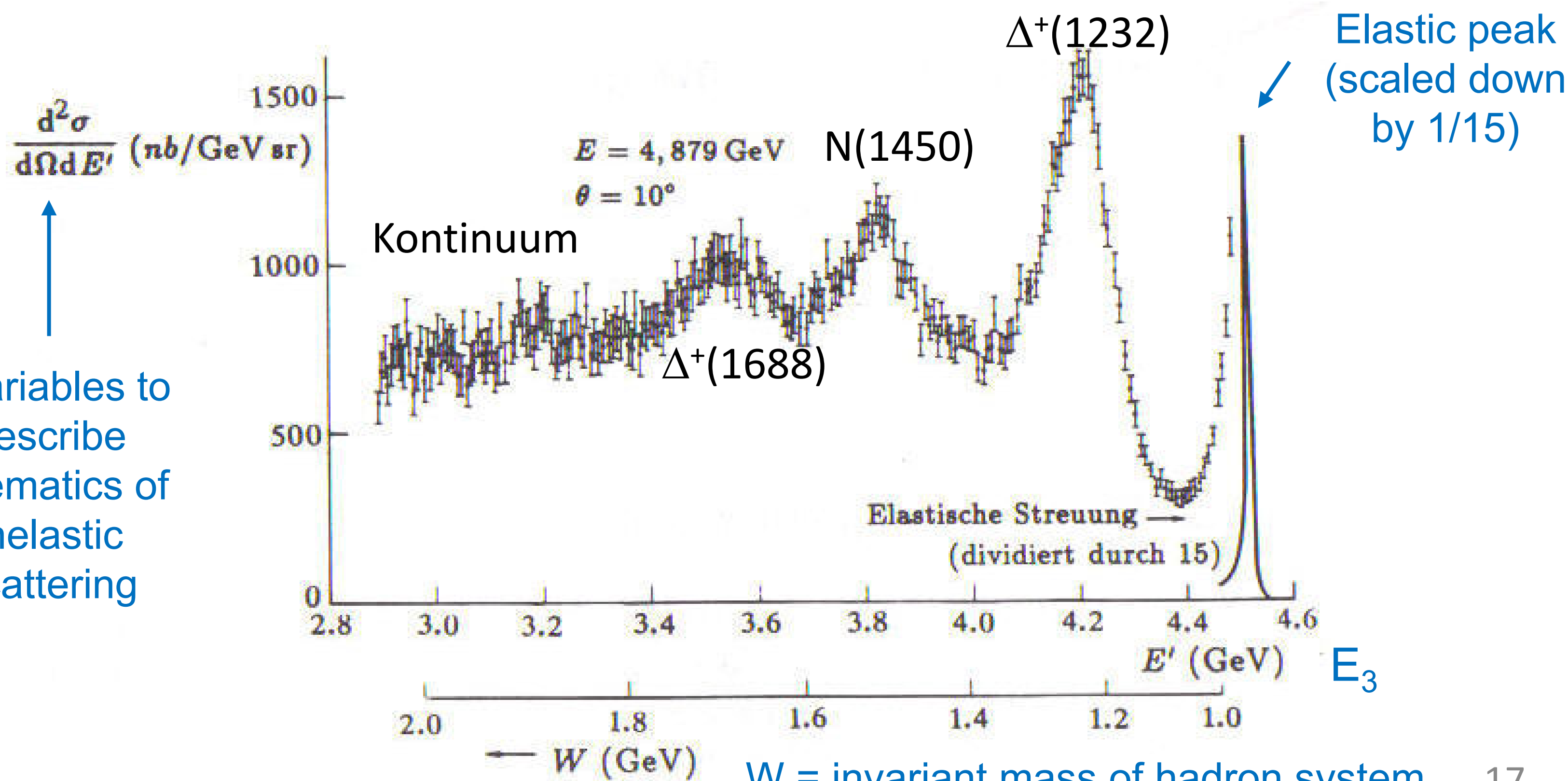
2. Deep-inelastic electron proton scattering

Elastic scattering: no excitation of inner degrees of freedom, no proton break-up

⇩ Increase energy transfer $\nu = E_1 - E_3$ from electron to proton beyond the level of the proton recoil ($q^2 \neq \vec{q}^2$)

<u>Inelastic scattering</u>:



2 variables to describe kinematics of inelastic scattering

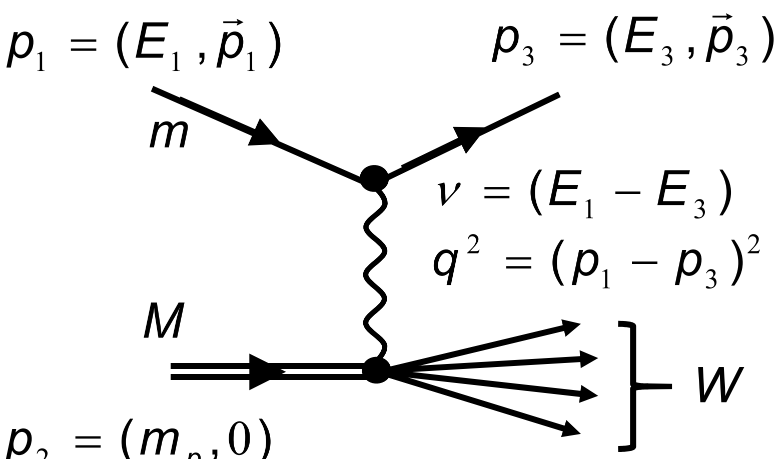
W = invariant mass of hadron system

Observations:

- Excitations (Δ^+(1232), N(1420), …) of the proton
- At higher energy transfer (smaller E_3) one observes a continuum, cannot be explained by the Q^2 dependence of a compact proton w/ $F(Q^2) \sim 1/Q^4$. This would lead to a strong suppression $\sim 1/Q^8$

→ here the proton instead breaks up.

Kinematics of inelastic scattering:



W = invariant mass of hadronic system

Elastic: $W = m_p$

with

$$q = (p_1 - p_3) = (\nu, \vec{p}_1 - \vec{p}_3)$$

$$\rightarrow \quad p_2 q = m_p \nu \quad \rightarrow \quad \boxed{\nu = \frac{p_2 q}{m_p}}$$

W always $\geq m_p$. Reason: baryon number conservation.

$$W^2 = (p_2 + q)^2 = m_p^2 + 2 p_2 q + q^2$$

Define a new Lorentz invariant dimensionless variable (important to describe parton distributions in the proton): **Bjorken x**

$$x = \frac{Q^2}{2p_2 q} = \frac{Q^2}{2m_p \nu}$$

Using the invariant mass W of hadronic system one can rewrite x:

$$x = \frac{Q^2}{Q^2 + W^2 - m_p^2} \quad \Rightarrow \quad 0 \le x \le 1$$

$x = 1$ for elastic scattering W=m_p

Another dimensionless variable is the inelasticity y:

$$y = \frac{p_2 q}{p_2 p_1}$$

In the rest frame of the proton $p_2 = (m_p, 0, 0, 0)$

($\rightarrow$ y is the electron energy fraction transferred to photon)

$$y = \frac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1}$$

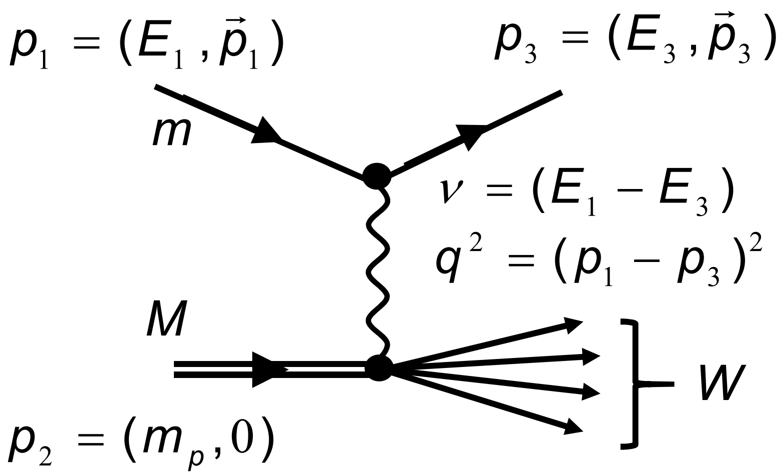
$$0 \le y \le 1$$

One finds with $s = (p_1 + p_2)^2$ the following useful relations:

$$y = \left(\frac{2m_p}{s - m_p^2}\right)\nu \quad \text{and} \quad Q^2 = (s - m_p^2)xy$$

Out of Q^2, x, y, ν
2 variables needed to define kinematics!

