Higgs Properties II

Higgs couplings can be read off from gauge boson and fermion mass terms

$$\mathcal{L}_{\text{Higgs}} \supset D_\mu \Phi (D^\mu \Phi)^\dagger \ni M_V^2 \left(1 + \frac{h}{v}\right)^2$$

$$\mathcal{L}_{Yuk} = - y_e \bar{L} \Phi e_R - y_d \bar{Q} \Phi d_R - y_u \bar{Q} \tilde{\Phi} u_R + h.c.$$

$$\supset - m_f \left(1 + \frac{h}{v}\right)$$

in which

$$M_W = \frac{1}{2} v \cdot g \,, \quad M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2} \,, \quad m_f = \frac{y_f \cdot v}{\sqrt{2}}$$

Further, the Higgs potential gives rise to Higgs self couplings

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

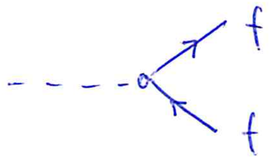
$$= \frac{\mu^2}{2} (0, v+h) \begin{pmatrix} 0 \\ v+h \end{pmatrix} + \frac{\lambda}{4} \left| (0, v+h) \begin{pmatrix} 0 \\ v+h \end{pmatrix} \right|^2$$

$$= -\frac{1}{2} \lambda v^2 (v+h)^2 + \frac{1}{4} \lambda (v+h)^4 \ni \lambda v h^3 + \frac{\lambda}{4} h^4$$

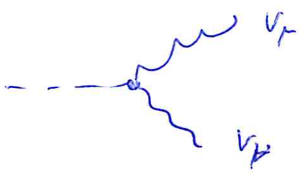
Minimization condition $v^2 = -\frac{\mu^2}{\lambda}$

$$, \quad M_h^2 = 2 \lambda v^2 = -2\mu^2$$

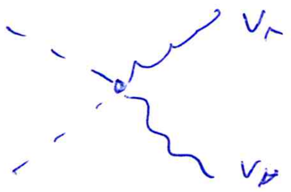
This gives rise to the Feynman rules



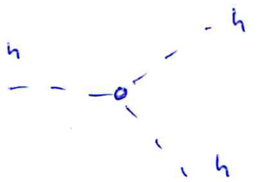
$$g_{hff} = \frac{m_f}{v} \cdot i$$



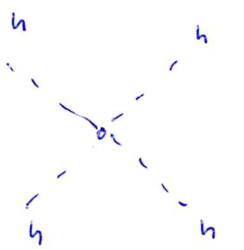
$$g_{hVV} = \frac{2 M_V^2}{v} \quad (-i g_{\mu\nu})$$



$$g_{hhVV} = \frac{2! M_V^2}{v} \, (-i g_{\mu\nu})$$



$$g_{hhh} = 3! \, \lambda v \cdot i = \frac{3 M_h^2}{v} \cdot i$$



$$g_{hhhh} = 4! \, \frac{\lambda}{4} \cdot i = \frac{3 M_h^2}{v^2} \cdot i$$

using that

$$\Gamma = \frac{1}{16\pi} |M|^2 \frac{1}{M_h} (1 - \frac{4M_x^2}{M_h^2})^{1/2}$$

and

$$M(h \to f\bar{f}) = -\frac{m_f}{v} \delta^{ab} \bar{u}^{s_1}(p_1) v^{s_2}(p_2)$$

$$|M|^2 = \left(\frac{m_f}{v}\right)^2 \sum_{a,b} \delta^{ab} \sum_{s_1, s_2} u^{s_1} \bar{u}^{s_1} v^{s_2} \bar{v}^{s_2}$$

$$= \left(\frac{m_f}{v}\right)^2 N_c \, \mathrm{Tr} \{ (\not{p}_1 + m_1)(\not{p}_2 - m_2) \}$$

$$= \left(\frac{m_f}{v}\right)^2 N_c \; 4 \, (p_1 p_2 - m^2)$$

$$= \left(\frac{m_f}{v}\right)^2 N_c \; 4 \left(\frac{M_h^2}{2} - \frac{4m^2}{2} \right)$$

