QCD Test at Colliders

1. DIS and scaling violation and determination of PDFs
2. Hadronic final states
3. Jets at colliders
4. Hadron collider physics
5. Results from LHC

References (other than T. Plehn's scripts):

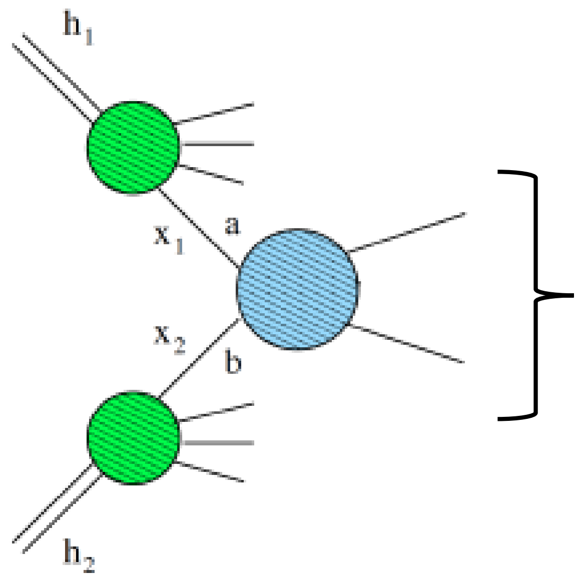
- P. Z. Skands, Introduction to QCD, https://arxiv.org/abs/1207.2389
- Alexander Mitov, Lectures on QCD in the LHC precision era https://indico.global/event/2342/attachments/16319/26285/A_Mitov-QCD-lectures.pdf
- G. Salam, Basics on QCD, ICTP–SAIFR school on QCD and LHC physics, https://gsalam.web.cern.ch/repository/talks/2015-SaoPaulo-lecture3.pdf

1. DIS and scaling violation and determination of PDFs

Theory wrap-up: factorization and DGLAP evolution of PDFs

Drell-Yan process (lepton-pair production) in hadron-hadron collisions:

$$pp \rightarrow \mu^+\mu^- + X$$



Hard process: DY, Higgs production, vector boson production

Inclusive cross section: $$\sigma = \iint dx_1 dx_2\, f_1(x_1,\mu^2) f_2(x_1,\mu^2)\,\hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$$

Soft and collinear gluon emission → divergences: absorbed in pdfs f_i

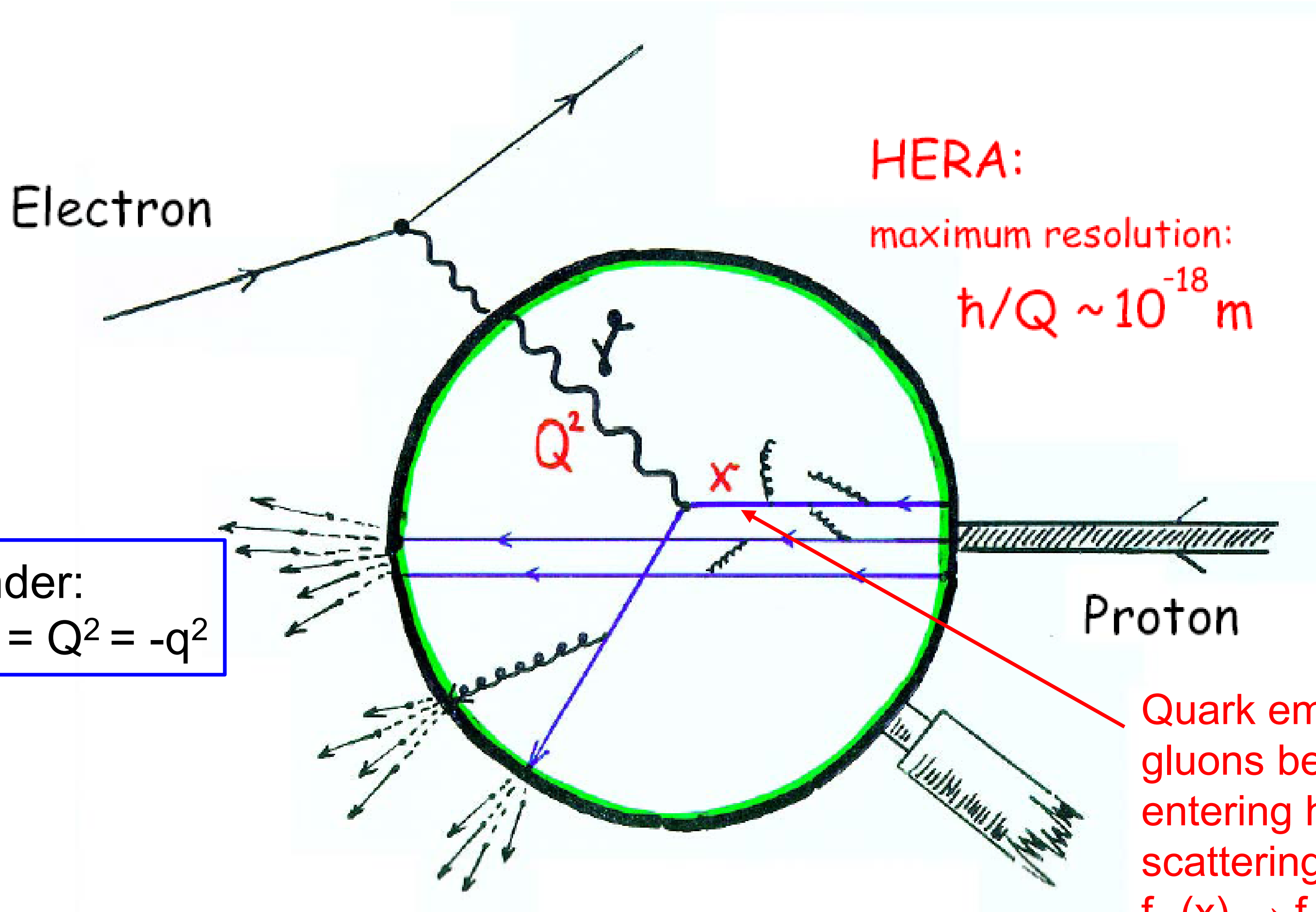
Factorization scale

DGLAP evolution for parton distribution functions:

$$\frac{df_i(x,\mu_F^2)}{d\log\mu_F^2} = \sum_j \int_x^1 \frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,P_{i\leftarrow j}(z)\, f_j\left(\frac{x}{z},\mu_F^2\right) = \frac{\alpha_s}{2\pi}\sum_j \left(P_{i\leftarrow j}\otimes f_j\right)(x,\mu_F^2)\,.$$

While DGLAP describes the scale dependence of pdfs it does not tell anything about the pdfs themselves: need to be determined in experiment – e.g. in DIS.

Deep inelastic scattering (DIS)

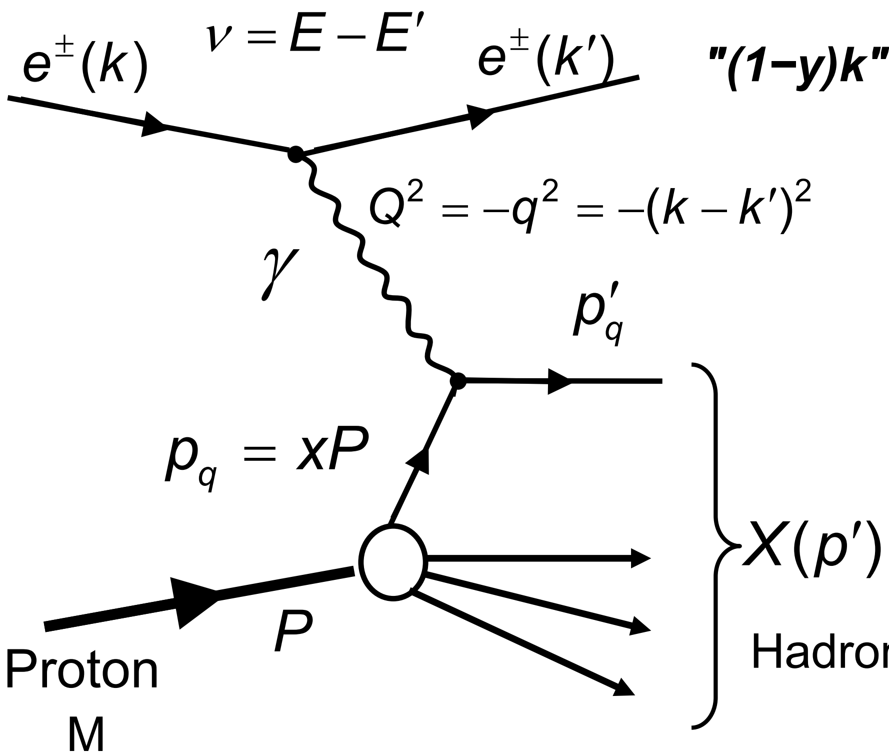


Reminder:
$-t = \mu^2 = Q^2 = -q^2$

Quark emits gluons before entering hard scattering:
$f_q(x) \rightarrow f_q(x, Q^2)$

Courtesy: H.C. Schultz-Coulon

Recap: Deep-inelastic scattering - kinematics



x = fractional momentum of struck quark
y = Pq/Pk = elasticity, fractional energy transfer in proton rest frame
ν = E - E′ = energy transfer in lab

$$y = \frac{P \cdot q}{P \cdot k}$$

fixed target

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$$ (Bjorken x)

$$Q^2 = sxy$$

s = CMS energy

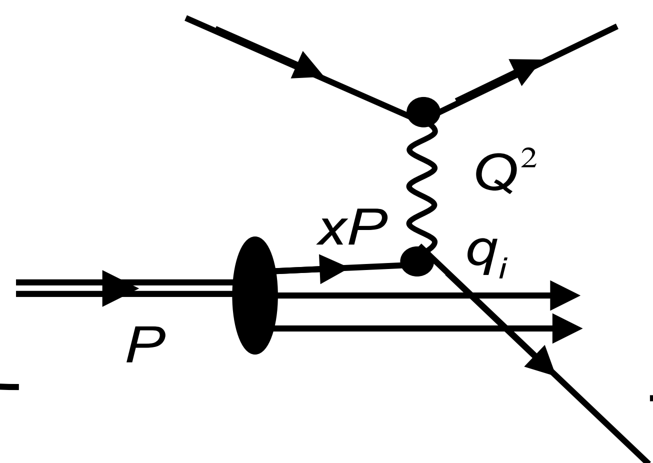
Recap: DIS ep scattering in the parton model

= elastic-scattering at quasi free point-like quarks:
only possible if momentum fraction of quark $\xi = x_{Bj}$

$$\xi = -\frac{q^2}{2Pq} = \frac{Q^2}{2Pq} = x$$

Symbolic form of cross section in parton model:

$$\sigma_{inel}^{ep} \sim \sum_{q_i} \int dx\, f_{q_i}(x)\ \hat{\sigma}\left[\ \right]$$

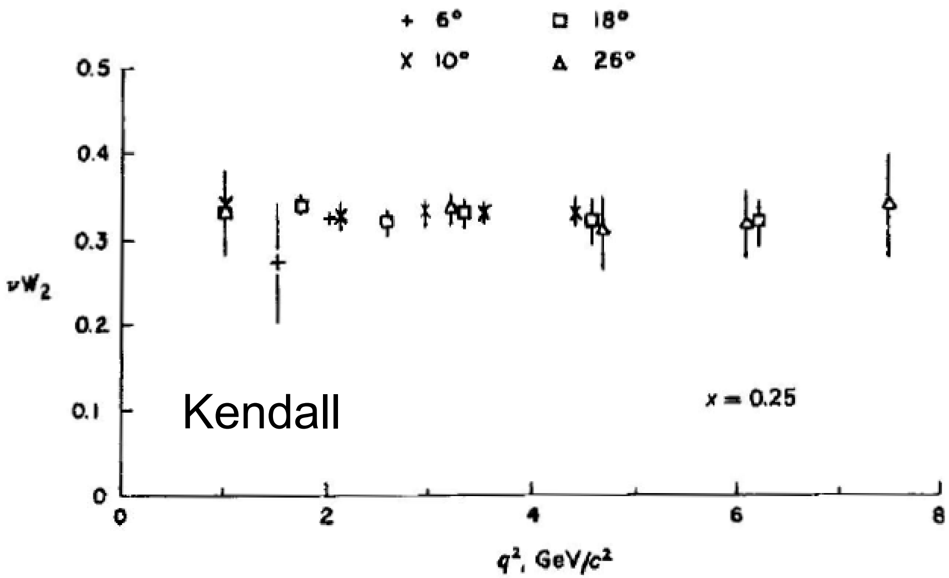


No Q^2 dependence

$$\frac{d\sigma_{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\cdot\sum_{q_i} Q_i^2 f_{q_i}(x) = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\cdot\frac{F_2^{ep}(x)}{x}$$

w/ process dependent structure function: $F_2^{ep}(x) = x\sum_i Q_{q_i}^2 f_{q_i}(x)$

Quasi-free point-like partons and Bjorken scaling:



From first DIS measurements: The Q^2-independence of the measured structure function $F_2(x)$ was interpreted as a confirmation of the parton model: quarks are quasi-free (non-interacting) point-like spin ½ constituents of the proton.

