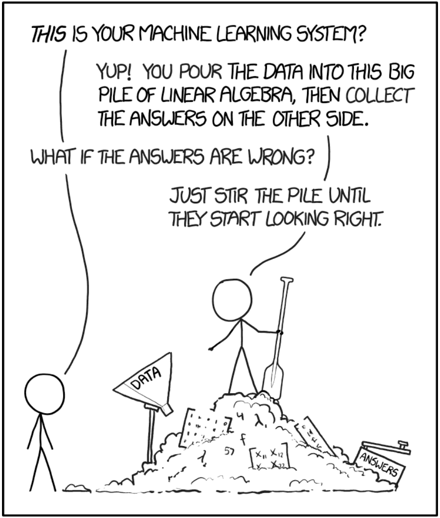

Source: https://xkcd.com/1838/

Introduction to Feed-forward and Convolutional Neural Networks

Adrian Braemer

23.04.2019

Outline

What is Machine Learning?

- Goal: Make predictions based on given data

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$\to$ "Training the model"

Comparison to Fitting

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$\rightarrow$ Subtle differences, very different algorithms!

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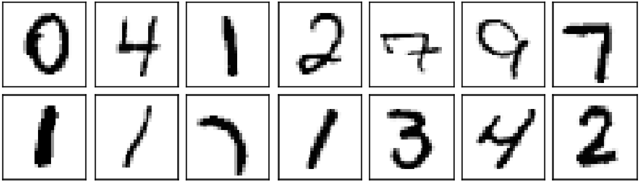

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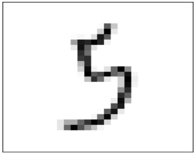

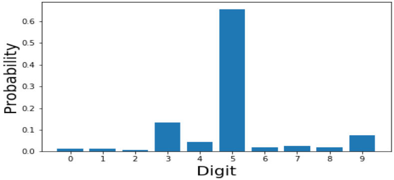

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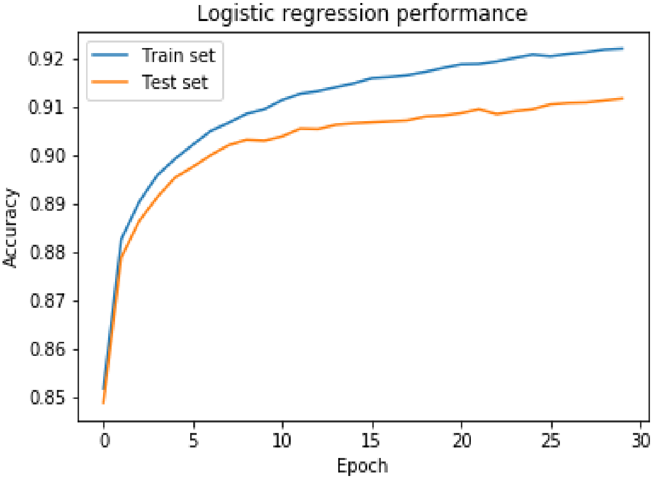

→ Caps out at around 91%

Can we do better with a more complex model?

Neural Networks

Neural Networks

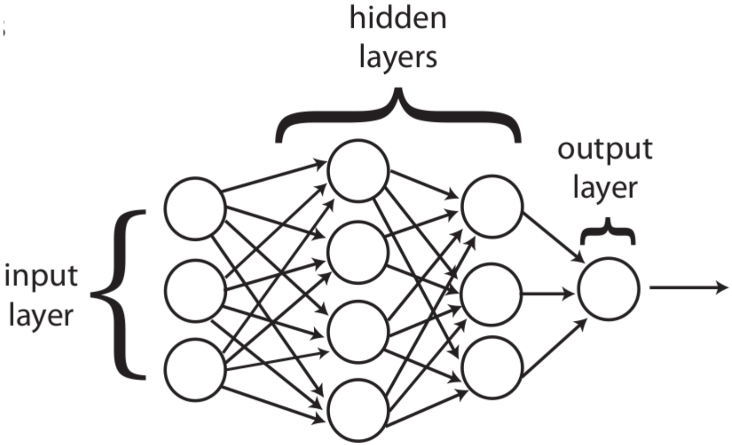

Figure: General architecture

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$\rightarrow$ Problems at every step!

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- Do we even converge?