$$= \left(\frac{m_f}{v}\right)^2 2 M_h^2 N_c \left(1 - \frac{4m^2}{M_h^2}\right)$$

and

$$M(h \to VV) = \frac{2M_V^2}{v} \epsilon_\mu^i \epsilon_\nu^j$$

$$\sum_{i,j} |M|^2 = \frac{4M_V^4}{v^2} \left(-g^{\mu\nu} + \frac{p_1^\mu p_1^\nu}{M_V^2}\right)\left(-g_{\mu\nu} + \frac{p_{2\mu} p_{2\nu}}{M_V^2}\right)$$

$$= \frac{4M_V^4}{v^2} \left(2 + \frac{(p_1 \cdot p_2)^2}{M_V^2}\right) = \frac{4M_V^4}{v^2} \left(3 + \frac{1}{4}\frac{M_h^4}{M_V^4} - \frac{M_h^2}{M_V^2}\right)$$

These couplings determine the branching ratios of the Higgs (depending on its mass):

For fermions

$$\Gamma(h \to \bar{q} q) = \frac{1}{8\pi} N_c M_h \left(\frac{m_q}{v}\right)^2 \left(1 - \frac{4 m_q^2}{M_h^2}\right)^{3/2}$$

$$\Gamma(h \to \ell^+ \ell^-) = \frac{1}{8\pi} M_h \left(\frac{m_\ell}{v}\right)^2 \left(1 - \frac{4 m_\ell^2}{M_h^2}\right)^{3/2}$$

and for gauge bosons

$$\Gamma(h \to W^+ W^-) = \frac{1}{16\pi} \left(\frac{M_W^2}{v}\right)^2 \frac{M_h^3}{M_W^4} (1-4x)^{1/2} (1 - 4x + 12x^2)$$

$$\Gamma(h \to ZZ) = \frac{1}{32\pi} \left(\frac{M_Z^2}{v}\right)^2 \frac{M_h^3}{M_Z^4} (1-4x)^{1/2} (1 - 4x + 12x^2)$$

with $x = \frac{M_V^2}{M_h^2}$. For heavy Higgses, $x \to 0$

and

$$\frac{\Gamma(h \to W^+ W^-)}{\Gamma(h \to ZZ)} \longrightarrow 2$$

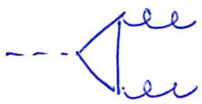
The reason is, that the longitudinal polarization ~~dominates~~ of the final states dominate in this limit

$$\frac{\Gamma_L}{\Gamma_L + \Gamma_T} = \frac{1 - 4x + 4x^2}{1 - 4x + 12x^2} \xrightarrow{x \to 0} 1$$

The Higgs does not couple "directly" to massless fields. Quantum effects (loops) induce such decays

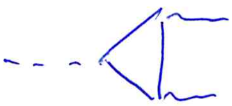
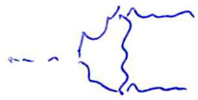
Decay into gluons

$$\Gamma(h \to gg) = \frac{\alpha_s^2}{72\pi^3} \frac{M_h^3}{v^2} \left| \frac{3}{4} \sum_q A_{1/2}(\tau_q) \right|^2$$



in photons

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2}{256\pi^3} \frac{M_h^3}{v^2} \left| \sum_f N_c Q_f^2 A_{1/2}(\tau_f) + A_1(\tau_w) \right|^2$$