Kinematics of inelastic scattering:



W = invariant mass of hadronic system

Elastic: W = m_p

with

$$q = (p_1 - p_3) = (\nu, \vec{p}_1 - \vec{p}_3)$$

$$\rightarrow \quad p_2 q = m_p \nu \quad \rightarrow \quad \boxed{\nu = \frac{p_2 q}{m_p}}$$

W always $\geq m_p$. Reason: baryon number conservation.

$$W^2 = (p_2 + q)^2 = m_p^2 + 2p_2 q + q^2$$

$$x = \frac{Q^2}{2p_2 q} = \frac{Q^2}{2m_p \nu} \qquad y = \frac{p_2 q}{p_2 p_1} = \frac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1}$$

$$Q^2 = (s - m_p^2)xy$$

Express the Rosenbluth formula of elastic ep scattering with the variables Q^2, x, y:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

with $f_2(Q^2) = \frac{G_E^2(Q^2) + 2\tau G_M^2(Q^2)}{1+\tau}$ and $f_1(Q^2) = G_M^2(Q^2)$ $\qquad \tau = \frac{Q^2}{4m_p^2}$

Remark: While y appears on the RH side, it is a function of Q^2 only as the scattering is elastic (x=1) !

Modified Rosenbluth formula can be generalized for inelastic scattering by replacing the two form factors f_1 and f_2 by so called structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$. (structure functions $F_{1,2}$ should depend on 2 variables to reflect the inelastic case)

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

For deep-inelastic scattering where $Q^2 >> m_p^2 y^2$

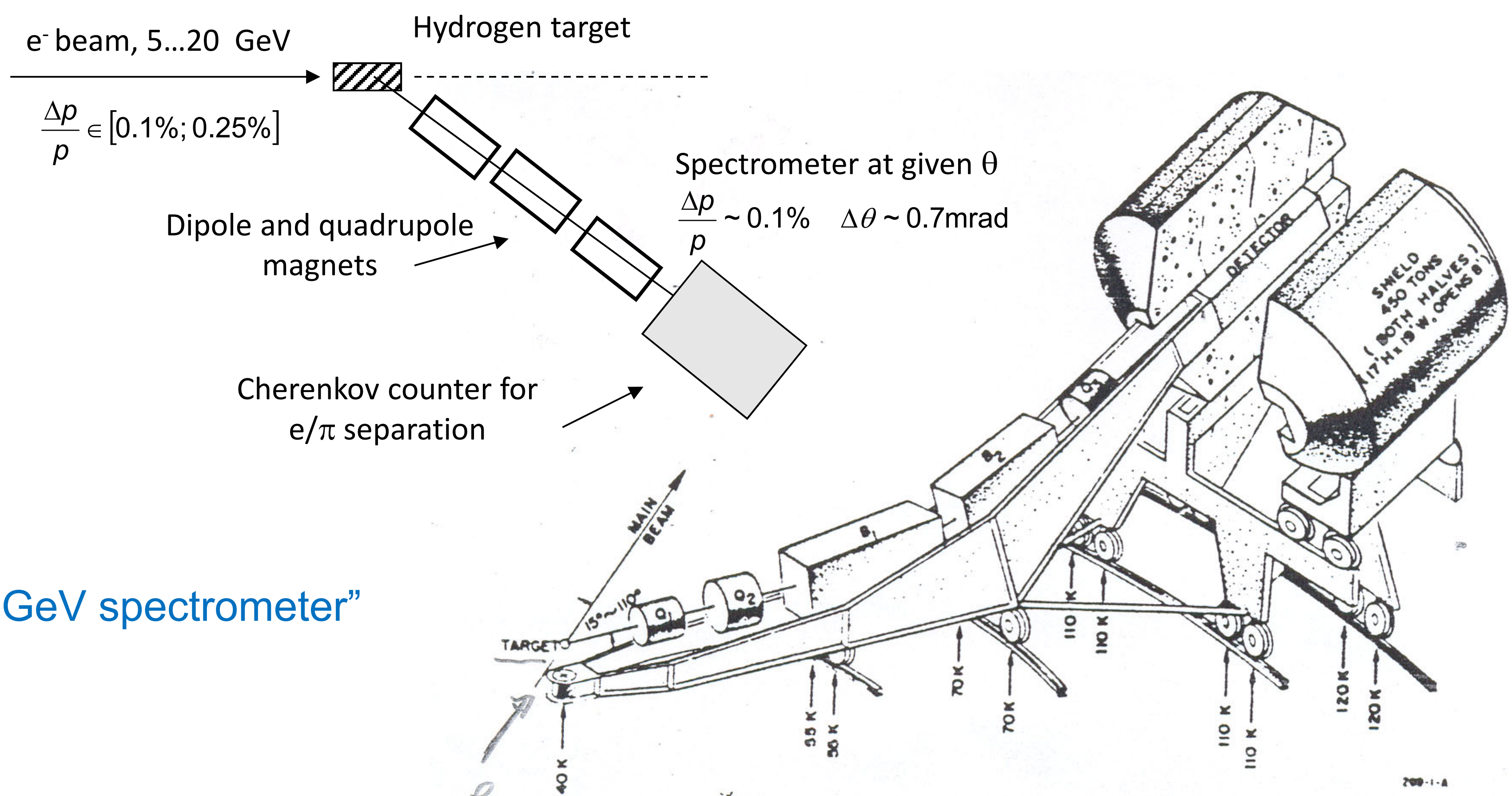
$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

First measurements of the structure functions (SLAC & MIT, 1969):

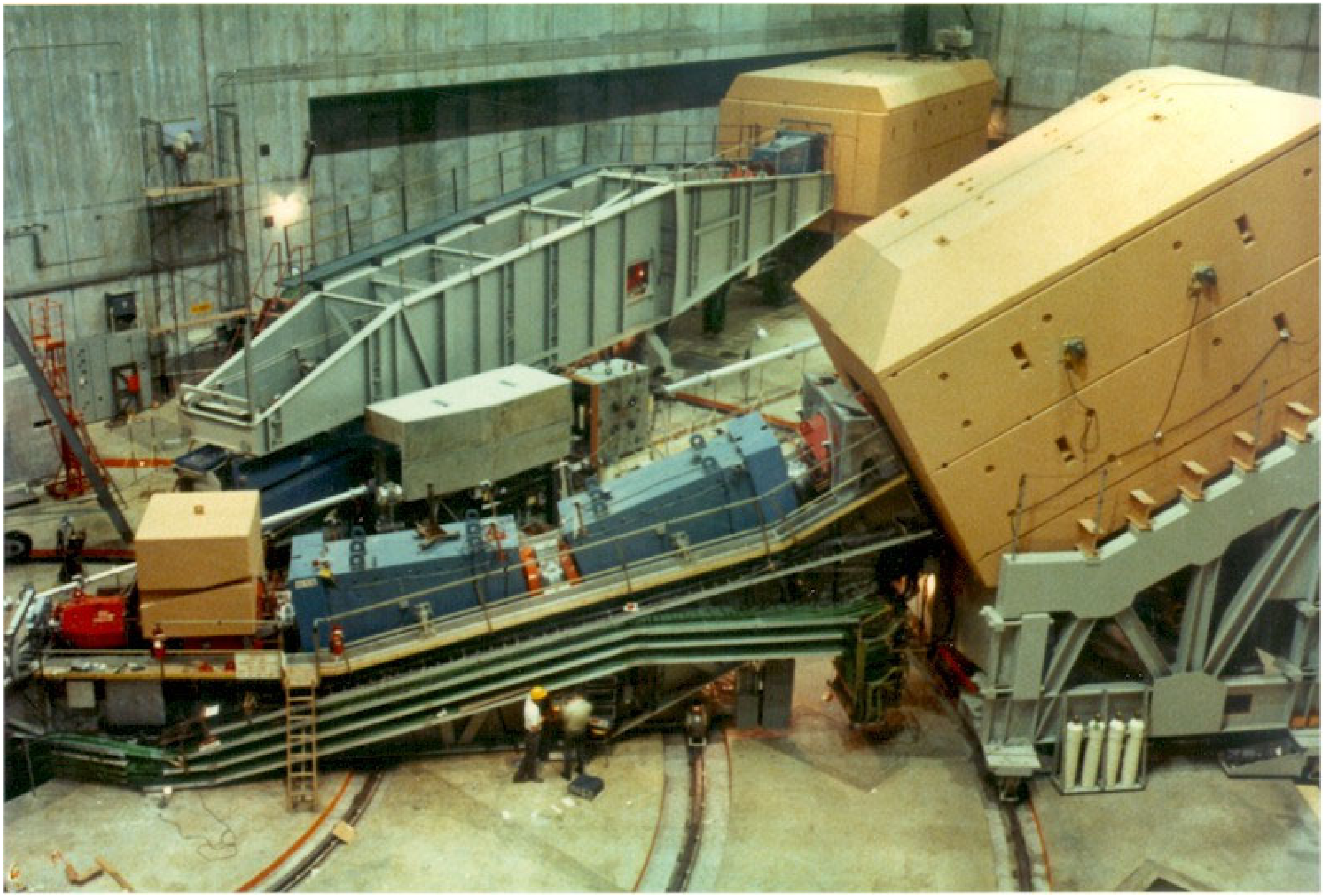
(J. Friedman, H. Kendall and R. Taylor, Nobel prize 1990)

