Introduction to the (Electro-) Weak Interaction

<u>Reminder</u>: helicity vs. chirality

- Chiral eigenstates are Lorentz-invariant but are <u>not</u> stationary states under time evolution

$$\left.\begin{aligned} P_R &= \tfrac{1}{2}(1+\gamma_5) \\ P_L &= \tfrac{1}{2}(1-\gamma_5) \end{aligned}\right\} P_R + P_L = \underline{1}$$

- Helicity eigenstates are <u>not</u> Lorentz-invariant in general, but are stationary states of the motion

$$\vec{\Sigma}\cdot\hat{p} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & \vec{0} \\ \vec{0} & \vec{\sigma}\cdot\hat{p} \end{pmatrix} = 2\vec{S}_4\cdot\hat{p}$$

(here $\vec{S}_4 \sim \frac{1}{2}\mathbb{1}_4 \otimes \vec{\sigma}$)

also written sometimes as $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$$\Sigma^i = \frac{1}{2}\epsilon_{ijk}\sigma^{jk}$$
$$= \gamma_5\gamma^0\gamma^i$$

do not confuse with relativistic 4-polarization:

$s^\mu = (0, \vec{S})$ in rest frame

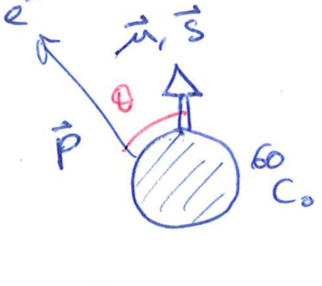
here, $\frac{1}{2}\vec{\sigma}$ (zero helicity)

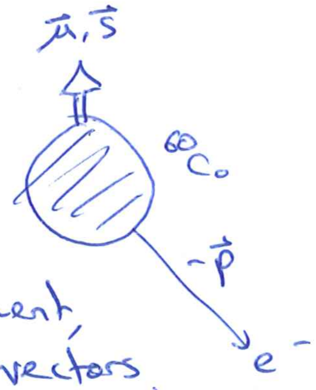
Lorentz boost:

$$s^0 \to \vec{\beta}\cdot\vec{S}, \text{ i.e. helicity}$$
$$\vec{S} \to \vec{S} + \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{S}) - \gamma\vec{\beta}s^0$$

<u>Reminder</u>: Wu experiment, 1957

$$^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$$
$$^{60}Ni^* \to {}^{60}Ni + 2\gamma$$





Positive magnetic moment, $\vec{B}$ and $\vec{\mu}$ are axial vectors (pseudovectors)

Experimentally check the direction of e^- emission and look for asymmetry:

$$I(\theta) \sim 1 + \alpha\left(\frac{\vec{\sigma}\cdot\vec{p}}{E}\right)$$
$$= 1 + \alpha\frac{v}{c}\cos\theta$$

contrast/visibility: $\dfrac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

^{60}Co has a spin $(S=5)$ and can be polarized (at low T) in a $\vec{B}$ field. $^{60}Ni^*$ has $S=4$

$$^{60}Co \longrightarrow {}^{60}Ni^* + e^- + \bar{\nu}_e \quad \Rightarrow \quad e^- \text{ and } \bar{\nu}_e \text{ emitted with spin up to conserve angular momentum}$$

$\Delta S = 1$ $\quad S=\frac{1}{2} \quad S=\frac{1}{2}$

Observation: electrons emitted preferentially with momentum opposite the ^{60}Co spin direction

$\Rightarrow$ negative helicity inferred from this momentum asymmetry

$\Rightarrow$ positive helicity for $\bar{\nu}_e$ (inferred)

Recall that for $m \to 0$ (or ultra-relativistic limit) the helicity states go over to the chiral ones:

$$\vec{\sigma}\cdot\hat{p} = +1 \text{ (positive)} \sim RH$$
$$\vec{\sigma}\cdot\hat{p} = -1 \text{ (negative)} \sim LH$$

$m_{\bar{\nu}} \approx 0 \Rightarrow$ "anti-neutrino is right-handed"

Goldhaber: "neutrinos are left-handed" (as we will see later on...)

The two experimental scenarios from the Wu measurement are also related by a parity symmetry $\Rightarrow$ the interaction which dominates this decay is not parity-conserving!