The fact that there is no explicit Q^2 dependence of the structure functions (Q2 scale invariance) in the simple quark parton model is called Bjorken scaling.

This is in contradiction with the expectation from QCD: DGLAP predicts an explicit Q^2 (scale) dependence of the pdfs: quarks are strongly interacting and permanently emit or reabsorb gluons.

Scaling violation

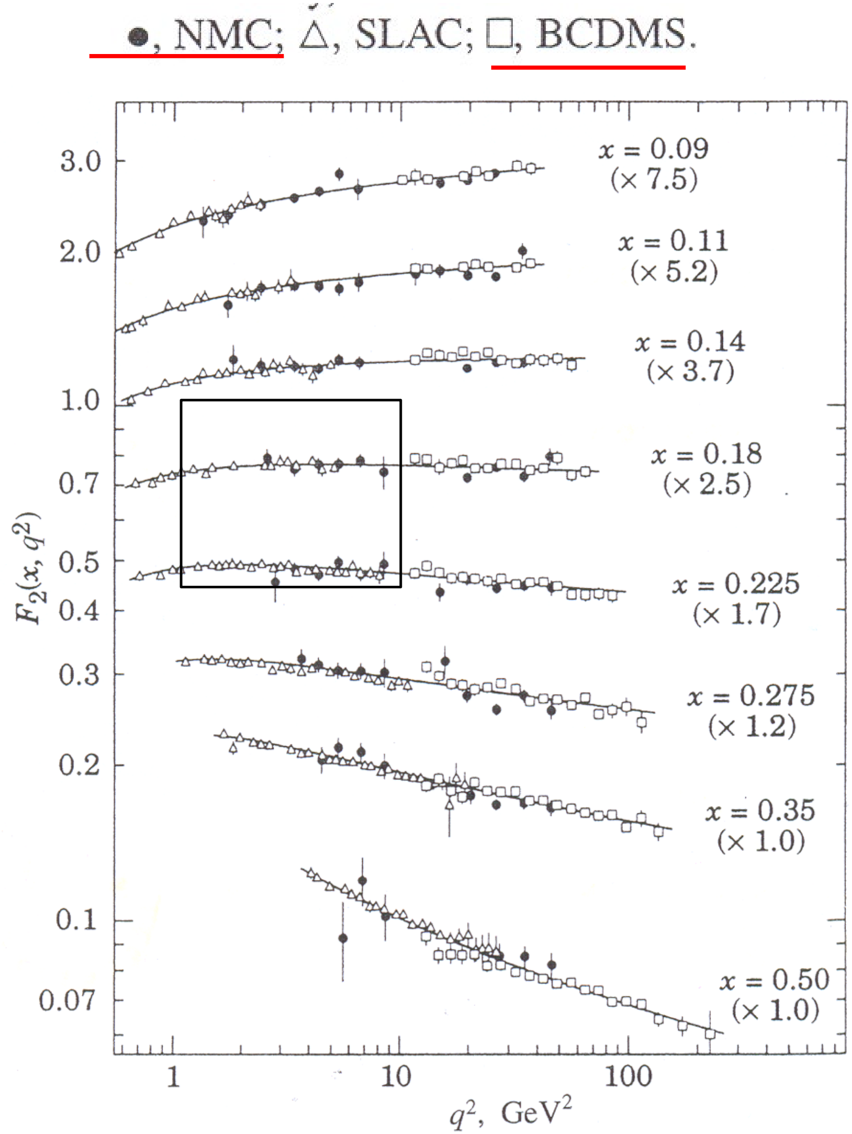
A summary of early F_2 measurements is shown in the plot: it covers a much extended Q^2 range and much different x-values than the early SLAC measurements (range given in the box)

Scaling violation:
What is clearly noticeable is that F_2 (scaled in the plot to avoid overlap) is has indeed very little Q^2 dependence for the early SLAC measurements (box). However at different Q^2 values and for different x-values the predicted "scaling behavior" is violated and F_2 is a clear function of both (x, Q^2).

The large dynamic effects between quarks ignored by simple parton model.

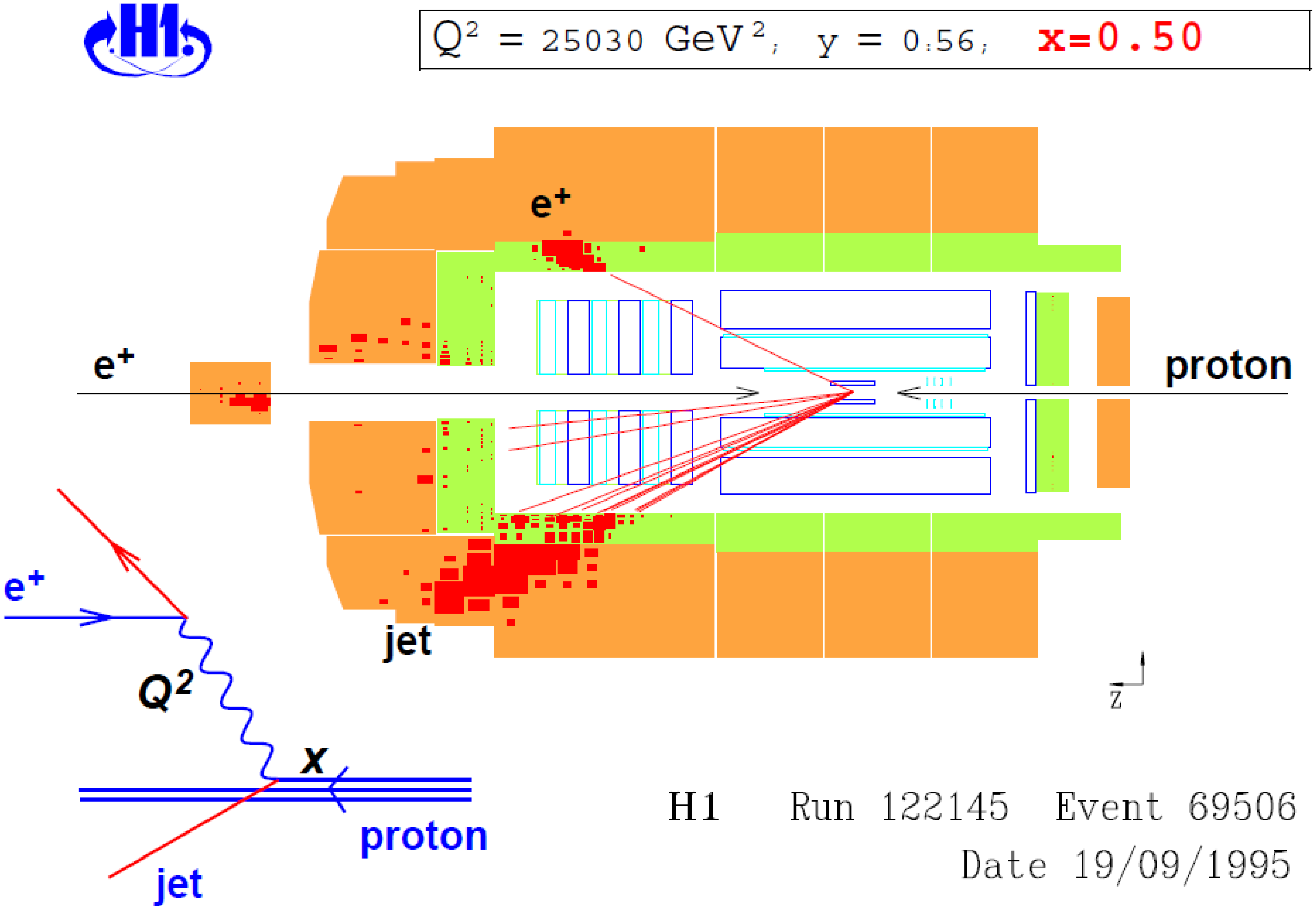
The effect becomes more pronounced if smaller x and larger Q^2 are investigated

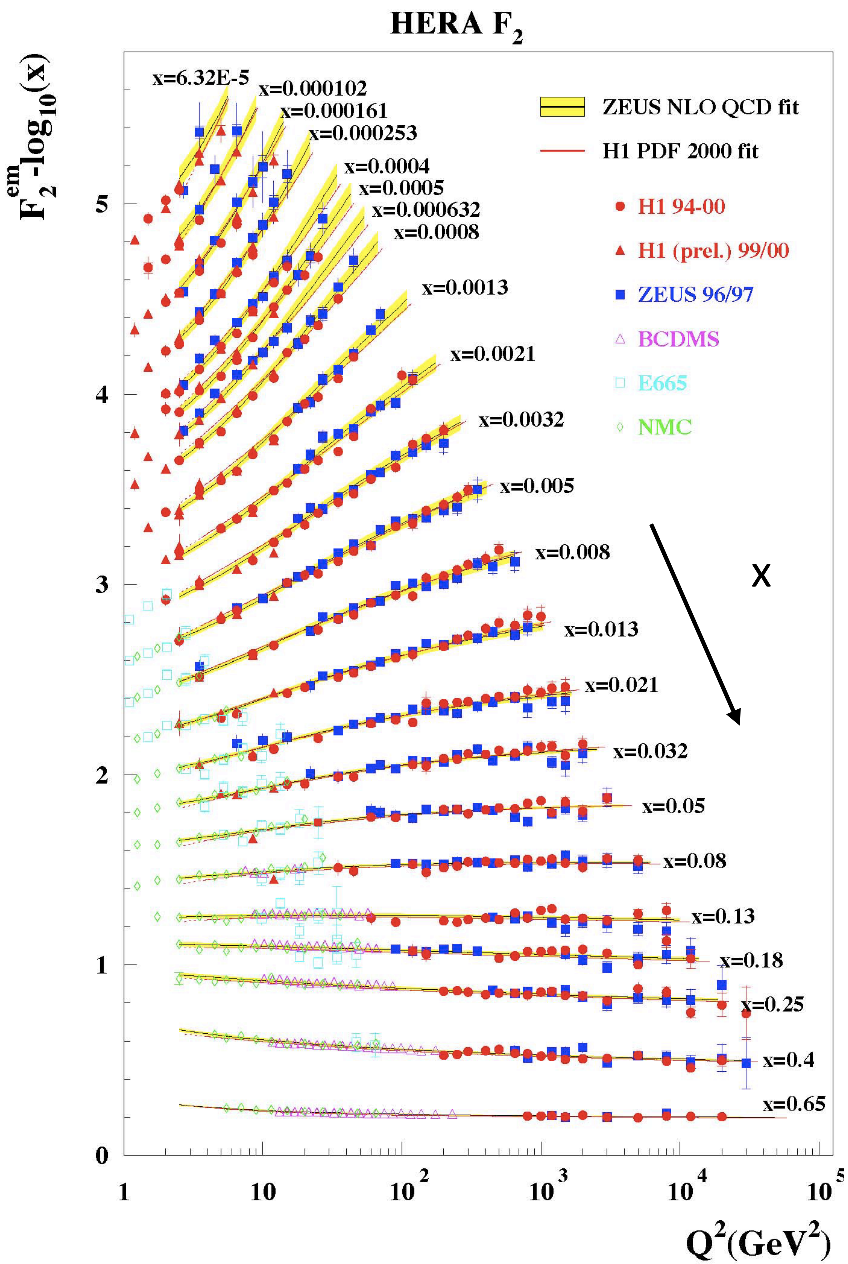
μ scattering

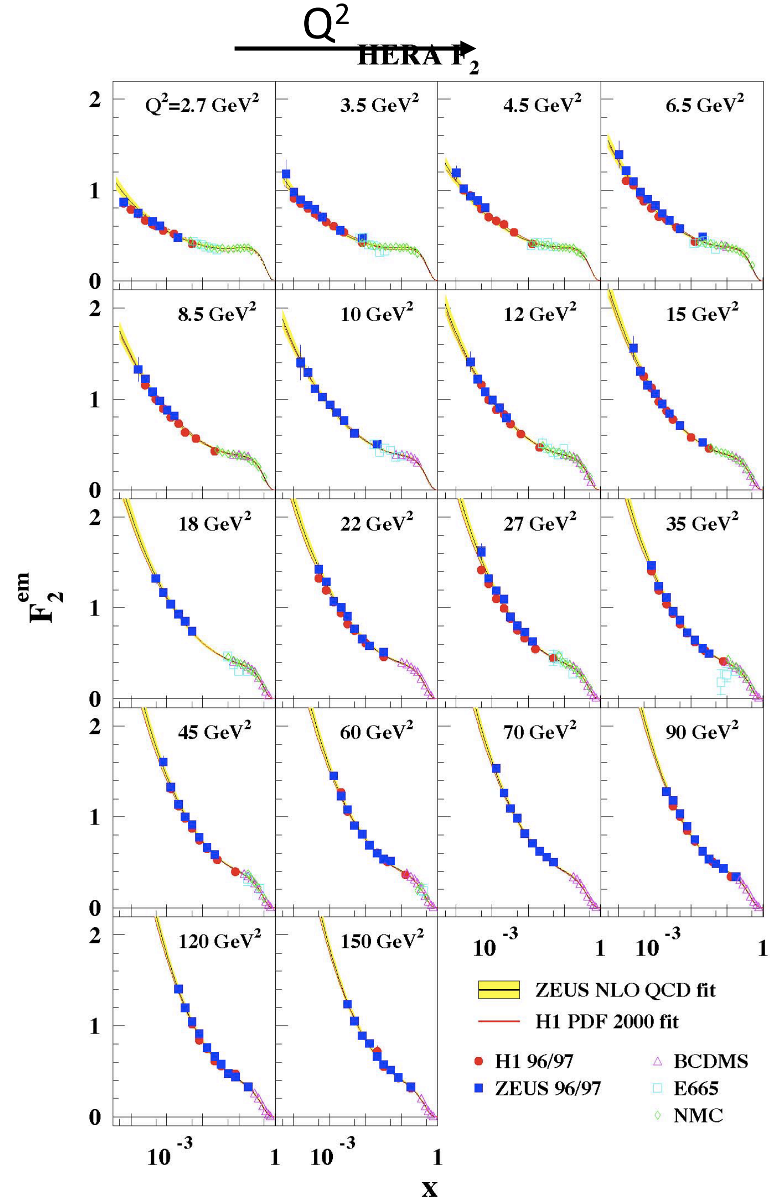


Precise measurement of PDFs at HERA

HERA e 30 GeV → ← p 900 GeV $s = 4E_eE_p \approx 10^5\,\text{GeV}^2$







Determination of PDFs relies on factorization

$$d\sigma \sim d\sigma_{eq} \times F_2$$

A word on PDF fits

Several groups are analyzing different data to determine pdfs:
ABMP, NNPDF, MMHT, HERA-PDF and CT perform global fits to available data (ep: DIS, hh: DY, inclusive jet production, etc.) and release their PDFs sets.

