Beyond the Standard Model

Several questions are left unanswered within the SM. For example

→ What triggers electroweak symmetry breaking?

→ What protects the electroweak scale?

→ What generates Neutrino masses?

→ What is Dark Matter?

→ Why is there more matter than antimatter?

→ Why are the fermion masses so different?

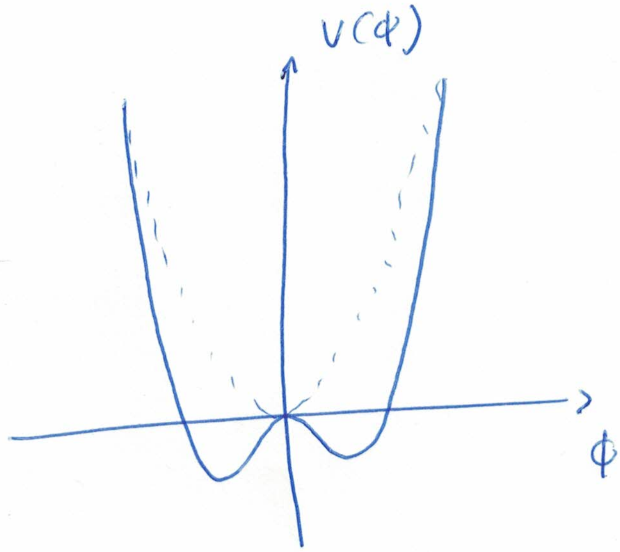
→ Is there a Grand Unified Theory?

→ What is the quantum theory of gravity?

The Standard model does not explain the electroweak scale. It is put in by hand

$$V = -\mu^2 \Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2$$

for $\lambda > 0$, $\mu^2 > 0$



$\langle\phi\rangle = \frac{v}{\sqrt{2}}$, $v = 246$ GeV

Sign and magnitude of $\mu^2 \simeq 88^2\,\mathrm{GeV}^2$ is crucial for the world we observe. <u>All</u> elementary particle masses are set by this scale:

$$M_W = \frac{g}{2}v \ , \quad M_Z = \frac{\sqrt{g'^2+g^2}}{2}v \ , \quad m_f = y_f\frac{v}{\sqrt{2}}$$

$$M_H = v\sqrt{\frac{\lambda}{2}}$$

What would a change in v mean? (All other couplings being equal)?

$M_p > M_N$

$M_{QCD} + m_d + 2m_u + \Delta_{EM}$

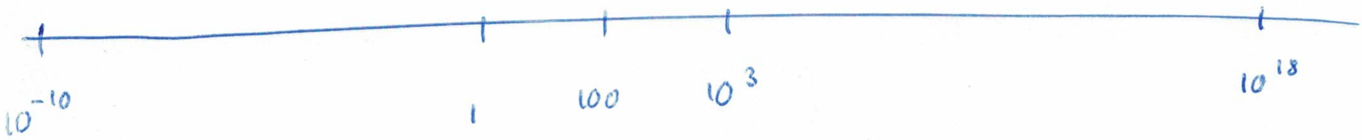
$> M_{QCD} + 2m_d + m_u$

$p \to N + e^+ + \nu_e$

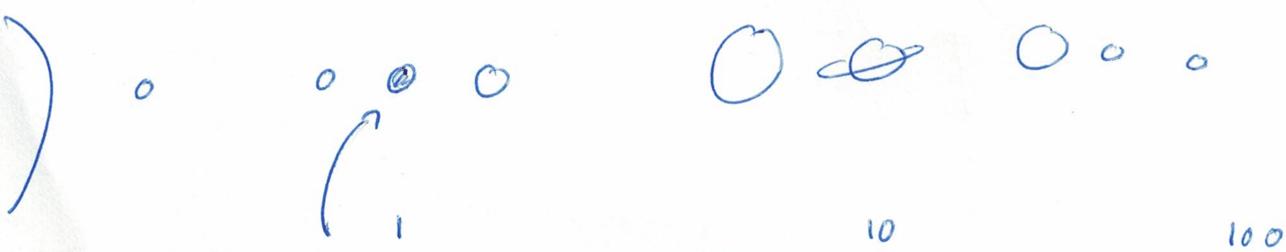
$\leftarrow \nu \rightarrow$

$M_N - M_p >$ Nuclear Binding energy 13 keV

$\Rightarrow$ Atoms would decay



Similar to the question: Why is there life on earth?



Water is liquid here!

But the hierarchy problem is worse, because the electroweak scale is unstable. What does that mean?

