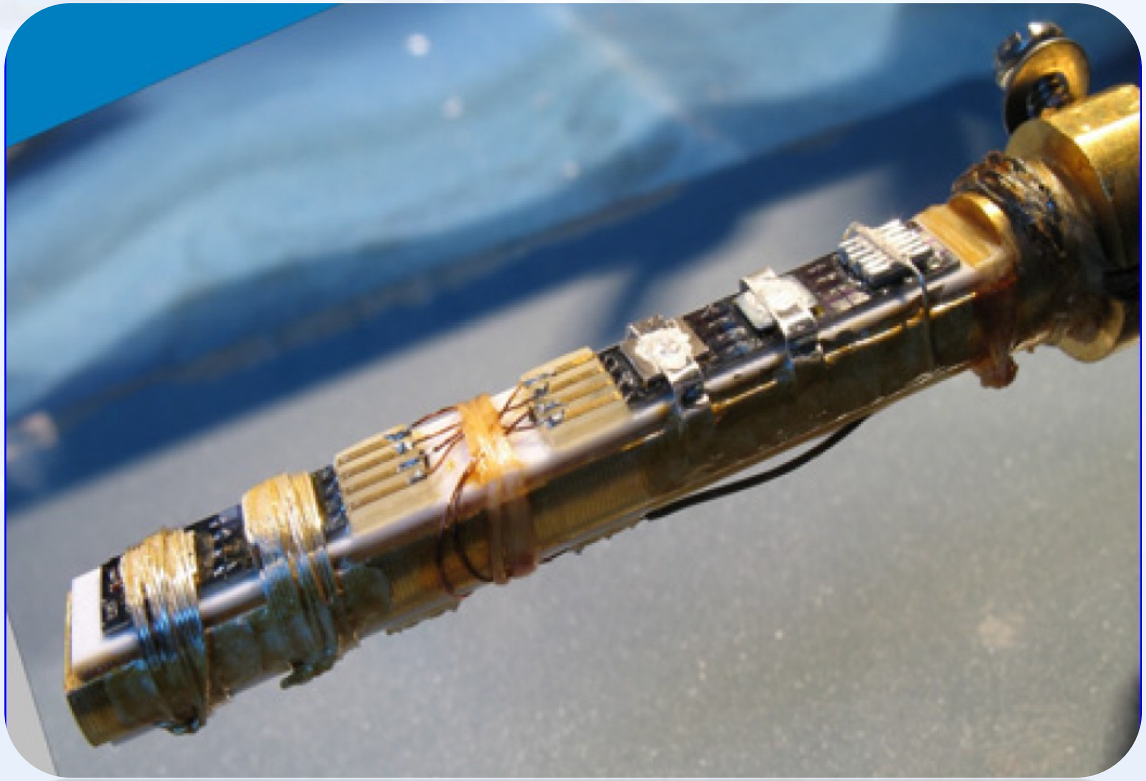
12. Thermometry at Low Temperature

Primary thermometers
Superconducting fixpoints
Current/flux noise
^{195}Pt NMR
Coulomb blockade
Nuclear orientation
^{3}He melting curve
....

Secondary thermometers
Resistance
Capacitance
Magnetic susceptibility
.....



12. Thermometry at Low Temperature

Temperature is a thermodynamic property of state

It can be defined by a reversible cycle, like a carnot cycle $\oint T^{-1}\mathrm{d}Q = 0$

not practical

- primary thermometers: can be used without any prior calibration
- secondary thermometers: must be calibrated against another thermometer