The data (typically differential cross sections) is described by: $\sigma = \hat{\sigma} \otimes f$

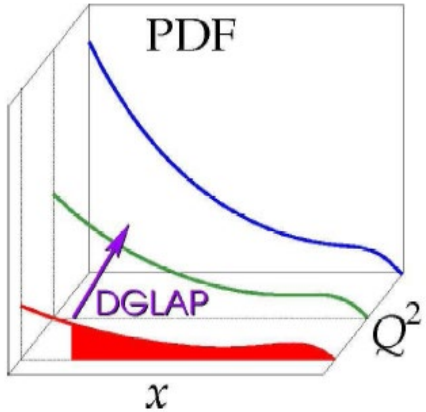
pdf (f)

Partonic xsect (LO, NLO,…) ($\hat{\sigma}$)

At a starting scale Q_0^2, typically below the charm mass threshold, quark and gluon PDFs are parametrized by polynomials in x, e.g. HERA-PDF:

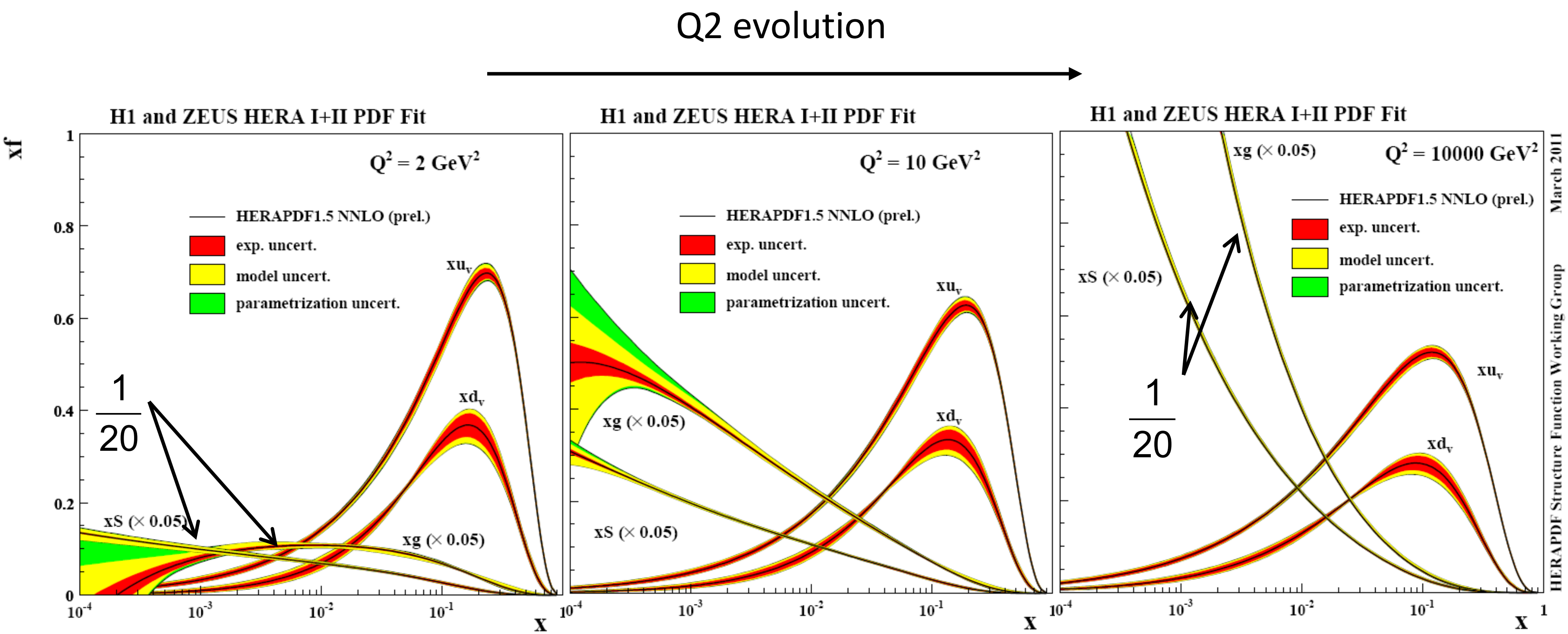
$$xg(x) = A_g x^{B_g} (1-x)^{C_g} (1 + D_g x),$$
$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2),$$
$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}},$$
$$x\overline{U}(x) = A_{\overline{U}} x^{B_{\overline{U}}} (1-x)^{C_{\overline{U}}} (1 + E_{\overline{U}} x^2),$$
$$x\overline{D}(x) = A_{\overline{D}} x^{B_{\overline{D}}} (1-x)^{C_{\overline{D}}}.$$

Coefficients are fitted



The Q^2 dependent is calculated using the DGLAP evolution.
Remark: DGLAP evolution and also calculation of partonic observables ($\hat{\sigma}$) can be done at LO, NLO, NNLO → leads to different pdfs (LO, NLO, NNLO).
These pdfs are different and should be used only for predictions of same order.

Parton distribution functions, e.g. HERA-PDF



https://www.desy.de/h1zeus/combined_results/

Parton distribution functions from NNPDF

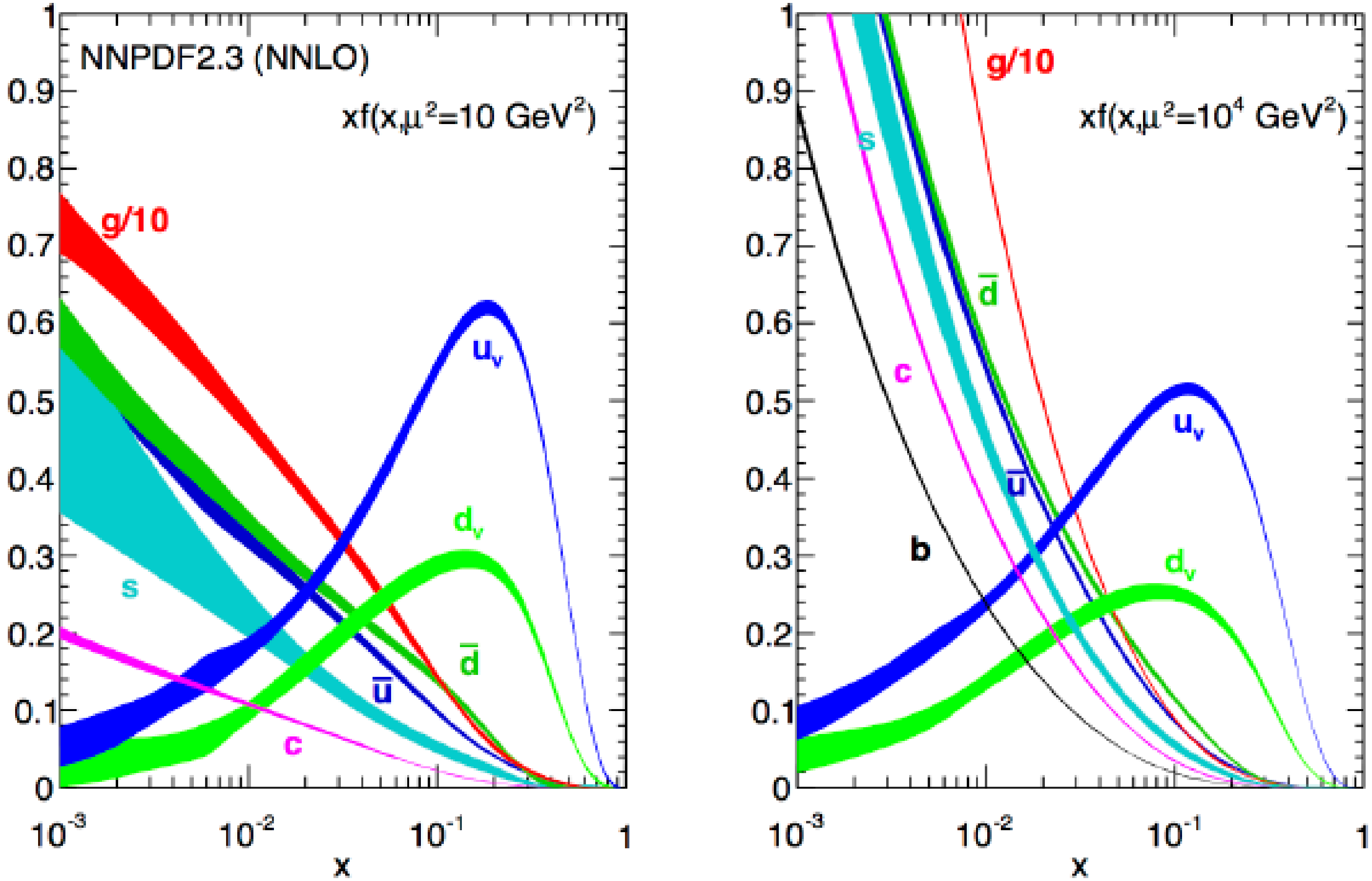


Illustration of scale dependence of PDFs

Hadrons are composite dynamic objects, with a time-dependent structure:

There are partons within clouds of further partons, constantly being emitted and reabsorbed.

Probing a hadron at a scale Q provides a snapshot of this dynamics at a characteristic resolution in time and space given by 1/Q

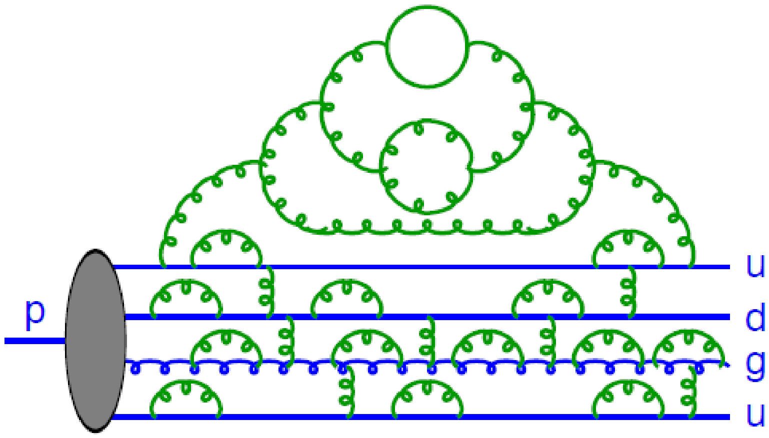
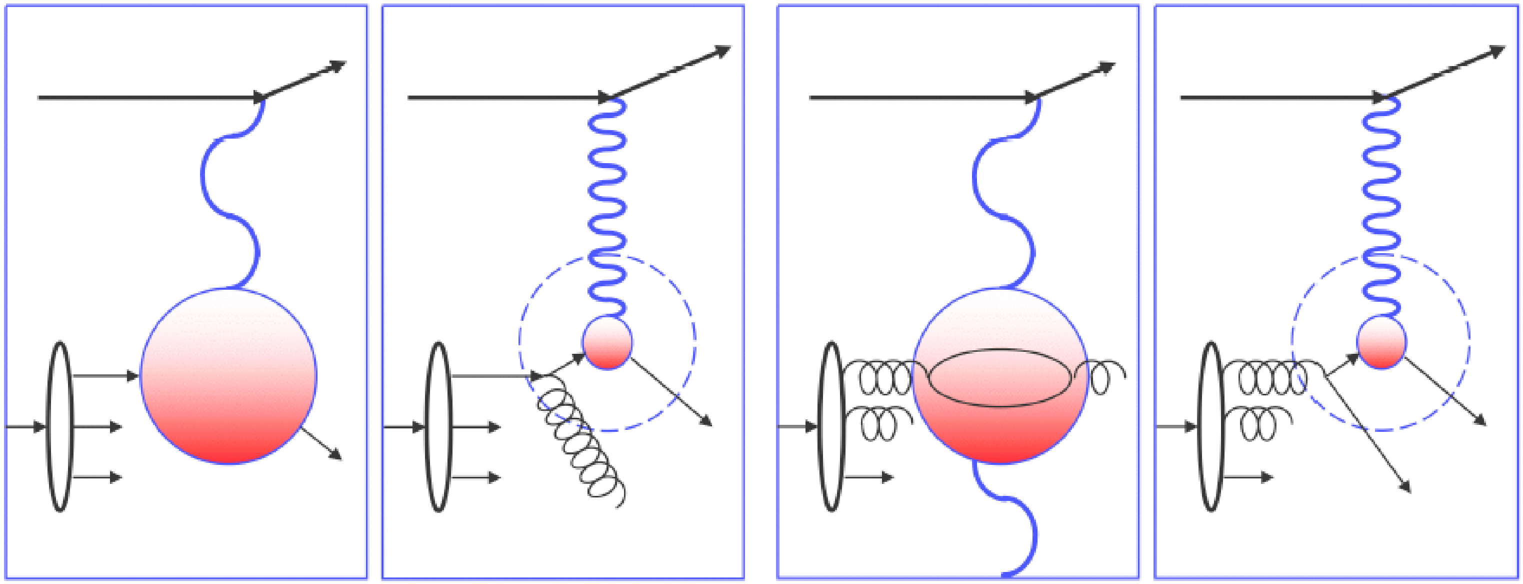


Illustration of scale dependence of PDFs

Large x: valence quark scattering   Small x: sea quark scattering



If we test quarks at large x (mostly valence quarks) an increase of Q will resolve more and more emissions and thus the pdfs (F_2) will decrease with increasing Q .