Quantum effects destabilize the electroweak scale, because it is not protected by a symmetry.

Each

The bare mass term in the Lagrangian receives quantum corrections

$$V = -\underbrace{(\mu^2 + \delta\mu^2)}_{\bar{\mu}^2}\Phi^\dagger\Phi + \dots$$

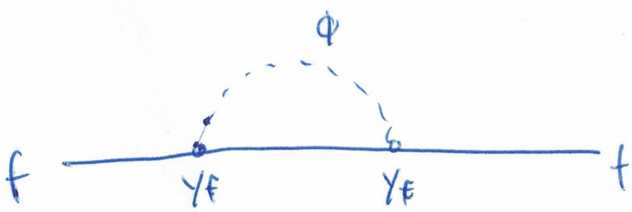
Already the quartic interaction induces such a correction

$$\propto \lambda \int^{\Lambda} d^4k \, \frac{1}{k^2 - M_H^2}$$

$$\delta\mu^2 \approx -\frac{\lambda\Lambda^2}{(4\pi)^2}$$

What about fermion (or gauge boson) masses?

For example $\quad y_f \bar{f} \Phi f$



$$\propto y_f^2 \int^{\Lambda} d^4k \, \frac{\not{k} + m_f}{[k^2 - m_f^2][k^2 - M_H^2]}$$

naively $\sim \Lambda$, but the mass cannot be proportional to y_f.

$$\delta m_f \approx \frac{y_f^2}{(4\pi)^2} m_f \log(\Lambda)$$

The deeper reason is that fermionic mass terms are protected by a symmetry, scalar mass terms are not.

If the SM is valid up to the Planck scale

[A particle can form a black hole if

Compton wavelength < Schwarzschild radius

$$\frac{h}{M} < 2MG$$

$$\Rightarrow \quad M > \sqrt{\frac{\hbar}{4\pi G}} = M_{PL} \approx 10^{19}\ \text{GeV} \quad]$$

$$\bar{\mu}^2 = \mu^2 - \frac{\lambda M_{PL}^2}{(4\pi)^2} \approx \mu^2 - 10^{33}\ \text{GeV}^2 = 88^2\ \text{GeV}^2$$

enormous fine-tuning

$$\bar{m}_f = m_f\left(1 + \frac{Y_f^2}{(4\pi)^2}\log\Lambda\right) \approx m_f \cdot (1 + 0.3) \approx m_f$$

If the SM is only valid up to a lower scale, the fine-tuning problem is ameliorated, but then one would expect new physics soon.

In the SM at one loop:

$\delta\mu^2 \propto$

$$\simeq \frac{1}{2v^2}\,\frac{\Lambda^2}{4\pi^2}\left(6M_W^2 + 3M_Z^2 - 12m_t^2 + 3M_h^2\right)$$

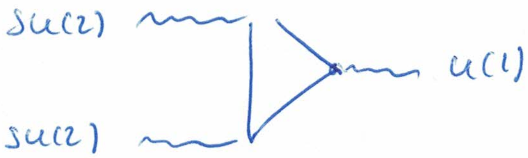
Can there be a symmetry that protects a scalar mass term? For each particle there would need to be an opposite spin partner that cancels its contribution... <u>Supersymmetry</u>

Each SM field has a superpartner

	bosons	fermions	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_i	$(\tilde{u}_L, \tilde{d}_L)_i$	$(u_L, d_L)_i$	3	2	$1/6$
$\bar{u}_i$	$\tilde{u}^*_{Ri}$	$\bar{u}_i = u^+_{Ri}$	3	1	$-2/3$
$\bar{d}_i$	$\tilde{d}^*_{Ri}$	$\bar{d}_i = d^+_{Ri}$	3	1	$1/3$
L_i	$(\tilde{\nu}_L, \tilde{e}_L)_i$	$(\nu_L, e_L)_i$	1	2	$-1/2$
$\bar{e}_i$	$\tilde{e}^*_{Ri}$	$\bar{e}_i = e^+_{Ri}$	1	1	1
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	1	2	$1/2$
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	1	2	$-1/2$
G	G^a_μ	$\tilde{G}^a_\mu$	8	1	0
W	$W^3_\mu, W^\pm_\mu$	$\tilde{W}_3, \tilde{W}^\pm$	1	3	0
B	B_μ	$\tilde{B}$	1	1	0

Supersymmetry predicts 2 Higgs doublets, otherwise anomalies don't cancel:

SM



$$\propto \quad 3\cdot\left(\tfrac{1}{6}\right)-\tfrac{1}{2} = 0$$

MSSM

$+$ 

$+$ 

$$= +\tfrac{1}{2}-\tfrac{1}{2}=0$$

Obviously, supersymmetry cannot be realized exactly in nature. By how much can it be broken?