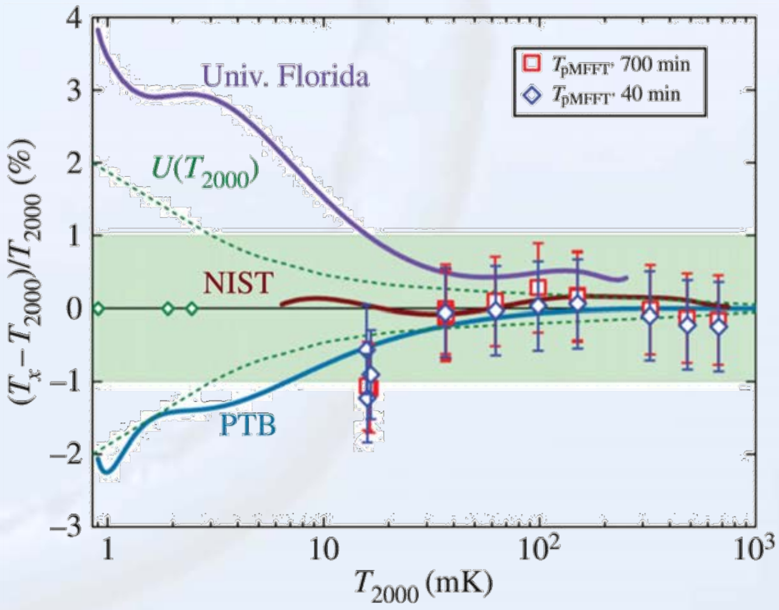
distinction is often somewhat arbitrary …

Temperature scales

defined by *Comité International des Poids et Messures*

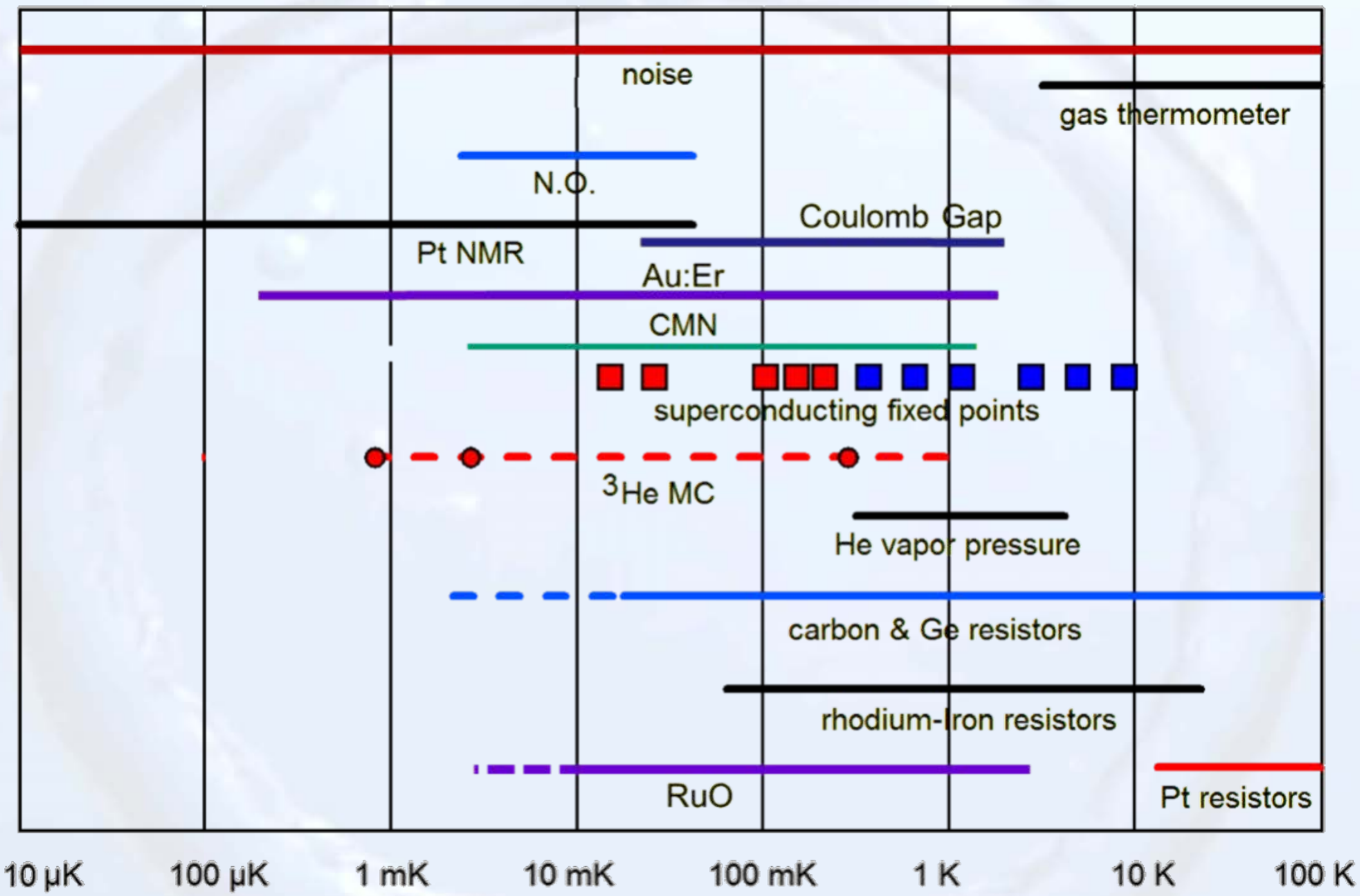
based on fixpoints like the triple point of water
and interpolation like Pt-100 resistance thermometry
or gas thermometry

