Higgs Properties

In the SM: Mass terms for fermions and gauge bosons are forbidden

* $\frac{1}{2} M^2 A_\mu A^\mu$ $\lightning$ $A_\mu \to A_\mu + \frac{1}{g} \partial_\mu \alpha$

* $m_e \bar{e}_R \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ $\lightning$ $L_L \to e^{i\alpha + i\alpha^a t^a} L_L$

$$e_R \to e^{i\alpha} e_R$$

In both cases, the reason is gauge invariance.
Remember the construction rules for the SM Lagrangian:
- Poincaré invariance
- Gauge invariance $(SU(3)_c \times SU(2)_L \times U(1)_Y)$
- Renormalizability

Why do we impose $SU(2)_L$ gauge symmetry, if the W and Z are massive and it forbids fermion masses?

In QED: $\mathcal{L}_{QED} \supset m_e \bar{e}_L e_R$

Can we just write down a massive Spin 1 particle? Lets consider $U(1)$:

$$\mathcal{L} = -\frac{1}{2}\partial_\nu A_\mu \partial^\nu A^\mu + \frac{1}{2}M^2 A_\mu A^\mu + \lambda A_\mu A^\mu A_\nu A^\nu$$
$$+ \kappa \, \partial_\mu A_\nu A^\mu A^\nu + \dots$$

EOM for the free field ($\lambda = \kappa = 0$)

$$(\Box + M^2) A_\mu = 0$$

These are 4 scalar degrees of freedom. A spin 1 particle has 3! We need to implement an additional condition that eliminates one dof:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A_\mu A^\mu$$

$$\Box A^\mu - \partial^\mu \partial_\nu A^\nu + M^2 A^\mu = 0$$

$$\Leftrightarrow \quad (\Box + M^2) A_\mu = 0$$
$$\partial_\nu A^\nu = 0 \quad \leftarrow \text{kills 1 dof.}$$

(follows from $\partial_\mu (\Box A^\mu - \partial^\mu \partial_\nu A^\nu + M^2 A^\mu) = 0$
$M^2 \partial_\mu A^\mu = 0$)

Any solution can be written as

$$A_\mu(x) = \sum_k \int \frac{d^3\vec{p}}{(2\pi)^3} a_k(\vec{p}) \, \epsilon^k_\mu(p) \, e^{ipx}$$

for some basis vectors $\epsilon_\mu^k(p)$.

We could take $\epsilon_\mu^k = \delta_\mu^k$, but we want to enforce $\partial_\mu A^\mu = 0 \iff p_\mu \epsilon_\mu^k(p) = 0$

$\Rightarrow$ 3 polarization vectors (normalized: $\epsilon_\mu^* \epsilon^\mu = -1$)

for example $p^\mu = (E, 0, 0, p_z)$, $E^2 - p_z^2 = M^2$

$$\epsilon_\mu^1 = (0, 1, 0, 0), \quad \epsilon_\mu^2 = (0, 0, 1, 0), \quad \epsilon_\mu^L = \left(\frac{p_z}{M}, 0, 0, \frac{E}{M}\right)$$

For $E \gg M$, any cross section ~~diverges~~ of the longitudinal modes diverges

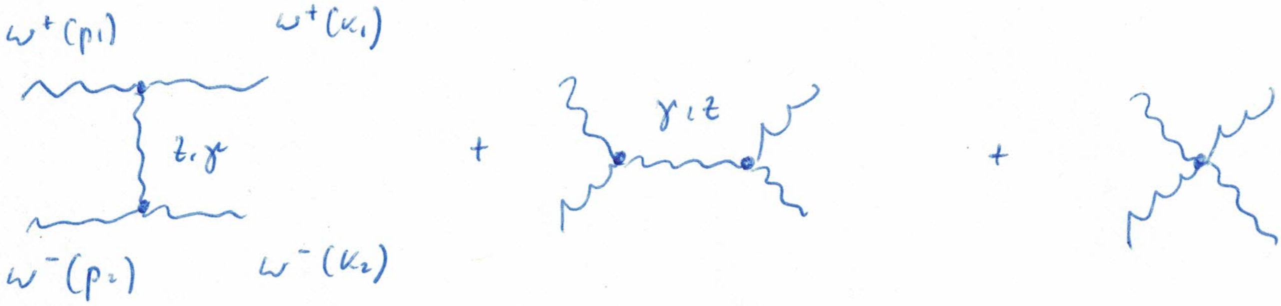
$$d\sigma \sim g^2 (\epsilon^L)^2 \sim g^2 \frac{E^2}{M^2}$$

but cross sections cannot be arbitrarily big! They are probabilities and bounded by 1.