For fixed target electron-proton scattering the necessary kinematic variables x, Q^2, y can all be determined form the electron system: E_1, E_3, e scattering angle θ

Electron beam from 2 miles LINAC

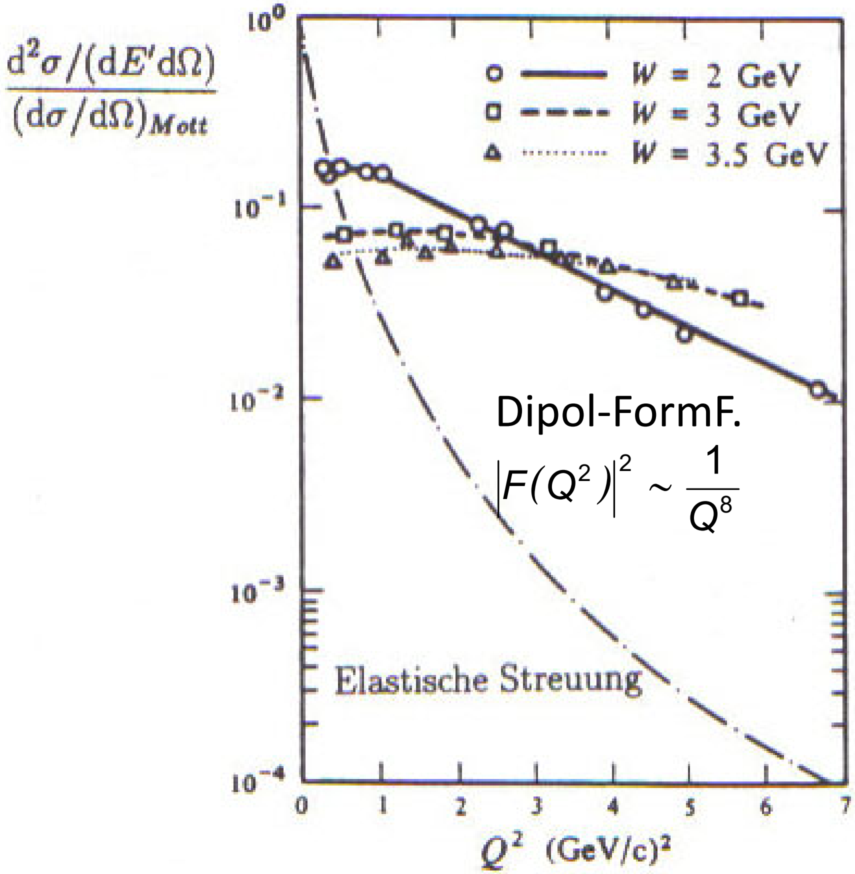


“8 GeV spectrometer”

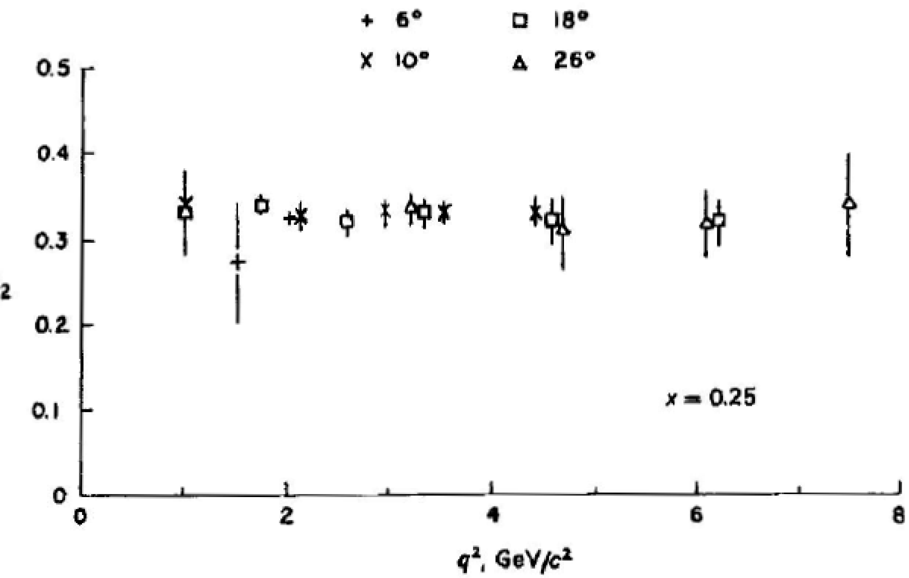


in front: 8 GeV spectrometer, in back: 20 GeV spectrometer

Cross section and structure function F_2



Breidenbach et al., 1969



Two observations/findings:

1) Bjorken scaling:
$F_{1,2}$ are functions of x and not (x,Q^2)

$$F_1 = F_1(x) \quad F_2 = F_2(x)$$

(no explicit Q^2 dependence)

The lack of Q^2 dependence suggests the scattering on point-like constituents. The Callan-Gross relation: Spin ½ constituents

2) F_1 and F_2 are not independent:

$$F_2(x) = 2x F_1(x)$$

= Callan Gross relation

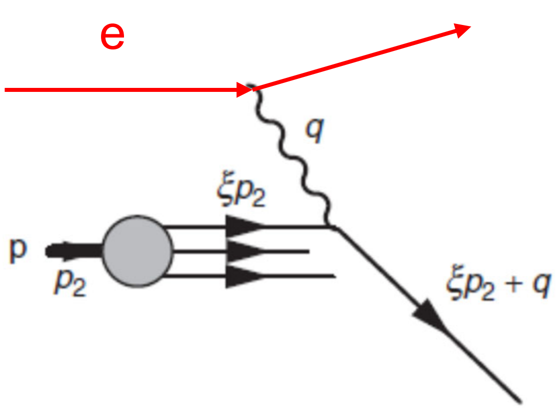
Confirms quarks as point-like constituents of the proton.

DIS ep-scattering in the parton model

Feynman, 1969

Both observations become clear if the scattering is discussed in the parton model.

Parton model is formulated in the infinite momentum frame: the proton has a very large (infinite) energy $E_P >> m_p$ and its mass can be neglected: $p_2 = (E_2, 0, 0, E_2)$



In this model the proton is a "stream of partons" (constituents). The transverse momentum of the partons can be neglected.

4-momentum of struck quark

$$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$$

ξ = proton 4-momentum fraction carried by quark

Invariant mass of the quark after interaction:

$$(\xi p_2 + q)^2 = \xi^2 p_2^2 + 2\xi p_2 q + q^2 = m_q^2$$

Quark mass before interaction $\xi^2 p_2^2$

$\Rightarrow$ Possible only if $2\xi p_2 q + q^2 = 0$

$$\xi = -\frac{q^2}{2p_2 q} = \frac{Q^2}{2p_2 q} = x$$

Definition of Bjorken x

Process possible only if momentum fraction carried by quark equals the Bjorken variable x!

$$\xi = x$$

Interesting finding:

defined by electron kinematics

Inelastic cross sections measured as function of the Bjorken variable x and the structure functions $F_{1,2}$ are related to the momentum distribution of the quarks.

The kinematic variables of the underlying e-quark scattering process are related to the kinematic variables of the electron-proton scattering process.

e-proton kinematics:

$$s = (p_1 + p_2)^2 \approx 2p_1p_2$$

$$y = \frac{p_2 q}{p_2 p_1} \qquad x = \frac{Q^2}{2p_2 q}$$

e-quark kinematics:

$$s_q = (p_1 + \xi p_2)^2 \approx 2xp_1p_2 = xs$$

$$\xi = x$$

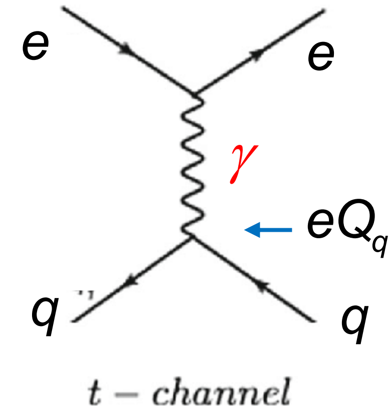
$$y_q = \frac{p_q q}{p_q p_1} = \frac{xp_2 q}{xp_2 p_1} = y \qquad x_q = 1 \quad \text{(elastic)}$$

To calculate the total electron-proton cross section in the parton model the fundamental e-quark cross section eq→eq is needed. However a similar cross section has already been calculated as t-channel contribution of $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering) ☺ .

