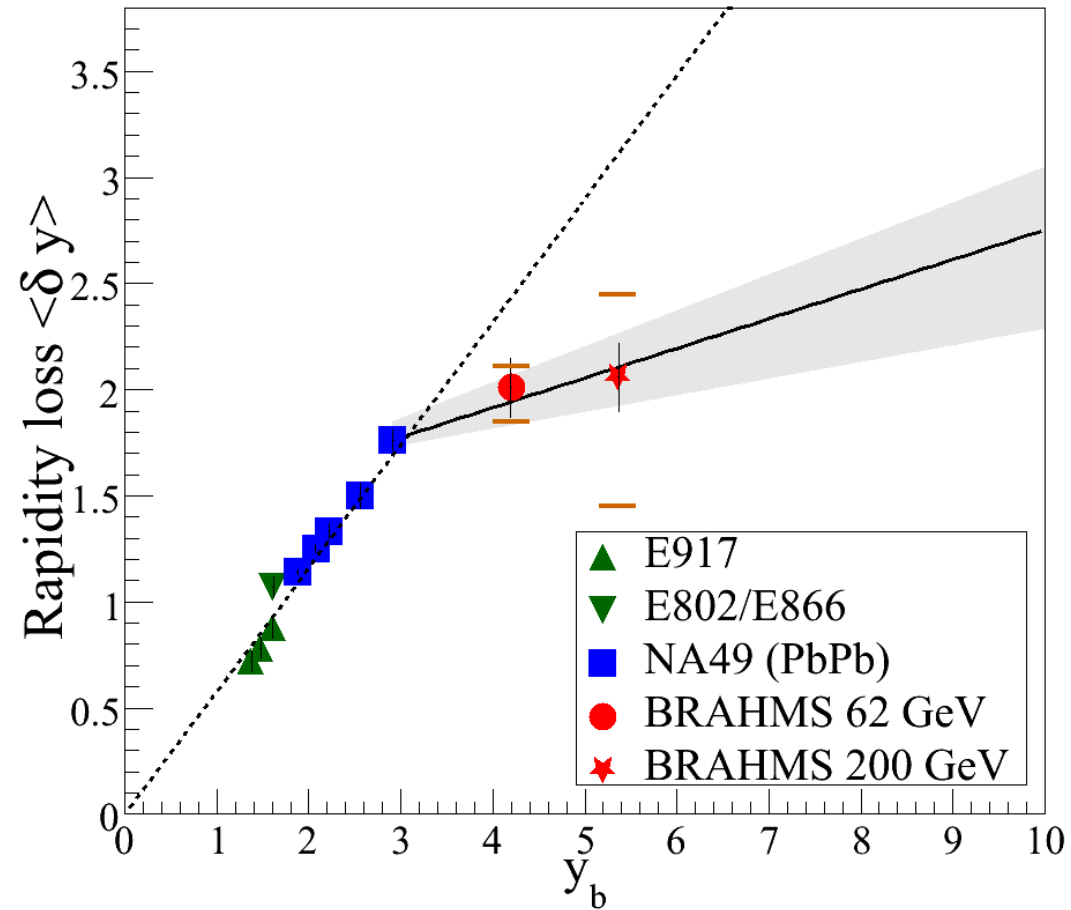
$$\langle \delta y_p \rangle = \frac{\int_{-\infty}^0 |y_p - y_{t(b)}| (dN_p / dy) dy}{\int_{-\infty}^{\infty} (dN_p / dy) dy}$$

# Excitation function of stopping

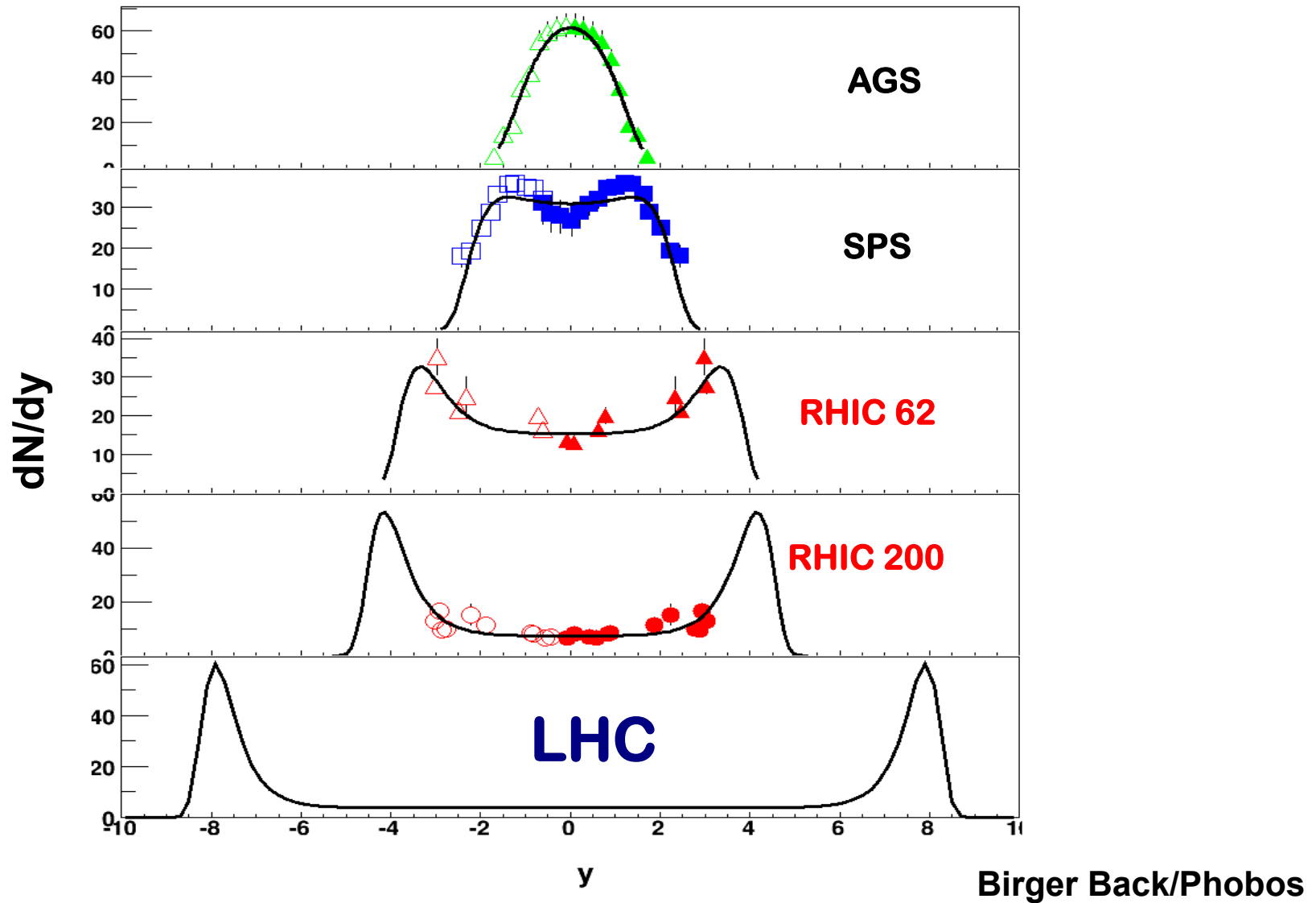


N.Herrmann,  
J.P. Wessels,  
T.Wienold,  
Ann.Rev.Nucl.Part.  
Sci.49,581 (1999)