If we test quarks at small x (mostly sea quark) an increase of Q will resolve more and more gluon-splitting and thus the pdfs (F_2) will increase with increasing Q .

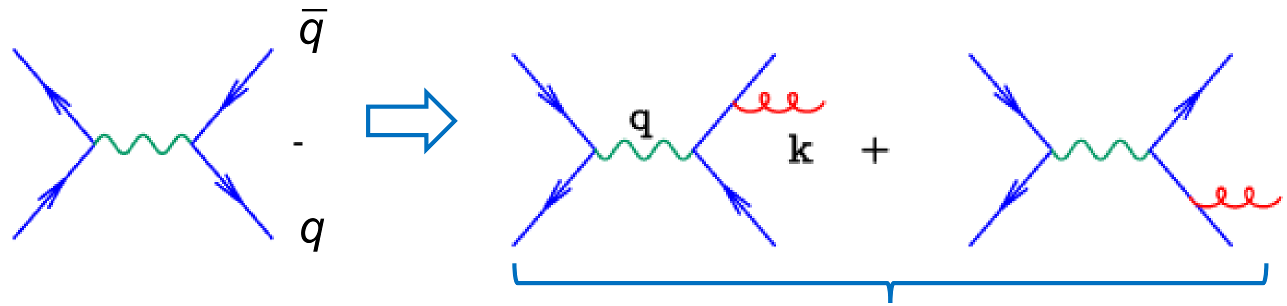
2. Hadronic final states

Deeper look at hadronic final states

We have seen that the quarks in the initial state undergo gluon emission
$\rightarrow$ soft (IR) and collinear divergences appear (absorbed in pdfs).

Quarks in the final state will also undergo gluon emission:

Simplest process:
$e^+e^- \rightarrow q\ q$:

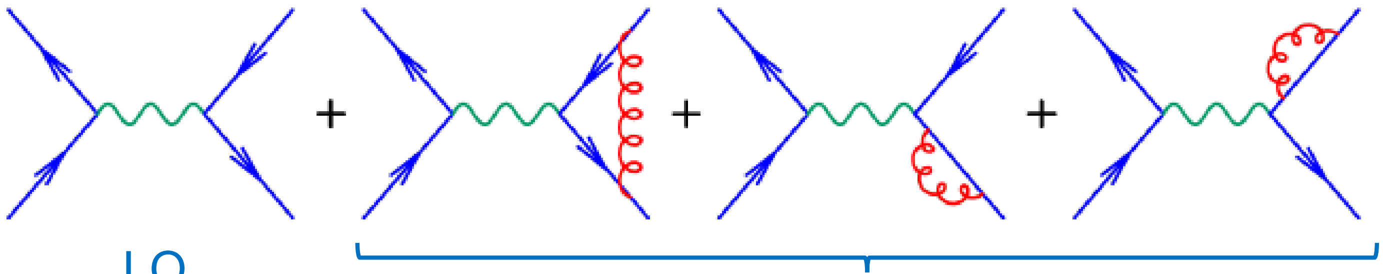


$\rightarrow$ soft and collinear divergent $\sim \dfrac{1}{p \cdot k}$

Somehow we have ignored this point when we have discussed 3-jet and R_{had} in the last semester! Moreover we need to understand when the additional gluon leads to a signature which we would identify as a third jet (not the case for collinear gluons)

The latter question raises a conceptual point: a state w/ a collinear gluon is indistinguishable from a state w/o that gluon (detector have finite resolution).
And: we can have many of these collinear gluons leading to the same state.
A "measurable final state" might therefore contain many "basic" states – complication for collider phenomenology.

Another complication arises when looking at the NLO correction to the "basic" $e^+e^- \rightarrow q\, q$ process (this was partially discussed in the QCD lecture last semester):



LO

NLO

Reminder: Loops are UV divergent
after regularization → renormalization absorbs UV divergences

But, even after renormalization the loop diagram are IR divergent.
However it turns out, that the IR divergent diagrams of the proceed ee→qqg exactly cancel the divergent part of the loops: the sum of the two contributions is finite and one obtains the well known NLO-order result for the inclusive hadron cross section:

$$\sigma(q^2) = \sigma_0(q^2)\left(1 + \frac{\alpha_s(q^2)}{\pi}\right)$$

In the limit of collinear or very soft photons, the two "basic processes" are formally different but not distinguishable experimentally.

(P. Skands)

Notations: fixed order QCD predictions

Consider final-state F (e.g. H, tt-pair, W, Z, DY). Schematically one may express the (perturbative) all-orders differential cross section for an observable O as

$$\left.\frac{d\sigma_F}{d\mathcal{O}}\right|_{ME} = \underbrace{\sum_{k=0}^{\infty} \int d\Phi_{F+k}}_{\Sigma \text{ legs}} \left| \underbrace{\sum_{\ell=0}^{\infty} \mathcal{M}^{(\ell)}_{F+k}}_{\Sigma \text{ loops}} \right|^2 \delta\left(\mathcal{O} - \mathcal{O}(\Phi_{F+k})\right)$$

(no pdfs, no flux normalization):

Remark: delta-function projects out a hypersurface of constant value of O (w/o it would be the total cross section). Φ_{F+k} is the phase-space.

$\mathcal{M}^{(\ell)}_{F+k}$ is the amplitude for producing F in association with k additional final-state partons, "legs" and with ℓ additional loops. Sum starts with k=0 and ℓ = 0 w/ the LO for producing F

Fixed order truncation of the full perturbative QCD result obtained by limiting k+ℓ :

k=0, ℓ =0 $\rightarrow$ LO (usually tree-level) for F production
k=n, ℓ =0 $\rightarrow$ LO for F + n jets
k + ℓ $\leq$ n $\rightarrow$ N^nLO for F (includes N^{n-1}LO for F + 1-jet, N^{n-2}LO for F + 2-jet .. up to LO for F + n-jets)

Exclusive final-states and differential observables

So far we have discussed only inclusive observables for the process ee→hadrons (total cross section). We have not looked at differential distributions. In particular we have not paid any attention to the difference between partons q, g and hadrons. However, only the latter are measurable in our detector.

By measuring differential distributions we start to ask questions about the structure of the final-state (e.g momenta of the partons). However, we have access only to hadrons: Can we associate individual hadrons to individual partons? That would mean that we could distinguish individual partons - however we know that with IR and collinear gluon emission this is hardly possible.

Contrary to inclusive observables for which IR/collinear divergences cancel, differential observable, e.g. diff cross section as function of energy of a quark in NLO, are divergent (shown by Tilman).

How can that be? Well, we are calculating partons however we are measuring hadrons:

$$\frac{d\sigma}{dx}(e^+e^- \to q + X) \quad \text{versus} \quad \frac{d\sigma}{dx}(e^+e^- \to H + X)$$

Fragmentation functions

Similar to the hadronic initial-state where we have used parton distribution functions which have absorbed the divergences we need to do the analog for the final states: **fragmentation functions $D_{q\to H}$ (z)** are used to describe the probability that a quark q produced a hadron carrying the momentum fraction z of the quark (**$D_{q\to H}$** absorb divergent terms).

We can then express the differential cross-section that a hadron with energy x (fraction of beam energy) is produced – result is finite:

$$\frac{d\sigma}{dx}(e^+e^- \to h + X) = \frac{d\hat{\sigma}}{dx} \otimes D_{q\to h}(x)$$

$$e^+e^- : x = 2E_h/s$$

Known convolution:

$$\int_x^1 \frac{dz}{z}\frac{d\hat{\sigma}}{dx} D_{q\to h}\left(\frac{x}{z}, \mu^2\right)$$

Like the pdfs the fragmentation functions are universal (process independent). They need to be determined from experiments (depend on scale and on the definition of the counter terms they absorb).

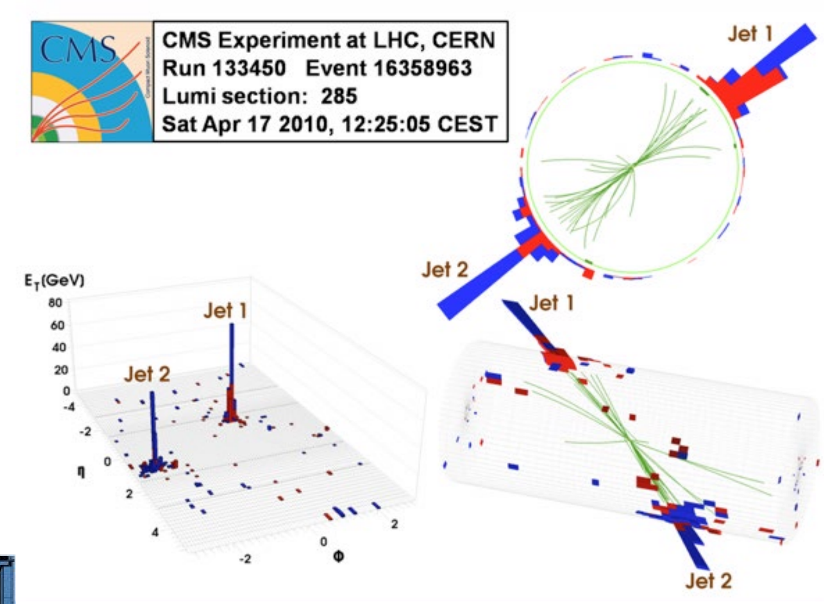
Fragmentation functions have been determined using electron-positron data. For collider physics, If one stays at the level of jets (at LHC), they are however not important. Only if you consider specific hadron production (e.g. LHCb).

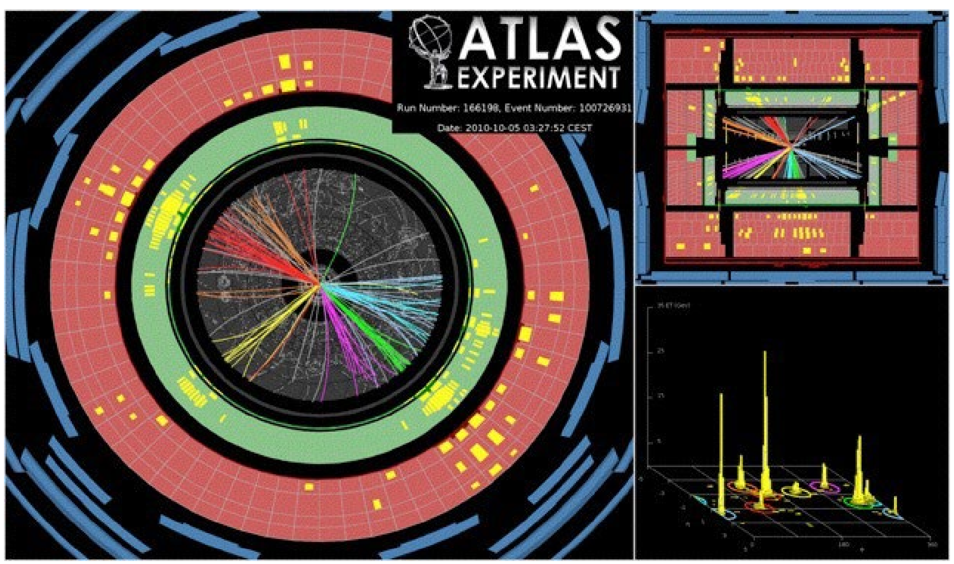
3. Jets at colliders

Jets at Colliders

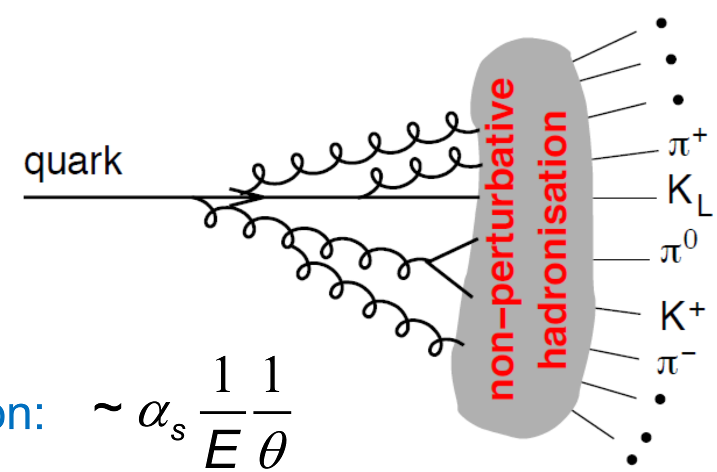
We have talked a lot about jets, w/o really explaining what they are!