Cross section of the fundamental eq→eq process:

$$\langle |M_{fi}|^2 \rangle = 2Q_q^2 e^4 \left(\frac{s_q^2 + u_q^2}{t_q^2} \right)$$

(see lecture on ee annihilation)



Diff. cross section in CMS frame (θ* is scattering angle in CMS frame) :

$$\frac{d\sigma_{eq}}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s_q} \frac{1 + \frac{1}{4}\left(1+\cos\theta^*\right)^2}{\left(1-\cos\theta^*\right)^2}$$

$$\frac{d\sigma_{eq}}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

$$\boxed{\frac{d\sigma_{eq}}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]}$$

Lorentz invariant form – use:

$$\frac{d\sigma_{eq}}{dq^2} = \frac{d\sigma_{eq}}{d\Omega^*}\left|\frac{d\Omega^*}{dq^2}\right|$$

with $q^2/s_q = -x_q y_q = -y$

and

$$\left[1 + (1-y)^2\right] = 2\left[(1-y) + \frac{y^2}{2}\right]$$

To calculate the deep-inelastic electron-proton cross section from the fundamental (elastic) electron-quark cross section one needs to sum over all possible quark flavor and weight the contribution with the probability to find a corresponding quark with the correct parton momentum fraction x.

The probability density $q_i(x)$ for a quark of flavor i is defined such that $q_i(x)dx$ gives the probability to find a quark of flavor i carrying a proton momentum fraction $\in [x, x+dx]$.

The DIS electron-proton cross section in the parton model is then given by:

$$\frac{d\sigma_{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\cdot\sum_i e_i^2 q_i(x)$$

charge of quark i

Comparison w/ the phenomenological result defines the structure functions:

$$\frac{d\sigma_{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)\frac{F_2(x)}{x}+y^2F_1(x)\right]$$

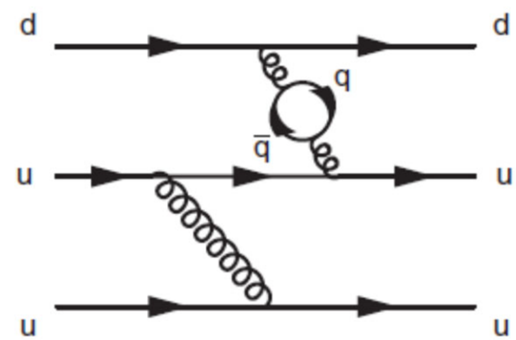
$$\boxed{F_2(x) = 2xF_1(x) = x\sum_i e_i^2 q_i(x)}$$

Parton model predicts Bjorken scaling (elastic scattering on point-like constituents (no explicit Q^2 dependence) and Callan-Gross relation (spin ½ partons).

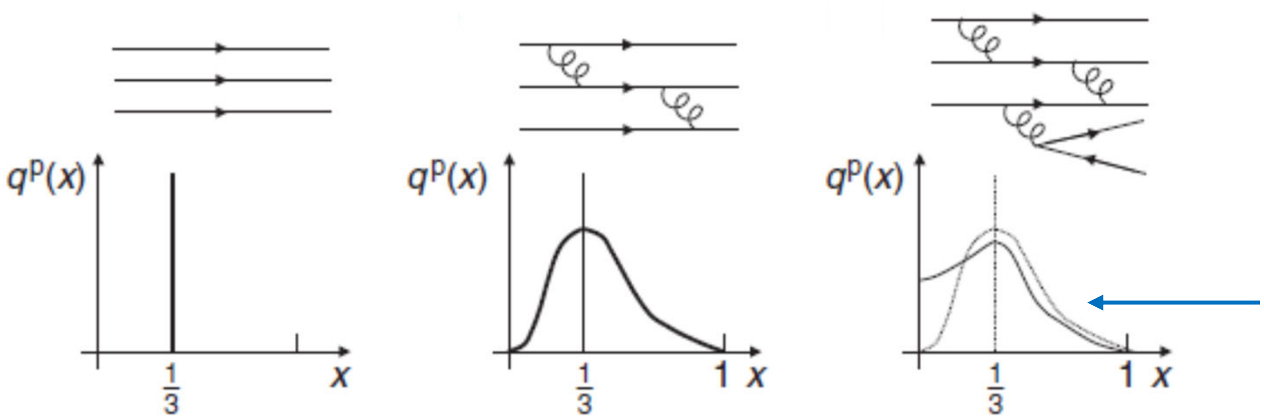
Parton distributions / parton densities $q_i(x)$

In static quark model, proton is made-up from 2 u-quarks and 1 d-quark (=**valence quarks**). If there were no interaction between the quarks one simply would assume that each quark carries 1/3 of the proton momentum.

In reality the proton is a dynamic system: quarks are bound strongly by exchanging gluons. Gluons could also – shortly – convert into additional $q\bar{q}$ pairs.



This leads to the presence of additional **$q\bar{q}$** pairs (in addition to the 3 valence quarks): **sea quarks** – most frequently **$u\bar{u}$** and **$d\bar{d}$**, but also **$s\bar{s}$** and even $c\bar{c}$ and $b\bar{b}$ pairs (strongly suppressed).



Dynamic effects lead to modified quark momentum distributions q(x).

Please note that the peak at 1/3 ignores that the gluons also carry momentum (see below).

Structure functions for e-nucleon scattering:

For the e-proton scattering the structure function $F_2(x)$ is thus given by:

$$F_2^{ep}(x) = x\sum_i e_i^2 q_i(x) \approx x\left[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right]$$

neglect s-quarks

where $u, \bar{u}, d, \bar{d}$ are the parton density distributions of the u, d quark and anti-quarks of the proton (sum of valence and sea quarks).

A similar expression could also be written down for DIS electron-neutron scattering (measurement done using deuterons and correcting for proton)

$$F_2^{en}(x) = x\sum_i e_i^2 q_i(x) \approx x\left[\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\bar{u}^n(x) + \frac{1}{9}\bar{d}^n(x)\right]$$

Isospin symmetry relates the parton densities of proton and neutron:

$$u^n = d^p = d, \quad \bar{u}^n = \bar{d}^p = \bar{d},$$
$$d^n = u^p = u, \quad \bar{d}^n = \bar{u}^p = \bar{u}$$

To calculate the proton / neutron momentum carried by the quarks one should integrate the structure functions over x:

$$\int F_2^{ep}(x)dx = \frac{4}{9}f_u + \frac{1}{9}f_d \qquad \text{with} \qquad f_u = \int \left[u(x) + \bar{u}(x)\right]dx$$

$$f_d = \int \left[d(x) + \bar{d}(x)\right]dx$$

and for the neutron

$$\int F_2^{en}(x)dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

Experimentally one finds for the two integrals:

$$\int F_2^{ep}(x)dx \approx 0.18 \qquad\qquad \int F_2^{en}(x)dx \approx 0.12$$

Solving for the integrals of the u and d quarks: one gets:

$$f_u \approx 0.36 \quad \text{and} \quad f_d \approx 0.18 \quad \Longrightarrow \quad f_u + f_d \approx 0.54$$

This means that the sum of the quarks (u,d) carry only ~50% of the proton momentum fraction: rest is carried by …???? Mostly by the gluons!