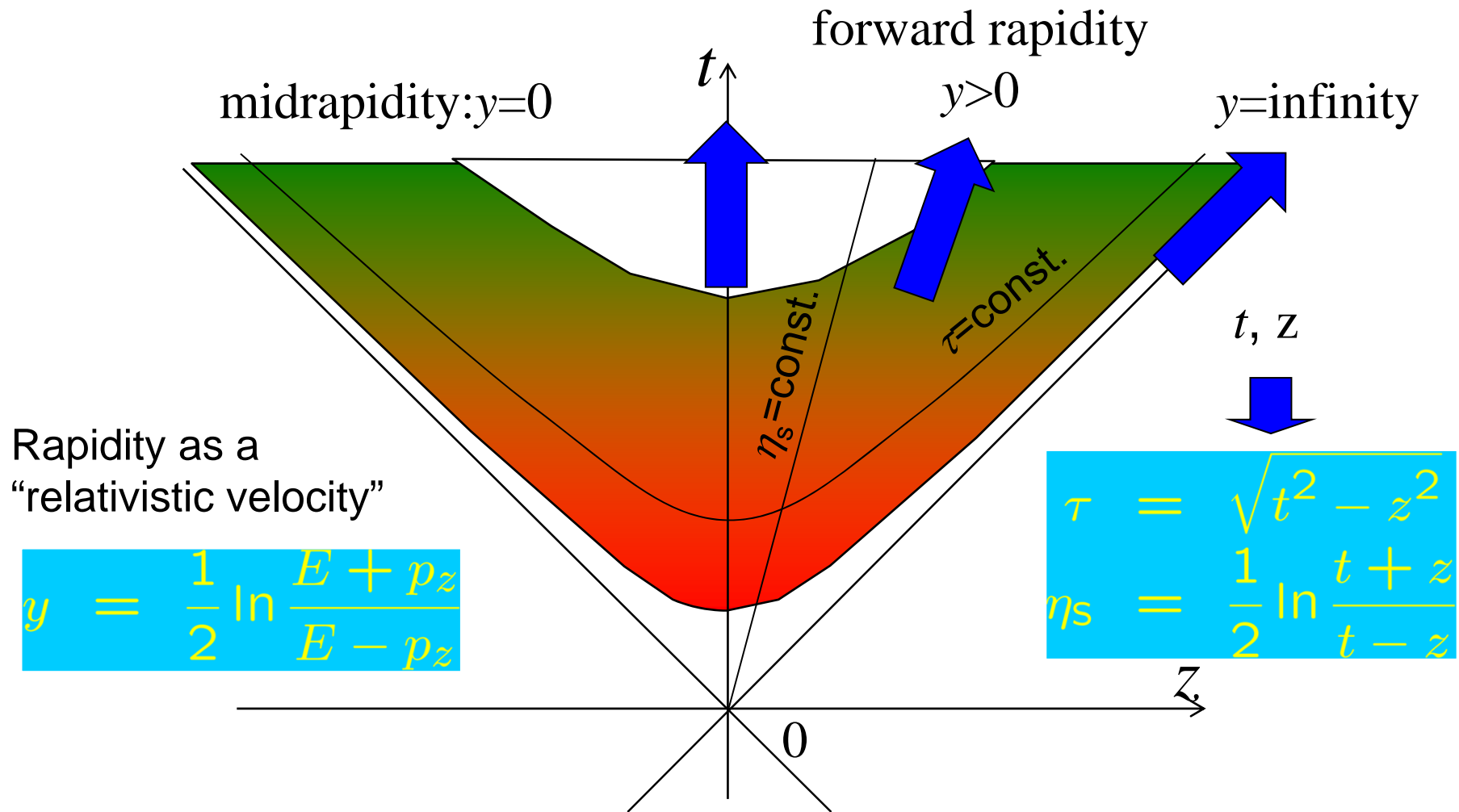
# Brahms – rapidity loss



# Evolution of rapidity distribution of protons



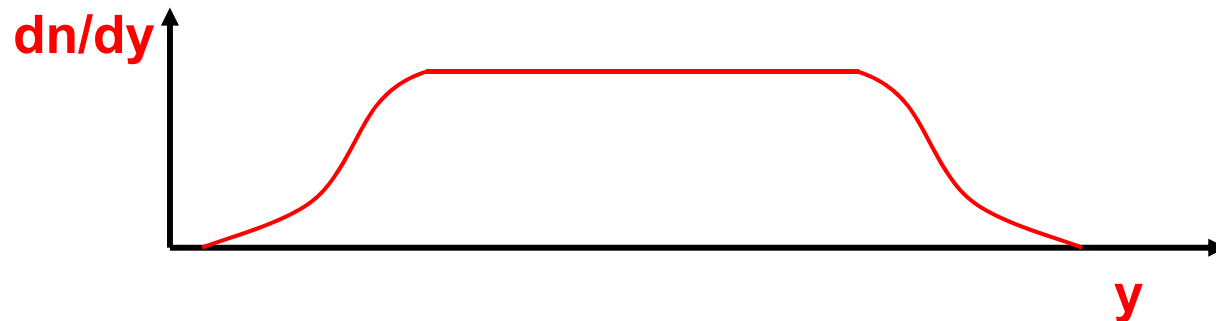
# Rapidity and Boost invariant expansion



Boost invariant ansatz Bjorken ('83)  
 → Dynamics depends on  $\tau$ , not on  $\eta_s$ .

## Bjorken Scaling

Bjorken Ansatz: “..... at sufficient high energy there is a ‘central-plateau’ structure for the particle production as a function of the rapidity variable.”



Physics must be invariant under Lorentz-boost:

1) Local thermodynamic quantity must be a function of proper time  $\tau = \sqrt{t^2 - z^2}$  only.

2) Longitudinal velocity

$$v_z = z/t \quad \text{or} \quad y = 0.5 \ln ((t+z)/(t-z))$$



$$\text{Energy density } \varepsilon = \frac{E \times \Delta N}{A \times \Delta z}$$

$E \rightarrow$  average energy per particle

$A \rightarrow$  transverse area of the collision volume

$\Delta z \rightarrow$  longitudinal interval

$\Delta N \rightarrow$  number of particles in  $\Delta z$  interval

$$v_z = z/t = \tanh y; \quad z = \tau \sinh y$$

$$\Delta z = \tau \cosh y \Delta y$$

$$E = m_T \cosh y$$

$$\varepsilon = \frac{m_T \cosh y \Delta N}{A \tau \cosh y \Delta y} \rightarrow \frac{m_T dn/dy}{A \tau}$$

# Energy and Baryon density density

Assumption: Bjorken expansion (homologous 1 +1 D expansion)

## Estimate of energy density

J.D. Bjorken, PRD 27,140 (1983)

$$\varepsilon = \frac{1}{A_T} \cdot \frac{dE_T}{d\eta} \cdot \frac{d\eta}{dz} = \frac{1}{A_T} \cdot \frac{dE_T}{d\eta} \cdot \frac{1}{\tau_0}$$

$$\rho_B = \frac{1}{A_T} \cdot \frac{dN_B}{dy} \cdot \frac{dy}{dz} = \frac{1}{A_T} \cdot \frac{dN_B}{dy} \cdot \frac{1}{\tau_0}$$

Initial time

$$\tau_0 = 0.2 - 1 \text{ fm/c}$$

Note:

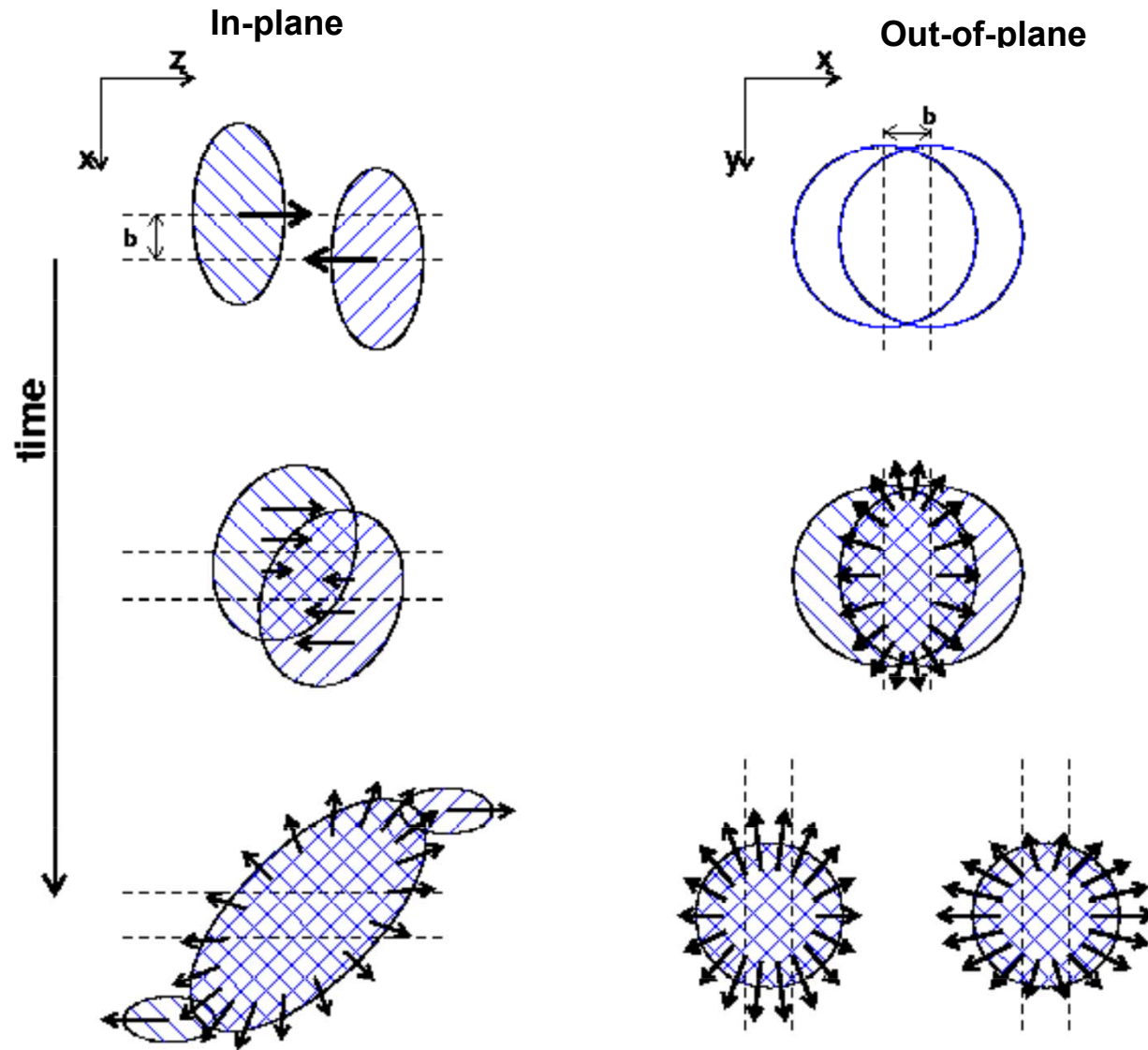
(from Lattice – QCD)

$$\varepsilon_{crit}^{QGP} \Big|_{\mu_B=0} = 0.8 \text{ GeV/fm}^3$$

Numerical estimates:

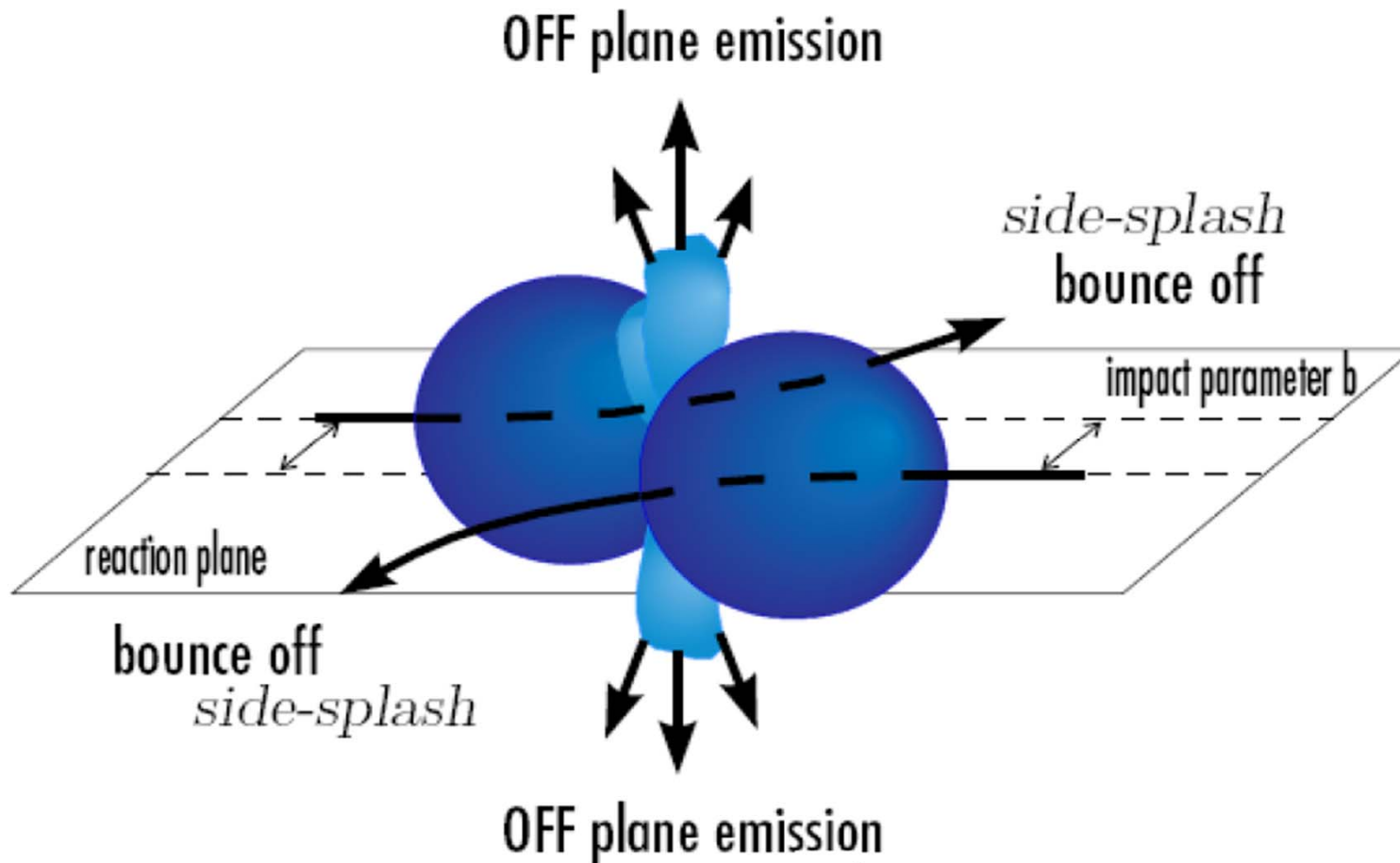
	SIS	AGS	SPS
$\varepsilon(\text{GeV/fm}^3)$	0.5	1.3	3
$\rho(1/\text{fm}^3)$	0.35	1.1	0.65