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- $\Delta^l_k = \sum_k \frac{\partial \mathcal{C}}{\partial z^{l+1}_k}\frac{\partial z^{l+1}_k}{\partial z^l_j} = \sigma'(z^l_j)\left(\sum_k w^{l+1}_{jk}\Delta^{l+1}_k\right)$
- Gradient: $\frac{\partial \mathcal{C}}{\partial w^l_{jk}} = \frac{\partial \mathcal{C}}{\partial z^l_j}\frac{\partial z^l_j}{\partial w^l_{jk}}$

Backpropagation Algorithm

- Weigths w_{jk}^l between j-th neuron in layer $l-1$ and k-th neuron in layer l
- Biases b_j^l
- Weighted inputs to neuron $z_k^l = \sum_j w_{jk}^l a_j^{l-1} + b_k^l$
- Activation levels a_k^l:
    - a_k^1 are the inputs
    - $a_k^l = \sigma(z_k^l) = \sigma\left(\sum_j w_{jk}^l a_j^{l-1} + b_k^l\right)$
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- Gradient: $\frac{\partial \mathcal{C}}{\partial w_{jk}^l} = \frac{\partial \mathcal{C}}{\partial z_j^l}\frac{\partial z_j^l}{\partial w_{jk}^l} = \Delta_j^l a_k^{l-1}$

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1. Calculate the activation levels z_k^l and a_k^l iteratively from front to back

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$\rightarrow$ problem of vanishing or exploding gradients

Repairing the gradient

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- Truncate too high values

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- Use non-saturating activation functions, f. e. ReLU (rectified linear unit)

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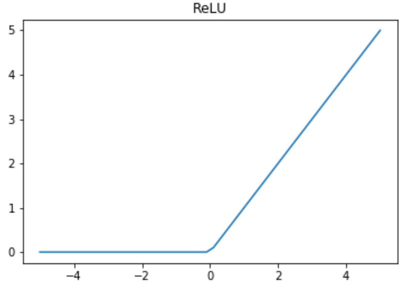

Repairing the gradient

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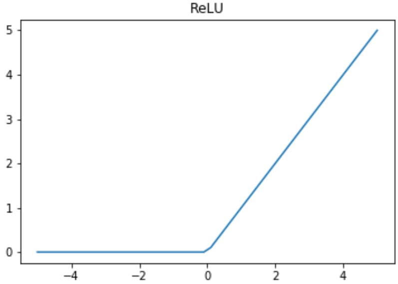
- Regularization also helps



Regularization

- Stochastic Gradient Descent

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  - Randomly divide training data into m minibatches

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  - Randomly divide training data into m minibatches
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  - Randomly neglect neurons during training steps

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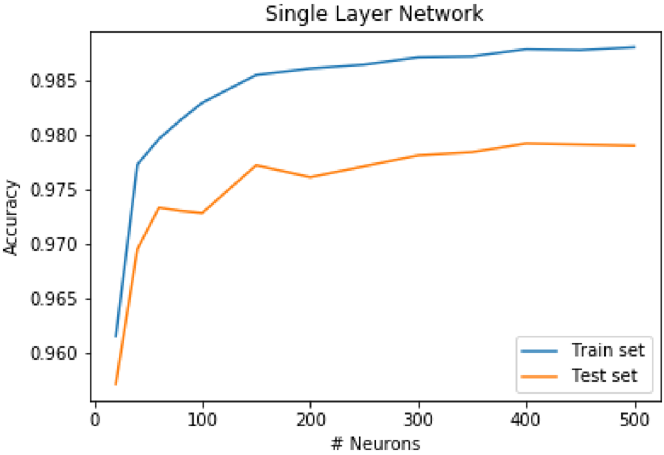
→ Prevent overfitting

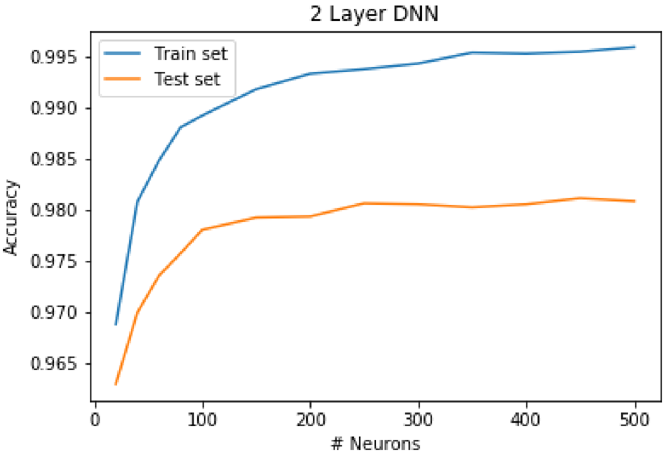
MNIST example revisited



→

Caps at 97.9%!