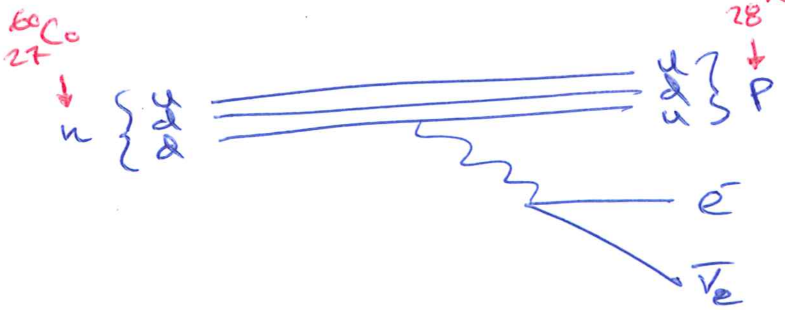
Recall from the theory part, that the vector interaction vertex from QED is parity-conserving ($j^\mu \sim \bar{u}\gamma^\mu u$)

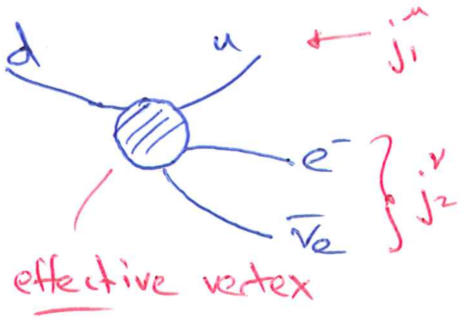
$\Rightarrow$ basic difference of weak interaction to QED (and QCD)

Question: what if we took $j_A^\mu \sim \bar{u}\gamma^\mu\gamma_5 u$

... would this interaction violate parity?

- Measured quantities involve <u>squared</u> amplitudes $\gamma_5^2 = \mathbb{1}$
- Already at the level of the matrix element, we are considering an interaction of 2 <u>currents</u>:





suppose for both, we have

$$j^\mu_{V-A} \sim \bar{u}(\gamma^\mu - \gamma^\mu \gamma_5) u$$

or $\nu, \bar{\nu}$

insert g_V g_A to scale the relative strength (just numbers)

$$M \sim j_1 \cdot j_2 \sim \underbrace{g_V^2 + g_A^2}_{\text{conserves parity}} + \underbrace{g_V g_A + g_A g_V}_{\text{violates parity}}$$

$\Rightarrow \dfrac{2 g_A g_V}{g_A^2 + g_V^2}$ gives the relative strength of parity-violation in relation to the parity-conserving part

- "maximal" for $|g_A| = |g_V|$
- vanishing for $g_A = 0$ <u>or</u> $g_V = 0$

The SM charged-current interaction (mediated by W) turns out to have equal weighting of the V and A contributions.
→ can be modified, e.g., in bound states and where NC is important

The neutral-current (mediated by Z) has weighting that depends on electric charge and <u>weak</u> <u>isospin</u>: ~~a new~~ quantum number.

Standard Model symmetries and particle content:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \qquad \text{discrete symmetries } C, P, T$$

↑ like spin

<u>Spin</u>:
- spin-0 → singlet (pion)
- spin-$\frac{1}{2}$ → doublet $(e, p, n, \dots)$ $\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$ ← by convention
- spin-1 → triplet $(d, J/\psi, \rho, \dots)$

Electroweak: experimentally there is a triplet: $\underbrace{W^-, W^0, W^+}_{\vec{W}}$

... and all quarks/leptons are observed to be either a singlet or member of a doublet

Take e^- as example, spinor $\psi_{e^-} = (P_R + P_L)\psi_{e^-} = e^-_R + e^-_L$

singlet

part of doublet with LH neutrino

key point: L and R transform differently under SU(2) electroweak

$e^-_R \sim$ SU(2) singlet

$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \sim$ SU(2) doublet

Note ν_e on top (cf. ↑) unlike on the current wikipedia particle chart!

I.e., when L points "up" in SU(2) electroweak space, it represents ν_{eL} while "down" $\sim e^-_L$. Rotations in this space can produce transitions: $\nu_{eL} \leftrightarrow e^-_L$

(just like strong isospin: $p \leftrightarrow n$)

... except better!

Raising/lowering operators can be defined, like for angular momentum: but for e^-_R, electroweak transitions don't connect it to any other state.

Weak isospin: fermions with negative (LH) chirality have $T = \frac{1}{2}$ and can be grouped in doublets with $T_3 = \pm\frac{1}{2}$

e.g. $\begin{pmatrix} u \\ d \end{pmatrix}$ or $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ $\begin{matrix} \leftarrow +\frac{1}{2} \\ \leftarrow -\frac{1}{2} \end{matrix}$

The right-handed ones are singlets: u_R, e^-_R, etc.