Jets = collimated, energetic
(Salam) bunches of particles



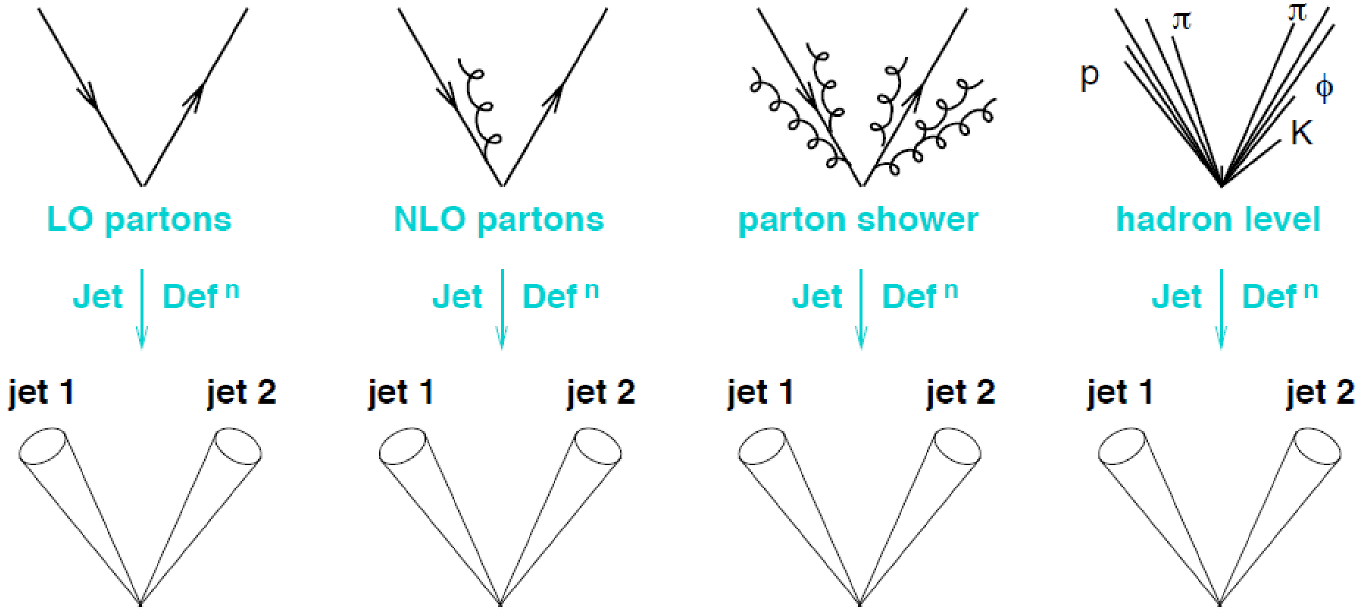


Why do we see jets?



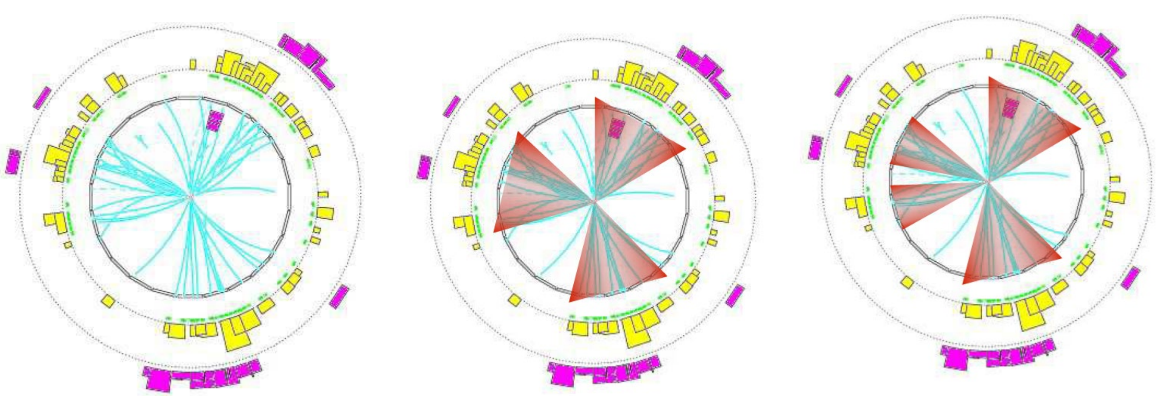
Gluon emission: $\sim \alpha_s \frac{1}{E}\frac{1}{\theta}$

Jet finding – kind of particle projection



Jets are defined through algorithms which group the measured particles
Many such definitions exists. Very popular: ant-k_T algorithms (see below).
"Jet definition" should be resilient to QCD effects – IR safe.

Jet definition
is ambiguious:



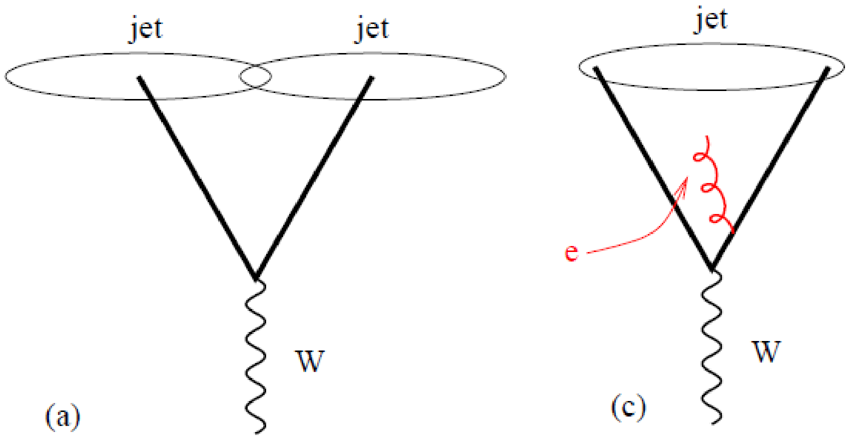
Some comments of the jet-finding procedures

1) Jet algorithm should be infrared and collinear safe:

An observable is infrared and collinear safe if, in the limit of a collinear splitting, or the emission of an infinitely soft particle, the observable remains unchanged.

<u>Jets and jet algorithms are not necessary IRCS</u>

e.g. simple cone algorithms are not IR safe: additional gluon could turn 2 jet signature into a single jet.



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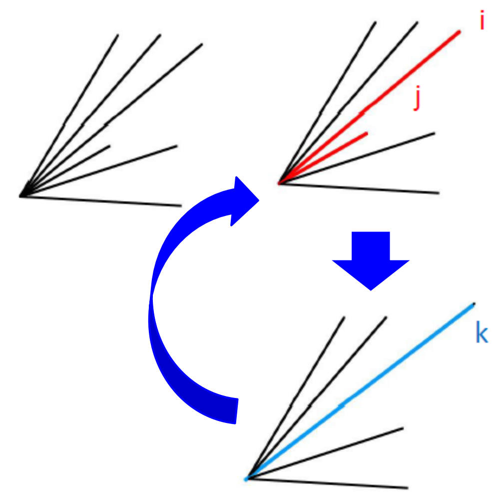
2) Jet algorithms should also be applicable also at the parton level:
Required to make theoretical predictions.

Jet Algorithms

Iterative jet algorithms ("Jade"-type, developed for $e^+ e^-$)

1) for all pairs of particles i, j calculate distance parameter y_{ij}
2) find pair i, j with smallest $y_{ij,\,\mathrm{min}}$
3) add 4-momenta: $p_i + p_j = p_k$ replace p_i, p_j by p_k
4) iterate till distances of remaining objects $y_{ij} > y_{cut}$



Distance measures (at e^+e^-):

$$y_{ij} = 2\,\frac{E_i\,E_j\,(1-\cos\theta_{ij})}{s}$$

Jade algorithm IRCS but theoretically difficult; large higher order correction.
(invariant mass squared)

$$y_{ij} = 2\,\frac{\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{s}$$

k_T – algorithm:
better higher order behavior
(relative transverse momentum squared)

$\min(E_i^2, E_j^2) \rightarrow \min(E_i^{-2}, E_j^{-2})$

anti-k_T – algorithm: often used nowadays
($\rightarrow$jets w/ soft radiation are conical)

=1 - no energy weight

Aachen-Cambridge algorithm

Anti-k_T adaptation for hadron colliders

Due to kinematics the distance measure at hadron collider needs adaptation:

Use rapidity $y_i = \frac{1}{2}\ln\frac{E_i + p_{z,i}}{E_i - p_{z,i}}$ and azimuthal angle ϕ_i:

Often use pseudo rapidity η

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right)$$

(same for massless particles)

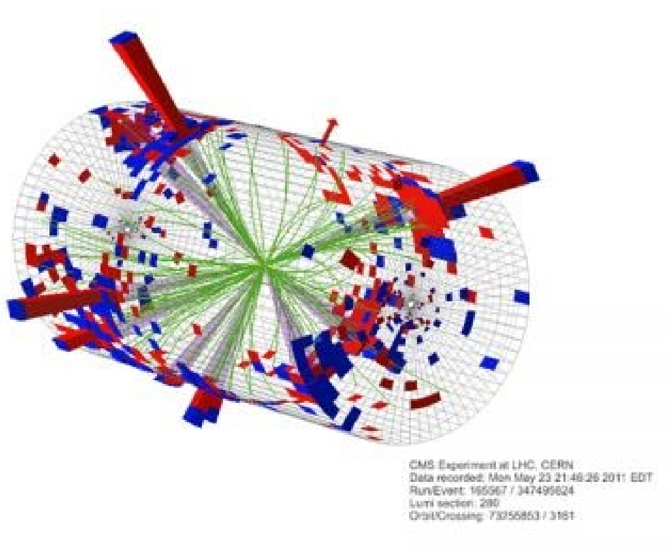
→ angular distance of 2 particles: $\Delta R_{ij}^{\ 2} = (y_i - y_j)^2 - (\phi_i - \phi_j)^2$

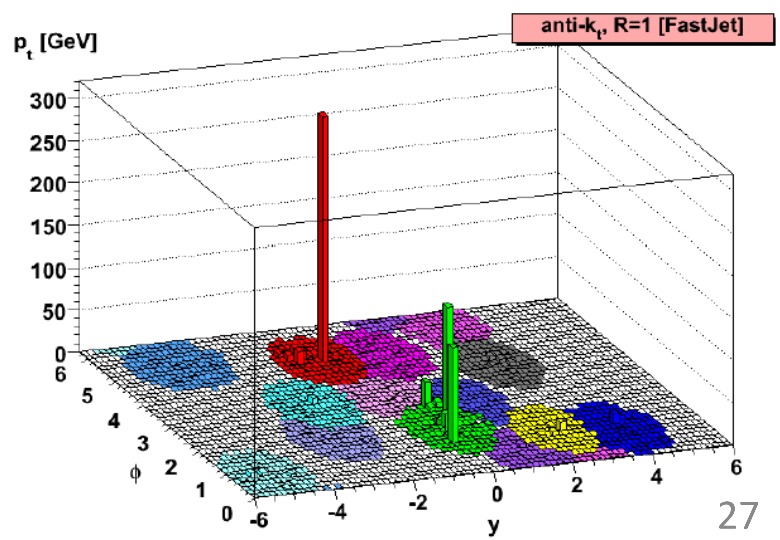
(anti)-k_t algorithm:

Distance measure (renamed $y_{ij} \rightarrow d_{ij}$)

$$d_{ij} = \min(p_{t,i}^{2l-2}, p_{t,i}^{2l-2}) \cdot \frac{\Delta R_{ij}^{\ 2}}{R^2}$$

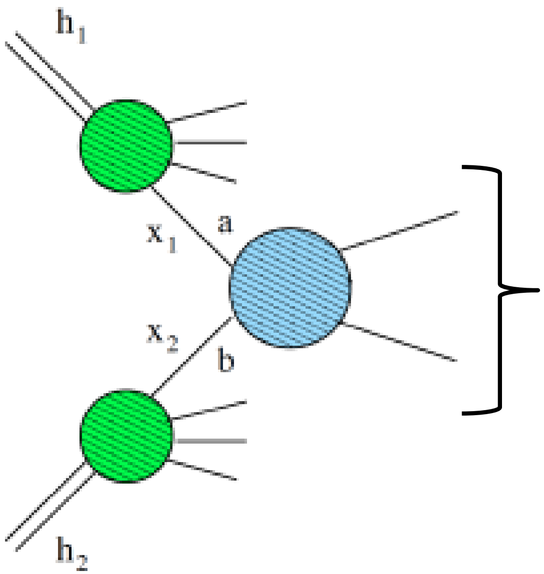
Parameter to describe typ. jet opening: R=0.4…0.7 (see below)





3. Hadron collider physics

Hadron Collider Physics (LHC physics)



Hard process:
Drell-Yan, Higgs- , vector boson, jj, tt production.

$$\sigma = \iint dx_1 dx_2\, f_1(x_1,\mu^2) f_2(x_1,\mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$$

W/ hard probes in final state, no need to care about single hadrons (no fragmentation functions).