So at some scale, this theory cannot be trusted. It becomes non-perturbative.

Explicit example:

$$W^+(p_1)\,W^-(p_2) \to W^+(k_1)\,W^-(k_2) \qquad [1303.6335]$$



$$i\mathcal{M}_t^{\gamma+Z} = -i\left(\frac{g_{ZWW}^2}{t-M_Z^2}+\frac{g_{\gamma WW}^2}{t}\right)\frac{t}{4M_W^4}$$
$$\left[(s-u)t - 3M_W^2(s-u) + \frac{8M_W^2}{s}u^2\right]$$

$$i\mathcal{M}_s^{\gamma+Z} = -i\left(\frac{g_{ZWW}^2}{s-M_Z^2}+\frac{g_{\gamma WW}^2}{s}\right)\frac{s}{4M_W^4}$$
$$\left[s(t-u) - 3M_W^2(t-u)\right]$$

$$i\mathcal{M}_4 = i\,\frac{g_{WWWW}^2}{4M_W^4}\left[s^2+4st+t^2-4M_W^2(s+t)-\frac{8M_W^2}{s}u\cdot t\right]$$

use that $\quad g_{ZWW} = c_W^2\, g_{WWW}$

$\qquad\qquad g_{\gamma WW} = s_W\, g_{WWW}$

and $\quad E=\sqrt{s} \gg M_W, M_Z$

$$i\mathcal{M}_{gauge} = i\mathcal{M}_t^{\gamma+Z} + i\mathcal{M}_s^{\gamma+Z} + i\mathcal{M}_4$$

$$= \frac{-i g_{WWV}^2}{4 M_W^4} \Big[(s-u)t + s(t-u) - 3 M_W^2 (s-u+t-u) + \frac{8 M_W^2}{s} u^2 \Big]$$

$$+ \frac{i g_{WWWW}}{4 M_W^4} \Big[s^2 + 4st + t^2 - 4 M_W^2 (s+t) - \frac{8 M_W^2}{s} u \cdot t \Big]$$

if $g_{WWWW} = g_{WWV}^2 = g^2$ as guaranteed by a gauge theory:

$$\mathcal{O}\left(\frac{E^4}{M_W^4}\right) = \frac{i g^2}{4 M_W^4} \Big[-\cancel{st} + ut - \cancel{st} + us + (s+t)^2 + \cancel{2st} \Big]$$

$$= \frac{i g^2}{4 M_W^4} \Big[u(4M_W^2 - u) + (4 M_W^2 - u)^2 \Big]$$

$$= \frac{i g^2}{4 M_W^4} \Big[-\underline{4 M_W^2 u} \Big]$$

$$\mathcal{O}\left(\frac{E^2}{M_W^2}\right) = \frac{i g^2}{4 M_W^4} \Big[3 M_W^2 (s+t-2u) - \frac{8 M_W^2}{s} u (u+t) - 4 M_W^2 (s+t) \Big]$$

$$= \frac{i g^2}{4 M_W^2} \Big[3(-3u) + 8u + 4u - 4u \Big]$$

$$= -\frac{i g^2}{4 M_W^2} u$$

still divergent : $\mathcal{M} \propto E^2$!

The Matrix elements are bound by ~~[illegible]~~ the demand that the theory does not violate probabilities (since this is tied to the unitarity of the S Matrix, it is often called unitarity constraint.)

~~[illegible] energy scale~~

$$|\mathcal{M}| < 8\pi$$

Therefore, at an energy scale below

$$E^2 < \frac{32\pi M_W^2}{g^2} \approx (1.2 \text{ TeV})^2$$

Some new physics must come in to "unitarize" WW scattering!

In other words, the theory tells us, that (explicitly) massive gauge bosons are not consistent up to arbitrarily high energies.

This is the real reason the LHC was build!

Add a scalar h

$$i\mathcal{M}^h = -i \frac{g_{hww}^2}{4M_w^4}\left[\frac{(t-2M_w^2)^2}{t-M_w^2} + \frac{(s-2M_w^2)^2}{s-M_w^2} \right]$$

$$\approx i \frac{g_{hww}^2}{4M_w^4}\, u$$

if $g_{hww} = g \cdot M_w$, it cancels exactly the gauge contribution.

This is exactly the coupling of the Higgs, if it is responsible for the W mass.