# Collective Flow



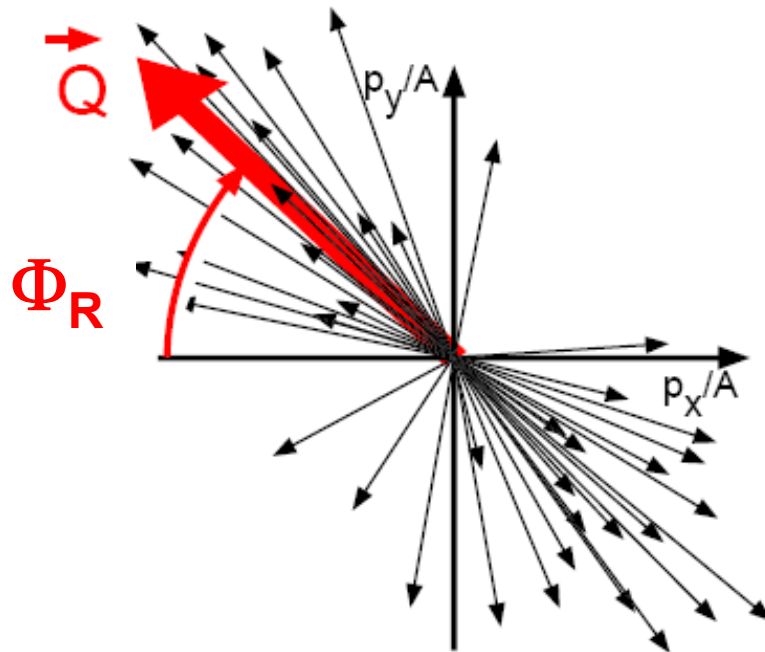
N.Herrmann,  
 J.P. Wessels,  
 T.Wienold,  
 Ann.Rev.Nucl.Part.  
 Sci.49,581 (1999)

## Flow in heavy ion collisions



# Reaction Plane

Transverse momentum method: P. Danielewicz, G. Odyniec, *Phys. Lett.* 157B, 146 (1985)



$$\vec{Q} = \sum_i \omega(v) \cdot \vec{p}_t(v),$$

$$\omega(v) = \begin{cases} 1 & y(v) > y_{CM} \\ -1 & y(v) < y_{CM} \end{cases}$$

**Generalisation:**

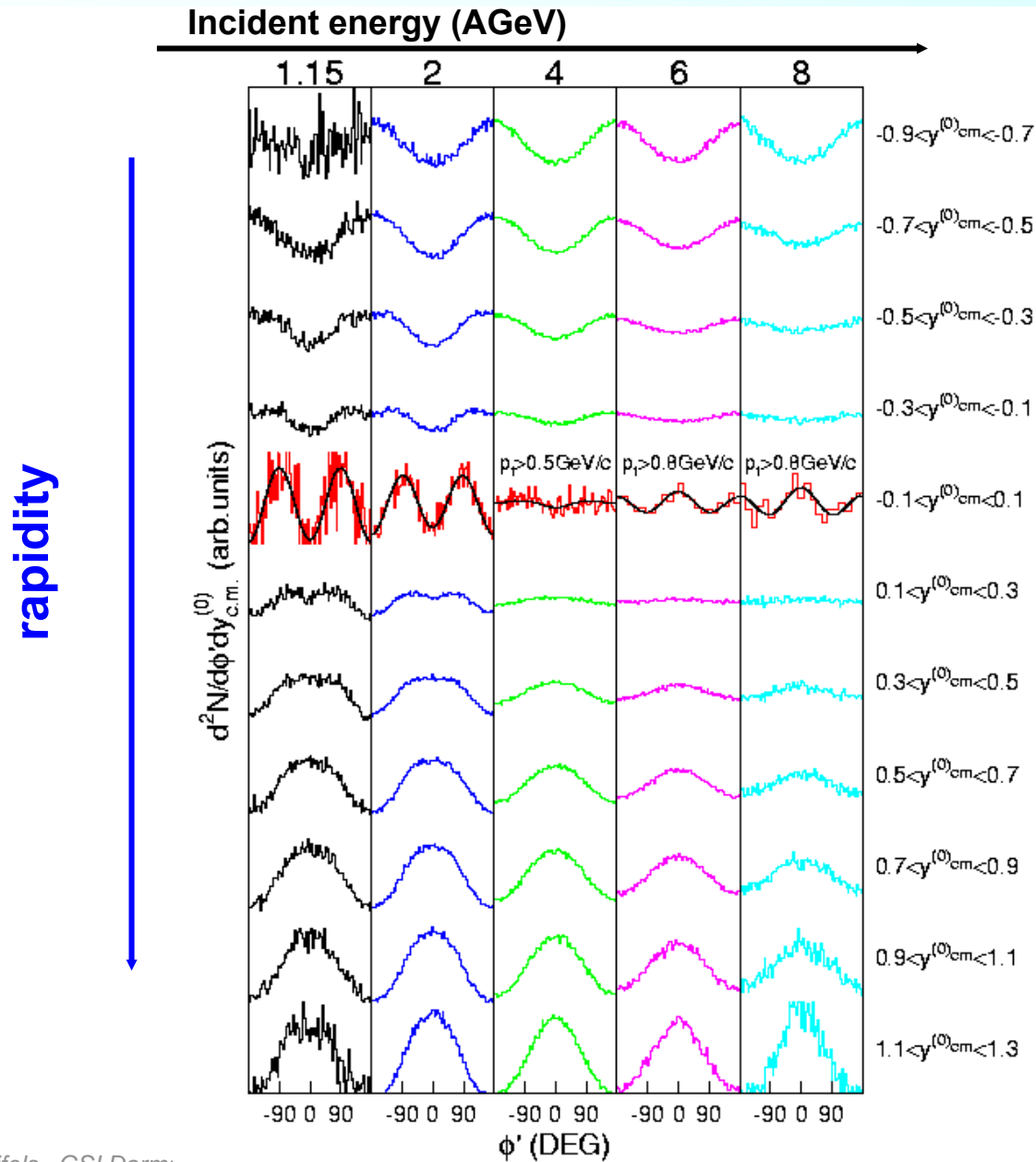
$$\vec{Q}^{(n)} = \sum_i \omega^{(n)} \cdot |\vec{p}_t| \cdot \begin{pmatrix} \cos(n \cdot \varphi) \\ \sin(n \cdot \varphi) \end{pmatrix}, \quad n = 1, 2, 3, \dots$$

$\omega^{(n)}$  has different sign in forward/backward hemisphere for odd values of  $n$ .

**Reaction plane angle:**

$$\Phi_R^{(n)} = \arctan(Q_y^{(n)}, Q_x^{(n)}) / n$$

# Azimuthal distributions



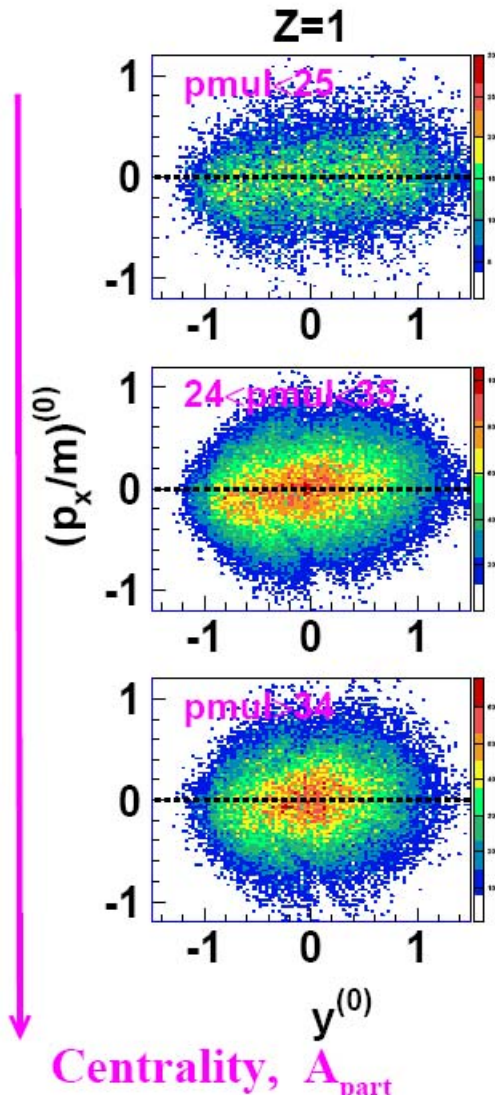
Azimuthal distributions  
with respect to  
reaction plane

