Neutrino mixing for Majorana neutrinos:

The weak eigenstates ν_α which by default are the states produced in the weak CC interaction of a charged lepton ℓ_α (flavor eigenstates) are the linear combinations of the mass eigenstates ν_i determined by the PMNS mixing matrix U:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^{*} |\nu_i\rangle$$

While for Dirac neutrinos the PMNS mixing matrix is given by three mixing angles and one phase δ,

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

for Majorana neutrinos there are two additional Majorana phases which cannot be absorbed in the redefinition of the neutrino states:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(there are different conventions; here the PDG convention)

4. Neutrino oscillations

If neutrinos have masses and lepton flavors are mixed by weak CC interactions, then lepton flavor is not conserved in neutrino propagation.

This phenomenon is usually referred to as neutrino oscillations. In brief, a weak eigenstate ν_α which by default is the state produced in the weak CC interaction of a charged lepton ℓ_α, is the linear combination determined by the mixing matrix *U*

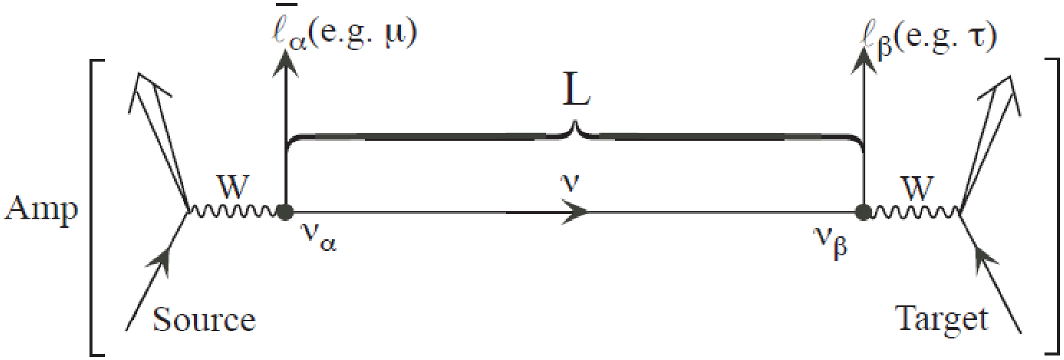
$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}^{*} |\nu_i\rangle$$

Where ν_i are the mass eigenstates

Neutrino oscillations in vacuum

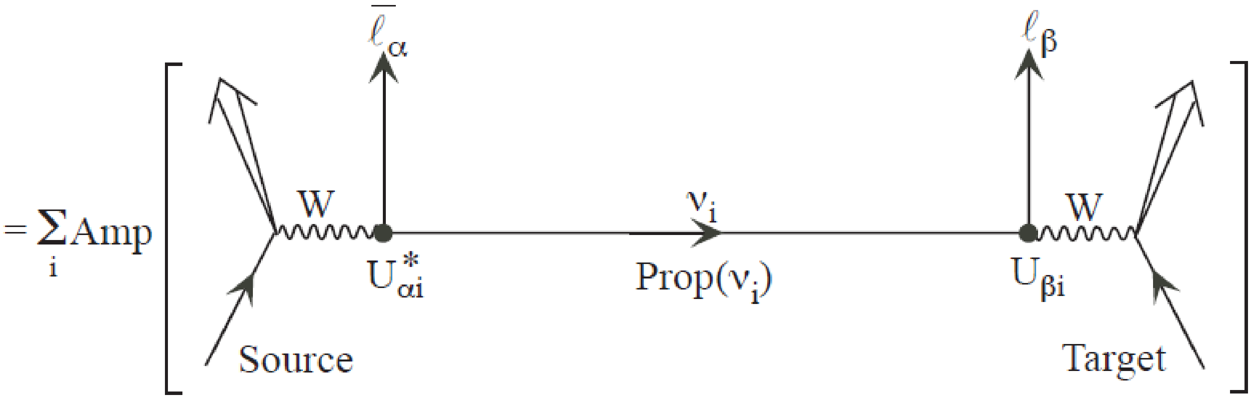
(follows a derivation by Boris Kayser)

A neutrino produced with flavor ν_α in the source can thus interact as ν_β in the target:



B.Kayser

Neutrino propagation: mass states



B.Kayser

Assumption:

Coherent mass states propagate as plane waves: $|\nu_i(t,x)\rangle = |\nu_i(0)\rangle \exp(-ip_\mu x^\mu)$

The amplitude $A(\nu_\alpha \to \nu_\beta)$ for oscillation, i.e. the amplitude that a ν_α produced in the source is detected as ν_β in the target is given by:

$$A(\nu_\alpha \to \nu_\beta) \sim \sum_i A(W \to \bar{\ell}_\alpha \nu_i)\text{Propagator}(\nu_i) A(\nu_i \to \ell_\beta W)$$

$$\sim \sum_i U^*_{\alpha i} \cdot \text{Propagator}(\nu_i) \cdot U_{\beta i}$$

The propagator $\sim \exp(ip_\mu x^\mu)$ describes the neutrino propagation along some distance L and is given in the lab frame by $\exp(-i(E_i t - p_i L))$, where t is the flight time from the source to the target at distance L. E_i and p_i are the energy and the momentum in the lab frame. That is, each mass eigenstate ν_i picks up the phase factor $\phi_i = -i(E_i t - p_i L)$

As the oscillation probability is given by $\mathrm{P}_{\alpha\beta} = |\mathrm{A}(\nu_\alpha \rightarrow \nu_b)|^2$ only the <u>relative</u> phase differences between the different propagation phases are relevant:

$$\Delta\phi_{ij} = -(E_i t - p_i L) + (E_j t - p_j L) = \left(p_i - p_j\right)L - \left(E_i - E_j\right)t$$

In practice experiments do not measure t. Instead t is replaced by $L/\bar{v}$ where $\bar{v}$ is the average velocity of the 2 neutrino mass states.

$$\bar{v} = \frac{p_1 + p_2}{E_1 + E_2}$$

(c=1)

One obtains then for the phase difference:

$$\Delta\phi_{ij} = \frac{p_i^2 - p_j^2}{p_i + p_j}L - \frac{E_i^2 - E_j^2}{p_i + p_j}L = \frac{m_j^2 - m_i^2}{p_i + p_j}L = \frac{\Delta m_{ij}^2}{2E}L$$

where we have used that for highly relativistic neutrinos p_1 and p_2 can be approximated by the neutrino beam energy $E_1 \approx E_1 \approx E$ (minor differences play no role in the sum) .

Thus the relative phases in $\mathrm{A}(\nu_\alpha \to \nu_\beta)$ between the neutrinos are correct if we take as propagator:

$$\mathrm{Propgator}(\nu_i) = \exp\left(i m_i^2 \frac{L}{2E}\right)$$

For the transition amplitude $\mathrm{A}(\nu_\alpha \to \nu_\beta)$ one thus obtains:

$$\mathrm{A}(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^* \exp\left(i m_i^2 \frac{L}{2E}\right) U_{\beta i}$$

The oscillation probability $\mathrm{P}(\nu_\alpha \to \nu_\beta)$ is then obtained from $\left|\mathrm{A}(\nu_\alpha \to \nu_\beta)\right|^2$ and exploiting unitarity:

$$\mathrm{P}(\nu_\alpha \to \nu_\beta) = \left|\mathrm{A}(\nu_\alpha \to \nu_\beta)\right|^2 = \delta_{\alpha\beta} - 4 \sum_{i,j\,:\,i>j} \Re\left\{U_{\alpha i}^* U_{\beta i} U_{\alpha j}^* U_{\beta j}\right\} \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

$$+2 \sum_{i,j\,:\,i>j} \Im\left\{U_{\alpha i}^* U_{\beta i} U_{\alpha j}^* U_{\beta j}\right\} \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

While for anti-neutrinos:

where $\Delta m_{ij}^2 = m_j^2 - m_i^2$

$$\mathrm{P}(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \ldots$$

with “–” sign
(complex conj. PMNS matrix elements)

Excursus: Majorana-Phases:

For Majorana neutrinos:

$$U_{PMNS} = V_{PMNS} \cdot \mathrm{diag}\left(e^{i\eta_1}, e^{i\eta_2}, 1\right) = \begin{pmatrix} e^{i\eta_1}U_{e1} & e^{i\eta_2}U_{e2} & U_{e3} \\ e^{i\eta_1}U_{\mu1} & e^{i\eta_2}U_{\mu2} & U_{\mu3} \\ e^{i\eta_1}U_{\tau1} & e^{i\eta_2}U_{\tau2} & U_{\tau3} \end{pmatrix}$$

the additional phases leave the combinations $U^*_{\alpha i}U_{\beta i}U^*_{\alpha j}U_{\beta j}$ invariant.

The Majorana phases do not change mixing (no additional CP violation)

Majorana phases are only observable in processes which change the lepton number by two units. Neutrino mixing changes the flavor.

→ Majorana phases are not visible, and for now not constrained.

CP-violation in neutrino mixing:

Jarlskog invariant (cf. CKM for quarks):

$$J_{CP} = \Im\{U^*_{\alpha i} U_{\beta i} U^*_{\alpha j} U_{\beta j}\} \neq 0 \qquad \Longleftrightarrow \qquad \mathrm{P}(\nu_\alpha \to \nu_\beta) \neq \mathrm{P}(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

i.e. if U_{PMNS} is complex ($\delta_{CP} \neq 0, \pi$).

Usage of SI-units:

The expression $\left(\Delta m^2_{ij} \frac{L}{2E}\right)$ uses natural units. Use $\hbar$ and c to transform to SI-units:

$$\Delta m^2_{ij} \frac{L}{2E} \to 1.27 \cdot \Delta m^2_{ij}[\mathrm{eV}^2] \frac{L[\mathrm{km}]}{2E[\mathrm{GeV}]}$$

(an often-used expression)

Example: Experiments studying 1 GeV neutrinos travelling L≈10^4 km is sensitive to m_{ij}^2 – splitting as small as ~10^{-4} eV^2 (Sensitivity of atmospheric neutrinos passing the earth)

Some remarks on the derivation of the mixing formula:

Many text books use either equal energy or equal momentum assumption:

$$\Delta\phi_{ij} = -(E_i t - p_i L) + (E_j t - p_j L) = (p_i - p_j)L - (E_i - E_j)t$$

Equal energy: $E_i = E_j = E$ and $p_i = \sqrt{E^2 - m_i^2} = E - \frac{m_i^2}{2E}$

$$\Delta\phi_{ij} = (p_i - p_j)L - (E_i - E_j)t = \frac{m_i^2 - m_j^2}{2E} L = \frac{\Delta m_{ij}^2}{2E} L$$

Equal momentum: $p_i = p_j = p$ and $E_i = \sqrt{p^2 + m_i^2} = p - \frac{m_i^2}{2p}$

$$\Delta\phi_{ij} = (p_i - p_j)L - (E_i - E_j)t = \frac{-(m_i^2 - m_j^2)}{2p} t = \frac{\Delta m_{ij}^2}{2E} L$$

(where in the last equality L=ct and pc=E has been used)

It turns out that neither the equal momentum nor the equal energy ansatz is correct (see e.g. E. Akhmedov arXiv:1901.05232v1).

Most derivations (including ours) use a plane-wave treatment for the propagation of the neutrino - instead a wave-packet ansatz is needed (see arXiv:1901.05232v1) However, a correct treatment using wave-packages results in the same formula.

Plane wave: no spatial localization. Cannot describe creation at source and conversion at target.