ITS-90	0.65 K	to 1358 K
PLTS-2000	0.9 mK	to 1358 K



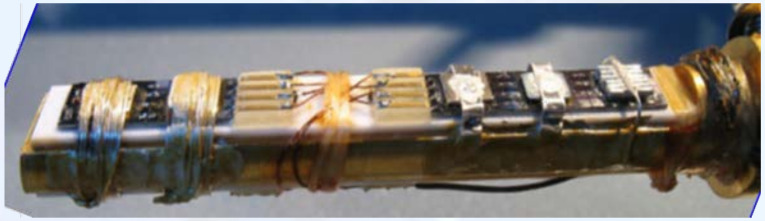
12. Thermometry at Low Temperature

Thermometer types and ranges

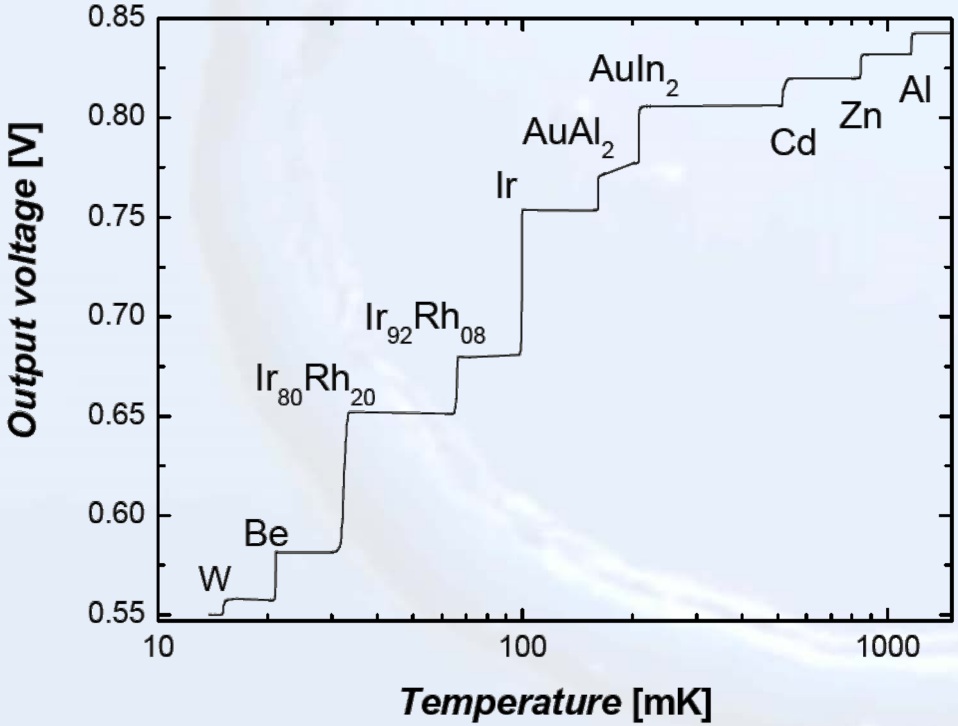


Superconducting fixpoint devices

inductive measurement of the superconducting transition



SRD 1000



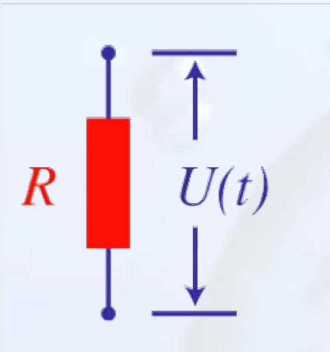
#	material	T_C [mK]	W_C [mK]	U_C [%]
1	W	15	< 0.2	< 0.26
2	Be	21	< 0.3	< 0.28
3	$Ir_{80}Rh_{20}$	30	< 0.5	< 0.34
4	$Ir_{92}Rh_{08}$	65	< 0.5	< 0.16
5	Ir	98	< 0.5	< 0.10
6	$AuAl_2$	145	< 0.5	< 0.06
7	$AuIn_2$	208	< 1	< 0.10
8	Cd	520	< 4	< 0.16
9	Zn	850	< 3	< 0.08
10	Al	1180	< 4	< 0.06

12. Thermometry at Low Temperatures

thermal voltage fluctuations across a conductor (predicted by A. Einstein 1905)