This is what makes it ugly

18 → 124 parameters

$+$

$$\delta\mu^2_{H_u} = -\frac{3y_t^2}{(4\pi)^2}\left[2m_{\tilde t}^2\left(\ln\frac{m_{\tilde t}^2}{Q^2}-1\right)-2m_t^2\left(\ln\frac{m_t^2}{Q^2}-1\right)\right]$$

If $\quad m_{\tilde t} = 500\ \mathrm{GeV} \qquad \dfrac{\mu^2_{H_u}}{\mu^2_{H_u}+\delta\mu^2_{H_u}} \simeq 0.1$

$\quad m_{\tilde t} = 1000\ \mathrm{GeV} \qquad\qquad \simeq 0.02$

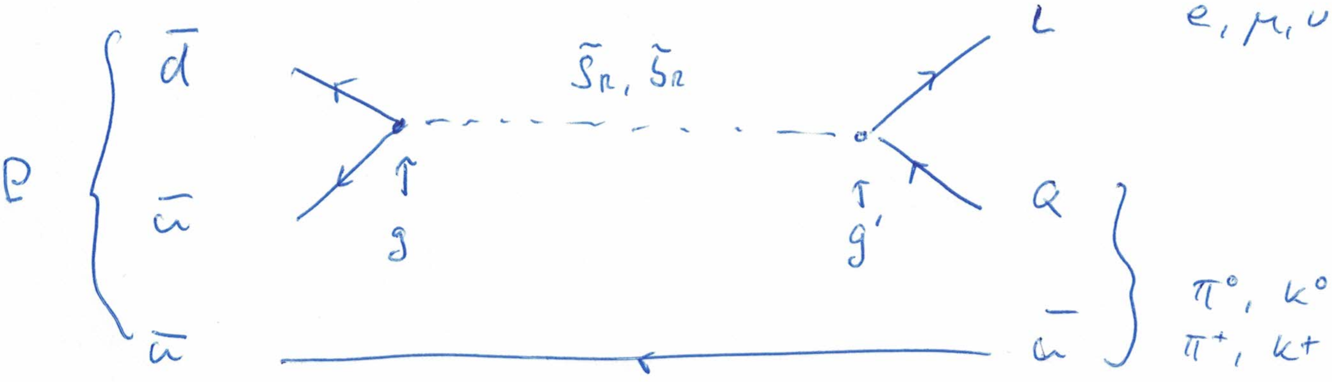
but for $\quad m_{\tilde t} = 10\,000\ \mathrm{GeV} \qquad \simeq 0.35$

$\Rightarrow$ we expect stops to be light!

gluinos

$$\delta m^2_{\tilde t} = \frac{2g_s^2}{3\pi^2}M_{\tilde g}^2 \log\frac{\Lambda}{M_{\tilde g}}$$

Supersymmetry alone is problematic, because



$$\Gamma_p \approx \frac{|g\cdot g'|^2}{m_{\tilde{q}}^4}\frac{M_p^5}{8\pi} \Rightarrow \tau_p = \frac{1}{\Gamma} \approx \frac{1}{|gg'|^2}\left(\frac{m_{\tilde{q}}}{\text{TeV}}\right)^4 \cdot 10^{-11}\,\text{s}$$

experimentally $\tau_p > 10^{34}$ yrs $\Rightarrow m_{\tilde{q}} \gtrsim 10^{13}$ TeV

A additional symmetry can make the proton stable:

R-parity: $q \to q \quad , \tilde{q} \to -\tilde{q}$

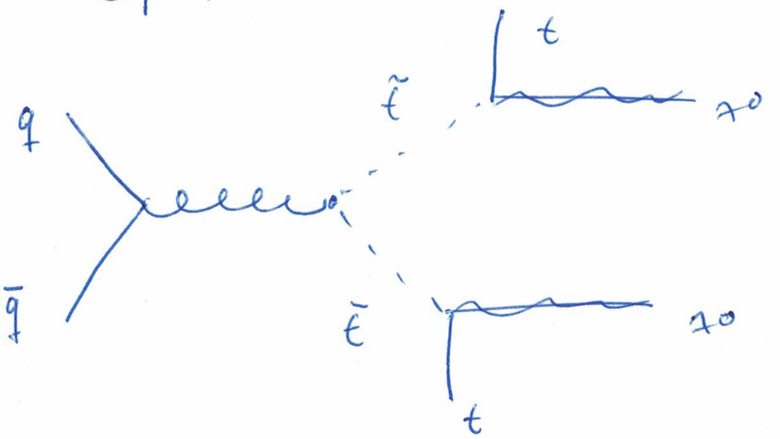
$$R = (-1)^{3(B-L)+F}$$

(Un-) intended consequences:

- superpartners are always produced in pairs
- the lightest supersymmetric particle is stable
- every sparticle decays into a final state containing the LSP

what are typical SUSY search channels?

we ~~have~~ expect light LSP, stops, gluinos



suppressed, because the stops must carry angular momentum (p-wave)

$\beta \to \beta^3$

and no spin projection sum!

↑ not true for 1st gen.

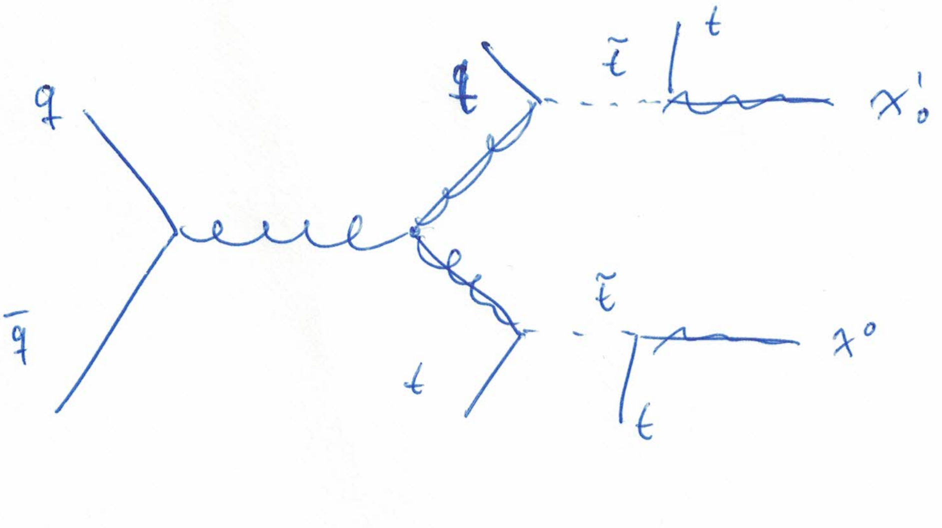
$\beta^2 = 1 - 4m^2/s$

$q\bar{q} \to \bar{t}t = 23\,pb$

$q\bar{q} \to \tilde{t}\tilde{t}^* \simeq 1.6\,pb$

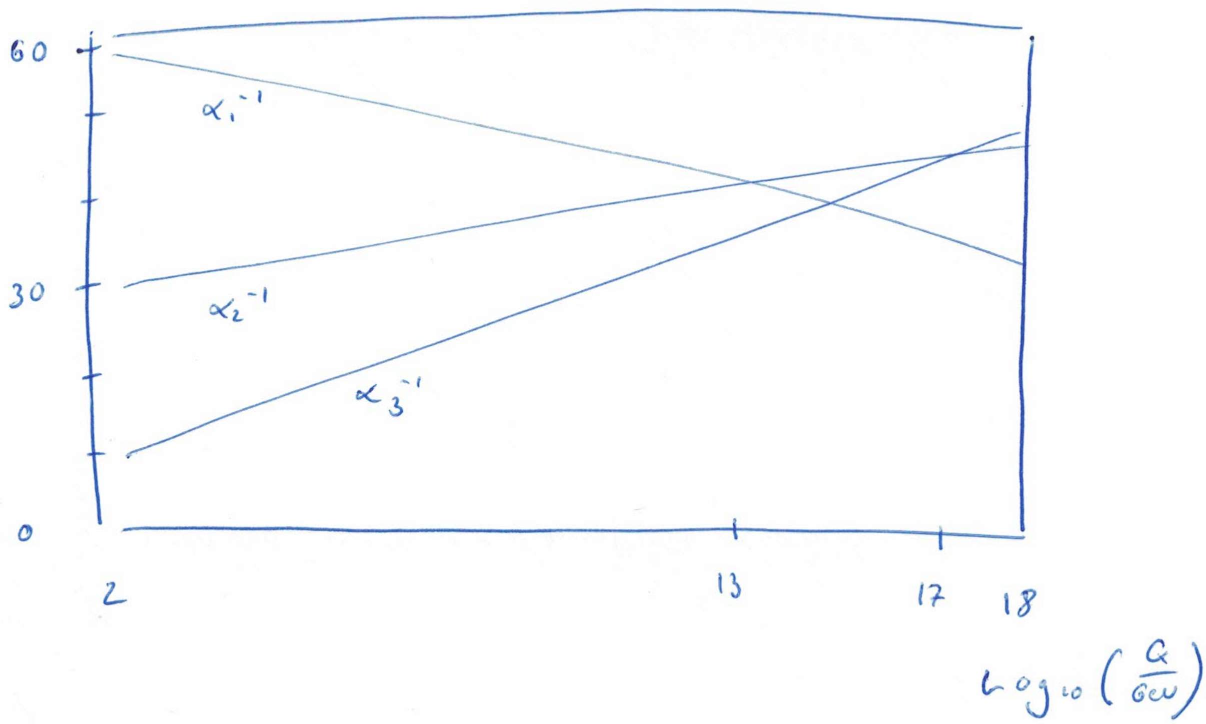
Gluon-production is log enhanced for tops (not for stops → no singularity in the t-channel)