- $\mathcal{O}\left(\frac{E^4}{M_w^4}\right)$ cancels in a gauge theory.
- $\mathcal{O}\left(\frac{E^2}{M_w^2}\right)$ cancel with a Higgs.

Notes:

× It does not need to be one Higgs, it could be several Higgses with

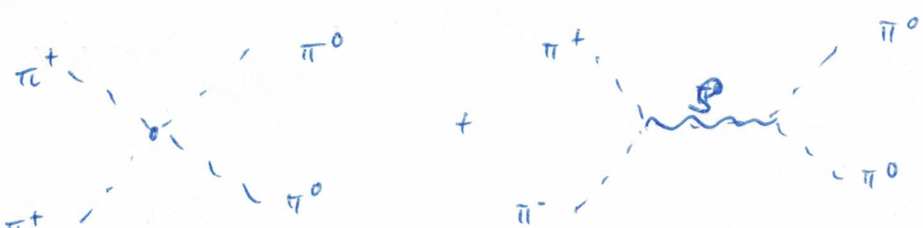
$$\sum_h g_{hWW} = g M_W$$

× It could also be a new Spin 1 resonance:

However, such a field would have its own unitarity problem at high energies. There would need to be a tower of states $Z', Z'', Z''', \ldots$

A similar situation occurs in QCD in pion-scattering

$$\mathcal{M}(\pi^+\pi^+ \to \pi^0\pi^0) = \frac{s}{F_\pi^2} \qquad f_\pi \approx 130 \text{ MeV}$$



↑ unitarized by ρ meson exchange

expected at

$$\Lambda^2 \lesssim 16\pi F_\pi^2 \approx (900 \text{ MeV})^2$$

$$M_\rho = 775 \text{ MeV}$$

More details in Chanowitz ELSB 1988

Assume there is a single Higgs

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger D_\mu \phi + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

After EWSB $\quad m_H = \sqrt{2}\,\mu^2 = \sqrt{\frac{\lambda}{2}}\, v$

The vacuum expectation value $v = 246$ GeV was known from the heavy gauge Boson masses $M_W = \frac{g \cdot v}{2}$, but the quartic coupling λ fixes the Higgs mass. From unitarity:

$$M_H < 1.2 \text{ TeV} \Rightarrow \lambda \leq 50$$

This is already clear from perturbativity:

$$\lambda < 4\pi$$

$$\lambda + \frac{\lambda^2}{16\pi^2} f(m_H) + \dots$$

Each additional loop needs to be smaller than the previous step, otherwise perturbativity breaks down.

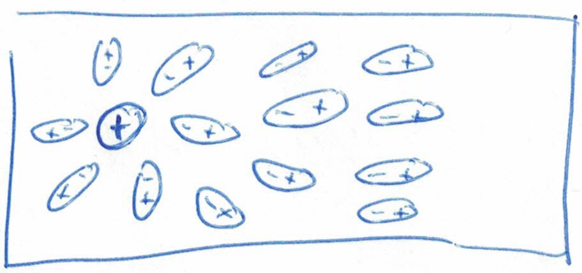
However, if the SM should remain valid up to a high scale Λ, the quartic coupling should stay small. But couplings run!

$=$

change with energy

This was discussed for QCD. Testing physics at higher and higher energies probes different couplings

Remember a dielectric medium



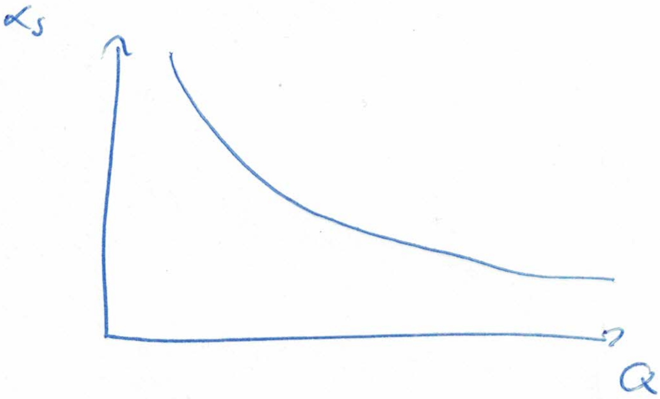
dielectric Screening

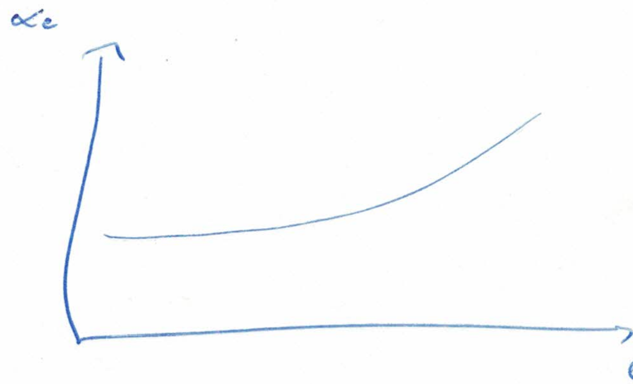
$$\vec{E} = \frac{\hat{e}_r}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\downarrow$$