Three neutrino oscillation:

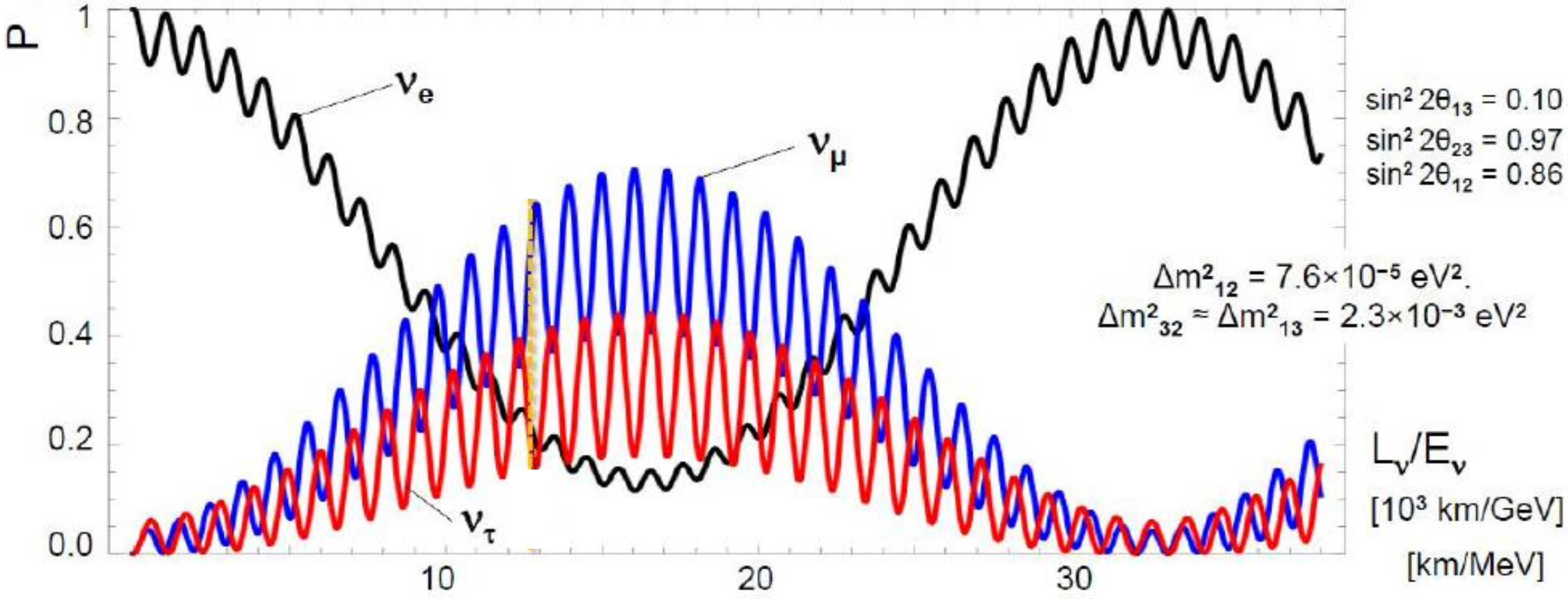
Formula is quite complex

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1}U_{\beta 1}^{*} + U_{\alpha 2}U_{\beta 2}^{*} \exp\left(-i\frac{\Delta m_{21}^2}{2E}L\right) + U_{\alpha 3}U_{\beta 3}^{*} \exp\left(-i\frac{\Delta m_{31}^2}{2E}L\right)\right|^2$$

It depends on two Δm^2 with three angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one CPV phase.
Assume:

$$\Delta m_{21}^2 \ll \Delta m_{31}^2 \approx \Delta m_{32}^2$$

initial electron neutrino beam.

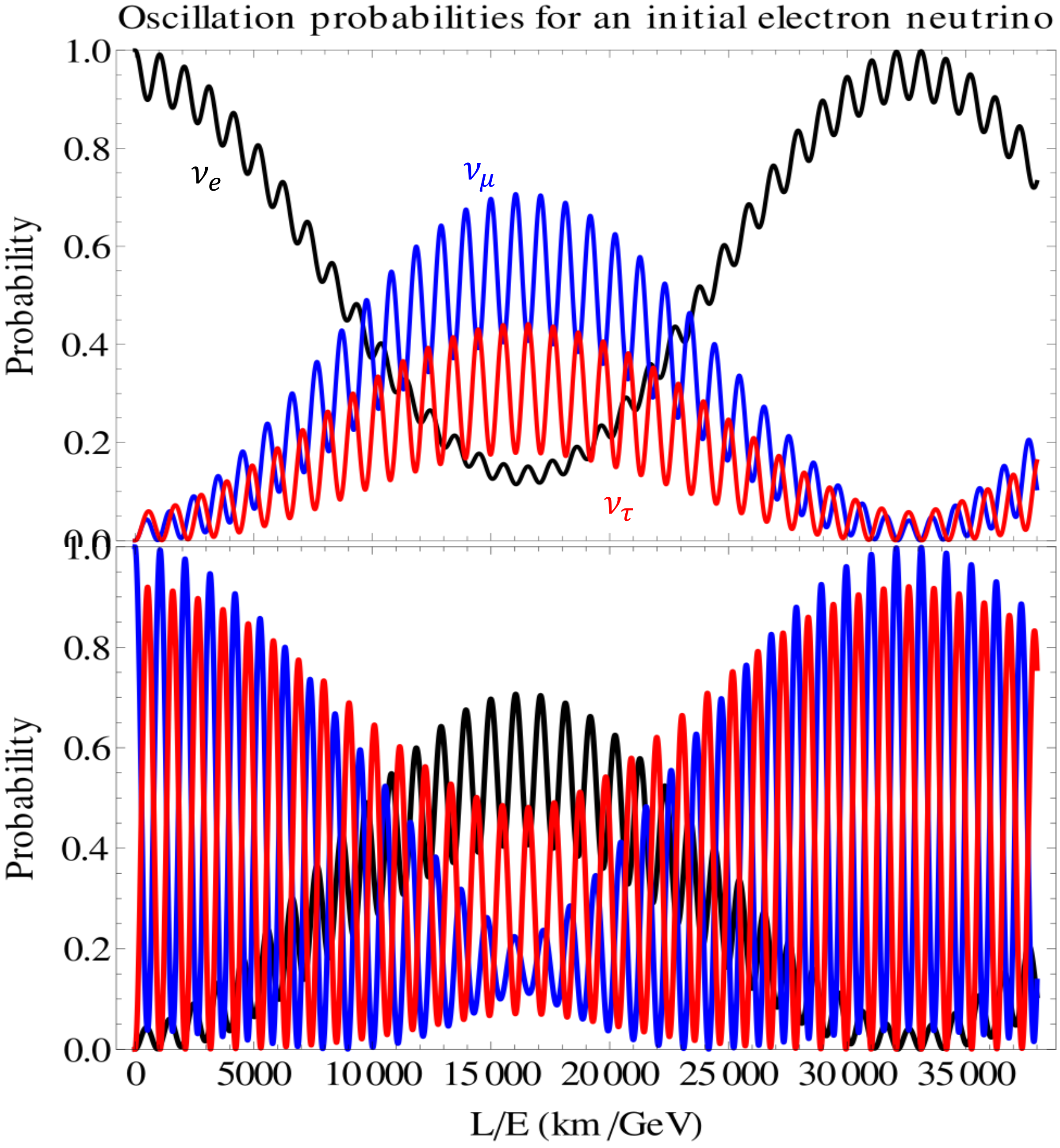


initial
electron
neutrino

Solar
neutrinos

initial muon
neutrino

Atmospheric
neutrinos



Summary of neutrino oscillations in vacuum:

- if we observe oscillations:
  - $\Rightarrow \Delta m^2_{ij} \neq 0 \Rightarrow m_i \text{ or } m_j \neq 0$
  - U_{PMNS} is non diagonal. (→ mixing)
- Oscillation provides access to very small Δm^2_{ij}
- Observation of neutrino oscillation in two ways: disappearance of ν_α or appearance of ν_β
- Neutrinos oscillation does not alter the total ν-flux: $\sum_{\nu_\beta} \mathrm{P}(\nu_\alpha \rightarrow \nu_\beta) = 1$
  - However, if some of ν_β are “sterile flavors” (no weak interactions,) then the total flux of the active neutrinos (ν_e , ν_μ , ν_τ) is reduced.

Measurement of θ_{13}

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{Atmospheric} \\ \text{neutrino mixing}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & & c_{13} \end{pmatrix}}_{\substack{\text{reactor neutrinos;} \\ \text{accelerator neutrinos}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{Solar neutrino} \\ \text{mixing; KamLAND}}}$$

$\Delta m^2_{atm} \sim 2.4 \cdot 10^{-3}\mathrm{eV}^2$

$\Delta m^2_{sol} \sim 7.5 \cdot 10^{-5}\mathrm{eV}^2$

$\theta_{23} \sim 49^o$

$\theta_{13} \sim 9^o$

$\theta_{12} \sim 33^o$

To observe CP violation in neutrino mixing a finite value of $\sin^2\theta_{13}$ is necessary.

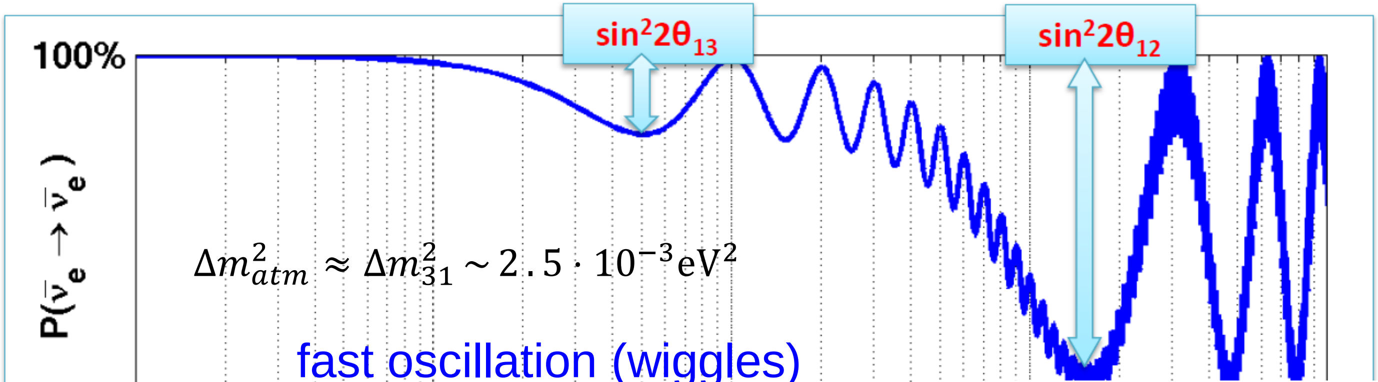
$|\Delta m^2_{32}| \sim |\Delta m^2_{31}|$ …but sign unknown

$\delta_{CP} \sim 200^o$ (indirectly)

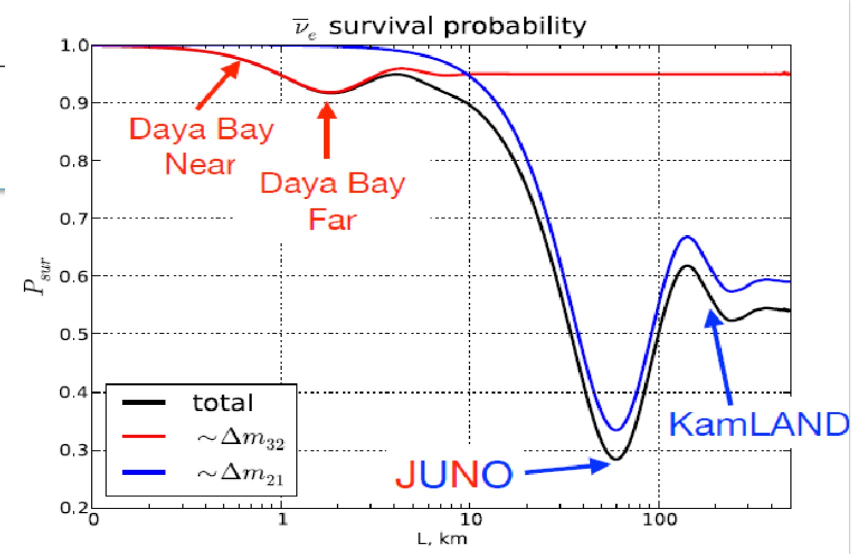
θ_{13} with reactor neutrinos:

Survival probability for 3-neutrino mixing:

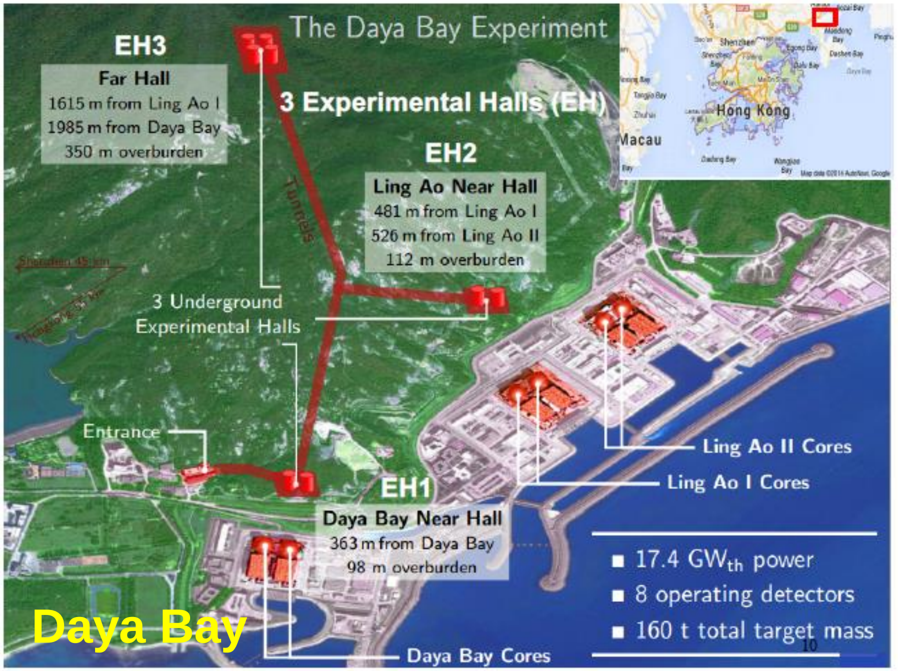
$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}{}^2 L}{4E_\nu}\right) - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}{}^2 L}{4E_\nu}\right)$$



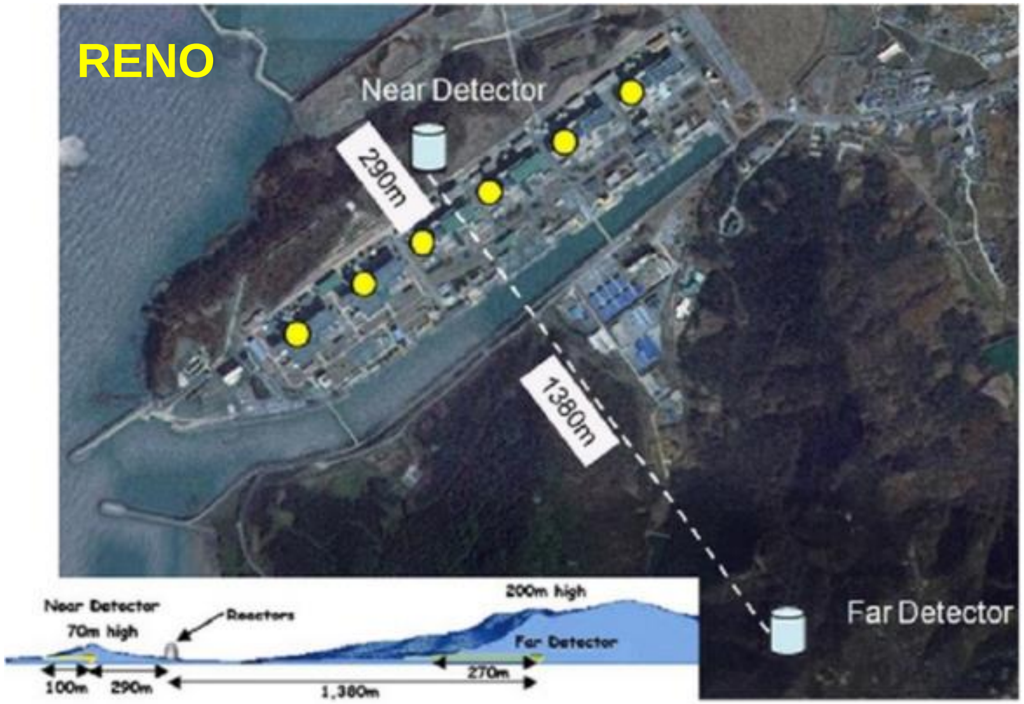
$\Delta m^2_{sol} \approx \Delta m^2_{21}$
$\sim 7 \cdot 10^{-5}\,\text{eV}^2$



The $\overline{\nu}_e$ survival probability as a function of the distance from the nuclear power plant (NPP).



(China)

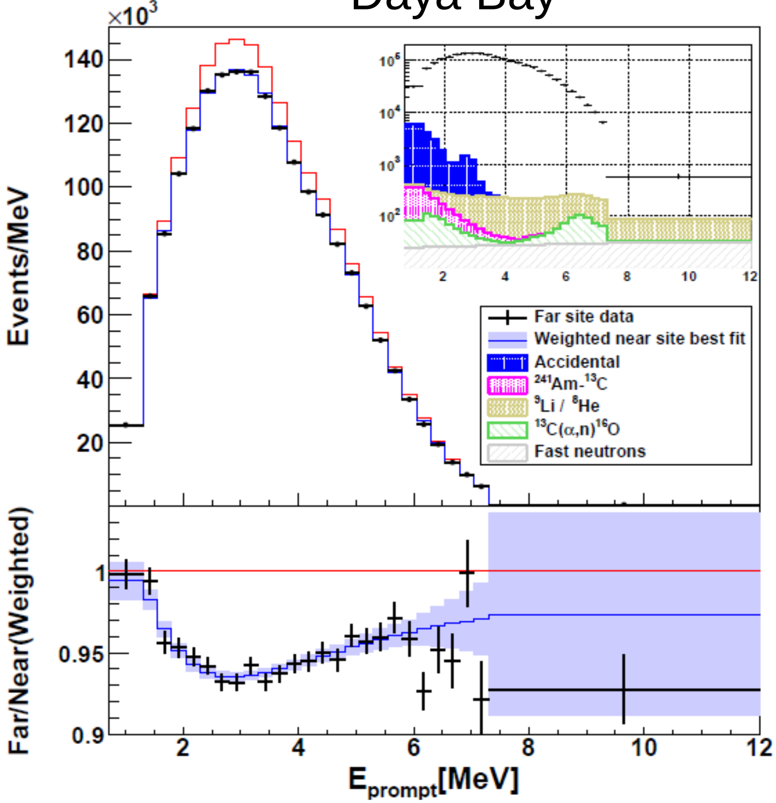


All experiments have a “near” detector to monitor the neutrino flux and a “far” (typ. Distance 1.5 km) to measure the deficit.

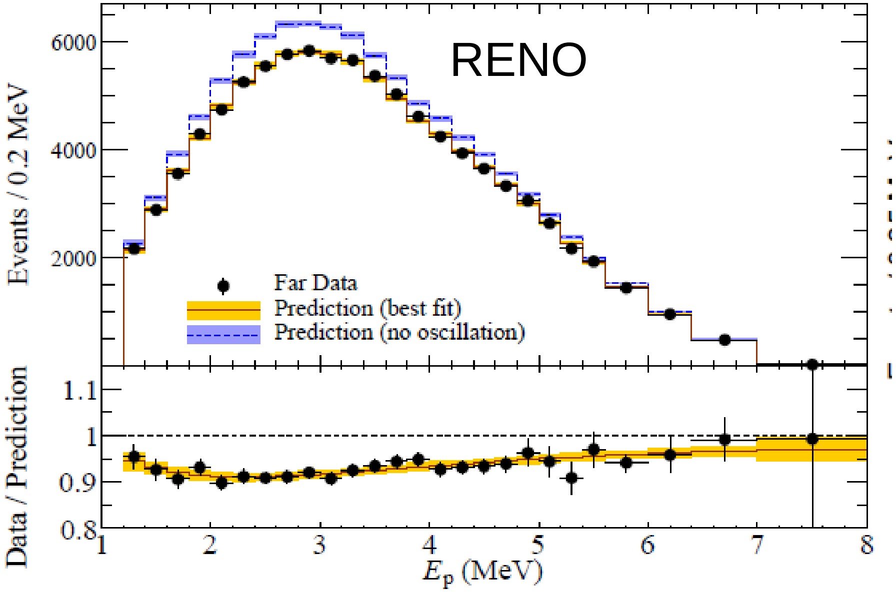
(South Korea)

The 3 reactor neutrino experiments published first results in 2012: Double Chooz reported an indication of electron antineutrino disappearance with the ratio of observed to expected events of R= 0.944±0.016 ±0.04 ruling out the no-oscillation hypothesis at 94.6% CL. Daya Bay observed of R= 0.940±0.011 ±0.004 corresponding to 5.2σ of a non-zero value of θ_{13}. RENO reported R= 0.920±0.000 ±0.014 indicating a non-zero value of θ_{13} with a significance of 4.9σ.

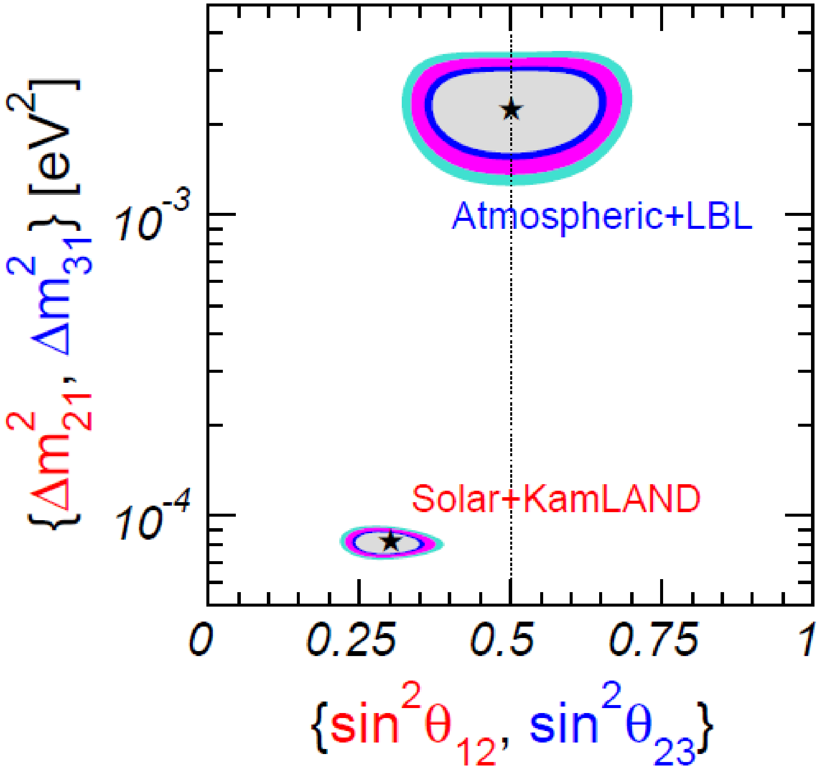
Daya Bay



PDG 2020



Summary: Neutrino Mixing



$\Rightarrow$

$\theta_{23} \sim 49^o$

$\theta_{12} \sim 33^o$

$\theta_{13} \sim 9^o$

T..Schwetz-Mangold

$$|U_{PMNS}| \approx \begin{pmatrix} 0.82 & 0.58 & > 0 \\ 0.64 & 0.58 & 0.71 \\ 0.64 & 0.58 & 0.71 \end{pmatrix}$$