Excitation function of  
Au+Au reactions

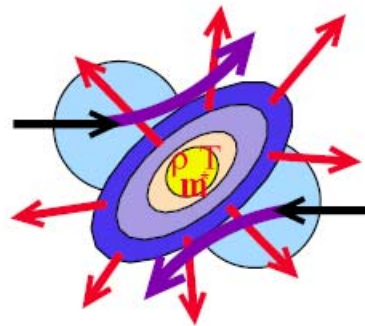
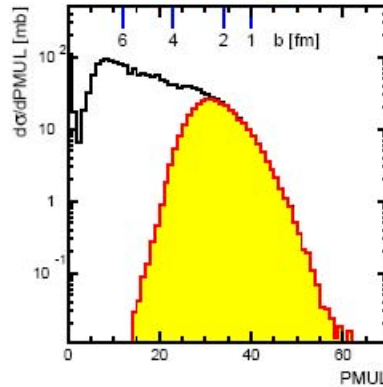
C.Pinkenburg et al., (E895),  
Phys.Rev.Lett. 83 (1999) 1295  
*nucl-ex/9903010*

# Flow observables (history)

Projections onto  
Reactionplane



Multiplicity



Integral sideflow

$$p_x^{dir} = \sum_{\nu} \omega \cdot \vec{p}_t(\nu) \cdot \vec{Q} / |\vec{Q}|$$

(Classical) 'Sideflow'  
(slope of mean  $p_x$  at midrapidity)

$$F_y = \frac{d\langle p_x \rangle / A}{dy}$$

Transverse momentum tensor

$$F_{ij} = \sum_{\nu} p_i(\nu) p_j(\nu) / 2m_{\nu},$$

$$i, j = x, y, z$$

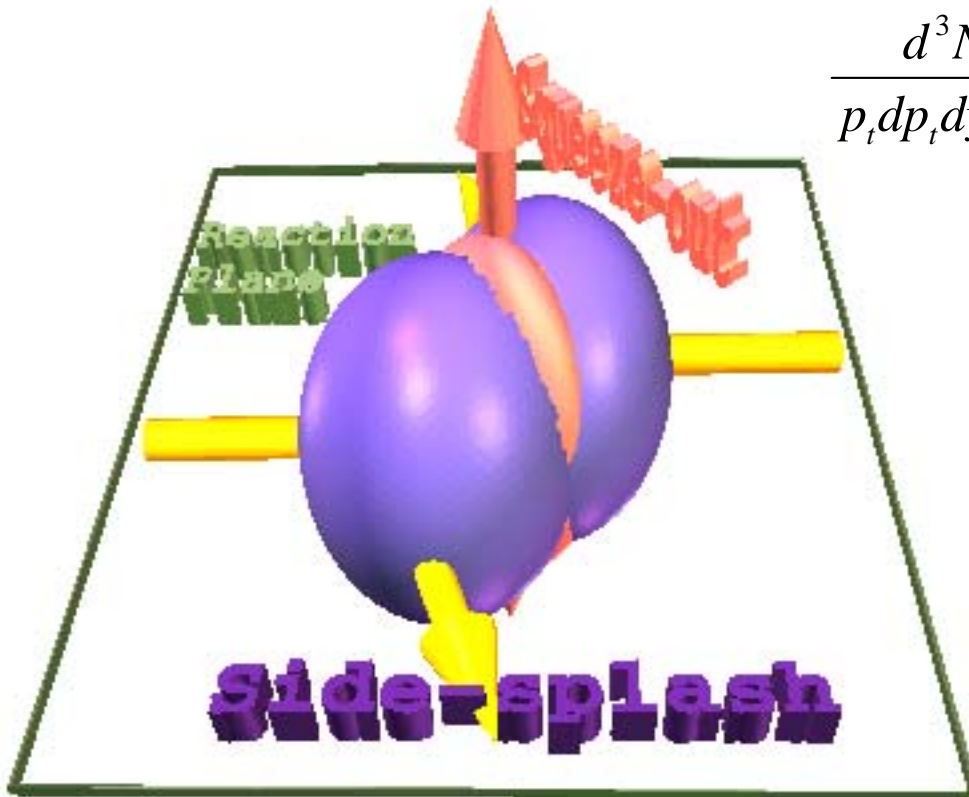
## 5.2.3

# Fourier Expansion of Azimuthal Distributions

Phase space distribution with respect to reaction plane  $\Phi_r$

$$\varphi' := \varphi - \Phi_R$$

$$\frac{d^3 N}{p_t dp_t dy d\varphi'} \propto (1 + 2v_1 \cos(\varphi') + 2v_2 \cos(2\varphi') + \dots)$$



Fourier expansion coefficients

$$v_1 = \left\langle \frac{p_x}{p_t} \right\rangle$$

sideflow

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

elliptic flow

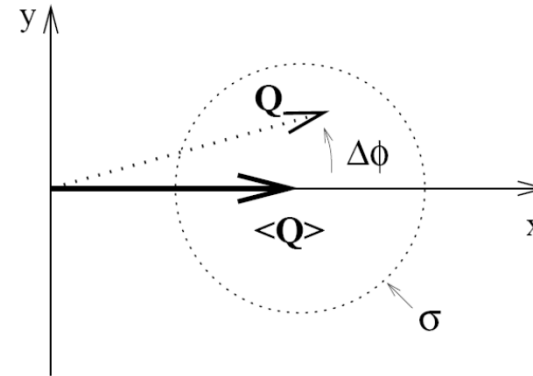
S. Voloshin, Y. Zhang, *hep-ph/9407082*

J.Y. Ollitrault, *nucl-ex/9711003*



# Reaction plane resolution

Reconstructed reaction plane is fluctuating around true reaction plane  
 => measured  $v_i$  are smaller than true  $v_i$ .



Subevent method:

$$\langle \cos(n(\Phi_A^{(n)} - \Phi_B^{(n)})) \rangle = \langle \cos(n(\Phi_A^{(n)} - \Phi_R)) \rangle \cdot \langle \cos(n(\Phi_B^{(n)} - \Phi_R)) \rangle$$

Estimate for correction factors of Fourier expansion coefficients:

$$v_m = v_m^{obs} / \sqrt{\langle \cos(n(\Phi_A^{(n)} - \Phi_B^{(n)})) \rangle}$$

Quantitative correction: J.Y. Ollitrault, arXiv:nucl-ex/9711003

# Ollitrault Formalism

## Determination of Fourier coefficients $v_i$ :

- 1) Fit of azimuthal distributions
- 2)  $v_i = \langle \cos n \Delta\phi \rangle$

J.Y. Ollitrault, arXiv:nucl-ex/9711003

Eq.(6) can be easily integrated over  $Q$  [21] to yield the distribution of  $\Delta\phi$ :

$$\frac{dN}{\Delta\phi} = \frac{1}{\pi} \exp(-\chi^2) \left\{ 1 + z\sqrt{\pi} [1 + \text{erf}(z)] \exp(z^2) \right\}. \quad (7)$$

where  $z = \chi \cos \Delta\phi$  and  $\text{erf}(x)$  is the error function. This distribution depends on  $\bar{Q}$  and  $\sigma$  only through the dimensionless parameter  $\chi \equiv \bar{Q}/\sigma$ . The Fourier coefficients are most easily calculated by integrating Eq.(6) first over  $\Delta\phi$  and then over  $Q$  [14]:

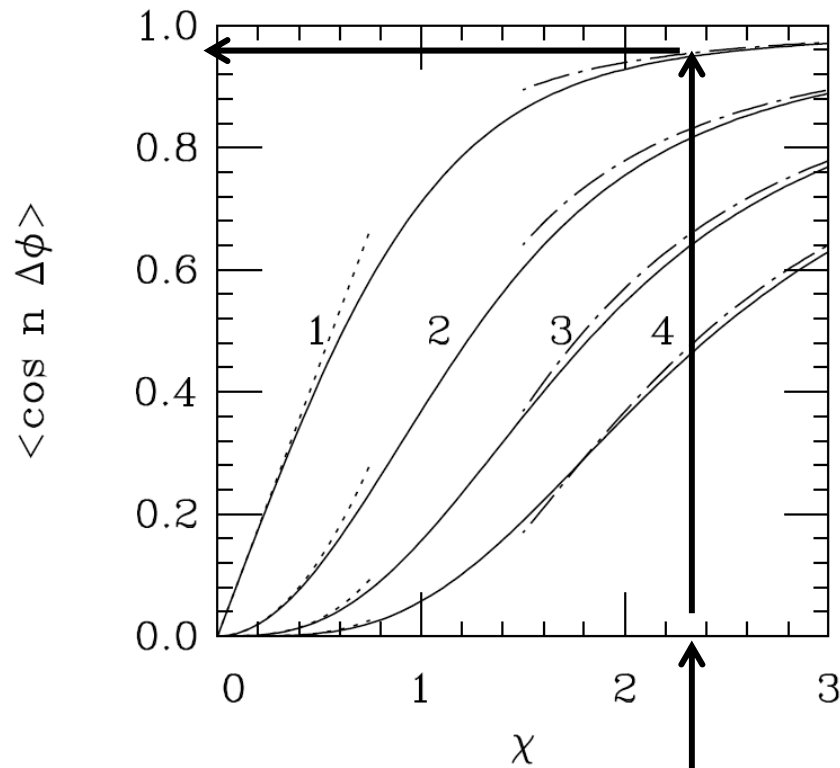
$$\langle \cos n\Delta\phi \rangle = \frac{\sqrt{\pi}}{2} \chi e^{-\chi^2/2} \left[ I_{\frac{n-1}{2}} \left( \frac{\chi^2}{2} \right) + I_{\frac{n+1}{2}} \left( \frac{\chi^2}{2} \right) \right] \quad (8)$$

where  $I_k$  is the modified Bessel function of order  $k$ . The variations of the first coefficients