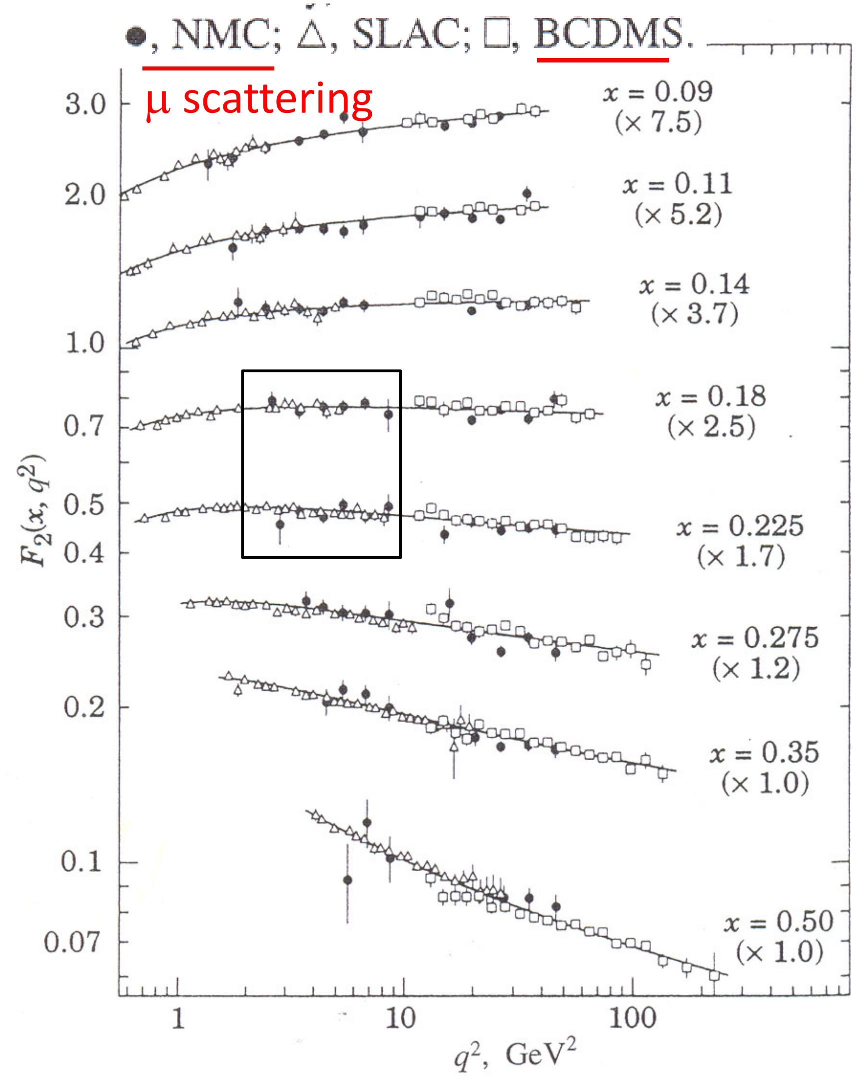
Precision determination of F_2 and of the parton distributions

After the first SLAC measurements many different DIS experiments have been conducted to determine F_2 and of the parton distribution of the proton:

Instead of electron also muons and neutrinos (CC interactions) have been used.

for νp scattering → new struct. func. $F_2^{\nu p}$, $F_3^{\nu p}$

A summary of early F_2 measurements is shown in the plot: it covers a much extended Q^2 range and much different x-values than the early SLAC measurements (range given in the box)



Scaling violation:

What is clearly noticeable is that F_2 (in the plot ×factor to avoid overlap) has indeed very little Q^2 dependence in range of early SLAC measurements (box). However at different Q^2 values and for different x-values the predicted "scaling behavior" is violated and F_2 is a clear function of both (x, Q^2).

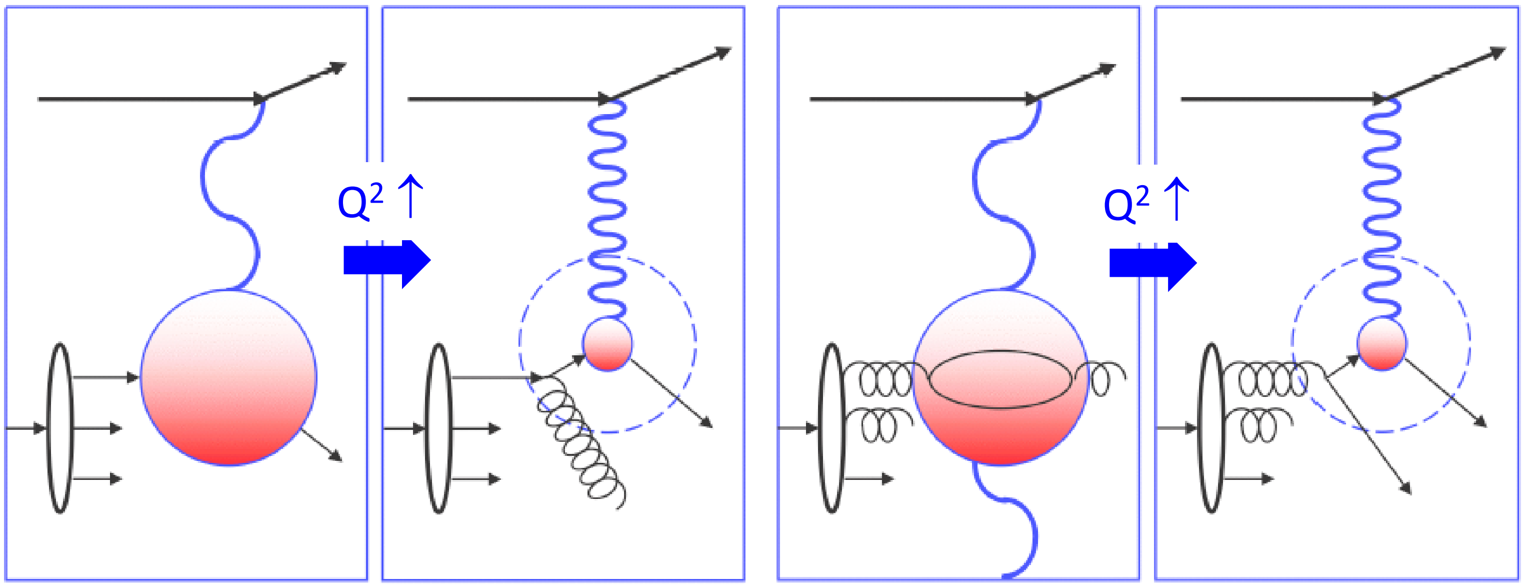
Reason: large dynamic effects between quarks ignored by simple parton model.

“Qualitative explanation” of observed scaling violation

Quantitative description (DGLAP evolution) is the topic of next semester!

Scattering at large x → mostly valence quark, at small x → mostly sea quark

Changing Q^2 one can change “the resolution” of the virtual photon (λ):



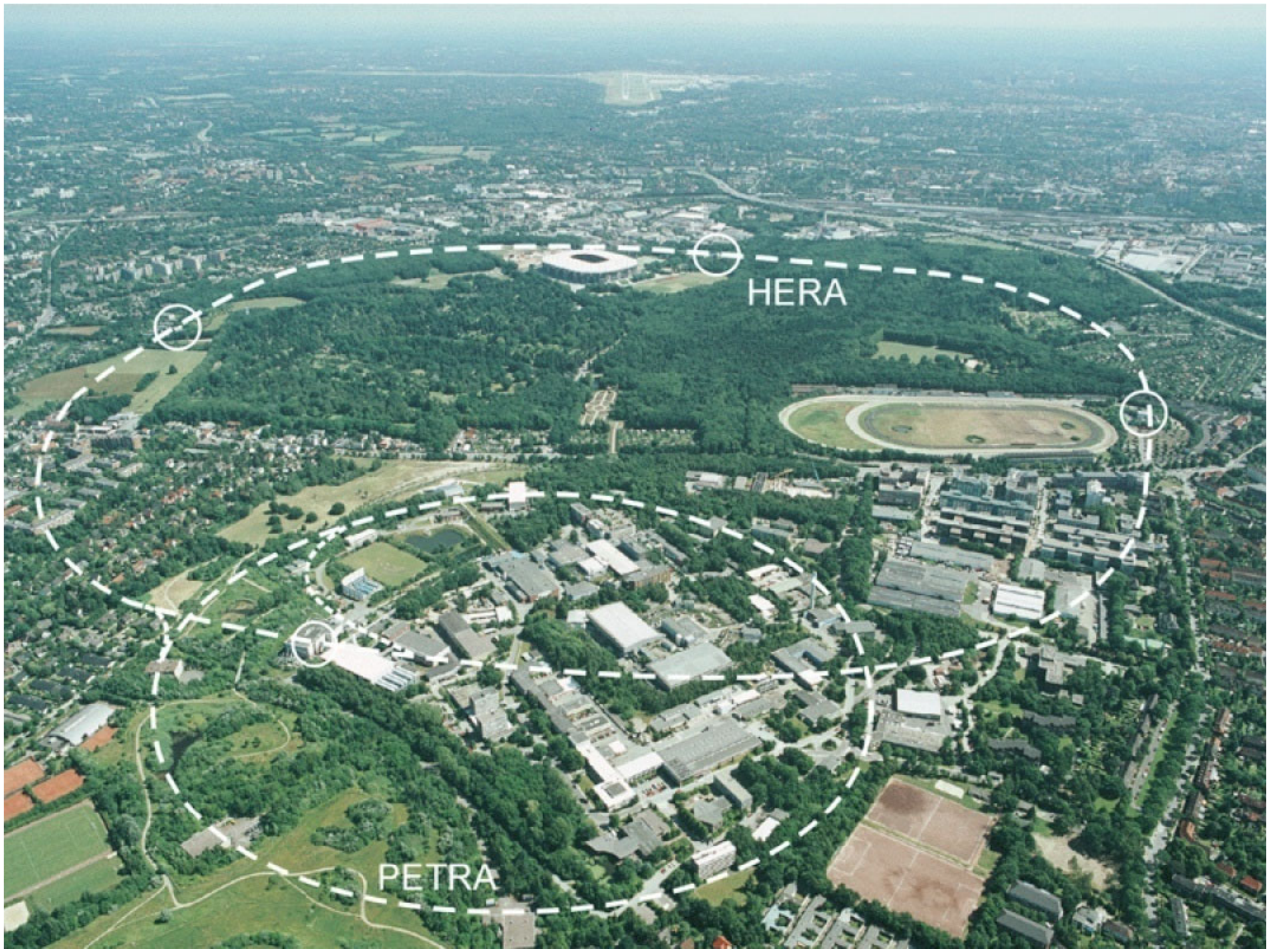
⇒ F_2 at fixed (large) x ↓ ⇒ F_2 at fixed (small) x ↑