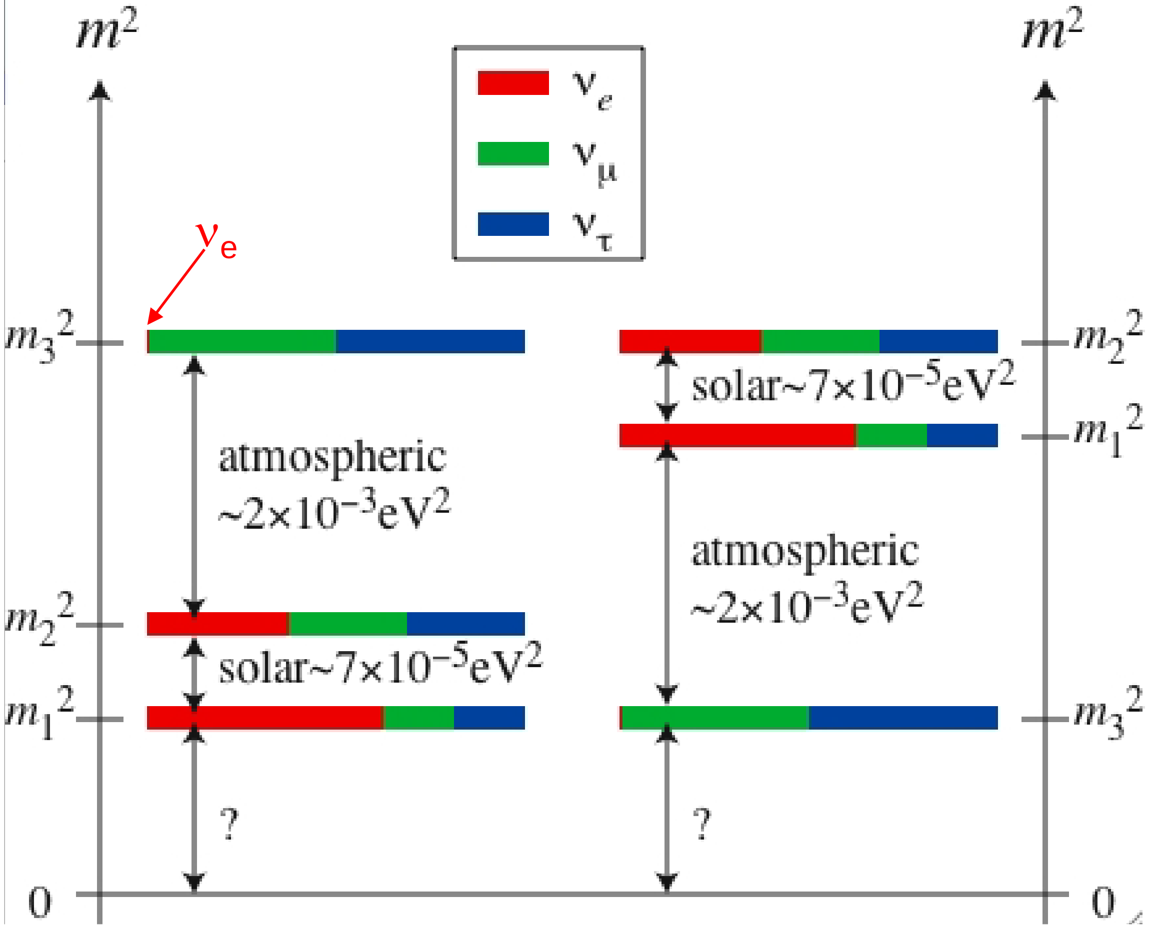
Summary: Neutrino masses

Normal

$\Delta m^2_{13} > 0$

Inverted

$\Delta m^2_{13} < 0$



5. Neutrino mass scale determination

For massive neutrinos the flavor states are linear combinations of the mass states. Mass limits can only be put on the effective mass of a neutrino with lepton flavor ℓ :

$$m^2_{\nu_\ell,eff} = \sum_i |U_{\ell i}|^2 m_i^2$$

Upper bounds on neutrino masses can be deduced from weak decays:

PDG 2024

${}^3\mathrm{H}_2 \to {}^3\mathrm{He}\,{}^3\mathrm{H}^+ + e^- + \bar{\nu}_e$ $(n \to p + e^- + \bar{\nu}_e)$	$m_{\bar{\nu}_e,eff} < 0.8\ \mathrm{eV}$
$\mu^\pm \to \nu_\mu + e^\pm + \nu_e$	$m_{\nu_\mu,eff} < 0.19\mathrm{MeV}$
$\tau^\pm \to n \cdot \pi + \nu_\tau$	$m_{\nu_\tau,eff} < 18.2\mathrm{MeV}$

Study energy distribution of visible final-state particles: “missing” invariant mass
→ neutrino mass

Upper bounds also exist from cosmology:

Large scale structure of galaxies, cosmic microwave background, type Ia supernovae, and big bang nucleosynthesis:

$$\sum m_i < 0.26\mathrm{eV}$$

arXiv:1811.02578v2

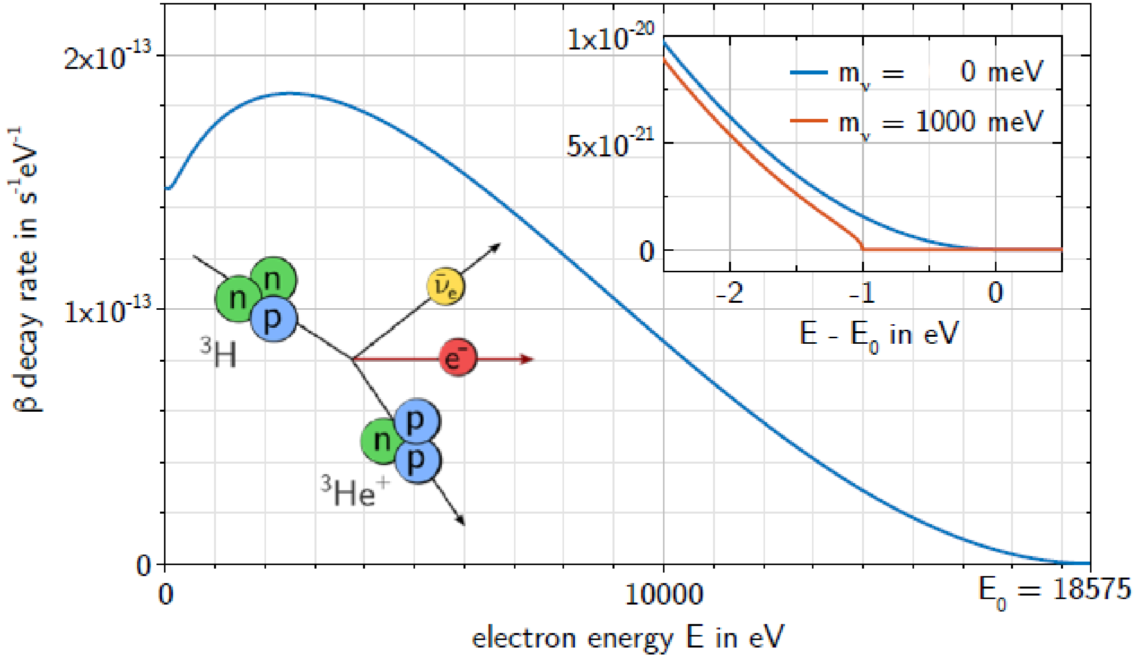
a) Effective electron anti-neutrino mass:

End-point method of a β-emitter (tritium, ^3H)

$$\frac{dN}{dE} = Cp(E+m_e)(E_0-E)\sqrt{(E_0-E)^2-\left(m_{\nu_e}^{eff}\right)^2}\cdot F(Z,E)$$

$$\equiv R(E)\sqrt{(E_0-E)^2-\left(m_{\nu_e}^{eff}\right)^2}$$

E_0 = Mass diff. of nuclei
E = kin. energy of electron
P = e^- momentum
F: Fermi function



Experimental requirements:

- High activity source
- Excellent energy resolution

"Direct" kinetic measurement: spectral distortion measures the "effective" mass squared:

Effective neutrino mass – consider mixing:

$$\frac{dN}{dE} = R(E)\sqrt{(E_0 - E)^2 - \left(m_{\nu_e}^{eff}\right)^2}$$

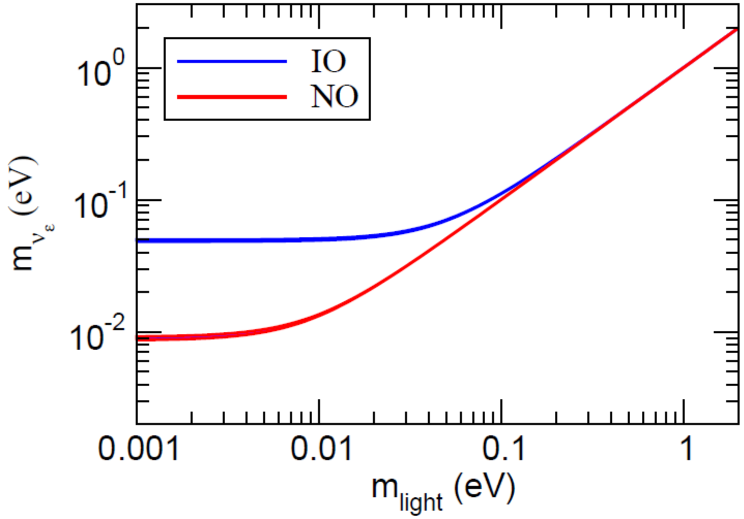
with:

$$m^2_{\nu_e,eff} = \sum_i |U_{ei}|^2 m_i^2$$

The KATRIN experiment has provided an upper bound for the effective neutrino mass:

$$0.8\text{ eV} \geq m_{\nu_e,eff} = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

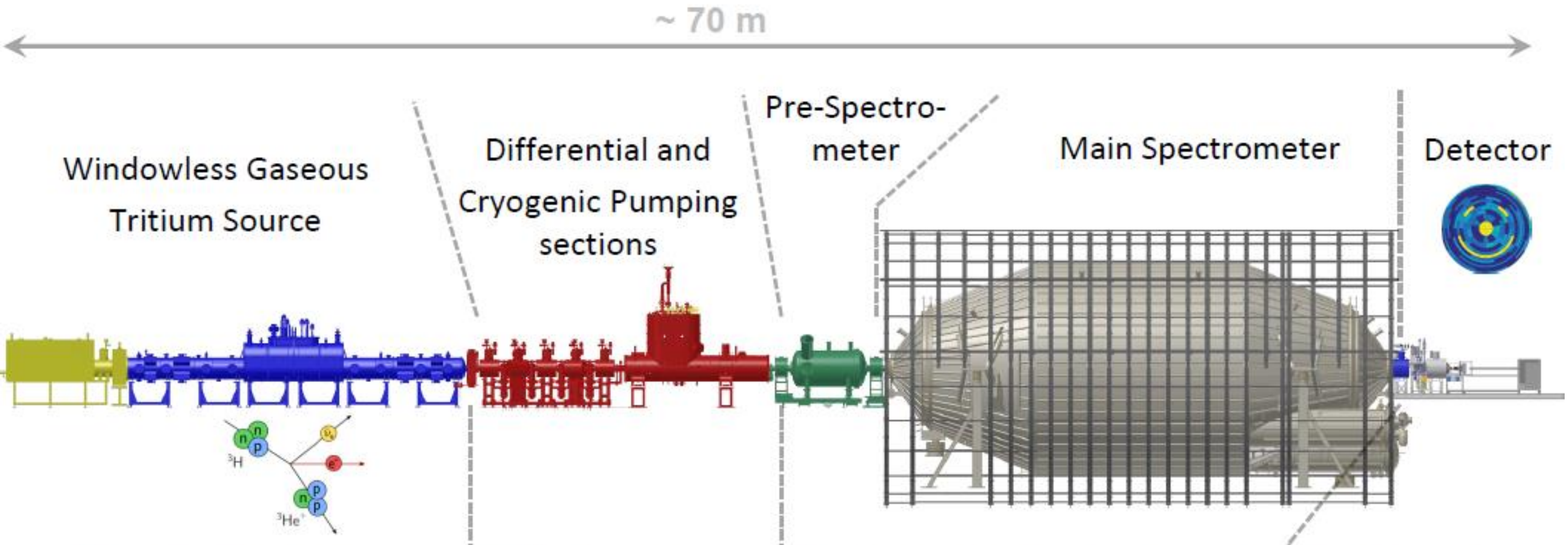
Depending on the neutrino mass hierarchy, this leads to a dependence on the light neutrino mass.



$$m_{\nu_e}^{\text{eff}} = \sqrt{\sum_i m_i^2 |U_{ei}|^2} = \begin{cases} \sqrt{m_0^2 + \Delta m_{21}^2(1 - c_{13}^2 c_{12}^2) + \Delta m_{32}^2 s_{13}^2} & \text{in NO}\,, \\ \sqrt{m_0^2 + \Delta m_{21}^2 c_{13}^2 c_{12}^2 - \Delta m_{32}^2 c_{13}^2} & \text{in IO}\,, \end{cases}$$

KATRIN = Karlsruhe Tritium Neutrino Experiment

Goal: measure neutrino mass w/ sensitivity of 0.2 eV (90%CL)



- tritium β-decay
- decay rate: 10^{11} 1/s
- T_2 pressure: 10^{-3} to 10^{-6} mbar

- β-electron transport
- tritium retention (factor > 10^{14})