1927

John Bertrand "Bert" Johnson



power spectral density

$$S_U = \frac{\langle U^2 \rangle}{\Delta f} = 4 k_\mathrm{B} T R$$

1928

Harry Nyquist

quantum corrections

$$S_U = \frac{\langle U^2 \rangle}{\Delta f} = 4 h f R \left[\frac{1}{2} + \frac{1}{e^{hf/k_\mathrm{B}T} - 1} \right]$$

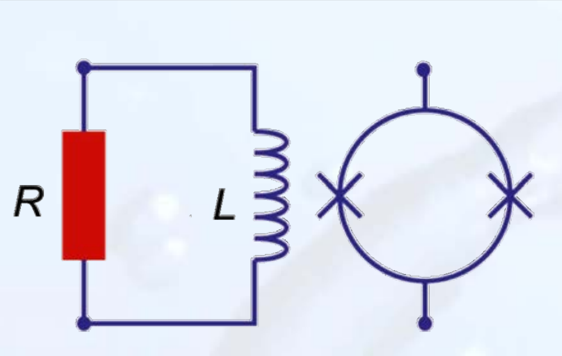
for $hf/k_\mathrm{B}T \ll 1$

$$\approx 4 k_\mathrm{B} T R \left[1 + \frac{1}{12} \frac{hf}{k_\mathrm{B}T} \right]$$

can be neclegted since
(T > 100 µK, f < 1 kHz)

$$\frac{hf}{k_\mathrm{B}T} < 5 \times 10^{-4}$$

12. Thermometry at Low Temperatures



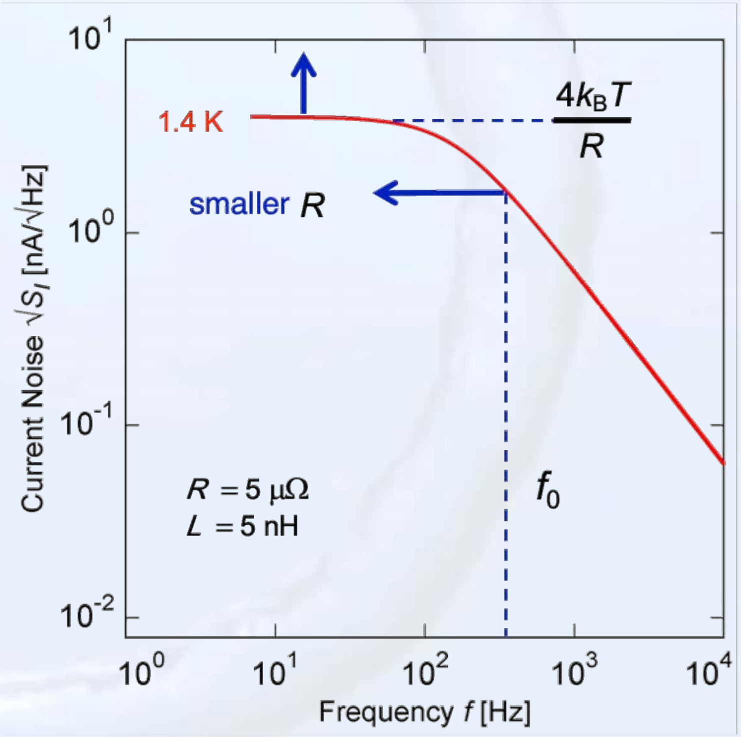
current noise $S_I = \frac{\langle I^2 \rangle}{\Delta f} = \frac{4k_\mathrm{B}T}{R}$