$$= \frac{\hat{e}_r}{4\pi\varepsilon_0}\frac{q/\varepsilon_r}{r^2}$$

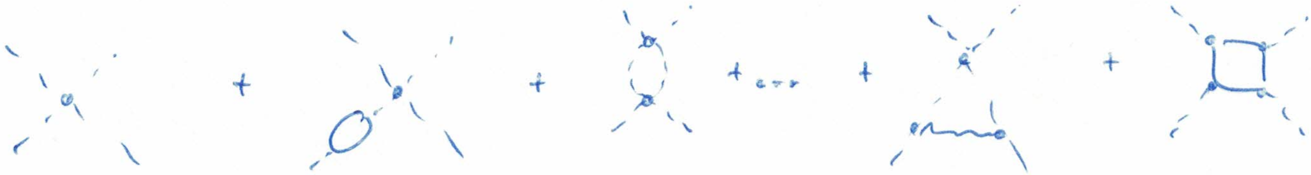
effective charge





gluons anti-screen

For the quartic coupling



$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\Big(12\lambda^2 + 12\lambda y_t^2 + \frac{3}{16}\left(2g^2 + (g^2+g'^2)^2\right) \qquad \text{Screening}$$

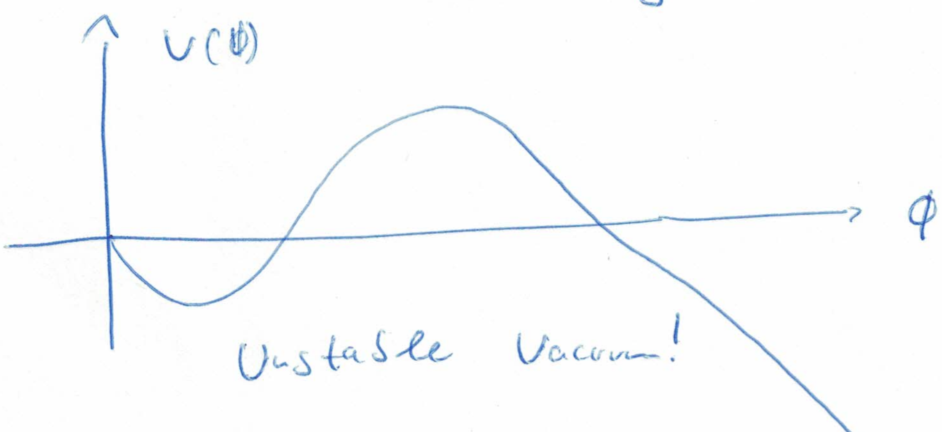
$$- 12 y_t^4 - \frac{3}{2}\lambda\left(3g^2+g'^2\right)\Big) \qquad \text{Anti-screening}$$