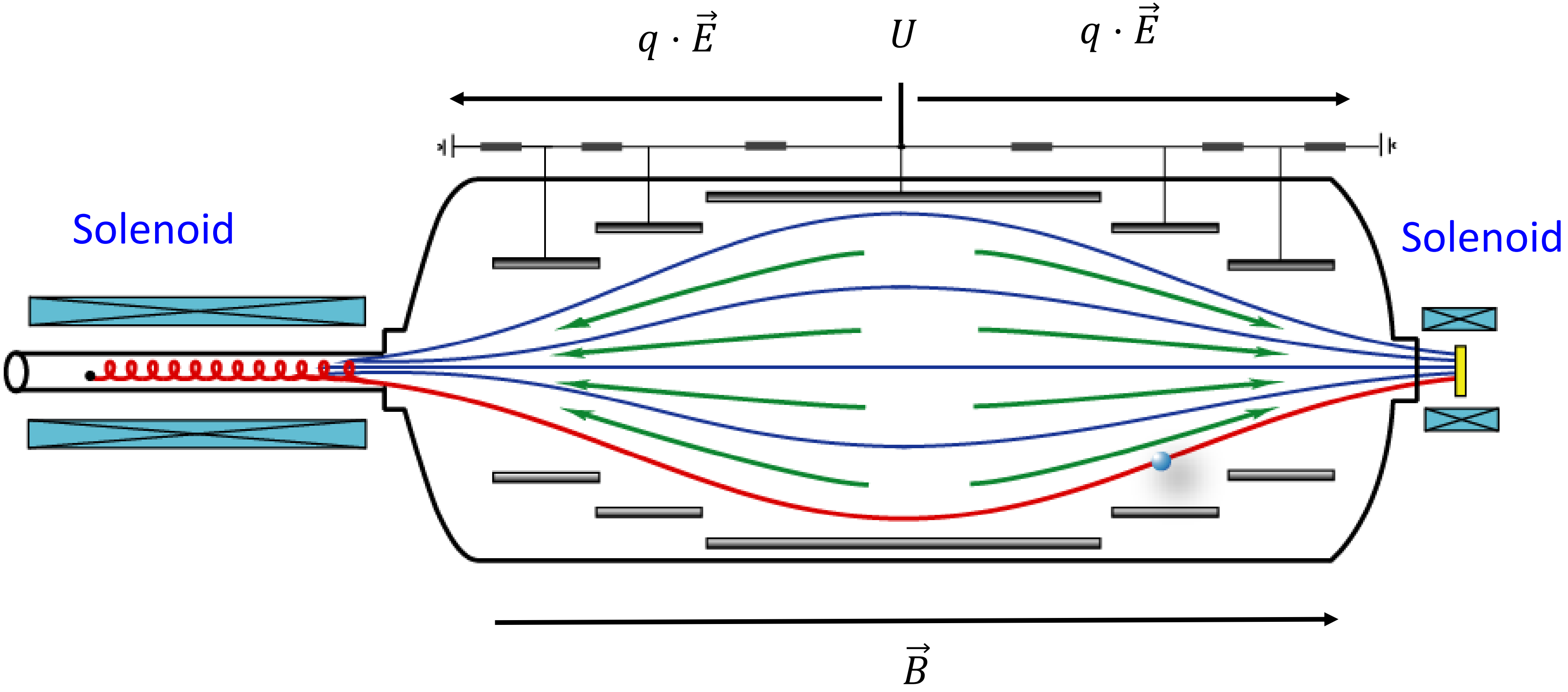
- energy analysis of β-electrons
- resolution 0.93 eV @ 18.6 keV
- pressure < 10^{-10} mbar

- counting of transmitted β-electrons < 1e / s

MAC-E Filter: Principle

"magnetic adiabatic collimation and electrostatic"

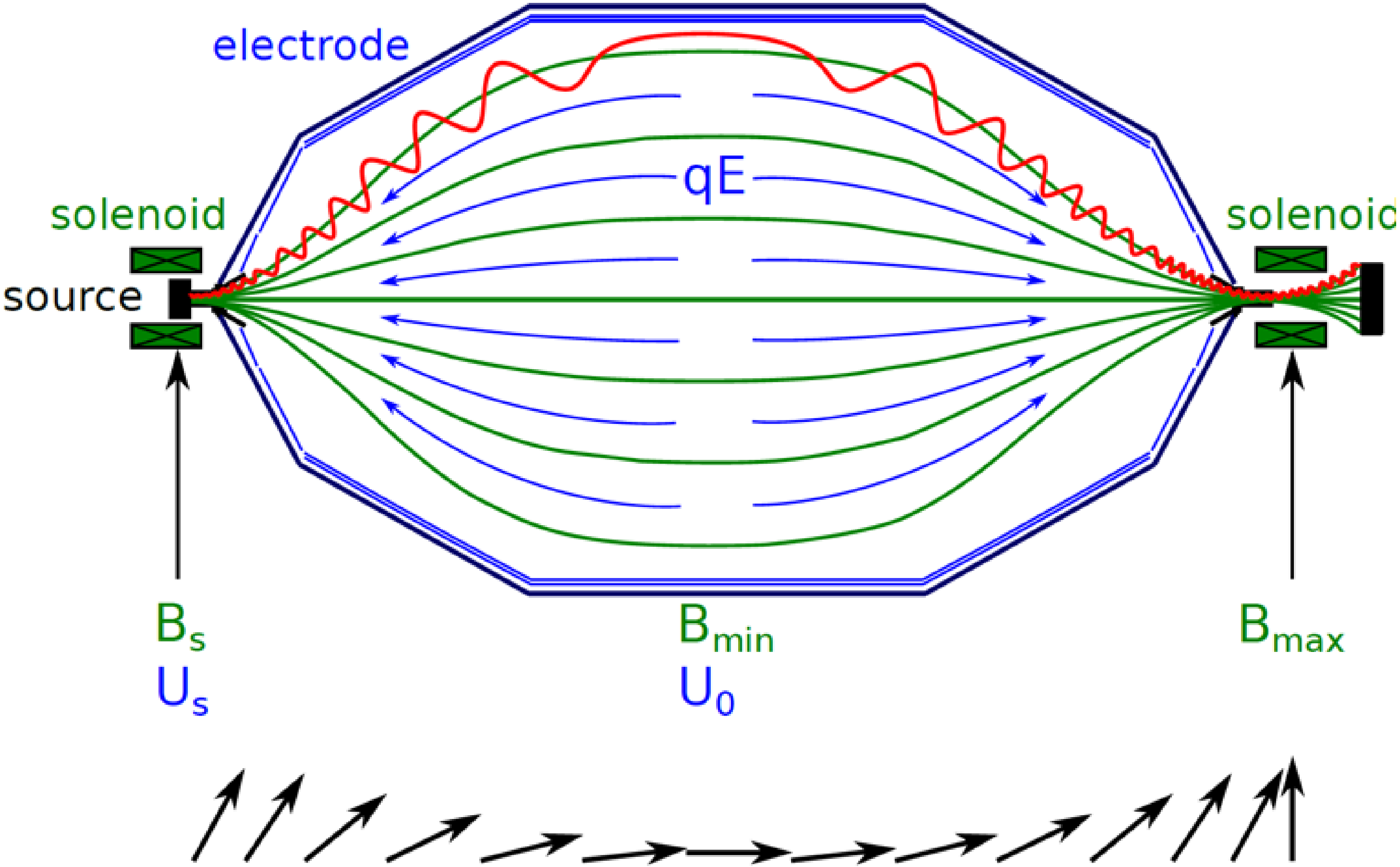
Electrostatic spectrometer:



No electron flux for: $E_{kin} = e \cdot U_{\max}$

MAC-E Filter: Principle

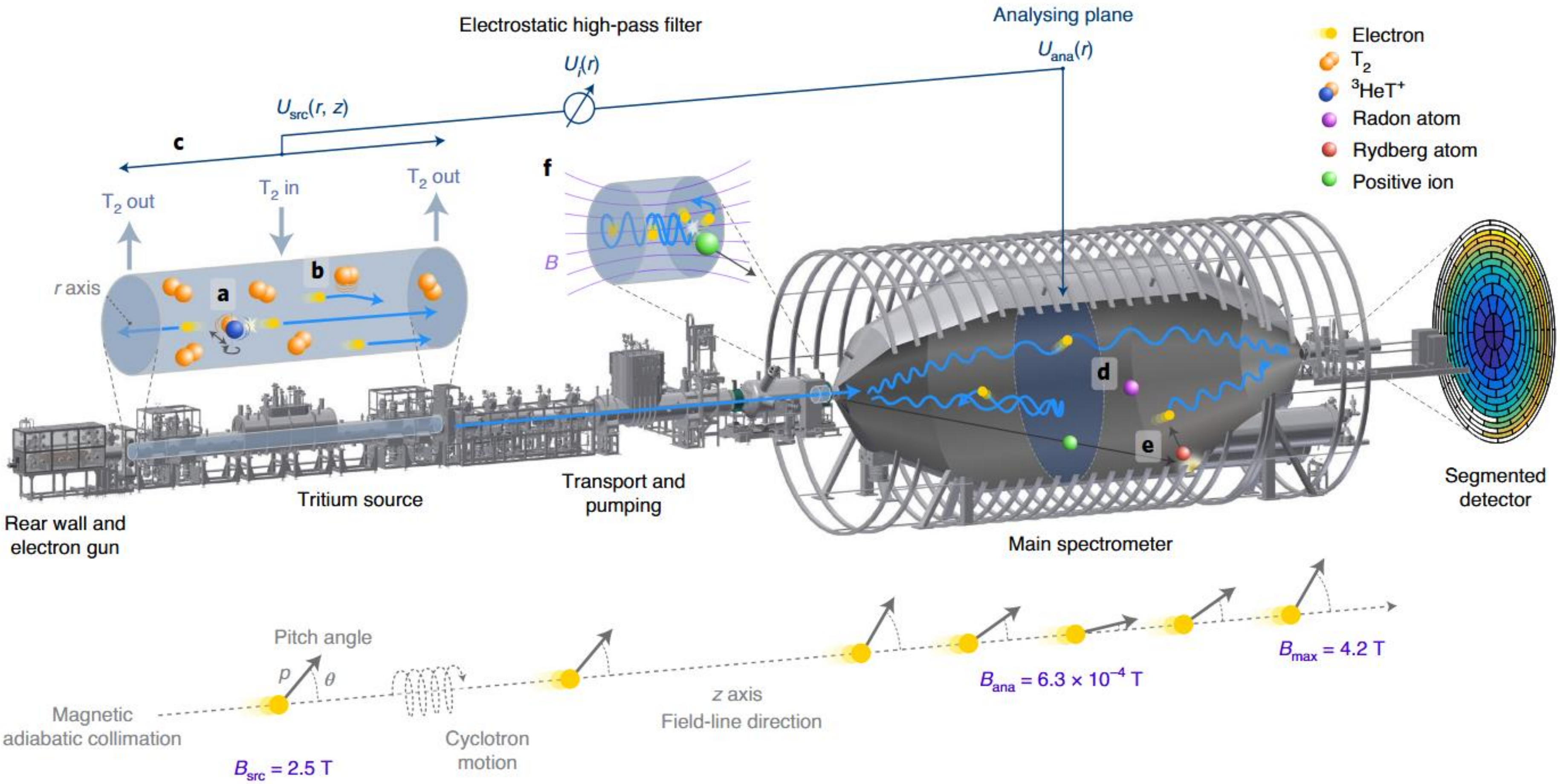
B fields serves to align the electron directions.



Adiabatic variation of B-field leads to alignment of momentum vector.

MAC-E Filter: Principle

B fields serves to align the electron directions.



Adiabatic variation of B-field leads to alignment of momentum vector.

Looks good on paper, but …

arXiv:1909.06048

KATRIN-Results:

First results from a 4 weeks measurement; Source activity 2.45 ×10^{10} Bq (Tritium density 1/5 of nominal).

Fit in the interval around the kinematic endpoint at 18.57 keV gives an effective neutrino mass square value of

$$m_{\nu,eff}^2 = (-1.0^{+0.9}_{-1.1})\mathrm{eV}^2$$

From this an upper limit of

$$m_{\nu,eff} < 1.1\mathrm{eV}\ (90\%\mathrm{CL})$$

on the absolute mass scale of neutrinos is derived.