Scaling violation is a clear manifestation of radiative effects predicted by QCD. PDFs (and structure functions) depend on Q^2 and x.

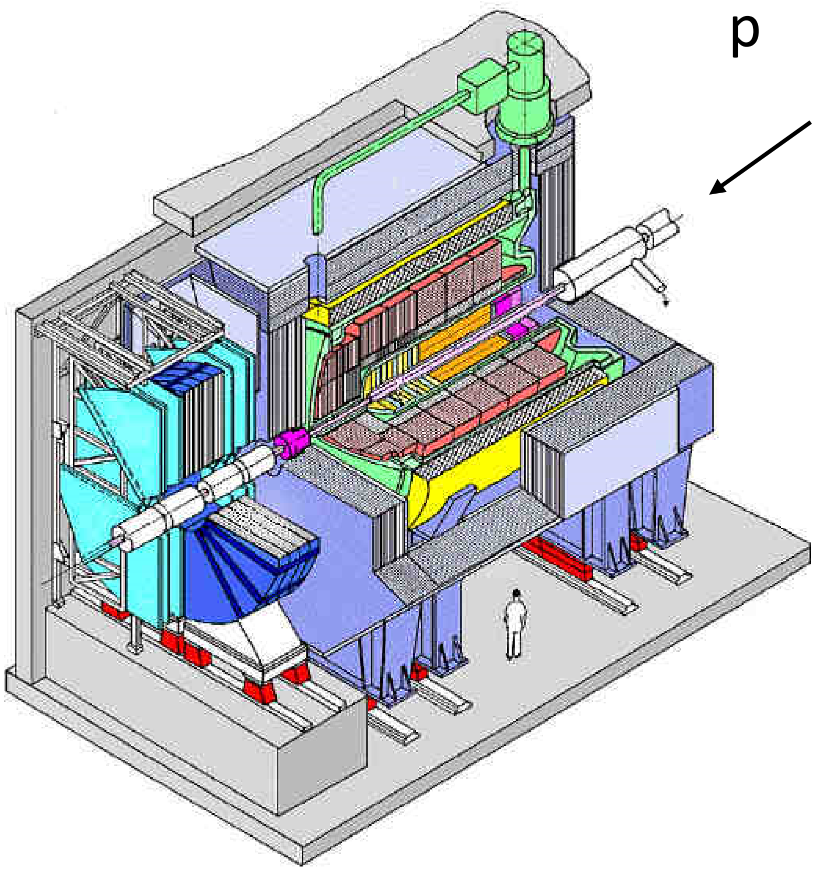
PDF = Parton distribution/density Functions

Precise measurement of PDFs at HERA

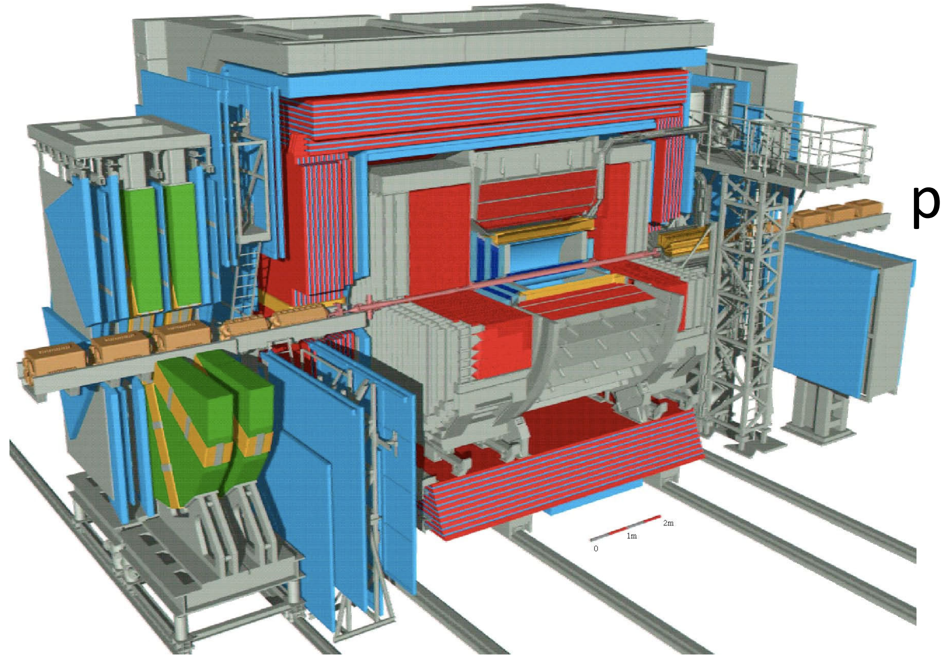
HERA collider: e → 30 GeV ← p 900 GeV $s = 4E_eE_p \approx 10^5 \,\text{GeV}^2$