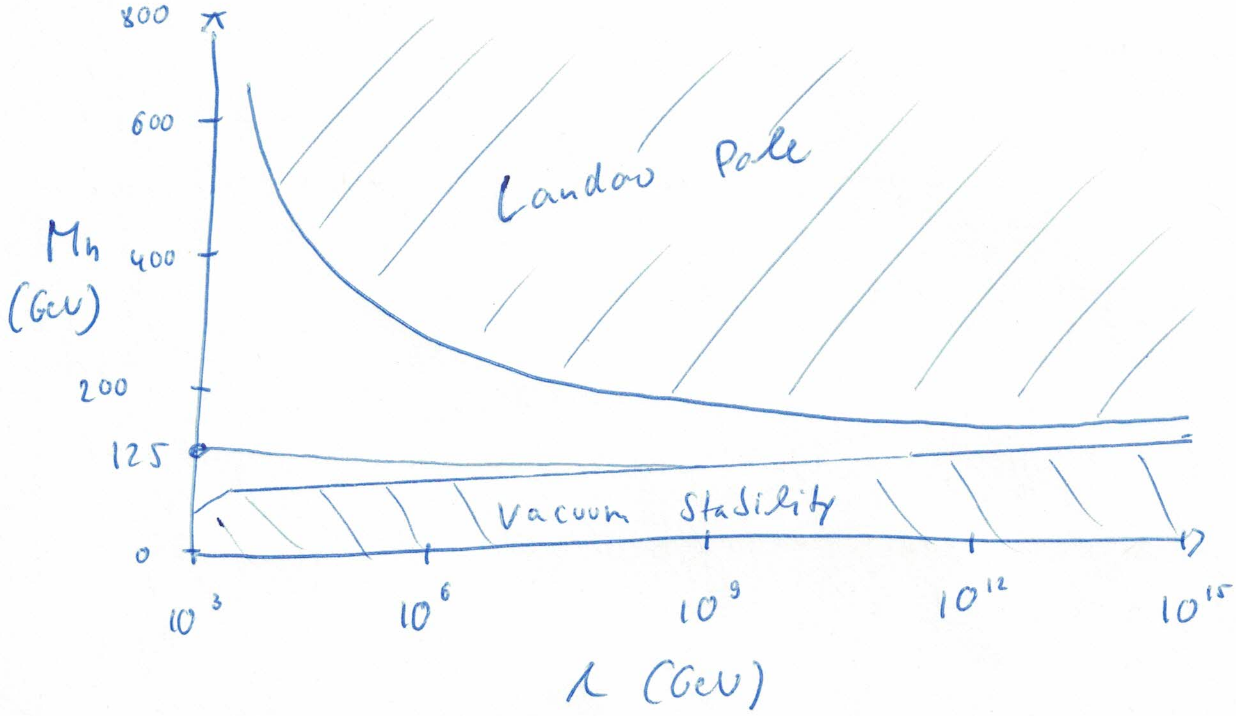
<u>Large λ</u>:
$$\frac{d\lambda}{dt} \approx \frac{3}{4\pi^2}\lambda^2$$

for high energy scales, λ explodes and developes a Landau pole.
Can only be avoided if the theory is trivial ($\lambda = 0$)

<u>Small λ</u>:
$$\frac{d\lambda}{dt} \simeq -\frac{3}{4\pi^2} y_t^4$$

for high energy scales, λ becomes smaller and turns negative:



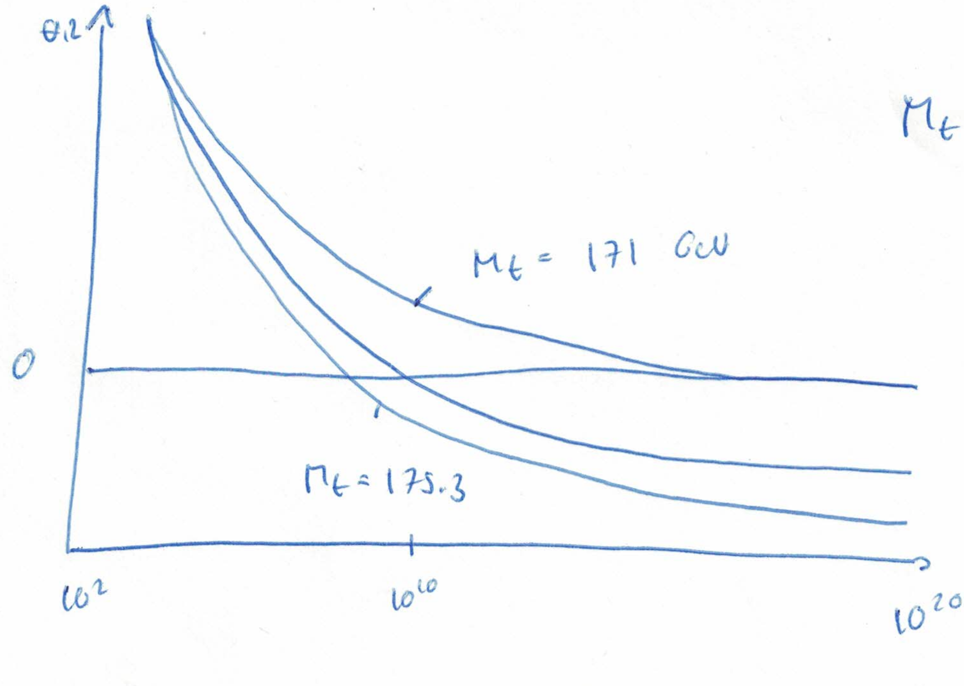


We measured the Higgs mass in 2012

↑

ATLAS & CMS

$M_h = 125$ GeV $\Rightarrow \lambda \simeq 0.5$



$M_t = 173.1 \pm 0.7$ GeV