Sensitivity after 1000 days of data-taking and nominal tritium density: 0.2 eV

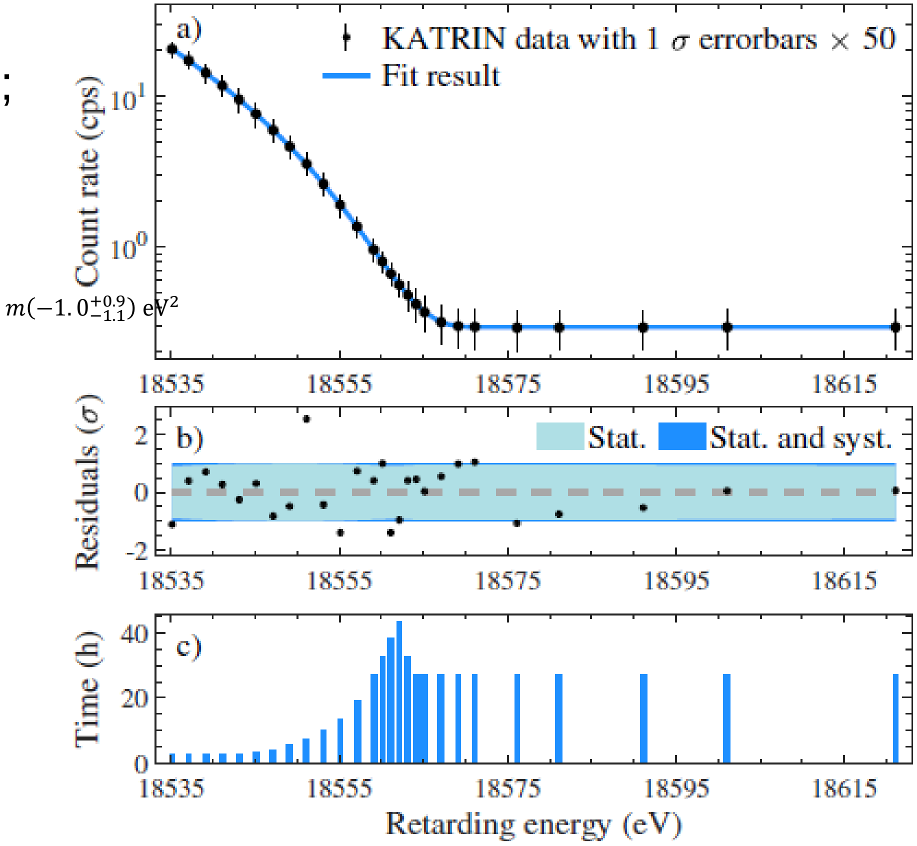


FIG. 3. a) Spectrum of electrons $R(\langle qU \rangle)$ over a 90 eV-wide interval from all 274 tritium scans and best-fit model $R_{\mathrm{calc}}(\langle qU \rangle)$ (line). The integral β-decay spectrum extends up to E_0 on top of a flat background R_{bg}. Experimental data are stacked at the average value $\langle qU \rangle_l$ of each HV set point and are displayed with 1-σ statistical uncertainties enlarged by a factor 50. b) Residuals of $R(\langle qU \rangle)$ relative to the 1-σ uncertainty band of the best fit model. c) Integral measurement time distribution of all 27 HV set points.

KATRIN-Results:

Nature Physics 18.160 (2022)

Latest results from 2022:
Source activity 9.5 $\times 10^{10}$ Bq
(Tritium density nominal).

Fit in the interval around the kinematic endpoint at 18.57 keV gives an effective neutrino mass-squared of:

$$m_{\nu,eff}^2 = 0.26^{+0.34}_{-0.34}\ \text{eV}^2$$

From this, an upper limit of

$$m_{\nu,eff} < 0.8\ \text{eV}\ (90\%\text{CL})$$

on the absolute mass scale of (electron anti-)neutrinos is derived.

Sensitivity after 1000 days of data-taking and nominal tritium density: 0.2 eV

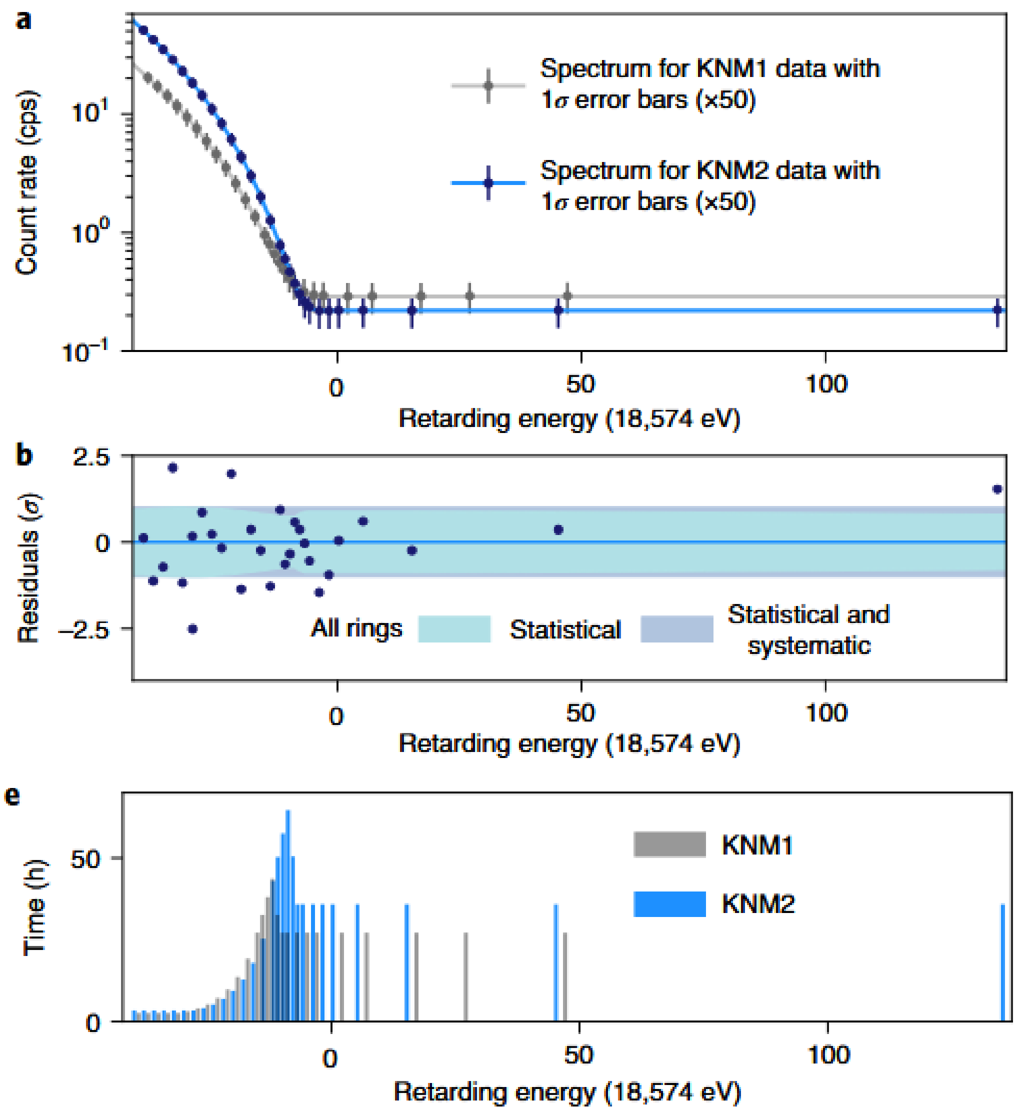


Fig. 2 | Measured rate at each retarding energy for KNM1 (refs. [17,18]) and KNM2 campaigns. a, Data points with statistical error (multiplied by a factor of 50) and best-fit model (blue and grey lines) individually shown for each campaign. The count rates are summed over all the detector rings.

KATRIN-Results: 2024 (preprint)

(result not yet peer-reviewed or published)

259 Days of running time, source activity (mostly) close to nominal

Fit in the interval around the kinematic endpoint at 18.57 keV gives an effective neutrino mass-squared of:

$$m^2_{\nu,eff} = -0.14^{+0.13}_{-0.15}\ \text{eV}^2$$

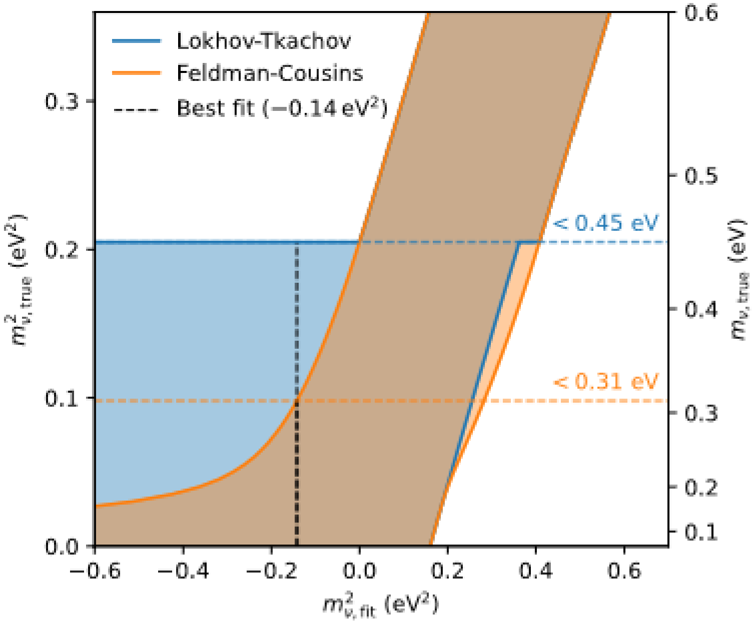
From this, an upper limit of

$$m_{\nu,eff} < 0.45\ \text{eV}\ (90\%\text{CL})$$

on the absolute mass scale of (electron anti-)neutrinos is derived.

Sensitivity after 1000 days of data-taking and nominal tritium density: 0.2 eV

arXiv:2406.13516



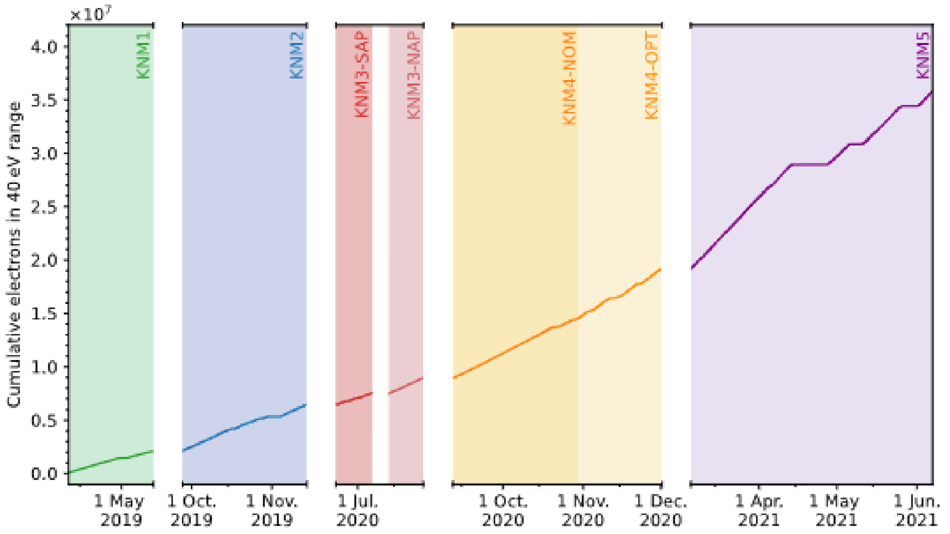


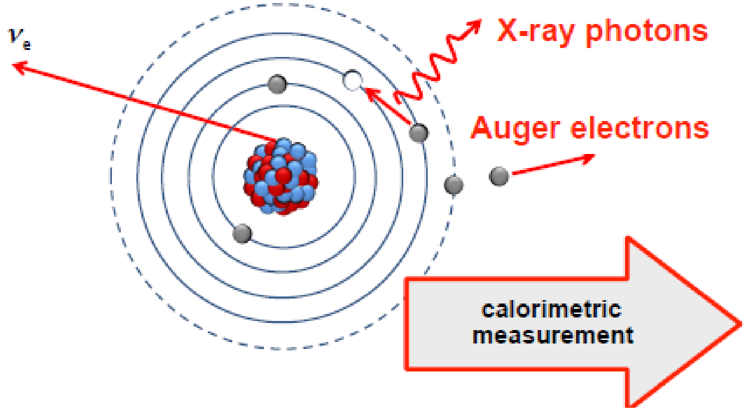
Figure 6: Cumulative counts collected in the $qU > E_0 - 40\,\text{eV}$ analysis window of the first five measurement campaigns. Each campaign is highlighted in the corresponding color.