For $R \sim$ mΩ even at $T \sim 1$ mK large compared to SQUID current sensitivity

Finite bandwidth due to reactance $i\omega L$:

$$S_I = \frac{\langle I^2 \rangle}{\Delta f} = \frac{4k_\mathrm{B}T}{R} \frac{1}{1+(f/f_0)^2}$$

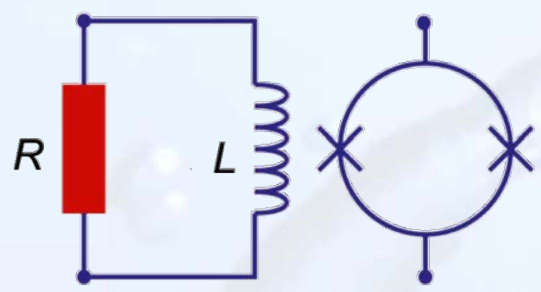
$$f_0 = (1/2\pi)L/R$$



Coil = one degree of freedom, thus

$$\overline{E} = \int_0^\infty \frac{1}{2} L\, S_I \,\mathrm{d}f = \frac{1}{2} k_\mathrm{B} T$$

12. Thermometry at Low Temperatures



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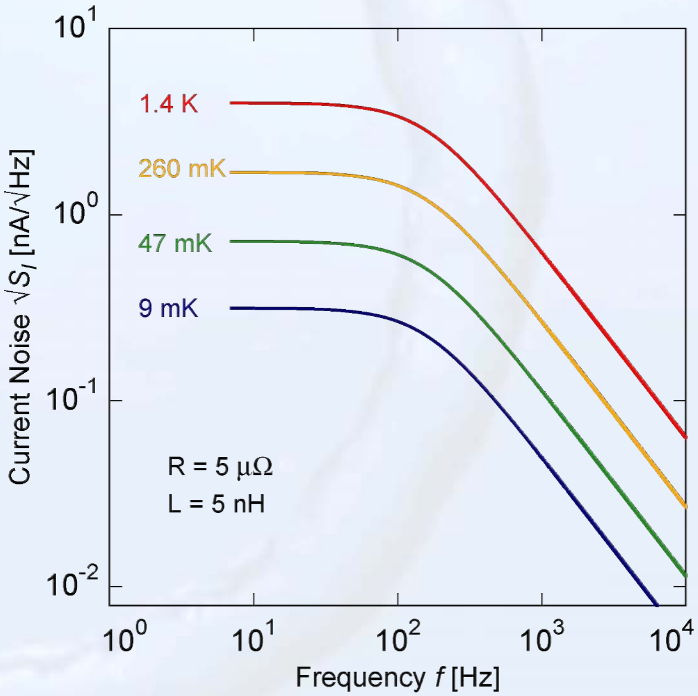

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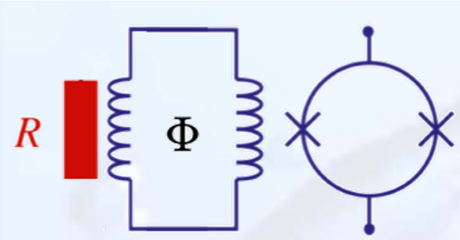
12. Thermometry at Low Temperatures