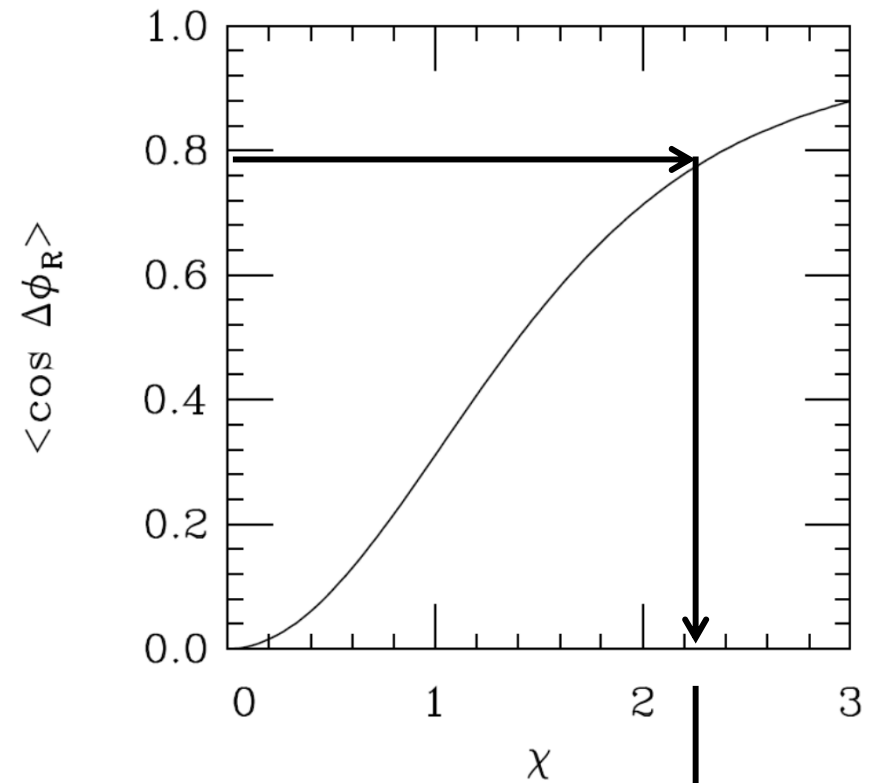
**Fourier coefficients  $v_i$  can be corrected consistently by evaluating dimensionless parameter  $\chi$ !**

# Ollitrault formalism

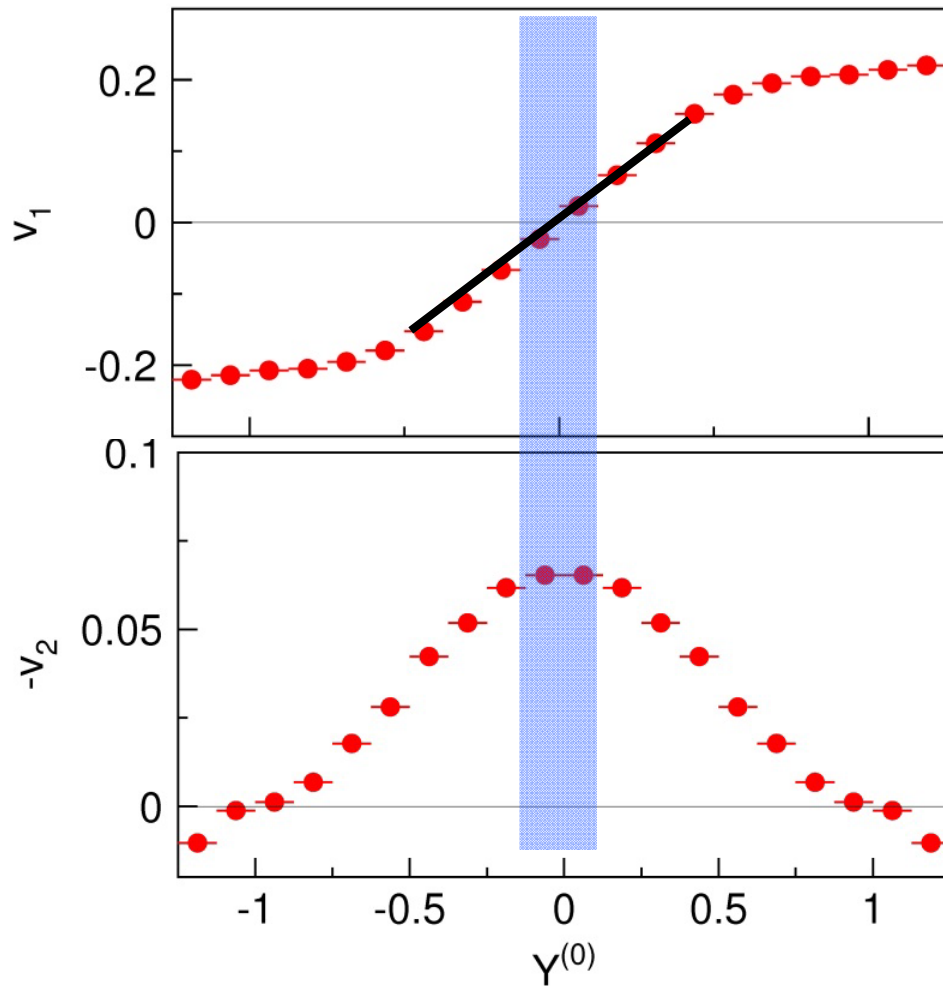
## Inverse correction factors



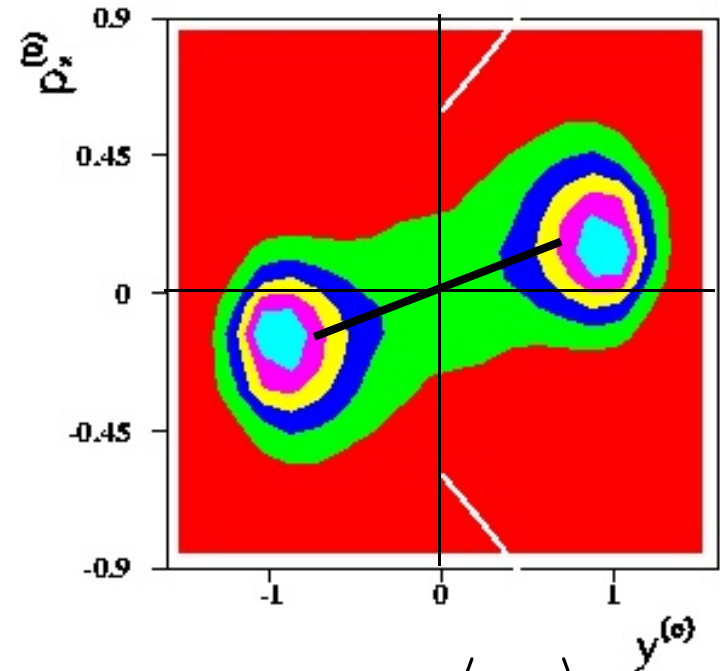
## Determination of $\chi$



# v1 and v2 as a function of rapidity



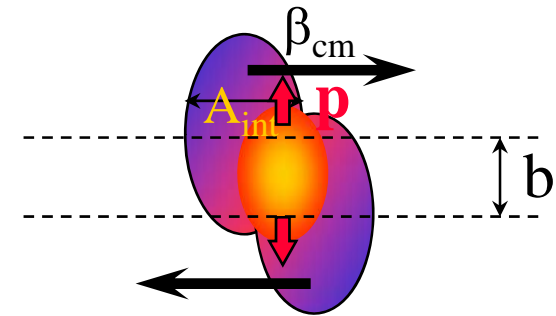
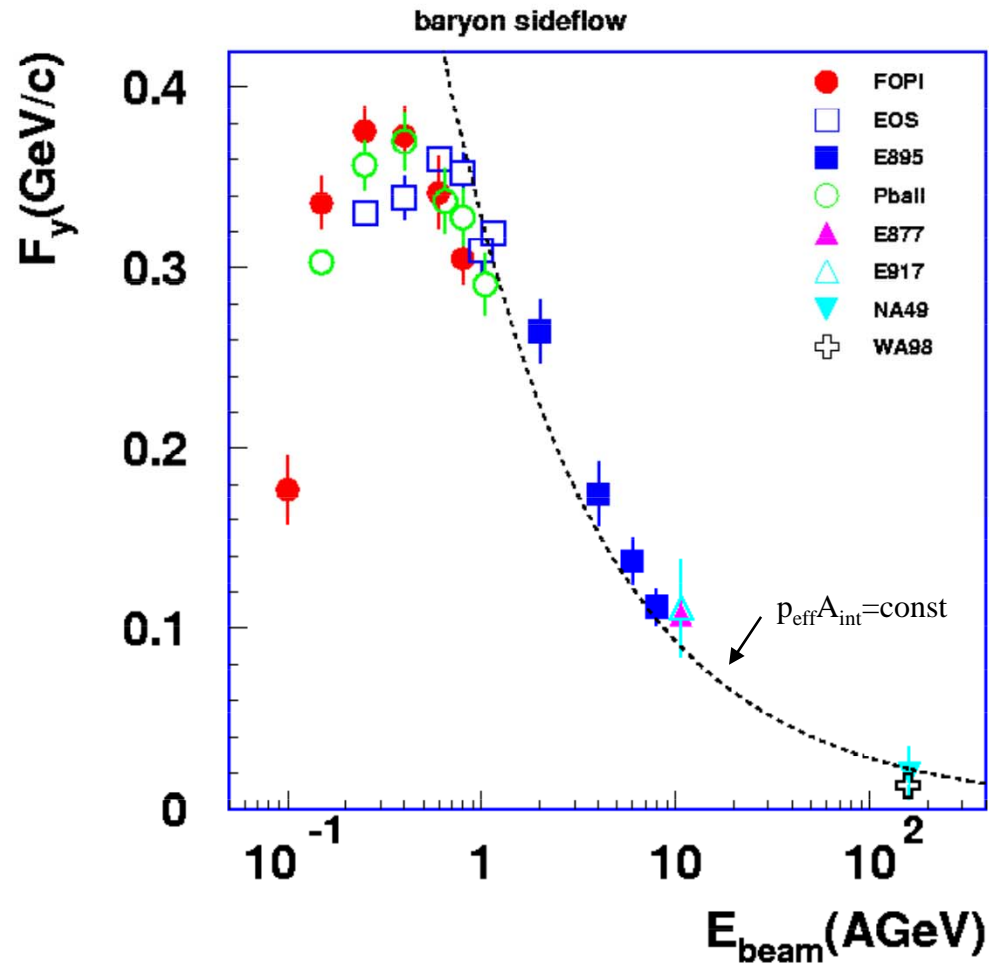
$p_x$  transverse momentum  
projected onto reaction plane