Holmium Electron Capture:

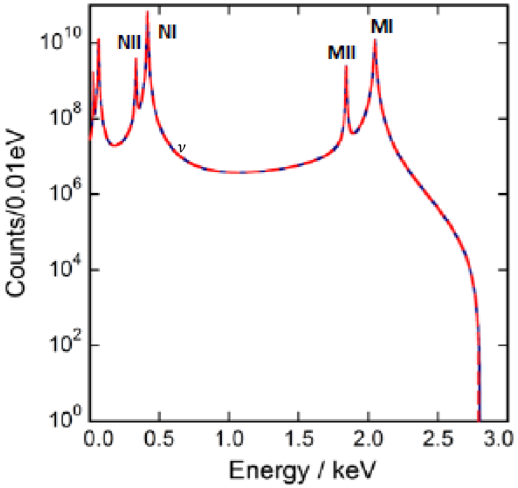
Slide by K. Valerius

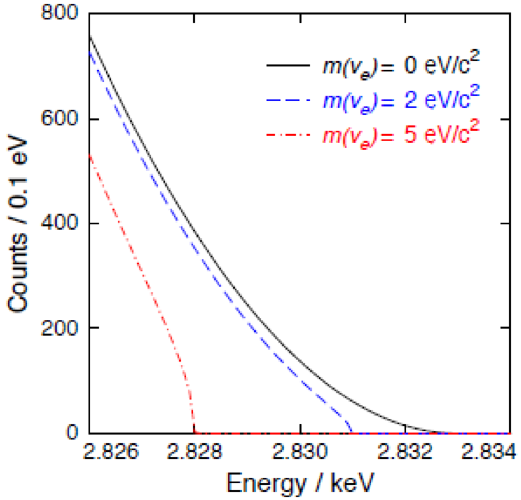
$^{163}\mathrm{Ho} \rightarrow {}^{163}\mathrm{Dy}^* + \nu_e$

Low Q_{EC} ~2.8 keV and $T_{1/2}$ ~ 4570 years



[De Rujula & Lusignoli 1982]





Challenges:

- production & purification of isotope ^{163}Ho
- incorporation of ^{163}Ho into high-resolution detectors
- operation & readout of large calorimeter arrays
- detailed understanding of calorimetric spectrum (nuclear & atomic physics + detector response)

How to measure 2.8 keV w/ high precision?

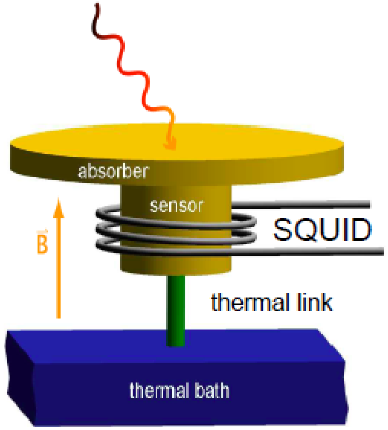
Dy = Dyprosium

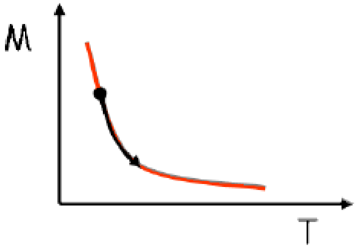
Micro Calorimeters: MMCs

Slide by K.Valerius

MMC: **m**etallic **m**agnetic **c**alorimeters
with paramagnetic sensor Au:Er

L. Gastaldo
C. Enss

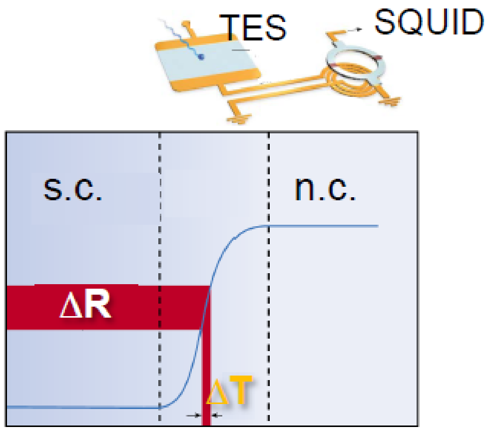




δT in absorber from EC-decay
⇨ change in magnetization M of sensor

signal: $$\delta\Phi_S \sim \frac{\partial M}{\partial T}\cdot\Delta T \sim \frac{\partial M}{\partial T}\cdot\frac{1}{C_{tot}}\cdot\delta E$$

thermal micro-calorimeters
with **t**ransition **e**dge **s**ensor (**TES**)



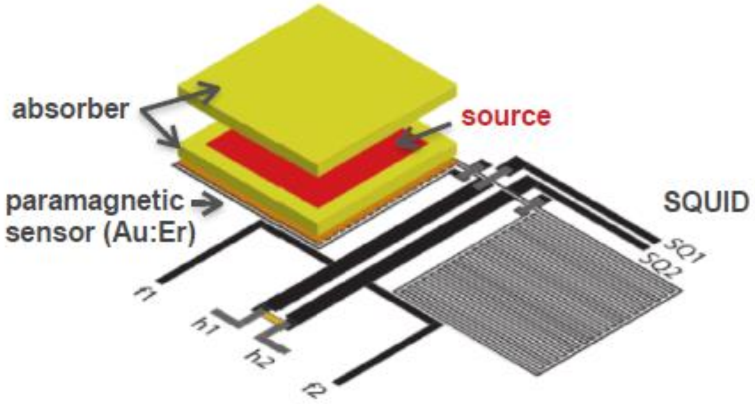
δT in absorber from EC-decay
⇨ change in temperature T and resistance R of thermistor

signal: current change measured by
SQUID array

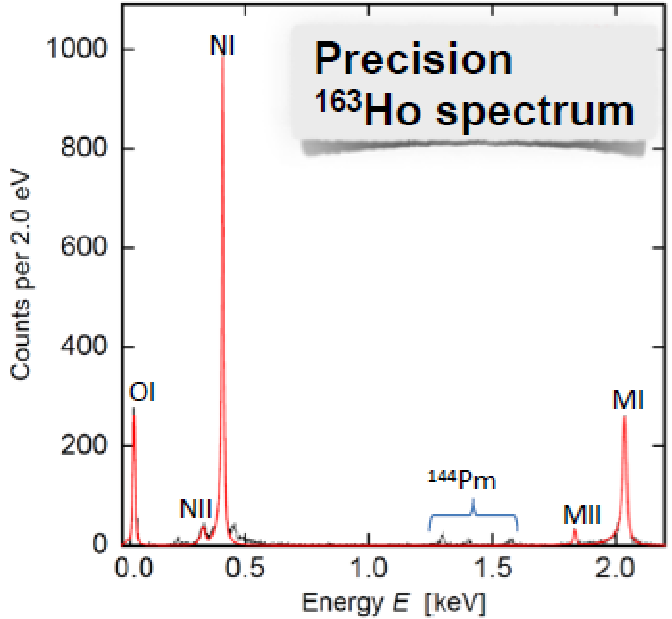
ECHo Experiment

Uni Heidelberg:
C. Enss, L. Gastaldo

MMC technology: ECHo



[Fleischmann *et al.* 2009; Gastaldo *et al.* 2013]



[Ranitzsch et al., arXiv:1409:0071; Gastaldo *et al.* 2014]

5. Dirac vs. Majorana: Neutrinoless Double-beta Decay

The problem of the nature of massive neutrinos ν_i (Dirac or Majorana?) is one of the most fundamental problems of neutrino physics. The answer to this question will have an important impact on the understanding of the origin of neutrino masses.

The Majorana mass term breaks lepton number by two units - the Majorana mass term is the lowest dimension operator which uses SM fields and obeys SM gauge symmetries and which breaks lepton number at tree-level. In order to reveal the nature of neutrinos with definite masses it is necessary to study processes in which the total lepton number L is violated by two units (i.e., neither neutrino oscillations nor CC interactions can reveal the neutrino nature).

Lepton flavor violation experiments:

- In case of Majorana particle $\nu = \nu^C$ the following process becomes possible:

$$\pi^+ \to \mu^+ + \nu_i \,; \qquad \nu_i + N \to \mu^+ + p \qquad \text{with} \qquad \mathrm{A}(\nu_i N \to \mu^+ N) \sim \frac{m_\nu}{E_\nu} \to \sigma \sim \left(\frac{m_\nu}{E_\nu}\right)^2$$

neutrino beam

Thus the cross section for observing this reaction in a collider experiment (E_ν larger than typ. 1 MeV, $m_\nu < 1$ eV) is suppressed by ($\times 10^{-12}$). This is much too small for observation with current experiments.

- Decays of B or K-mesons. E. g.: $K^+ \to \pi^- \mu^+ \mu^+$

Experimental bounds: $$\frac{\Gamma(K^+ \to \pi^- \mu^+ \mu^+)}{\Gamma(K^+ \to \text{ all})} \le 3 \cdot 10^{-9} \Longrightarrow$$

Limit on the effective mass $|m_{\mu\mu}| < 4 \cdot 10^4 \text{MeV}$ (not very strong)

(meaning of “effective mass” : see below)

- Processes such as $\mu^- + (A, Z) \to (A, Z-2) + e^+$

Experimental bounds: $$\frac{\Gamma(\mu^- \text{Ti} \to e^- \text{Ca})}{\Gamma(\mu^- \text{Ti} \to \text{ all})} \le 1.7 \cdot 10^{-12} \Longrightarrow$$

Limit on the effective mass $|m_{\mu e}| < 82 \text{MeV}$ (not very strong)

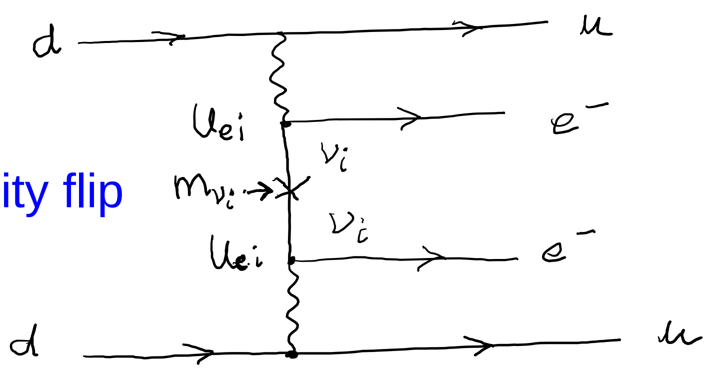
(meaning of “effective mass” : see below)

- The most sensitive probe to whether neutrinos are Dirac or Majorana states is the neutrinoless double-beta decay (0νββ) of a nucleus.