$gg \to \bar{t}t = 68\,pb$ $\quad gg \to \tilde{t}\tilde{t}^* = 11\,pb$

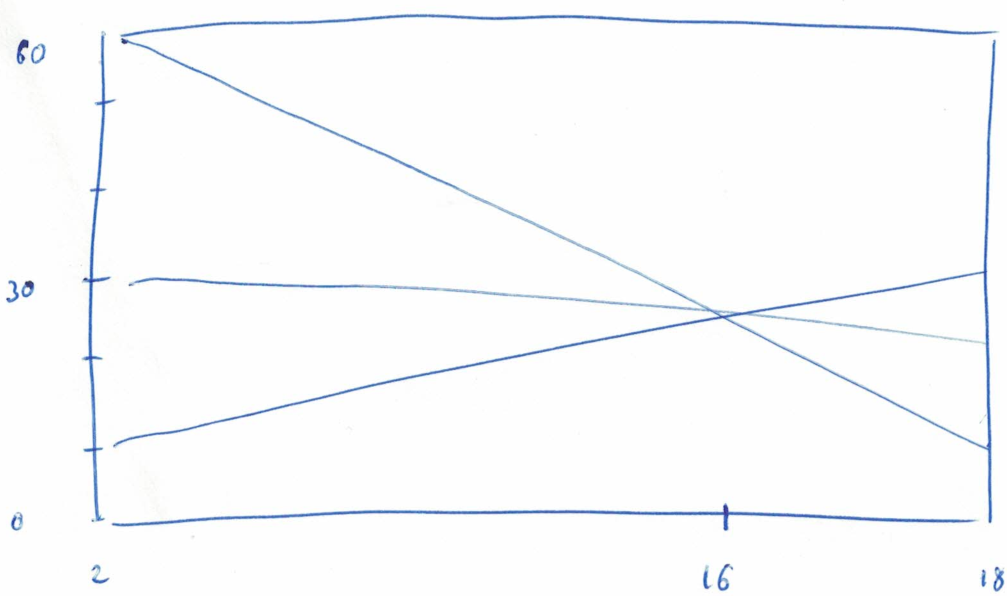


Another tantalizing feature of the supersymmetric SM is, that the gauge couplings seem to unify at a high scale

SM:



MSSM:



Is there a grand unified theory?

$$SU(N) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$$

A glimpse of GUTs:

It is a peculiar fact, that a full SM generation fits exactly in an $SU(5)$ multiplet.

$$\bar{5} \equiv \begin{bmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu \end{bmatrix} \begin{matrix} \\ \\ \\ \Big\} L \end{matrix} \qquad 10 \equiv \begin{bmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^c \\ & & & & 0 \end{bmatrix}$$

The generators are

$$\begin{bmatrix} t^a/2 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ 0 & \sigma^i/2 \end{bmatrix} \qquad \begin{bmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{bmatrix}$$

$\#$ 8   3   1

but $SU(5)$ has 24 generators:

$24 - 8 - 3 - 1 = 12$ additional gauge bosons with masses $M_X \approx M_{GUT}$

Consequences: → Anomalies cancel trivially

SM ($SU(3)^3$, $SU(2)^3$, $U(1)^3$, $SU(3)^2 \times U(1)$, $SU(2)^2 \times U(1)$, gravity $\times U(1)$)

GUT $SU(5)^3$: $\mathrm{Tr}\, T^a \{T^b, T^c\} = 0$

→ Charges are quantized!