$$F_y = \frac{d\langle p_x \rangle}{dy}$$

~ to slope at v1 at  
mid-rapidity

# Sideflow excitation function

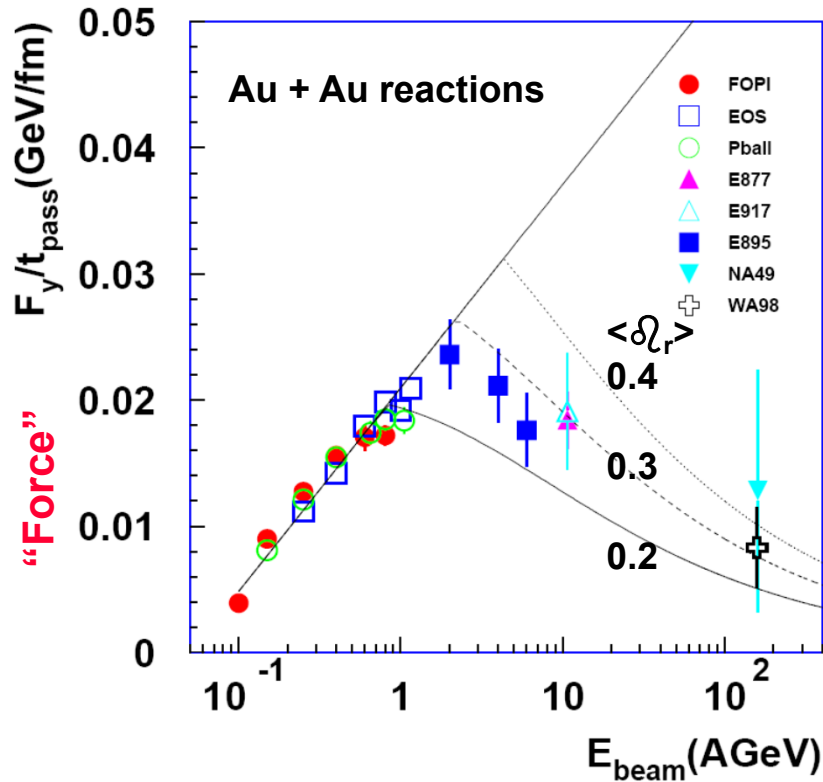


$$F_y \propto \int F dt \approx p_{\text{eff}} A_{\text{int}} t_{\text{pass}}$$

$$t_{\text{pass}} = \frac{2R}{\gamma_{\text{CM}}} \cdot \frac{1}{\beta_{\text{CM}}}$$

### 5.2.3

## Scaling properties of sideflow



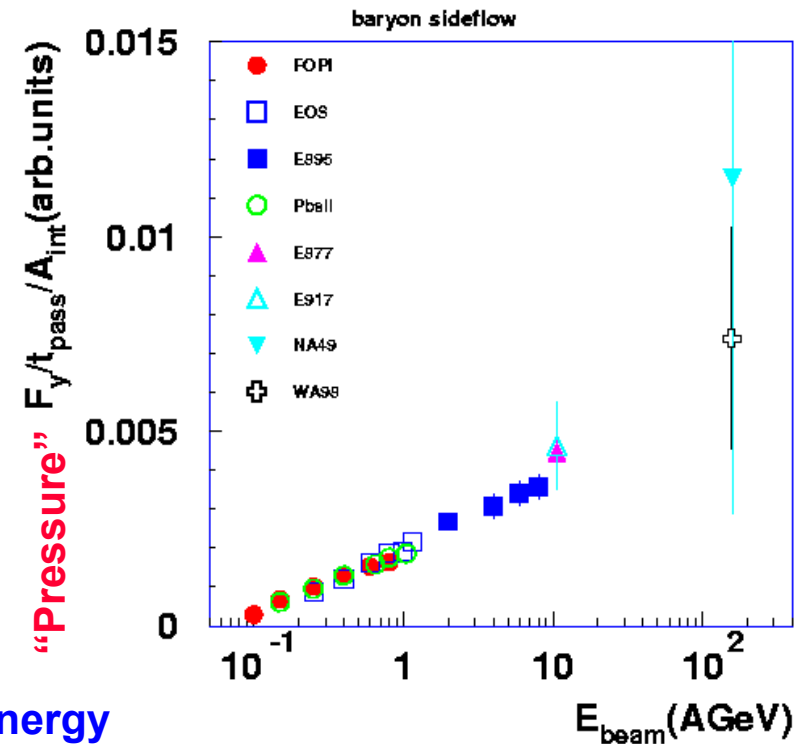
Smooth evolution of pressure with incident energy

$$F_y \propto \int F dt \approx p_{eff} A_{int} t_{pass}$$

$$t_{pass} = \frac{2R}{\gamma_{CM}} \cdot \frac{1}{\beta_{CM}}$$

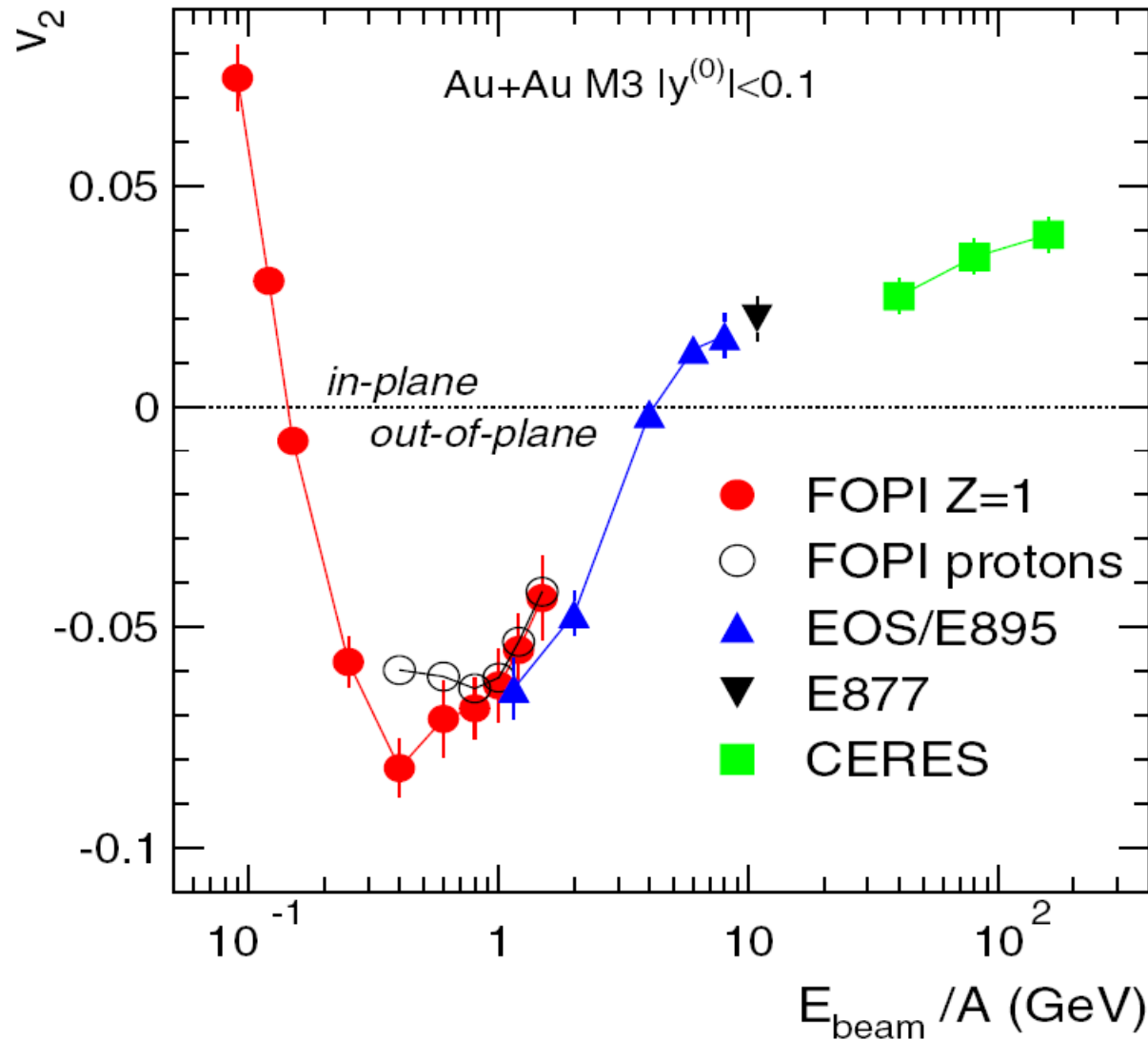
$$A_{int} = \frac{A(b)}{\gamma_{CM}}$$

$$p_{eff} \propto \frac{F_y}{A_{int} t_{pass}} = \frac{\gamma_{CM}^2 \beta_{CM}}{A(b) \cdot 2R} \cdot F_y$$