Hadron collider kinematics:

Suitable kinematical variables at hadron collider are a result of the cylindrical geometry of beam-detector system and the fact that the initial state momentum along the z-direction is unknown.

$$p^\mu = (E, p_x, p_y, p_z) = (m_T \cosh y, p_T \sinh\phi, p_T \cosh\phi, m_T \sinh y)$$

$$p_T^2 = (p_x^2 + p_y^2)$$ transverse momentum

$$m_T = (p_T^2 + m^2)$$ transverse mass

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$ Rapidity (Lorentz invar.)

$$\eta = -\ln\tan(\theta/2)$$

Pseudo rapidity:

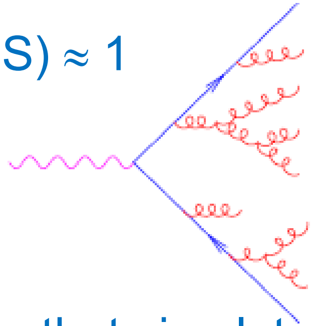
$$\eta = y\big|_{m=0}$$

Theoretical description: Monte-Carlo Generators

- **Hard process** (at scale Q) including the emission of hard gluons is described by the matrix element of the process in fixed order perturbation theory:
  - only few "hard" partons are produced
  - partons are off-shell and can still radiate → soft gluons

- **Soft gluon emission** at soft scale S is ubiquitous since $\alpha_S(S) \approx 1$

  (e.g.: in a typical Z → hadrons event down to ~GeV scale aorund7 gluons are emitted) ⇨



This stage is described by a "parton shower", i.e. a calculator that simulates soft and/or collinear emissions. Due to their universality and factorizability, parton shower calculations are much easier than FO calculations with many legs!

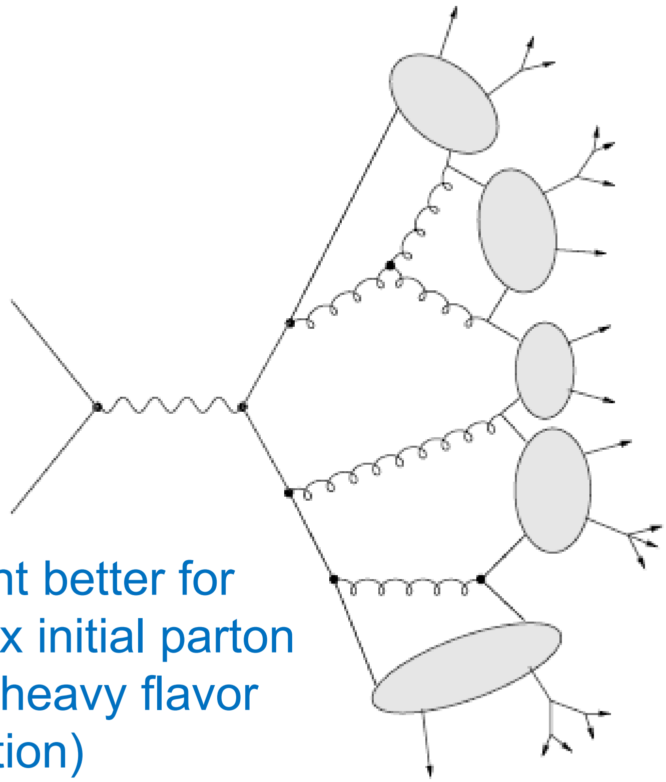
- **Hadronization:** Once the system has reached very low scales O(GeV), perturbation theory completely breaks down and enters the hadronization stage. Hadronization models: well defined hadrons w/ well defined kinematics in final state.

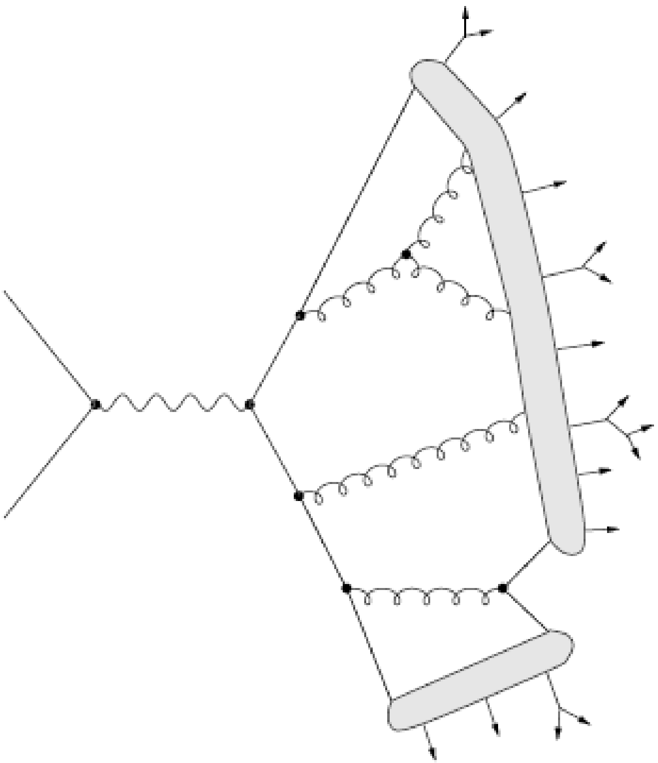
Programs that combine all steps are called event generators:
e.g. **PHYTIA, HERWIG**

For experts: main hadronization models

Cluster hadronization model

Lund string model





(account better for complex initial parton states, heavy flavor production)

(connection to color flow)

used ín HERWIG

used ín PHYTIA

The model parameters are tuned to describe type and kinematics of final state hadrons. After tuning both models do a very good job!

5. QCD Results from LHC

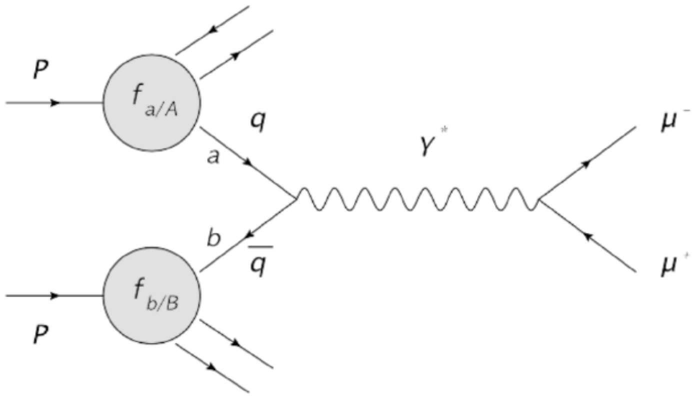
Results from LHC

In the following I am discussing a few selected LHC measurements which test the prediction of QCD (implemented in MC event generators) to an astonishing precision:

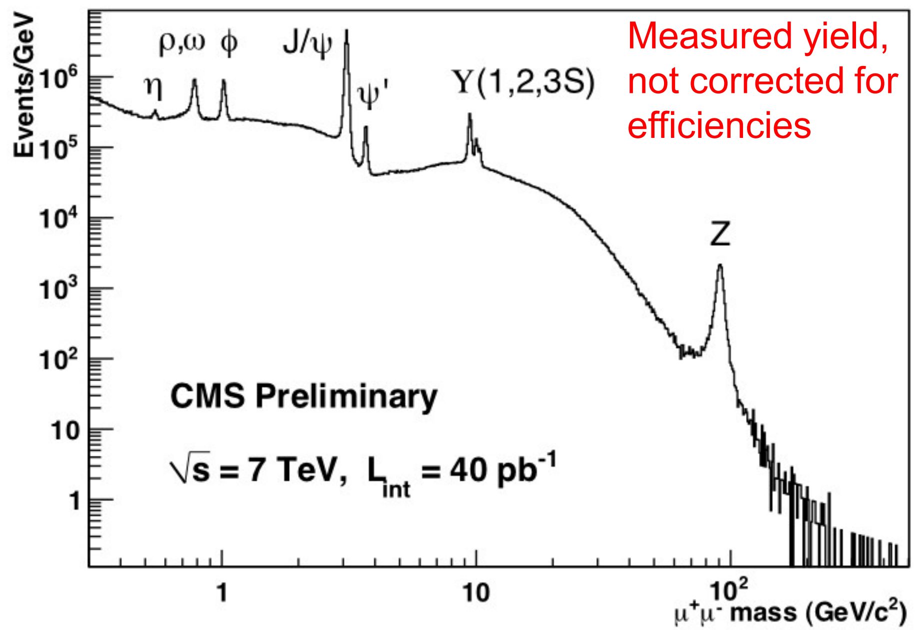
- Drell-Yan production
- Jet and Di-jet production
- W-production

A recent review of QCD tests at LHC can be found in:
T. Gehrmann and B. Malaescu "Precision QCD Physics at the LHC",
https://doi.org/10.1146/annurev-nucl-101920-014923

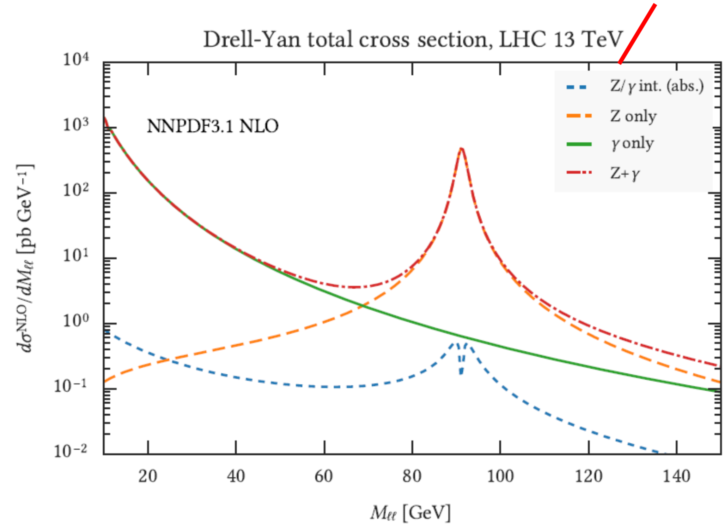
Simplest process: Drell-Yan production (“standard candle”)



$$\hat{\sigma}\left(q(p_1)\bar{q}(p_2) \to l^+l^-\right) = \frac{4\pi\alpha^2}{3\hat{s}}\frac{1}{N_c}Q_q^2$$

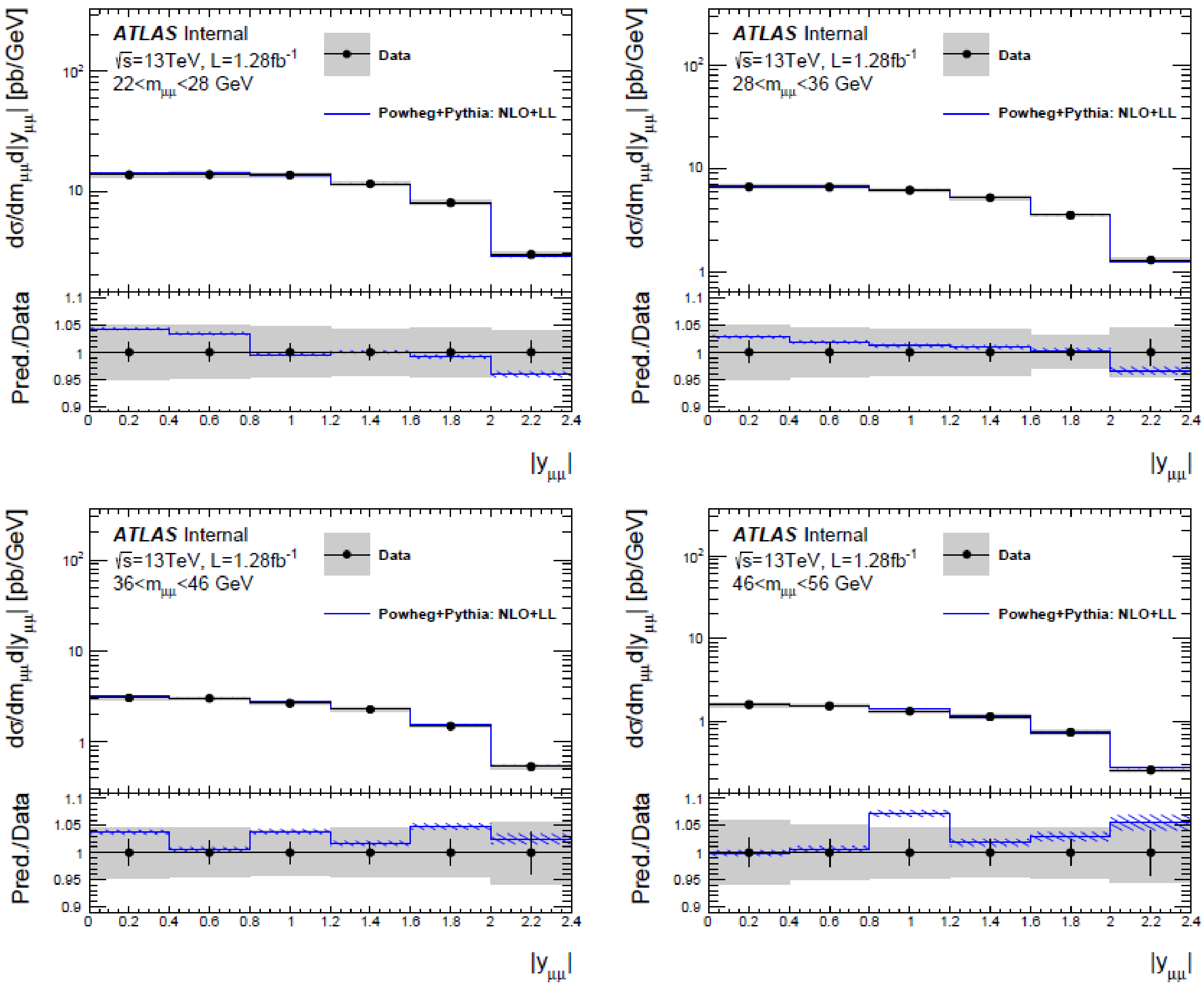


(we only look at the lepton final state and do not resolve the hadronic final-state)



Theoretical predictions:
NNPDF3.1 NLO (i.e. DY+ 1 jet)

pp $\rightarrow \mu\mu X$: measured double differential cross section shows excellent agreement w/ predictions.