MNIST example revisited



→

Caps at 98.1%!

Weakpoints of DNNs

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$\longrightarrow$ Can we reduce the network size?

Basic Idea

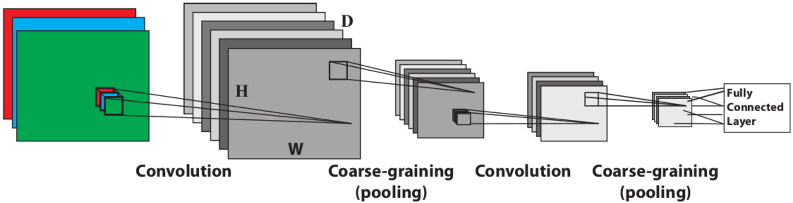
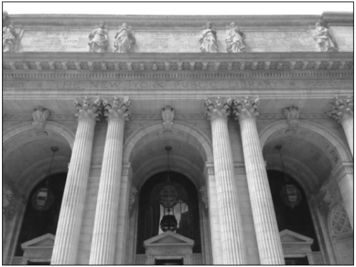
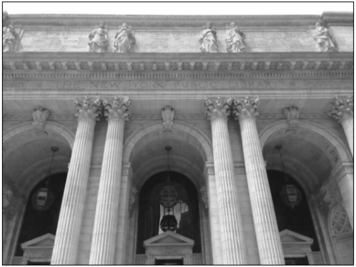


Figure: General Structure of CNNs

Example: Edge detection

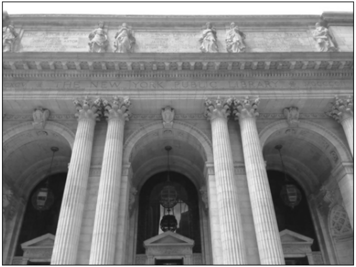


Example: Edge detection



$$\xrightarrow{\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}}$$

Example: Edge detection



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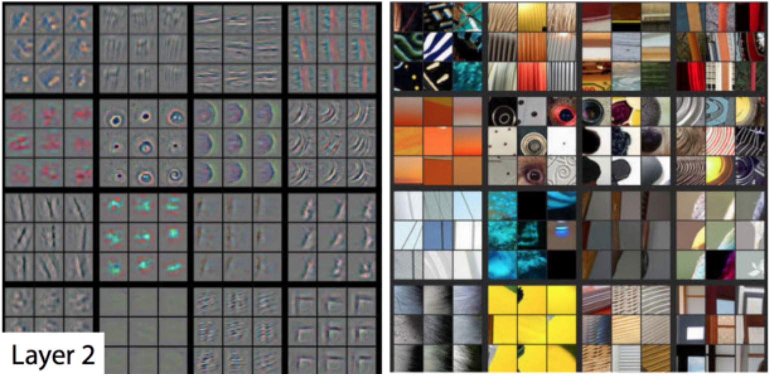
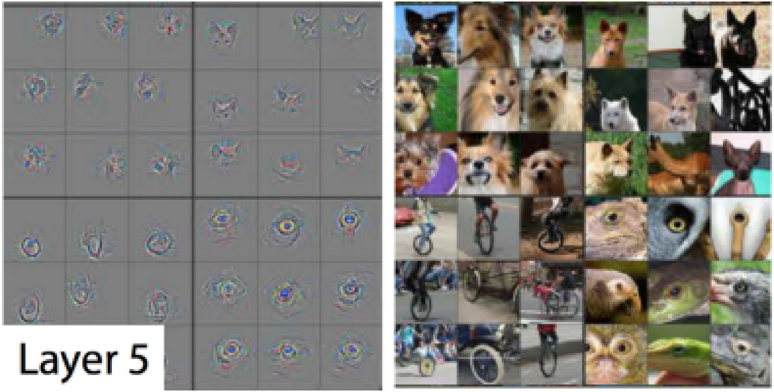
Low level convolutions



High level convolutions



Summary

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- How to train a neural network
- Short outlook to CNNs

Discussion/Question