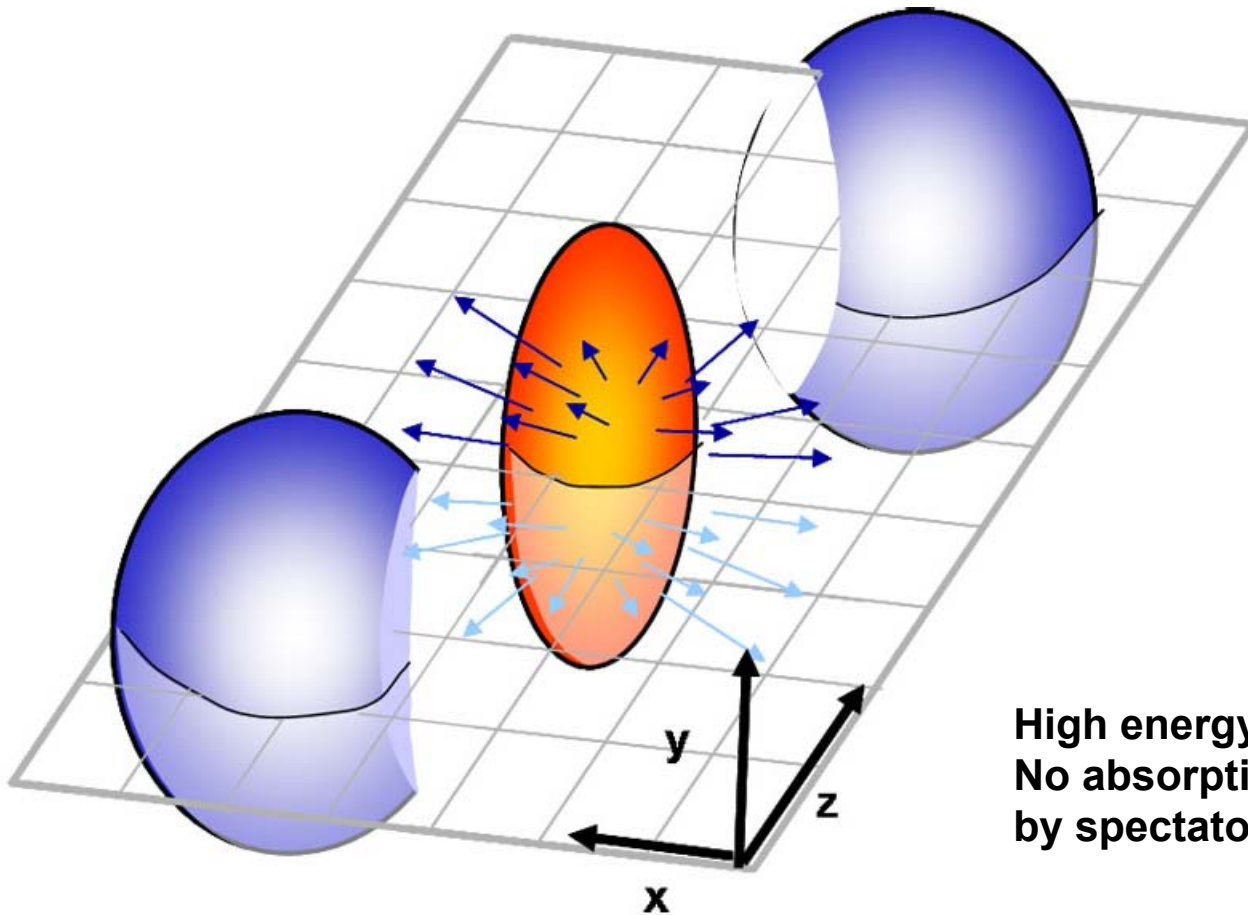


# Excitation function for elliptic flow

A.Andronic et al. (FOPI), PLB 612, 173 (2005)



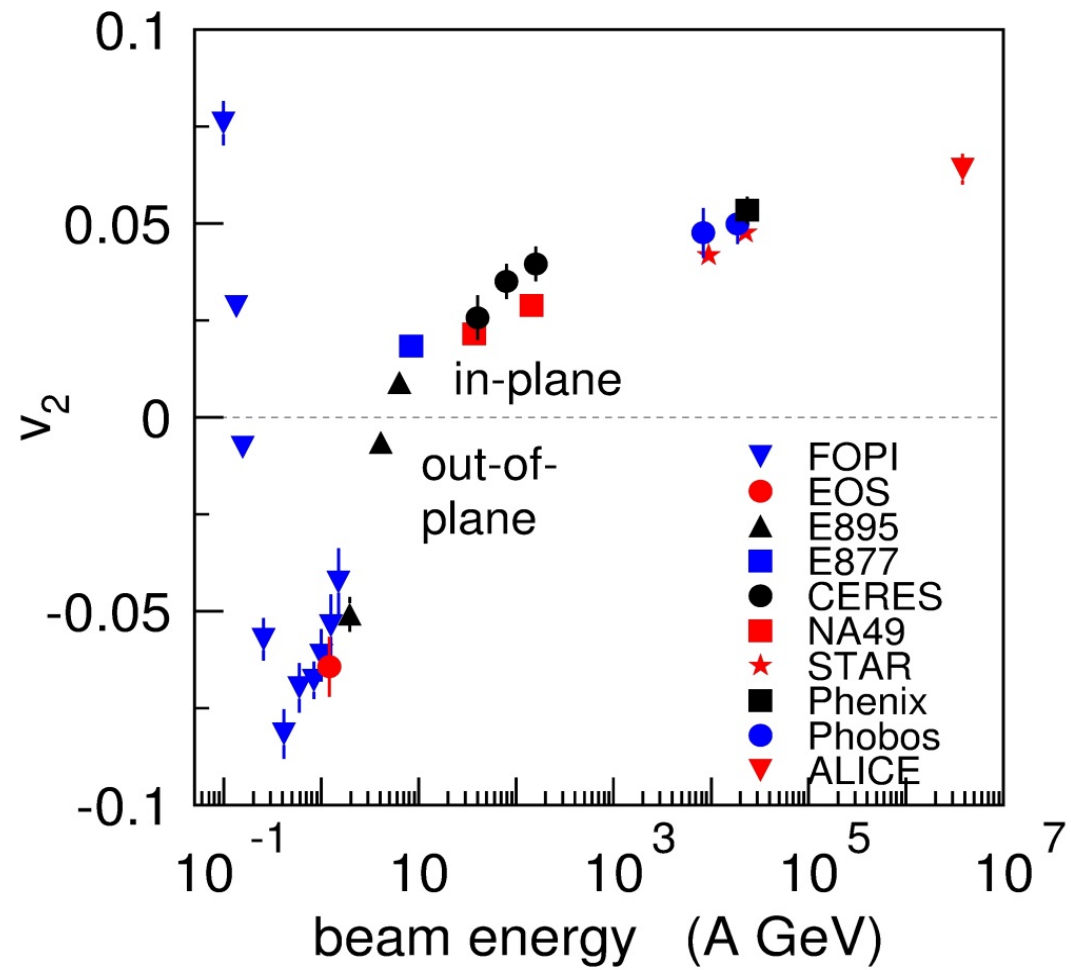
## Elliptic in-plane flow



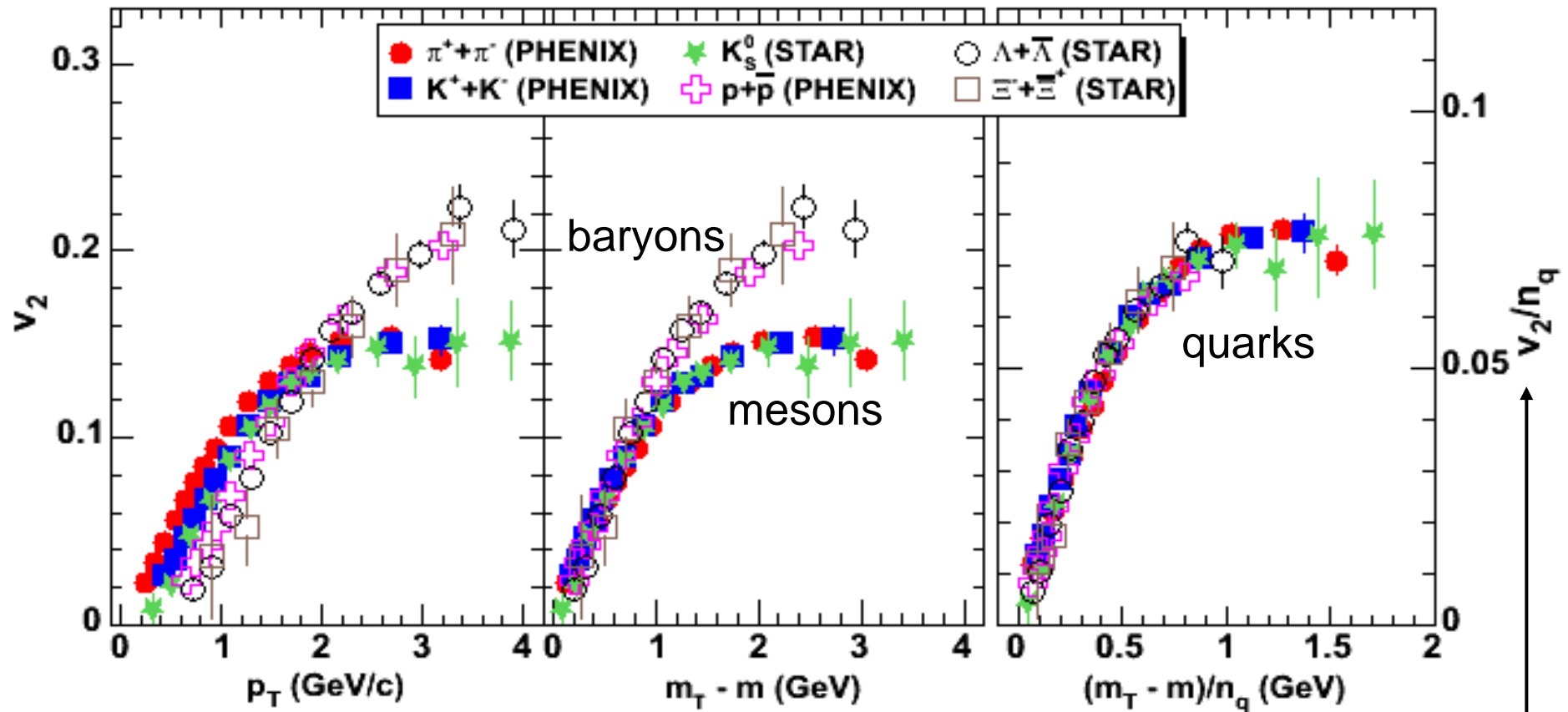
**High energy:  
No absorption of fireball particles  
by spectators**