POWHEG =
Parton Shower-NLO QCD:

NLO QCD calculations w/ parton shower MC progr.

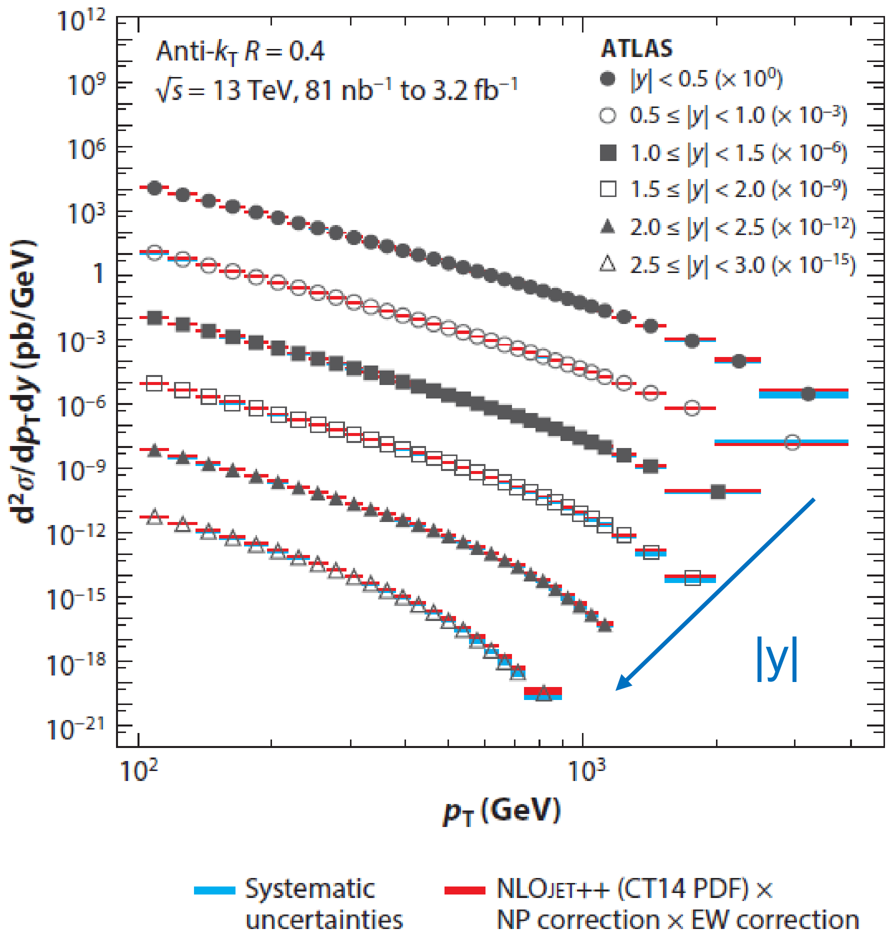
Alessandro Guida, PhD Thesis, 2022

https://cds.cern.ch/record/2681125/files/CERN-THESIS-2018-432.pdf

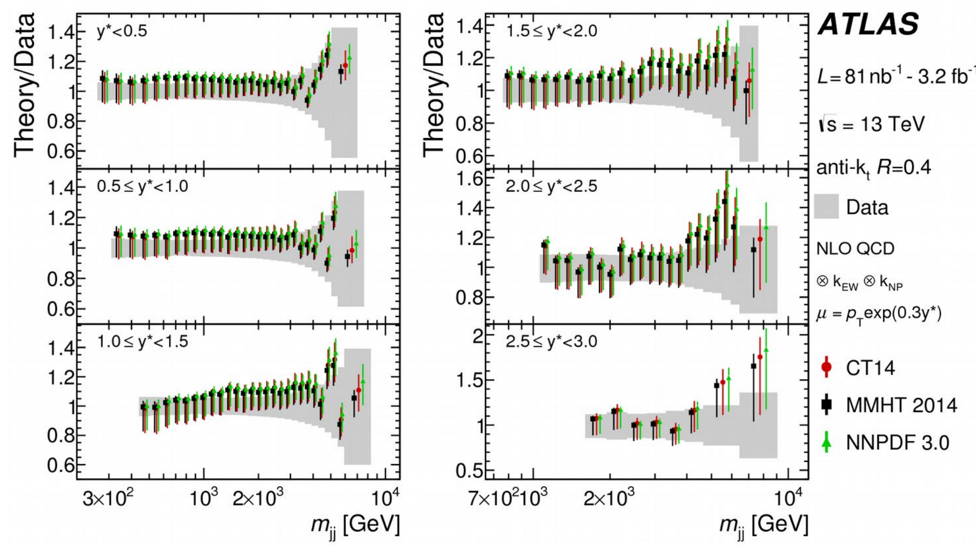
Di-jet and inclusive jet-production

Require min. 2-jets with both jets p_T(jet) > 75 GeV, use anti-k_T w/ R=0.4,

double differential incl. jet xsect: (p_T,y)
(every jet in the event enters)



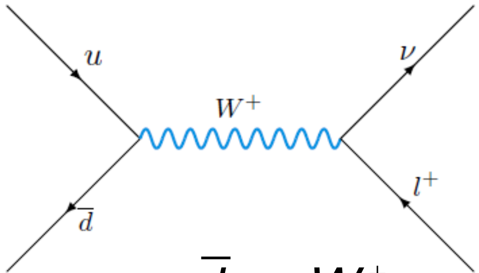
double differential xsect for di-jet (m_{jj},Δy)
(only 2 highest jets considered)



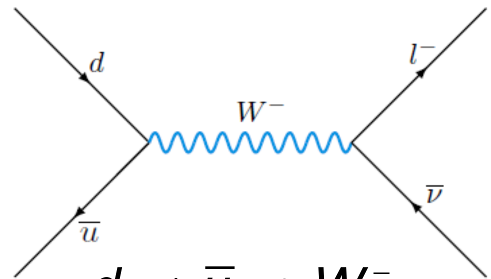
Excellent agreement over many orders of magnitudes between theory prediction and measurement!

W-production – sensitivity to valence quarks

LO



$u_v + \bar{d} \rightarrow W^+$



$d_v + \bar{u} \rightarrow W^-$

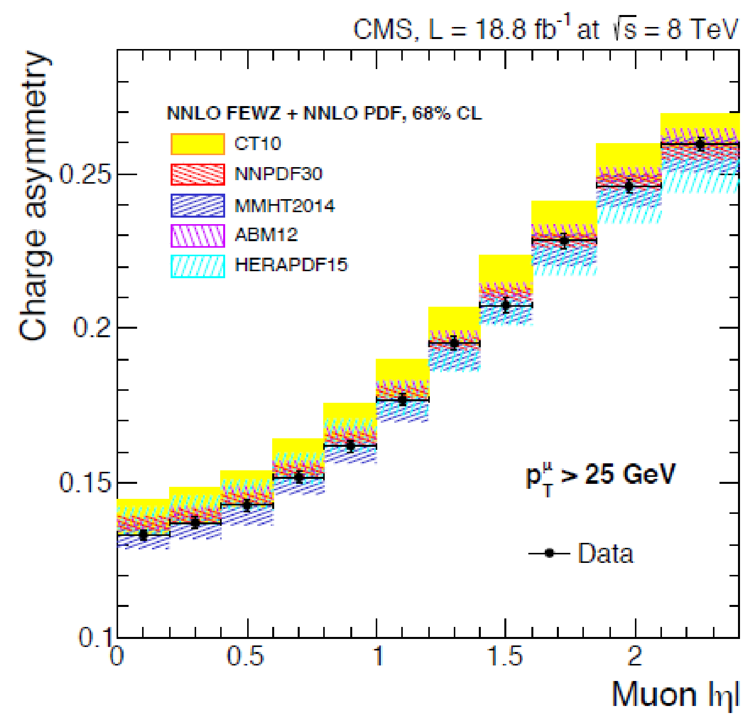
W are polarized!

W-charge Asymmetrie:

$$A(\eta) = \frac{\frac{d\sigma}{d\eta}(W^+ \rightarrow \ell^+\nu) - \frac{d\sigma}{d\eta}(W^- \rightarrow \ell^-\overline{\nu})}{\frac{d\sigma}{d\eta}(W^+ \rightarrow \ell^+\nu) + \frac{d\sigma}{d\eta}(W^- \rightarrow \ell^-\overline{\nu})}.$$

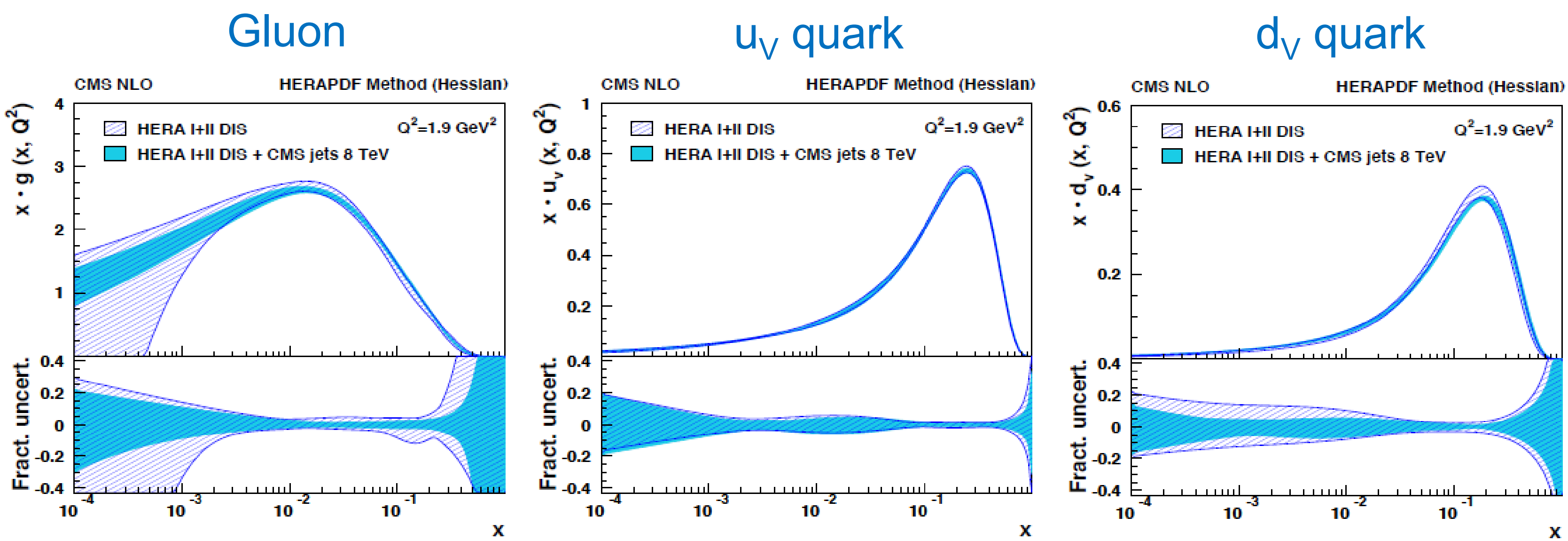
$$A(\eta) = \frac{\sigma_\eta^+ - \sigma_\eta^-}{\sigma_\eta^+ + \sigma_\eta^-} \approx \frac{u_v - d_v}{u_v + d_v + 2u_{\mathrm{sea}}},$$

Sensitive to difference between the u- and d-valence quark pdfs.