 + 

$$\Gamma(h \to Z\gamma) = \frac{\alpha}{128\pi^4} \frac{M_W^2 M_h^3}{v^4} \left| \sum_f N_f \frac{Q_f}{c_W} g_Z^v A_{1/2}(\tau_f, \lambda_f) + A_1(\tau_w, \lambda_w) \right|^2$$

$$\times \left(1 - \frac{M_Z^2}{M_h^2}\right)^3$$

with

$$A_{1/2}(\tau) = 2\left[\tau + (\tau - 1) f(\tau)\right] \tau^{-2}$$

$$A_1(\tau) = -\left[2\tau^2 + 3\tau + 3(2\tau - 1) f(\tau)\right] \tau^{-2}$$

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau}, & \tau \leq 1 \leftarrow \text{heavy fermion} \\ -\frac{1}{4}\left[\log \frac{1 + \sqrt{1-\tau^{-1}}}{1 - \sqrt{1-\tau^{-1}}} - i\pi\right]^2, & \tau > 1 \end{cases}$$

$$\tau_q \equiv \frac{M_h^2}{4 m_q^2}$$

in the limit of vanishing fermion, W boson mass

$$A_{1/2} \xrightarrow[\tau \to \infty]{} 0 \qquad \text{and} \qquad A_1 \xrightarrow[\tau \to \infty]{} -2$$

For the gauge bosons, the longitudinal component does not decouple.

in the opposite limit, $M_W^2, m_t^2 > M_h^2$

$$A_{1/2} \xrightarrow[\tau \to 0]{} \frac{4}{3} \qquad \text{and} \qquad A_1 \xrightarrow[\tau \to 0]{} -7$$

This excludes a forth generation!

$$\searrow -8.3 \text{ at } M_W$$

~~The production cross section can be written as~~

~~LET~~ Low energy theorem!

For a constant Higgs field (zero four momentum)

$$\lim_{p_h \to 0} \mathcal{M}(X \to Y + h) = \frac{1}{v} m_i \frac{\partial}{\partial m_i} \mathcal{M}(X \to Y)$$

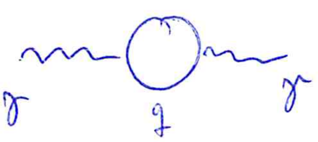
because the constant Higgs field only shifts the mass $m_i \to m_i \left(1 + \frac{h}{v}\right)$

Example: $\frac{1}{v} M_Z \frac{\partial}{\partial M_Z} \mathcal{M}(Z \to Z)$

$$= \frac{1}{v} M_Z \frac{\partial}{\partial M_Z} \frac{g_{\mu\nu}}{k^2 - M_Z^2} = \frac{g_{\mu\nu}}{k^2 - M_Z^2} \frac{2 M_Z^2}{v} \frac{1}{k^2 - M_Z^2}$$

$=$ 

For massless particles



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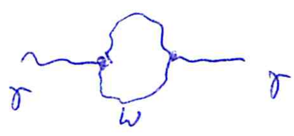
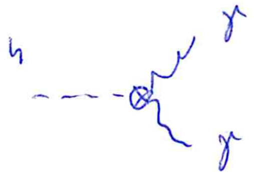


$$\to -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \sum_i \frac{b_i \alpha}{4\pi} \log \frac{\Lambda^2}{m_i^2}$$