Anti-particles get the opposite sign of T_3 (as for other quantum numbers) $\Rightarrow$ the antiparticle doublet is for RH and gets "flipped over".

RH particles and LH anti-particles have $T = 0$ (singlet)

To be more completely discussed in the EW theory block:

- charged current ($W^\pm$) only interacts with doublet states
- neutral current (Z^0) interacts with everything

Be careful: SM unifies $\underset{\text{weak isospin}}{SU(2)_L} \times \underset{\text{weak hypercharge}}{U(1)_Y}$

The initially massless neutral bosons mix,

$\vec{W} = (W^-, \underset{T=1,\, T_3=0}{W^0}, W^+)$ and $\underset{T=0,\, T_3=0}{B^0}$

"rotation" by θ_W,

$$\begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \Rightarrow \begin{aligned} A_\mu &= B_\mu \cos\theta_W + W^0_\mu \sin\theta_W \\ Z_\mu &= -B_\mu \sin\theta_W + W^0_\mu \cos\theta_W \end{aligned}$$

The massive Z^0 and massless γ (of QED) are observed below the electroweak scale of $\boxed{v \approx 246 \text{ GeV}}$
"Higgs vev"

Breaking of EW symmetry and acquisition of mass $\to$ Higgs

The correct vertex factors for $W^\pm$ and Z^0 turn out to be,

charged current

$$-i \frac{g_W}{\sqrt{2}} \underbrace{\frac{1}{2} \gamma^\mu (1-\gamma_5)}_{\gamma^\mu P_L}$$

neutral current

$$-i \frac{g_Z}{2} \gamma^\mu (c_V - c_A \gamma_5)$$

$c_L - c_R = T_3$

$c_L + c_R = T_3 - 2Q\sin^2\theta_W$

Compare to $-iQe\gamma^\mu$ for QED... but θ_W simplifies many things:

$$g_Z = \frac{g_W}{\cos\theta_W} = \frac{e}{\sin\theta_W \cos\theta_W} \quad \text{and} \quad \underset{80.4\text{ GeV}}{m_W} = \underset{91.2\text{ GeV}}{m_Z} \overbrace{\cos\theta_W}^{[-1,1]}$$

Once θ_W is known ($\theta_W \approx 0.5$ rad, $\sin^2\theta_W \approx 0.231$) the properties of the Z^0 are completely specified.

$$c_L = T_3 - Q\sin^2\theta_W \qquad c_V = c_L + c_R = T_3 - 2Q\sin^2\theta_W$$
$$c_R = -Q\sin^2\theta_W \qquad c_A = c_L - c_R = T_3$$

fermion	Q	T_3	Y_L, Y_R	c_L, c_R	c_V, c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	$-1, 0$	$+\frac{1}{2}, 0$	$+\frac{1}{2}, +\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	$-1, -2$	$-0.27, +0.23$	$-0.04, -\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}, +\frac{4}{3}$	$+0.35, -0.15$	$+0.19, +\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}, -\frac{2}{3}$	$-0.42, +0.08$	$-0.35, -\frac{1}{2}$

Now in general, we can construct fermion currents via any of the Lorentz tensor structures that were already introduced. But there are limited options, to maintain a Lorentz-invariant result. The propagator will connect currents, e.g. like in QED:

$$\mathcal{M} \sim Q_a Q_b \frac{(j_a \cdot j_b)}{q^2} \longrightarrow \text{built from scalar products}$$

Lorentz structures:

	name		parity	example
$\mathbb{1}$	scalar	"S"	+	mass, temperature
γ_5	pseudoscalar	"P"	−	helicity
γ^μ	vector	"V"	+	momentum, $\vec{E}$ field
$\gamma^\mu\gamma_5$	pseudovector	"A"	−	angular momentum, $\vec{B}$ field
$\frac{i}{2}[\gamma^\mu, \gamma^\nu]$	tensor	"T"	+	rel. 4-spin tensor

A general bilinear combination of two spinors:

or $\bar{v}, v$

$\bar{u}(p)\,\Gamma\,u(p') \longrightarrow$ e.g., (const.) $\times (\gamma^\mu \pm \gamma^\mu\gamma_5)$ $\begin{cases} \text{"V+A"} \\ \text{"V-A"} \end{cases}$

Γ: 4x4 matrix built from γ^μ, products etc. of above

*vertex factor drops external wavefunctions

Now for the propagator, recall in QED: $-i\dfrac{g_{\mu\nu}}{q^2}$ where the photon (γ) has spin-1 and this form is related to a sum over polarization states.