Precision LHC data also improves pdf

Precision LHC measurements of the DY process, W / Z production, inclusive jet production etc. allow to improve the precision of the pdfs:

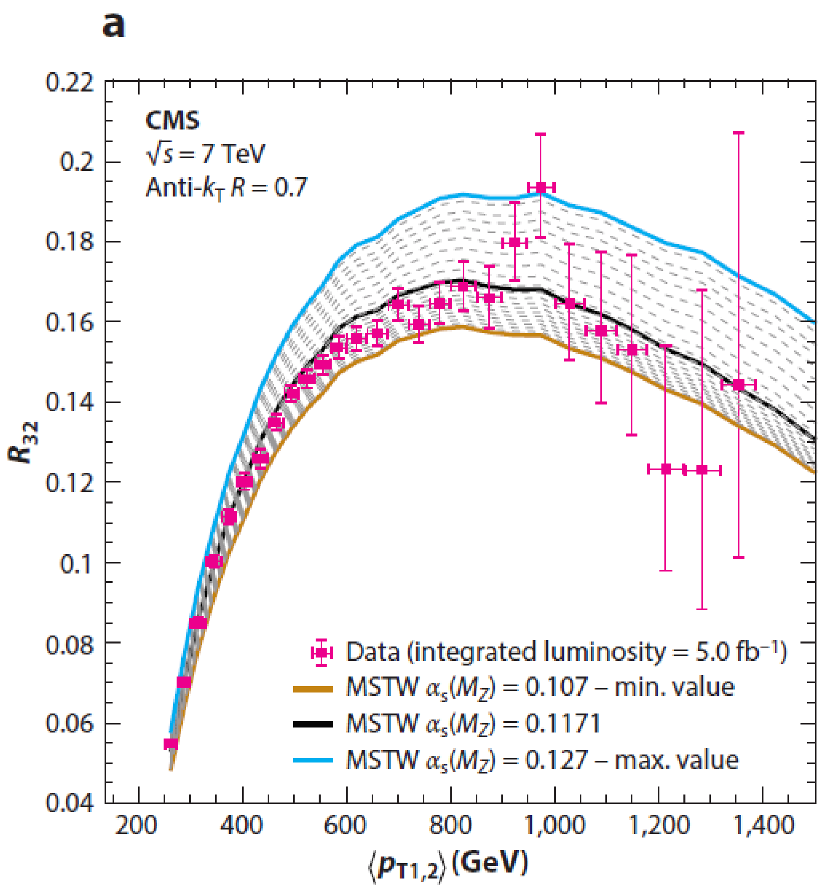


Shown are HERAPDF w/ only DIS data and after including LHC data

LHC data is complementing the DIS (HERA) with data at vey small x and large Q (and in a small corner at very small Q)

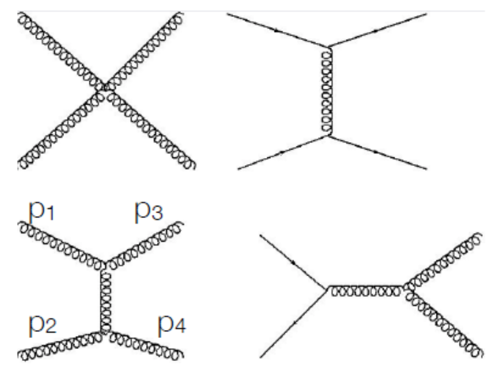
Determination of coupling α_s

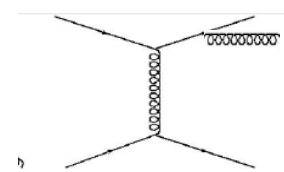
LHC data also provides sensitivity to the strong coupling α_s at different Q^2.
Similar to e^+e^- the ratio of 2-jet to 3-jet events (~ α_s) allows determination of $\alpha_S(Q^2)$



Larger α_S

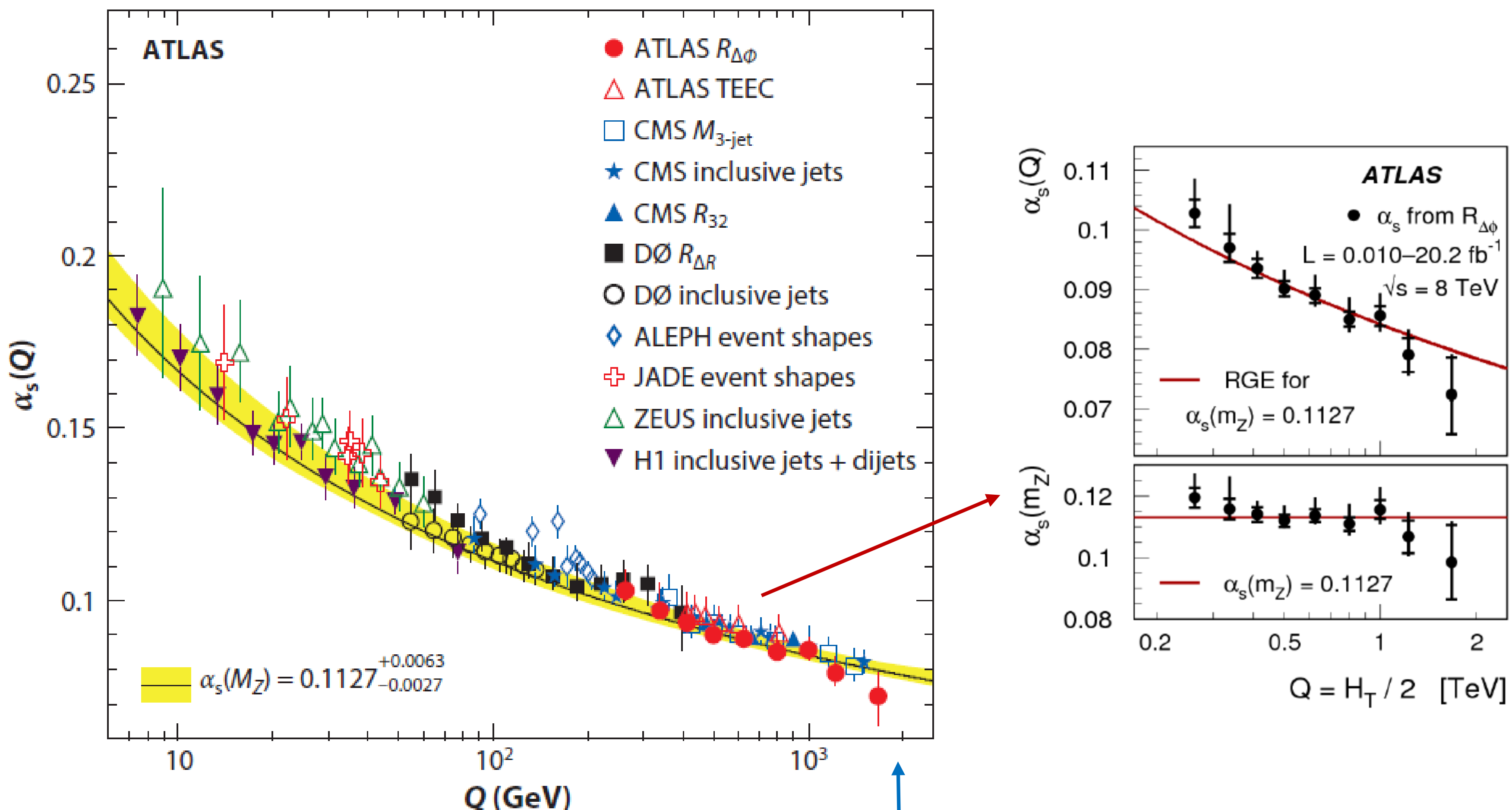
Q^2





Add gluon:
2jet -> 3jet
~ α_s

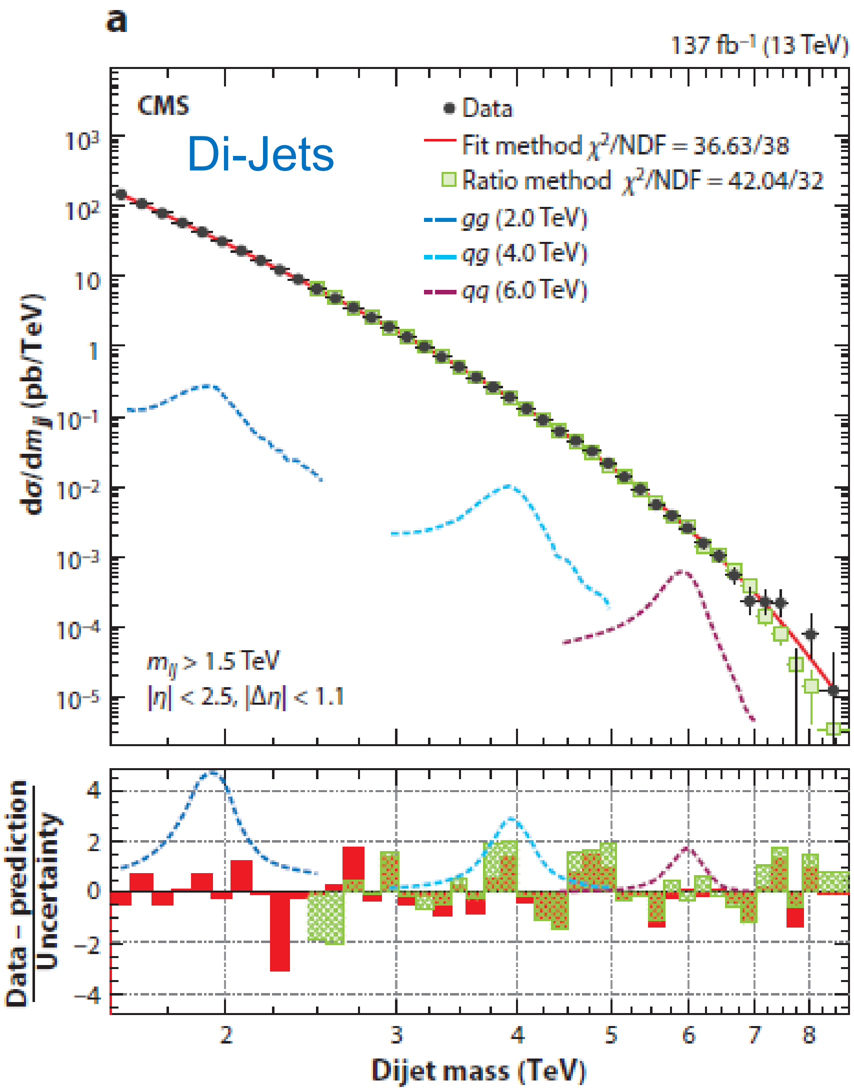
ATLAS and CMS have determined α_S from different measurements at different Q^2

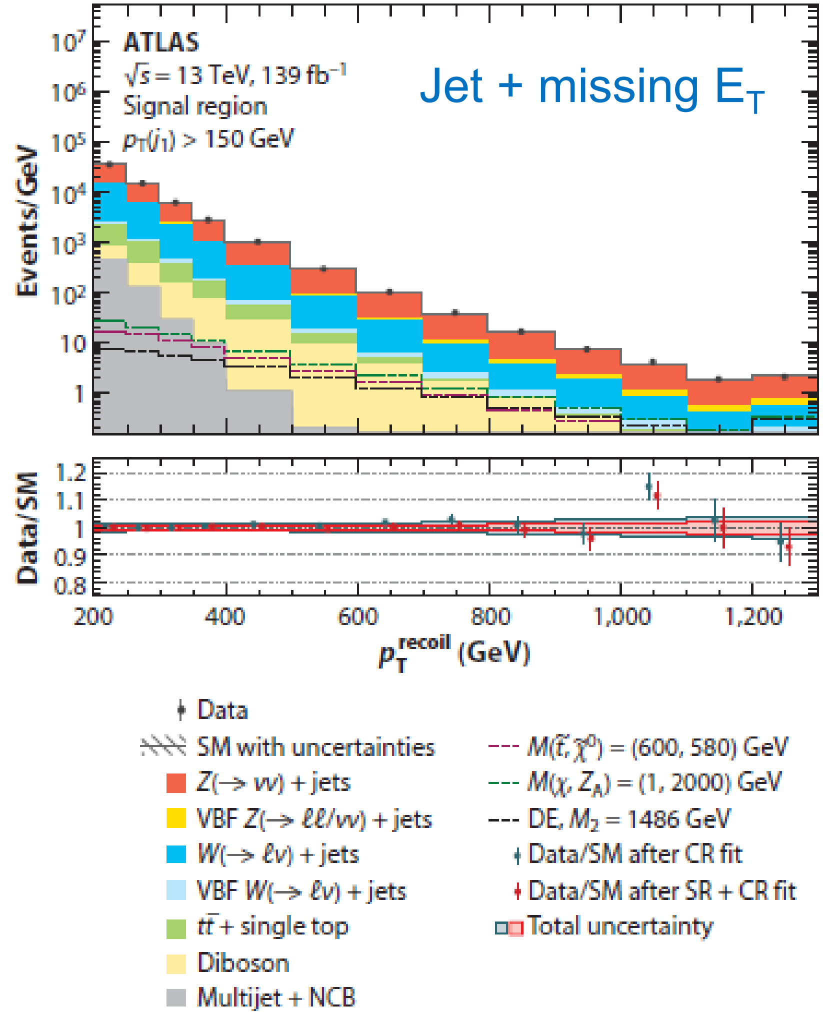


2 TeV

$R_{\Delta\phi}$ = ratio of dijet events for which $\Delta\phi<\Delta\phi_{max}$

BSM searches rely on precision QCD predictions





Look for NP decaying into 2-jets.

Look for invisible particle: recoil