$b_{1/2} = \frac{4}{3} N_c Q_f^2 \qquad b_1 = -7$

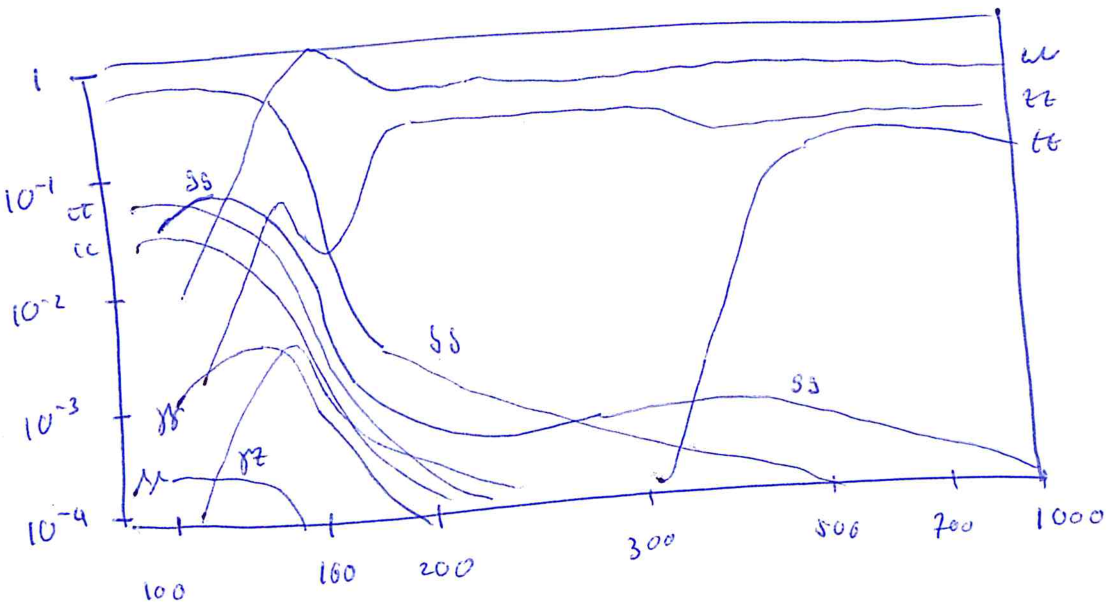
$$\mathcal{L}_{h\gamma\gamma} = \frac{\alpha}{16\pi} h F_{\mu\nu} F^{\mu\nu} \sum_i b_i \frac{M_i}{v} \frac{\partial}{\partial M_i} \log M_i^2 = \frac{\alpha}{8\pi} \frac{h}{v} F_{\mu\nu} F^{\mu\nu} (b_1 + b_{1/2})$$

induces the $h\gamma\gamma$ matrix element for $M_h^2 \to 0$



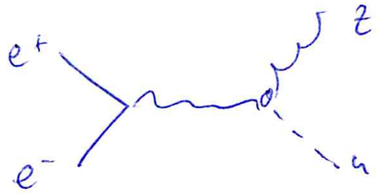
The Branching ratios for the SM Higgs are therefore

$$BR(h \to XX) = \frac{\Gamma(h \to XX)}{\sum_X \Gamma(h \to XX)}$$



At 125 GeV most decays can be seen!

At a lepton-collider, the Higgs is produced via Bremsstrahlung

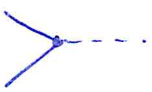


$$\sigma(e^+e^- \to Zh) = \frac{1}{192\pi \cdot s} \frac{M_Z^4}{v^4} (a_e + v_e)^2 \beta \frac{\beta^2 + 12 M_Z^2/s}{(1 - M_Z^2/s)^2}$$

$$a_e = -1 \quad , v_e = -1 + 4 s_w^2$$

$$\beta^2 = \left(1 - \frac{M_h^2 + M_Z^2}{s}\right)\left(1 - \frac{(M_h - M_Z)^2}{s}\right)$$

s-channel production is tiny due to the mass suppression



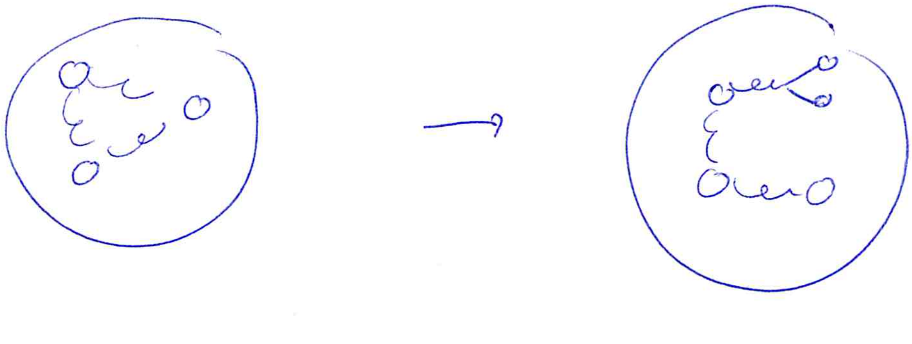
$$\propto \frac{m_e}{v}$$

At a hadron collider (at $s = M_h$)