# Excitation function of elliptic flow



# Scaling with Number of Quarks @ 200A GeV



**quarks have  $v_2$  before hadronization**

both axes scaled by number of constituent quarks

$n_q = 2$  for mesons

$n_q = 3$  for baryons

S. Voloshin, QM02, 379c (2003)

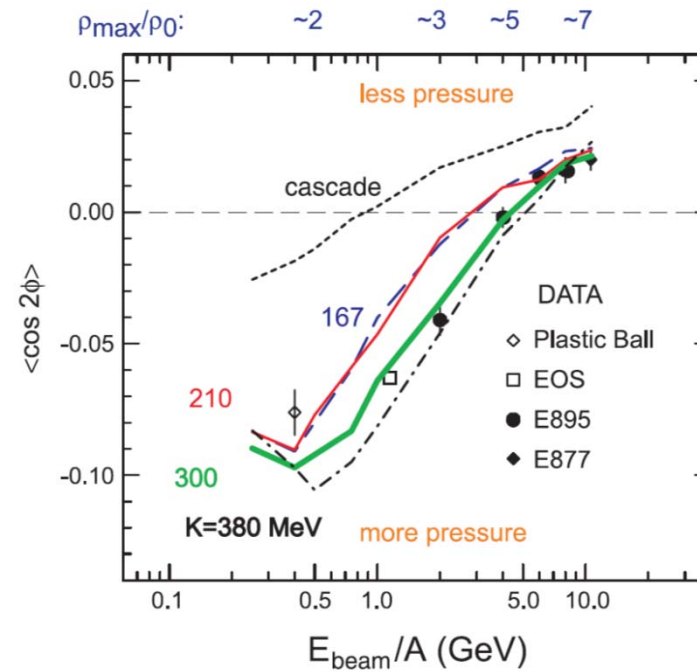
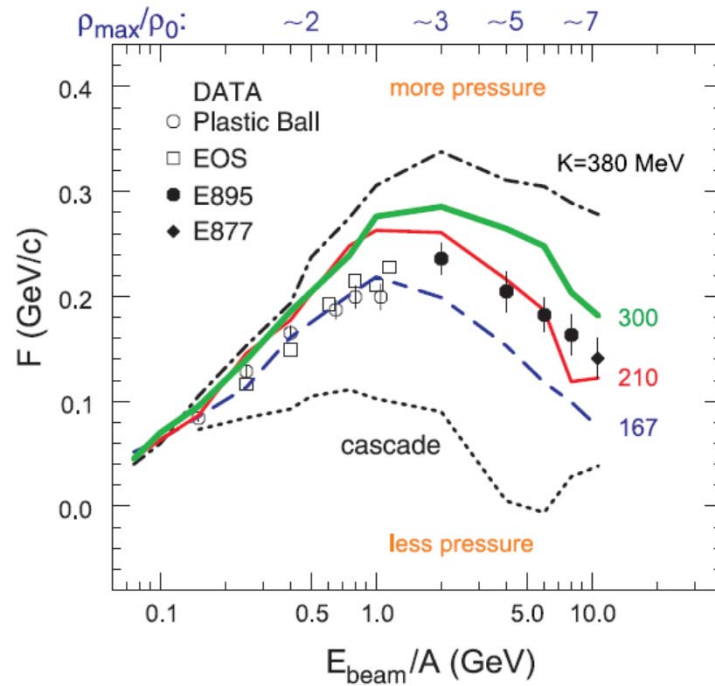
STAR, PRL **95**, 122301 (2005) PHENIX, PRL **98**, 162301 (2007)

5.2.3

# Excitation function of flow variables

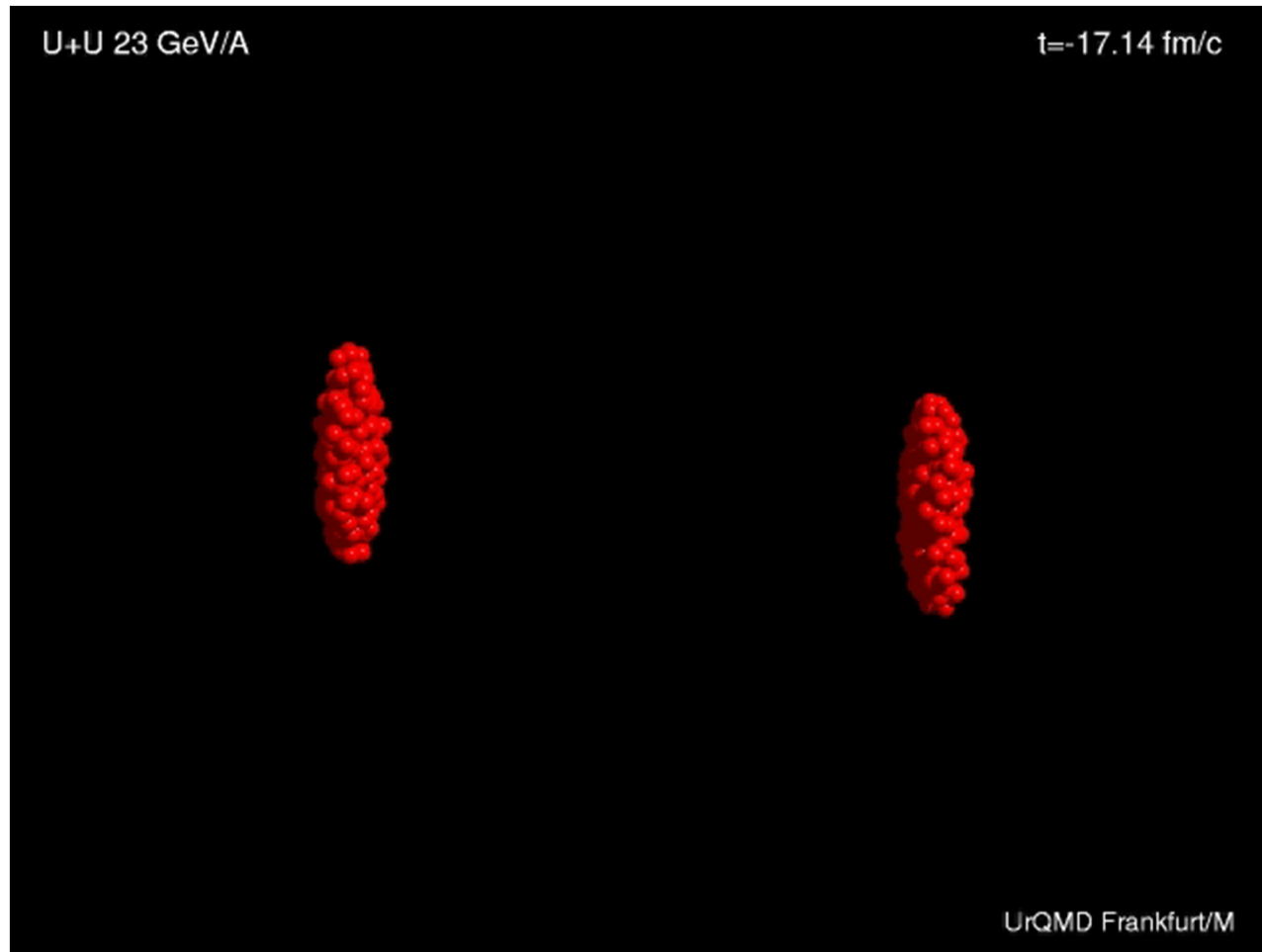
$$F = \frac{d\langle p_x / A \rangle}{d(y / y_{cm})}$$

P. Danielewicz et al.  
Science 298, 1592 (2002)

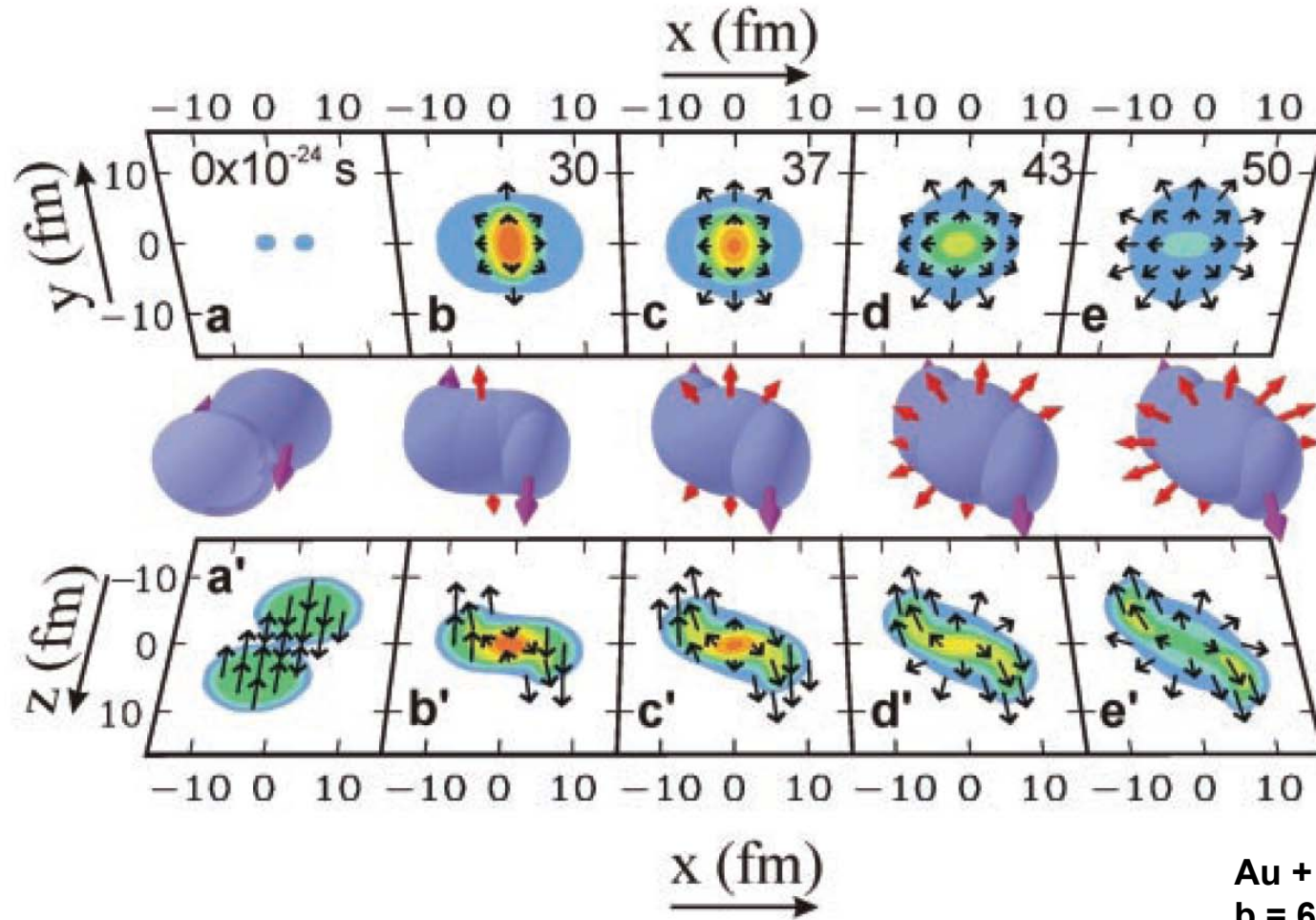


(depending on equation of state)

# Movie of a heavy ion collision



# Heavy-ion collisions



**Au + Au**  
**b = 6fm**

P. Danielewicz et al.  
 Science 298, 1592 (2002)

# Models for heavy ion collisions