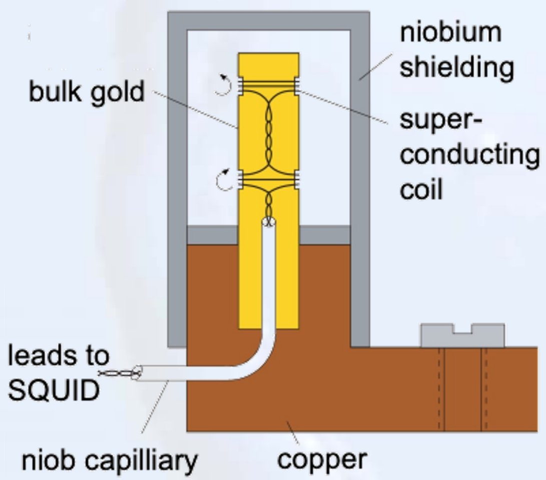


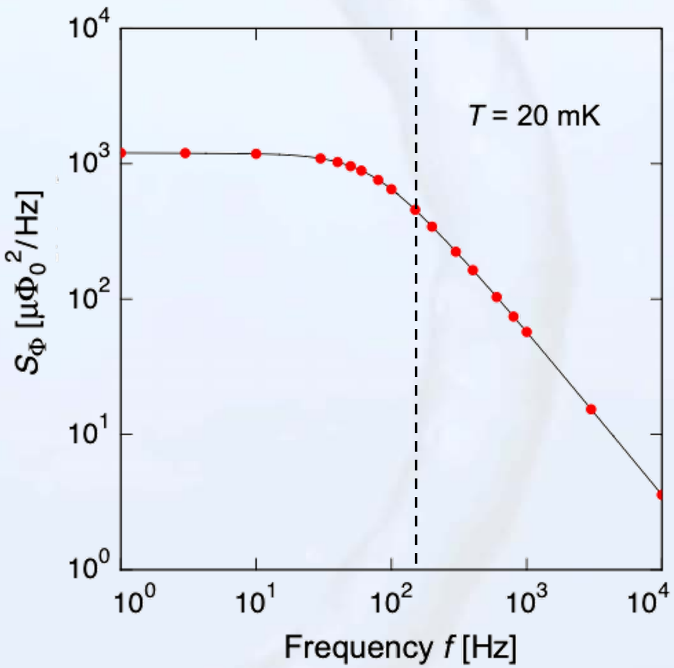
flux noise

$$S_\Phi(f,T) = \frac{4k_\mathrm{B}T\Re(Z(f))}{(2\pi f)^2}$$

$Z = R(\omega) + i\omega L(\omega)$

$f_\mathrm{c} \simeq 4.5/(\pi\mu_0\sigma r^2)$





12. Thermometry at Low Temperatures

Noise thermometry at ultralow temperatures

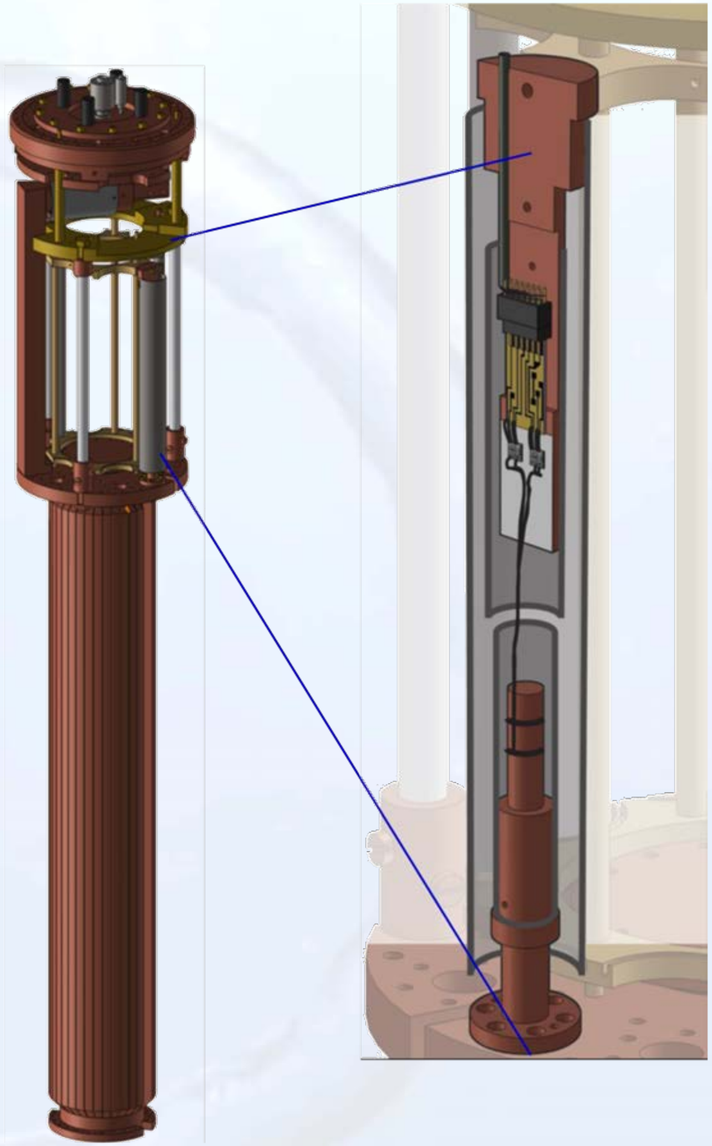
Problem:
noise amplitudes become very small