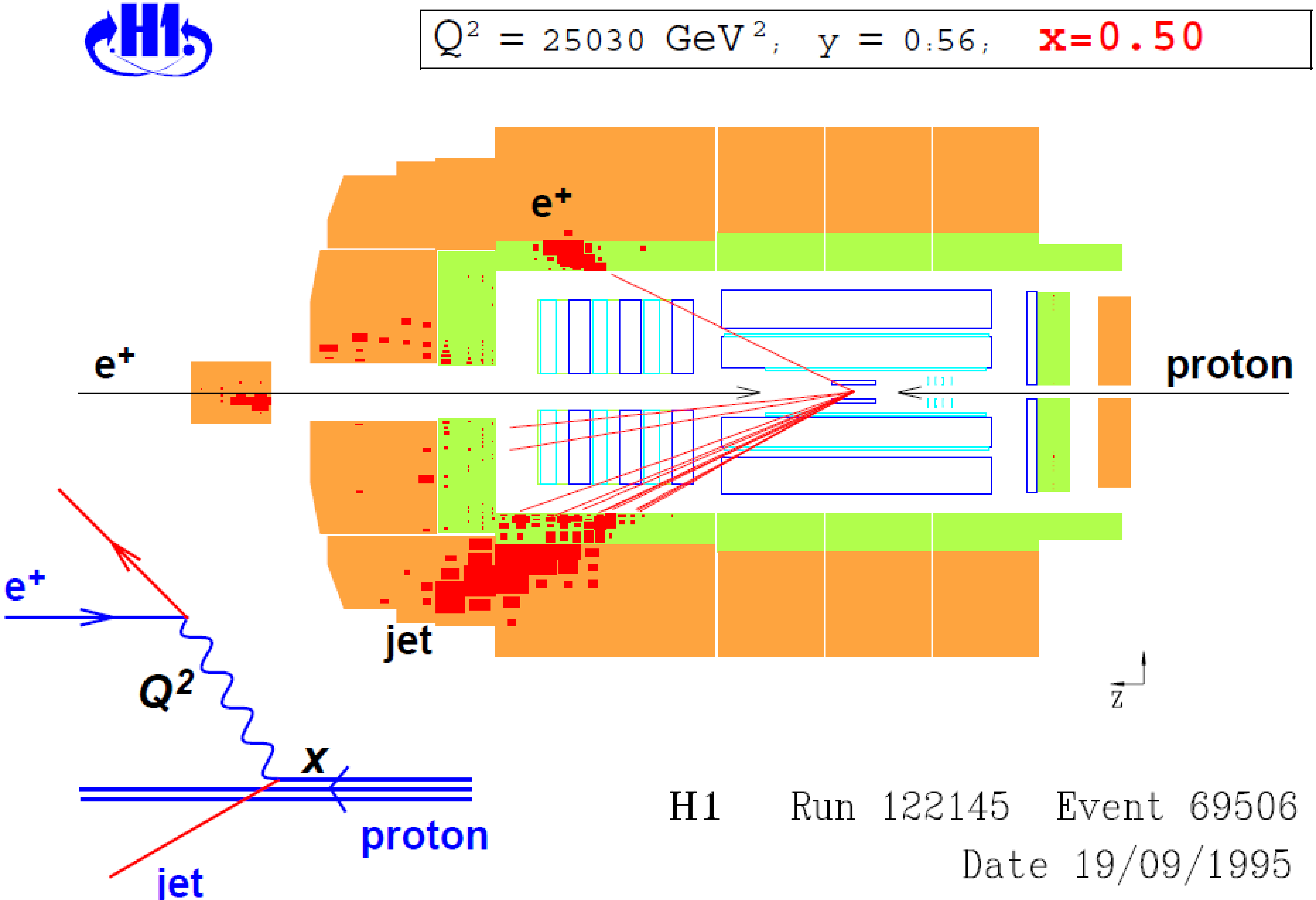


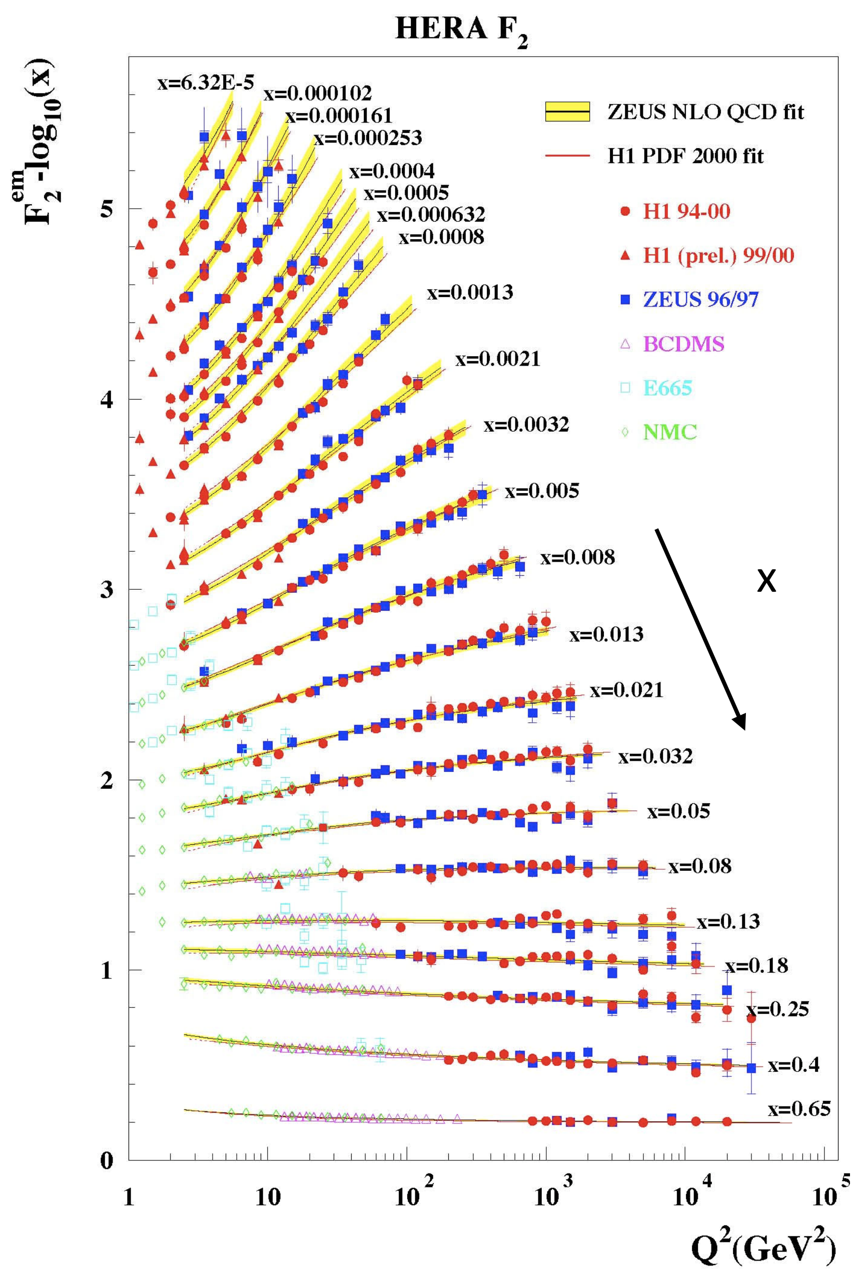
H1 detector

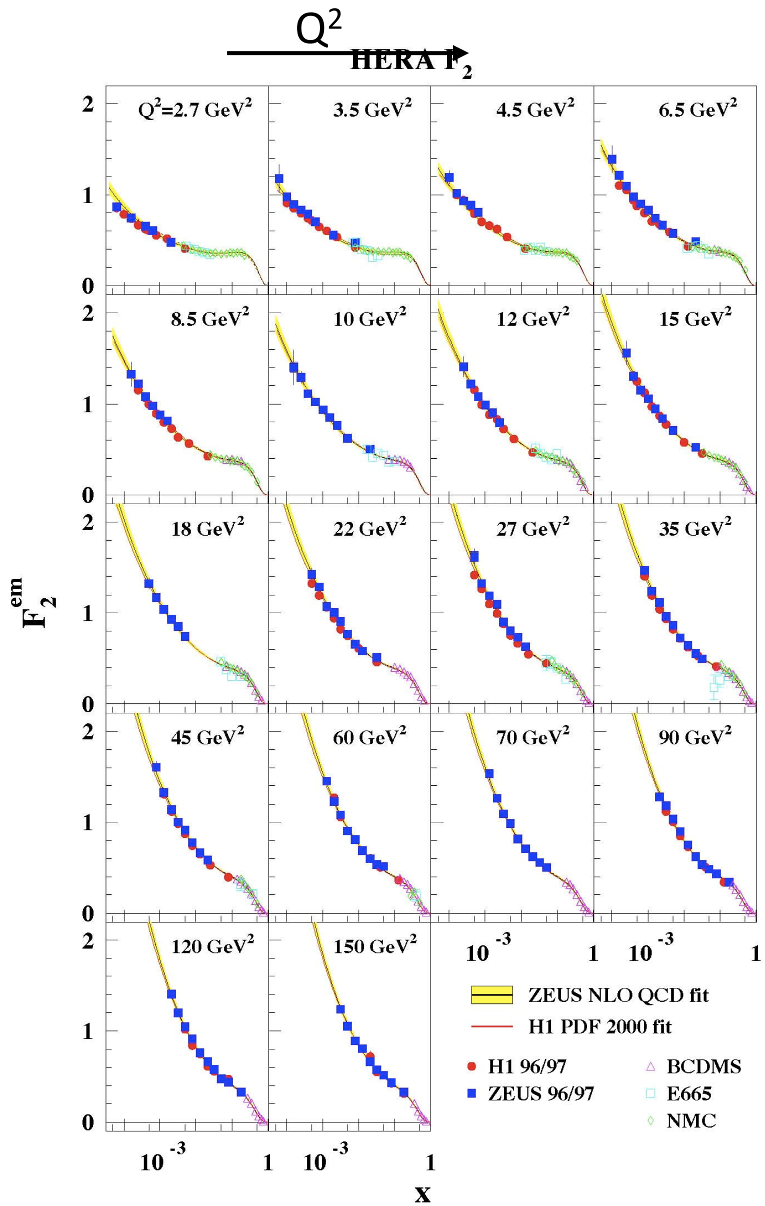


ZEUS detector




Q^2 = 25030 GeV^2, y = 0.56, x=0.50
e+
e+
proton
jet
z
e+
Q^2
x
proton
jet
H1 Run 122145 Event 69506
Date 19/09/1995