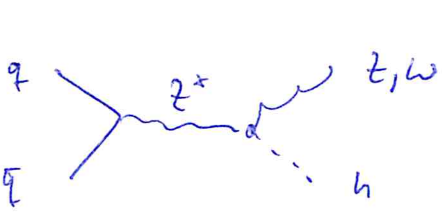
$$\sigma(pp \to h) = \frac{\pi^2}{8M_h} \sum_i f_{ii} \, \Gamma(h \to ii)$$

Here, $$f_{gg} = \frac{\pi^2}{8} \int_{M^2/s}^{1} \frac{dx}{x} \, g(x) \, g\left(\frac{M^2}{sx}\right)$$

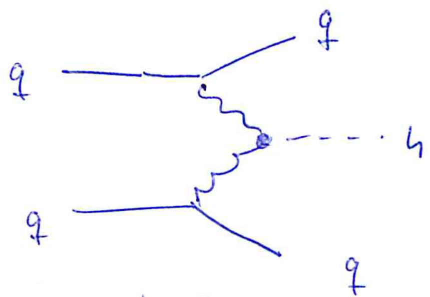
is the particle luminosity function. It quantifies the number of partons i in the proton at a energy s.



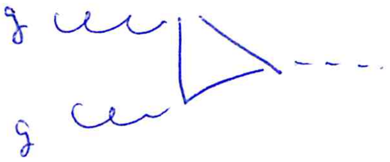
There are four main production processes at a hadron collider



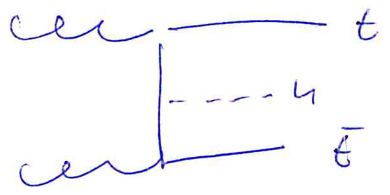
ass. production



VBF



gluon fusion



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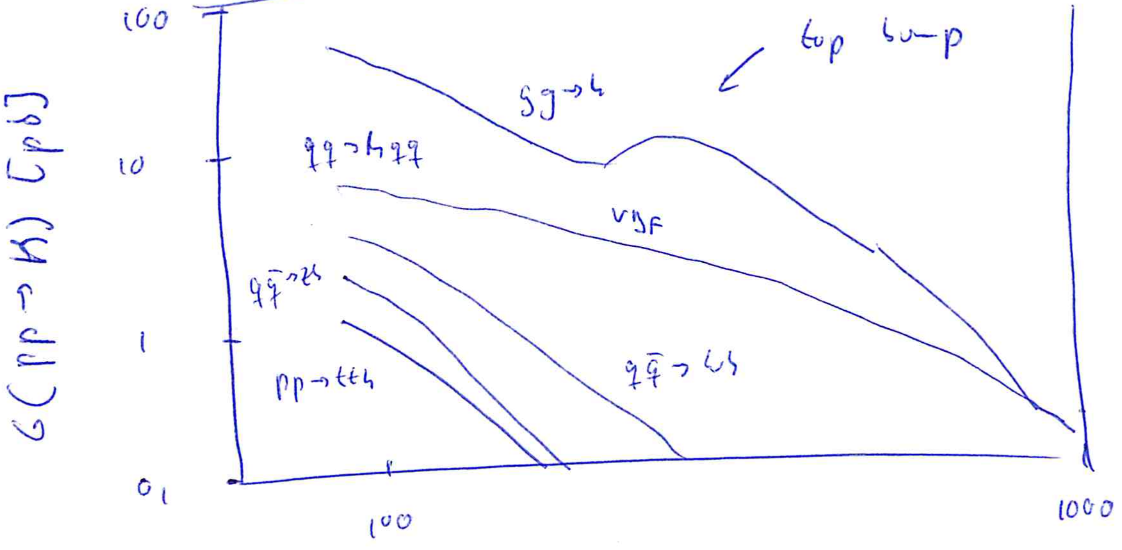
The pdfs for the LHC at 13 TeV are given by

It follows for the production cross sections



For example at 125 GeV $\sigma(gg \to h) \sim 40\,pb$

$BR(h \to \gamma\gamma) \sim 10^{-3}$

at $2\,fb^{-1}$ luminosity they have

$2000 \cdot 40 \cdot 10^{-3}$

≈ 80 events