In analogy to the completeness relation $\sum_s u_s\bar{u}_s = \not{p}+m$ for spin-sums of fermions, for a spin-1 boson

$$\sum_\lambda \epsilon_\lambda^{*\mu}\epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m^2} \longleftarrow \text{what about photons?}$$

- on-shell, $q^2=0$
- off-shell, $q^2\neq 0$ and $\frac{q^\mu q^\nu}{m^2} \to \frac{q^\mu q^\nu}{q^2}$

For the massless case, the equation of motion $\partial^2 A_\mu = 0$ and Lorenz condition $\partial_\mu A^\mu = 0$

$\Rightarrow$ 1 d.o.f. removed (ϵ^μ is a 4-vector, 3 remain)

and gauge-fixing removes one more, since $\epsilon_\mu \to \epsilon_\mu + a q_\mu$ must represent the same physical photon! (a: arb. const)

$\Rightarrow$ this means the $\frac{q^\mu q^\nu}{q^2}$ term doesn't matter anyway (by inspection of $\sum_\lambda j_\mu j_\nu \epsilon^{*\mu}\epsilon^\nu$)

$\Rightarrow$ only 2 polarization states remain! (Coulomb gauge $\to$ transverse)

For massive spin-1, the Lorenz condition is satisfied automatically from the equation of motion, and there is no further gauge freedom

$(\partial^2+m^2)A_\mu - \partial_\mu(\partial^\nu A_\nu) = 0$ "Proca eqn."

$\Rightarrow$ 3 d.o.f. remain (out of 4) $\Rightarrow$ there is an additional (longitudinal) polarization state for massive spin-1

The Lorentz-invariant form of the matrix element for 2 vertices can also be derived via perturbation theory:

$$M \sim \frac{g_a g_b}{q^2 - m^2}$$

$g_a g_b$ ← 2 vertices

$q^2 - m^2$ ← propagator for virtual particle with mass m coupling them

Putting it together,

$$P_W = \frac{-i}{q^2 - m_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right) \xrightarrow{q^2 \ll m_W^2} \frac{-i g_{\mu\nu}}{q^2 - m_W^2} \xrightarrow{q^2 \ll m_W^2} \frac{i g_{\mu\nu}}{m_W^2} \sim G_F$$

for P_Z just replace $m_W \to m_Z$

<u>Now</u> $m_W \approx 80$ GeV and $m_\mu \approx 100$ MeV $\Rightarrow \frac{m_\mu^2}{m_W^2} \approx 10^{-6}$

$$P_{QED} \sim \frac{1}{q^2} \qquad P_W \sim \frac{1}{q^2 - m_W^2} \begin{cases} \text{high-E} \\ \text{low-E} \end{cases}$$

In the "high-energy" limit, m_W is not important in comparison to q^2 $\Rightarrow$ similar strength of QED vs EW

In "low-energy" limit, W-mediated interactions are suppressed by q^4/m_W^4 $\Rightarrow$ this is the sense in which it is "weak"

We now have the Feynman rules (vertex and propagator), noting that only LH particles (RH anti-particles) participate in charged-current interactions.

- unlike QED and QCD, <u>all</u> SM particles participate in weak interactions (ok, all the fermions...)

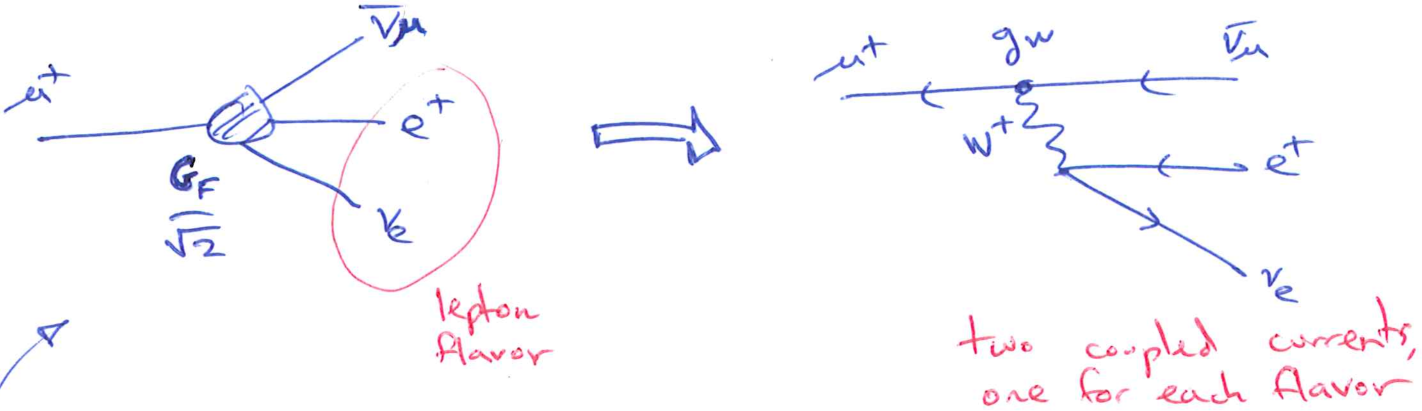
- every process appears consistent with a universal, dimensionless coupling*: compare