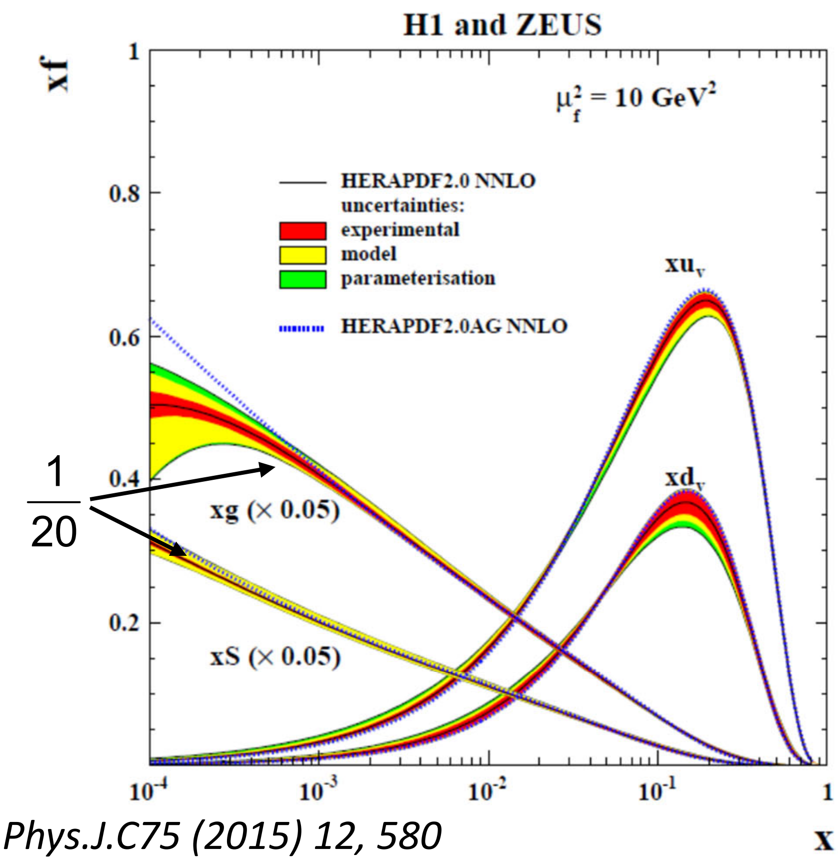




Determination of PDFs relies on factorization

$$d\sigma \sim d\sigma_{eq} \times F_2$$

QCD fit to the data – proton PDFs for a given Q^2 scale



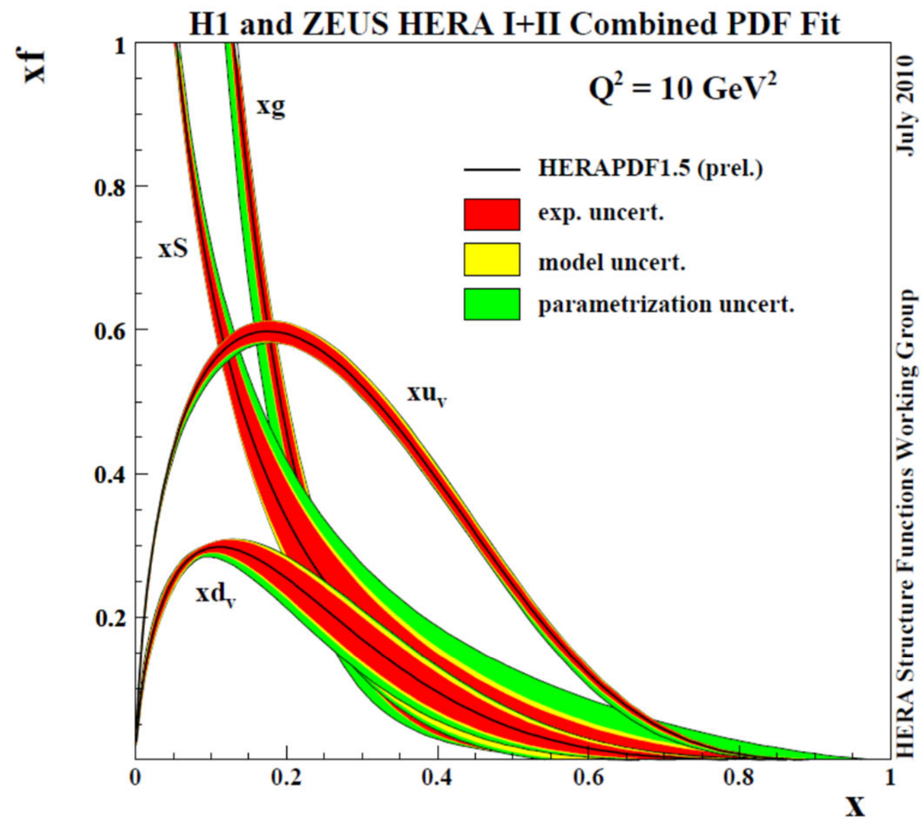


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Figure 23: The parton distribution functions xu_v, xd_v, $xS = 2x(\bar{U}+\bar{D})$ and xg of HERAPDF2.0 NNLO at $\mu_f^2 = 10\,\mathrm{GeV}^2$. The gluon and sea distributions are scaled down by a factor 20. The experimental, model and parameterisation uncertainties are shown. The dotted lines represent HERAPDF2.0AG NNLO with the alternative gluon parameterisation, see Section 6.8.

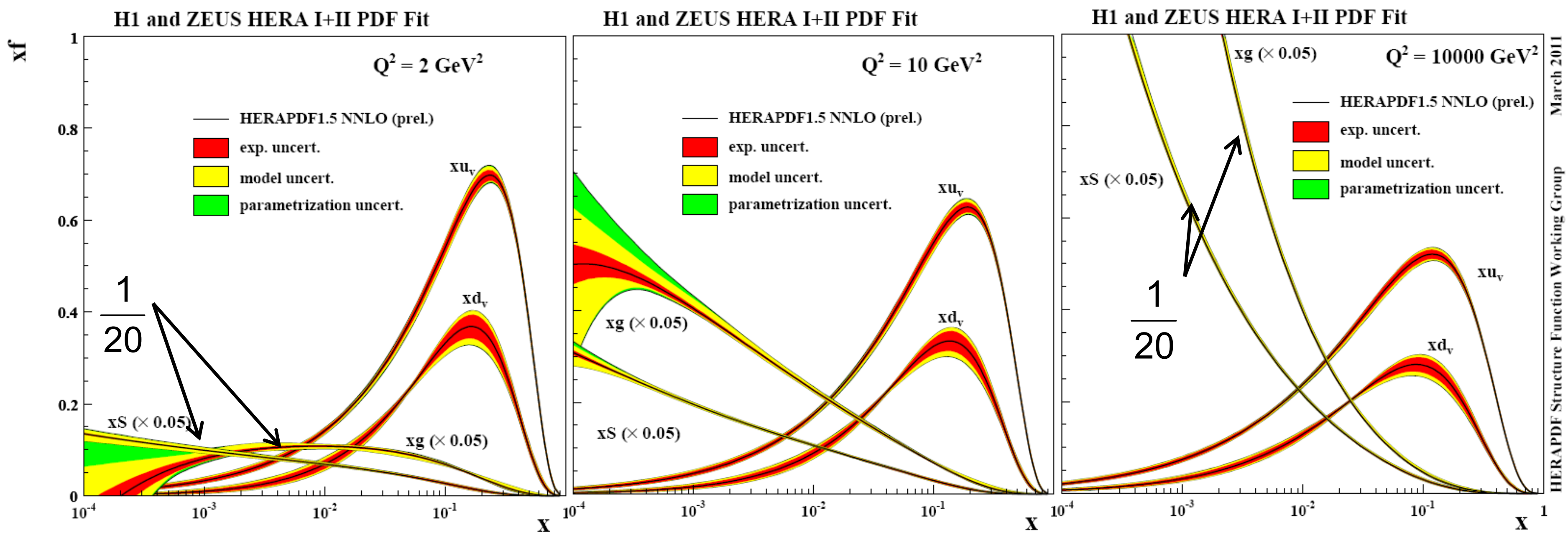
Linear scale for illustration (it is not exactly the same pdf set, but nearly)

Remarks:

- At low x, sea quarks dominate (xS in the plot) the scattering → huge gluon content
- While the proton C

Q2 evolution (predicted by QCD – DGLAP)

DGLAP = Dokshitzer, Gribov, Lipatoiv Altarelli, Parisi



https://www.desy.de/h1zeus/combined_results/

The most dramatic of these [experimental consequences] is that the protons viewed at ever higher resolution would appear more and more as field energy (soft glue), was only clearly verified at HERA ... F. Wilczek [Nobel Prize 2004]