→ the Weinberg angle is a prediction

$$g_1^2 \, Tr[T^2] = g_1^2 \left(\tfrac{1}{2}\right)^2 = g'^2 \, Tr[Y^2]$$

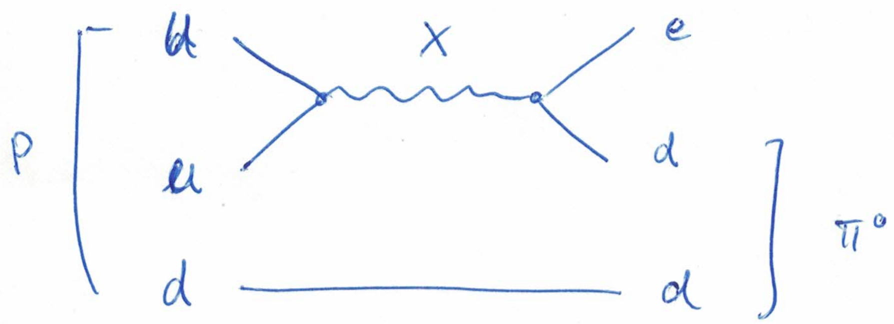
$$= g' \left(3 \left(\tfrac{1}{3}\right)^2 + 2 \left(-\tfrac{1}{2}\right)^2 \right)$$

index of the fundamental is fixed to be $Tr[T^a T^b] \equiv \frac{1}{2} \delta^{ab}$

$$\Rightarrow \qquad g' = \sqrt{\frac{3}{5}}\, g_1$$

and
$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3}{8}.$$

However, the proton decays



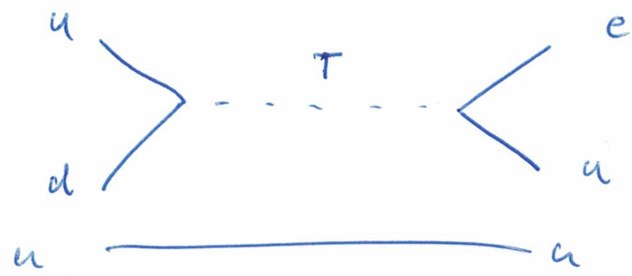
This time it is the new symmetry, that induces this decay. Lifetime:

$$\tau_p = \frac{1}{\Gamma_p} = \left(\frac{M_p^5}{8\pi M_X^4} \right)^{-1} \approx 10^{34} \, yrs \left(\frac{M_X}{10^{16}\, TeV} \right)^4$$

The GUT hypotheses predicts proton decay at the ~~limits of the~~ exp. limit.

There is also an $SU(3)_c$ triplet Higgs

$$5_H = \begin{bmatrix} T \\ H \end{bmatrix}$$



additional source of proton decay. While the Gauge bosons are naturally heavier than their SM counterparts, the scalar triplet gets its mass from the Higgs potential as well.

Introduce a scalar that breaks the GUT to the SM Adjoint rep. 24_Σ

with $$\langle \Sigma \rangle = \frac{v}{\sqrt{60}} \begin{pmatrix} 3 & & & & \\ & 3 & & & \\ & & -2 & & \\ & & & -2 & \\ & & & & -2 \end{pmatrix}$$

$$V = g\, 5_H^\dagger\, |24_\Sigma|^2\, 5_H - m^2 |5_H|^2 + \lambda |5_H|^4$$

$$\Rightarrow \quad m_H^2 \sim g\, 3^2 \langle \Sigma \rangle^2 + m^2$$
$$m_T^2 \sim g\, 2^2 \langle \Sigma \rangle^2 + m^2$$

$\hookrightarrow$ Doublet - Triplet splitting problem.

Are there other ways to protect the Higgs mass?

In supersymmetric models, the Higgs boson is in a multiplet with the Higgsino → fermion. It inherits the protection from that fermions have in the SM.

Chiral symmetry $\qquad \phi \Leftrightarrow \psi \Rightarrow$ MSSM

Gauge symmetry $\qquad A_M = (A_\mu, A_5) \quad A_5 = \phi$

$\Rightarrow$ ED theories

No scalars $\qquad \phi = \langle \bar{\psi}\psi \rangle$

$\Rightarrow$ Technicolor