$$\alpha_W = \frac{g_W^2}{\hbar c} \sim \frac{1}{30} \quad \text{vs} \quad \alpha_{QED} = \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$\frac{\alpha_{QED}}{\alpha_W} = \frac{e^2}{g_W^2} = \sin^2\theta_W \approx 0.23$

*NB: coupling "runs" with the energy → later theory + expt.s

Historically the weak interaction was understood bottom-up, i.e., decay via weak charged currents involved a nonlocal current-current coupling (4-fermion)



This is a convenient effective field theory (EFT) for charged-current weak interactions, and for defining G_F

(Note G_F was originally written for a pure vector current, so there is a factor of $\sqrt{2}$ to properly normalize the V-A theory.)

- V-A was an experimental conclusion

Matrix elements for leptons ($\ell \sim u_\ell$, etc. for $\ell \sim e, \mu, \tau$)

Effective (Fermi) Theory: $$\mathcal{M}_F = \frac{G_F}{\sqrt{2}}\left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right]\left[\bar{\nu}_{\ell'}\gamma^\mu(1-\gamma_5)\ell'\right]$$

Standard Model: $$\mathcal{M}_{SM} = \frac{g_w^2}{8}\left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right]\cdot\frac{-i\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{m_W^2}\right)}{q^2-m_W^2}\cdot\left[\bar{\nu}_{\ell'}\gamma_\nu(1-\gamma_5)\ell'\right]$$

(note the fermion currents are the same)

We might have expected $G_F \sim \frac{g_w^2}{m_W^2}$ from the propagator, but including the $\sqrt{2}$ and chiral projectors we get

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8m_W^2} = \frac{1}{2v^2}$$

where again $v \simeq 246$ GeV is the electroweak scale or "Higgs vev"

Note that in the $q^2 \to 0$ limit, $M_{SM} \to M_F$ and we recover the direct current-current coupling of the Fermi theory as a low-energy <u>effective</u> coupling (interaction).

$\Lambda \sim 246$ GeV is a typical energy where the full Electroweak theory would be required...

Recall in natural units, $[L] = [E]^{-1}$ i.e. high energy $\leftrightarrow$ short distance

... so we can also think of this as defining (vaguely) a resolution scale for the size of structures or range of interactions that can be resolved

The Fermi theory is not renormalizable ($\rightarrow$ later theory block) but:

- QED corrections to the leading Fermi interaction are finite to all orders ($\Rightarrow$ we can separately parameterize the weak interaction and electromagnetic corrections)

- G_F includes (in one number!) <u>all</u> weak interaction effects in the low-energy EFT (full dynamics of W etc., insofar as they impact low-energy observables)

The muon lifetime, experimentally, is a powerful way to determine G_F

$$\mu^\pm \to e^\pm \overset{(-)}{\nu_e} \overset{(-)}{\nu_\mu}$$

- no hadronic corrections until sub-ppm precision (via 2-loop QED) due to mass
- τ_μ well suited to precise experimental measurement
- clear theory interpretation

The decay rate is

corrections

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 + \sum_i \Delta q_i\right)$$

Δq_0 ~ phase space

Δq_1 ~ 1st order QED

Δq_2 ~ 2nd order QED

etc.

Sargent's rule: weak interactions have rate $\propto Q^5$

available energy

mass

we know $[\Gamma] \sim [E] \sim [M]^{+1}$

$[\tau] \sim [M]^{-1}$

matrix element

$\Gamma \propto G_F^2$ from $|M_F|^2$

$[G_F] \sim [M]^{-2}$

$\Rightarrow G_F^2 Q^5 \sim [M]^{+1}$

rest frame, $Q \sim m_\mu$