Neutrinoless Double-beta Decay

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$

chirality flip



$$\mathrm{A}_{0\nu 2\beta} \sim \underbrace{\sum_i m_i U_{ei}^2}_{m_{ee}}$$

(similar definition for $m_{\mu\mu}$ and $m_{\mu e}$)

Under the assumption that the Majorana neutrino mass is the only source of lepton number violation at low energies, the decay half-life is given by:

$$\Gamma^{0\nu}_{1/2} \sim \left(T^{0\nu}_{1/2}\right)^{-1} = G^{0\nu}\left|\mathrm{M}^{0\nu}\right|^2 \left(\frac{m_{ee}}{m_e}\right)^2$$

$G^{0\nu}$ is the phase space integral taking into account the final atomic state;
$\mathcal{M}^{0\nu}$ is the nuclear matrix element of the transition;
m_{ee} is the effective Majorana mass of ν_e:

$$m_{ee} = \left|\sum_i m_i U_{ei}^2\right|$$

Note that the term $\sum_i m_i U_{ei}^2$ is in general complex and depends on the phases of the PMNS elements (δ_{CP} and the two Majorana phases $\eta_{1,2}$)

Thus, in addition to the masses and mixing parameters the decay spectrum depends also on the leptonic CP violating phases (→ allows determination):

$$m_{ee} = \left|\sum_i m_i U_{ei}^2\right| = \left|m_1 c_{13}^2 c_{12}^2 e^{i2\eta_1} + m_2 c_{13}^2 s_{12}^2 e^{i2\eta_2} + m_3 s_{13}^2 e^{-i2\delta_{CP}}\right|$$

arXiv:1811.05487

One can discuss two different mass orderings:

(inspired by experimental data)

1. Normal ordering (NO): $m_1 < m_2 < m_3\,;\quad \Delta m_{12}^2 \ll \Delta m_{23}^2\,;$ $\quad\Rightarrow m_1 < m_2 \ll m_3$
2. Inverted ordering (IO): $m_3 < m_1 < m_2\,;\quad \Delta m_{12}^2 \ll \left|\Delta m_{13}^2\right|\,;$ $\quad\Rightarrow m_3 \ll m_1 < m_2$

$$m_{ee} = \left|\sum_i m_i U_{ei}^2\right|$$

with $m_0 = m_1$ (NO), m_3 (IO), smallest mass

$$= \begin{cases} \left|m_0 c_{12}^2 c_{13}^2 + \sqrt{\Delta m_{21}^2 + m_0^2}\, s_{12}^2 c_{13}^2 e^{2i(\eta_2-\eta_1)} + \sqrt{\Delta m_{32}^2 + \Delta m_{21}^2 + m_0^2}\, s_{13}^2 e^{-2i(\delta_{\rm CP}+\eta_1)}\right| & \text{in NO}\,, \\ \left|m_0 s_{13}^2 + \sqrt{m_0^2 - \Delta m_{32}^2}\, s_{12}^2 c_{13}^2 e^{2i(\eta_2+\delta_{\rm CP})} + \sqrt{m_0^2 - \Delta m_{32}^2 - \Delta m_{21}^2}\, c_{12}^2 c_{13}^2 e^{2i(\eta_1+\delta_{\rm CP})}\right| & \text{in IO}\,, \end{cases}$$

$$\begin{cases} \leq \left(\sin^2\theta_{12}\sqrt{\Delta m_{12}^2} + \sin^2\theta_{13}\sqrt{\Delta m_{23}^2}\right) \\ \simeq \sqrt{|\Delta m_{13}^2|}\ \left(1 - \sin^2 2\theta_{12}\ \sin^2\alpha\right)^{\frac{1}{2}}, \end{cases}$$

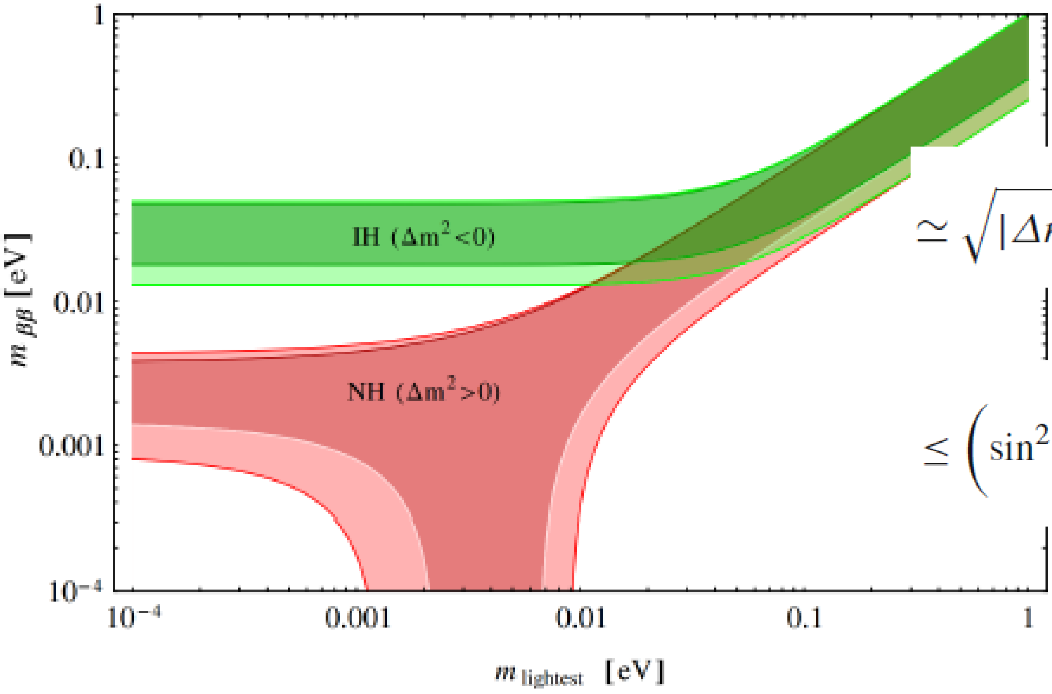
in NO: m_{ee} can me arbitrarily small

in IO: there is a lower bound on m_{ee}

α is Majorana phase diff.

S.Bilenky (2010)

Neutrinoless double beta decay can help to resolve the neutrino mass hierarchy (of course only if neutrinos are Majorana particles):



Inverted hierarchy:

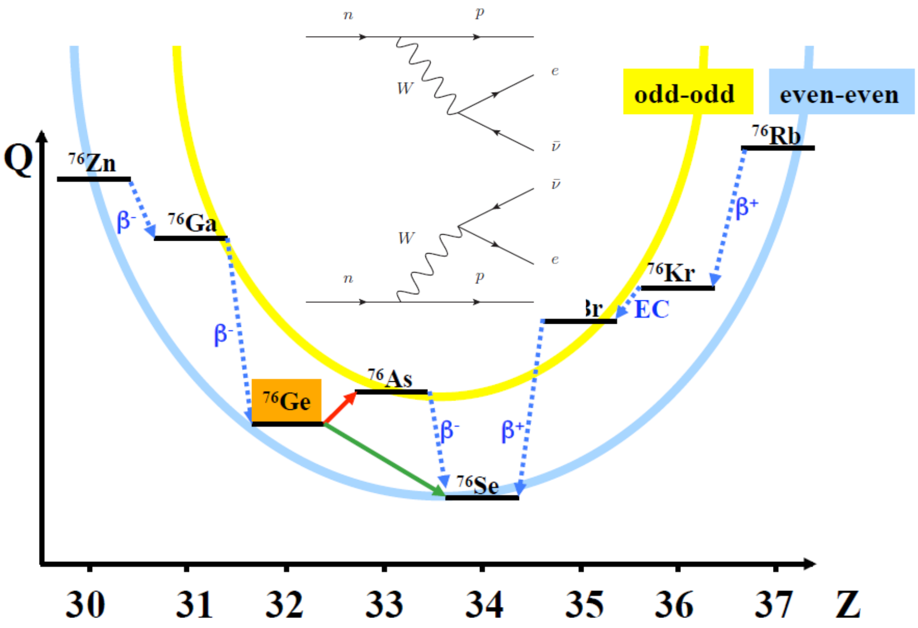
$$\simeq \sqrt{|\Delta m^2_{13}|}\ (1-\sin^2 2\,\theta_{12}\ \sin^2\alpha)^{\frac{1}{2}},$$

Normal hierarchy:

$$\leq \left(\sin^2\theta_{12}\sqrt{\Delta m^2_{12}}+\sin^2\theta_{13}\sqrt{\Delta m^2_{23}}\right)$$

Searching for neutrinoless double-beta decay:

2β decay:
mass parabola from Weizsäcker formula



Possible 2β candidates:

Transition	$T_0 = Q_{\beta\beta}$ (KeV)
$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$	$2,039.6 \pm 0.9$
$^{100}\mathrm{Mo} \rightarrow {}^{100}\mathrm{Ru}$	$3,934 \pm 6$
$^{130}\mathrm{Te} \rightarrow {}^{130}\mathrm{Xe}$	$2,533 \pm 4$
$^{136}\mathrm{Xe} \rightarrow {}^{136}\mathrm{Ba}$	$2,479 \pm 8$
$^{150}\mathrm{Nd} \rightarrow {}^{150}\mathrm{Sm}$	$3,367.1 \pm 2.2$
$^{82}\mathrm{Se} \rightarrow {}^{82}\mathrm{Kr}$	$2,995 \pm 6$
$^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Ti}$	$4,271 \pm 4$

$$T_{1/2}^{2\nu}(^{76}\mathrm{Ge}) =$$
$$(1.929 \pm 0.095) \cdot 10^{21}\mathrm{yr}$$

arXiv:1501.02345

Normal β-decay energetically forbidden for ^{74}Ge.
Double β-decay allowed: even-even nuclei.

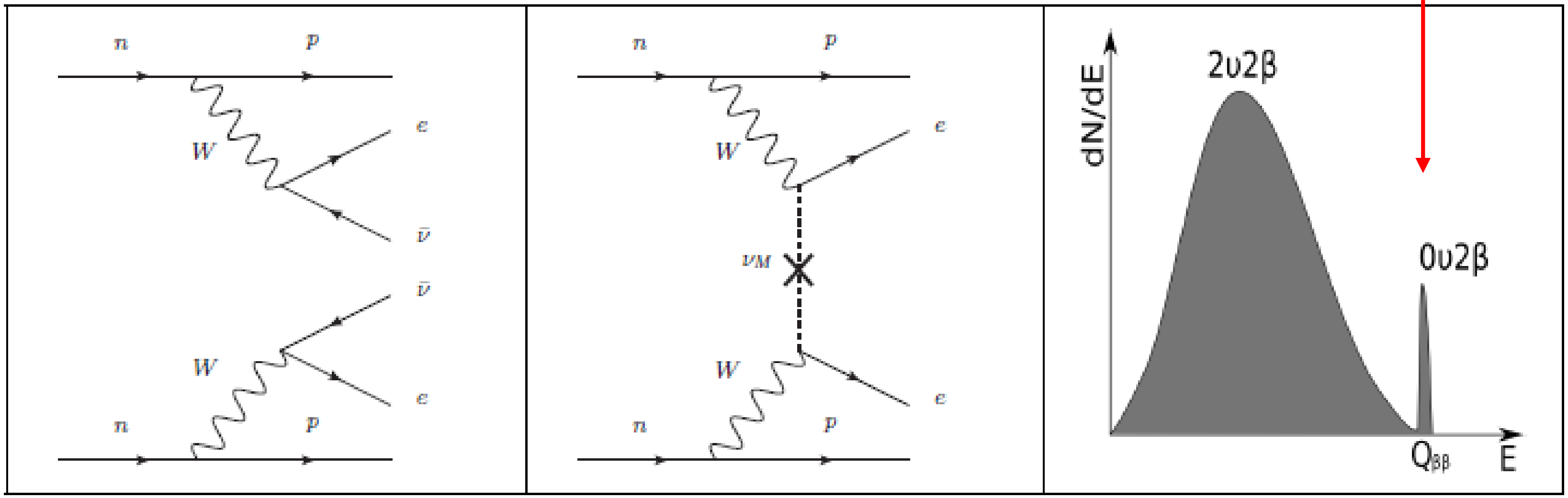
Two-neutrino double β decay is a process of second order in the Fermi constant G_F, which is governed by the standard CC Hamiltonian of the weak interaction. This decay was observed in more than ten different nuclei with half-lives in the range (10^{18} – 10^{24}) years.

Search technique:

Background: 2ν2β

Signal: 0ν2β

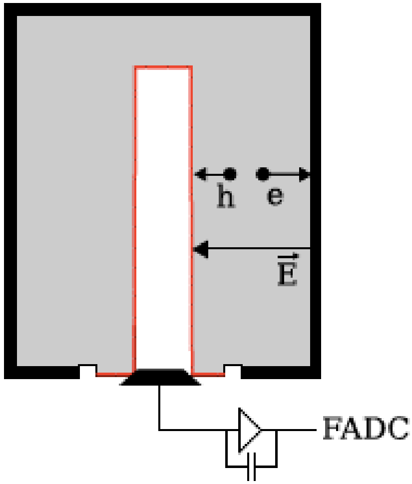
endpoint



Source = Detector

Decay & detection material ^{76}Ge:

- Ge is a 2β decay isotope
- Source material = detector material
- Germanium detectors (=semi-conductor) have excellent energy resolution: FWHM ~ 1.5 10^{-3} @ 2 keV
- Enrichment of ^{76}Ge up to 86%



Ge diode w/ reverse biasing