Requirements for noise source:
high conductivity → large signal
low conductivity → wide bandwidth
constant conductivity at low temperatures

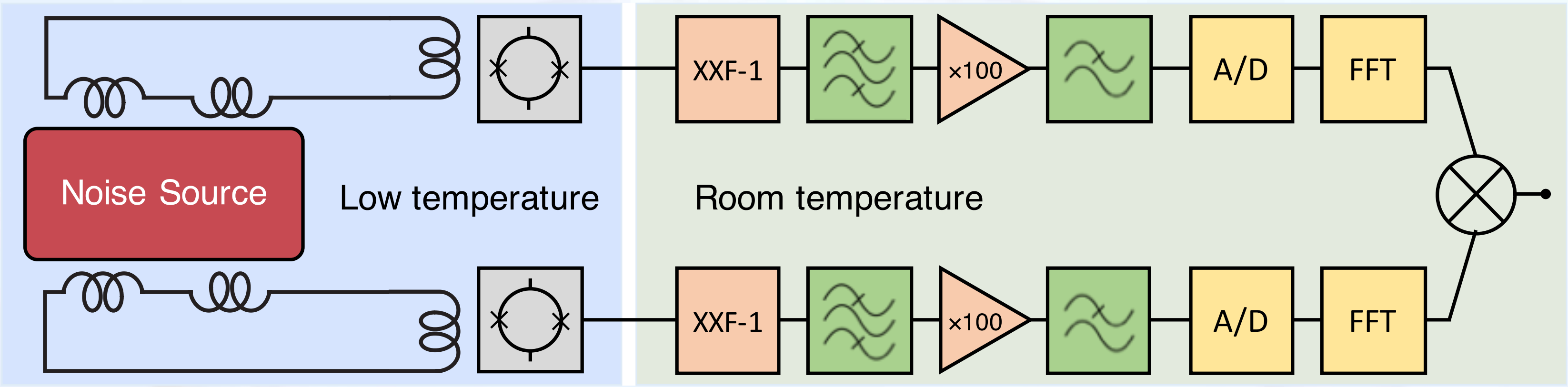
high purity copper (5N),
free of Kondo-impurities
additional heat treatment to release hydrogen
→ very high conductivity (RRR ~1000)

optimizing the RRR by cold working

→ cut-off frequency ~ 100 Hz



Noise thermometer readout scheme:



Channel 1: $A_1(t) = U(t) + N_1(t)$

Channel 2: $A_2(t) = U(t) + N_2(t)$

if $U(t), N_1(t), N_2(t)$ pairwise uncorrelated

Cross correlation:

$$C(t') = \lim_{T_W \to \infty} \frac{1}{T_W} \int_0^{T_W} A_1(t) A_2(t+t') \mathrm{d}t = R(t)$$

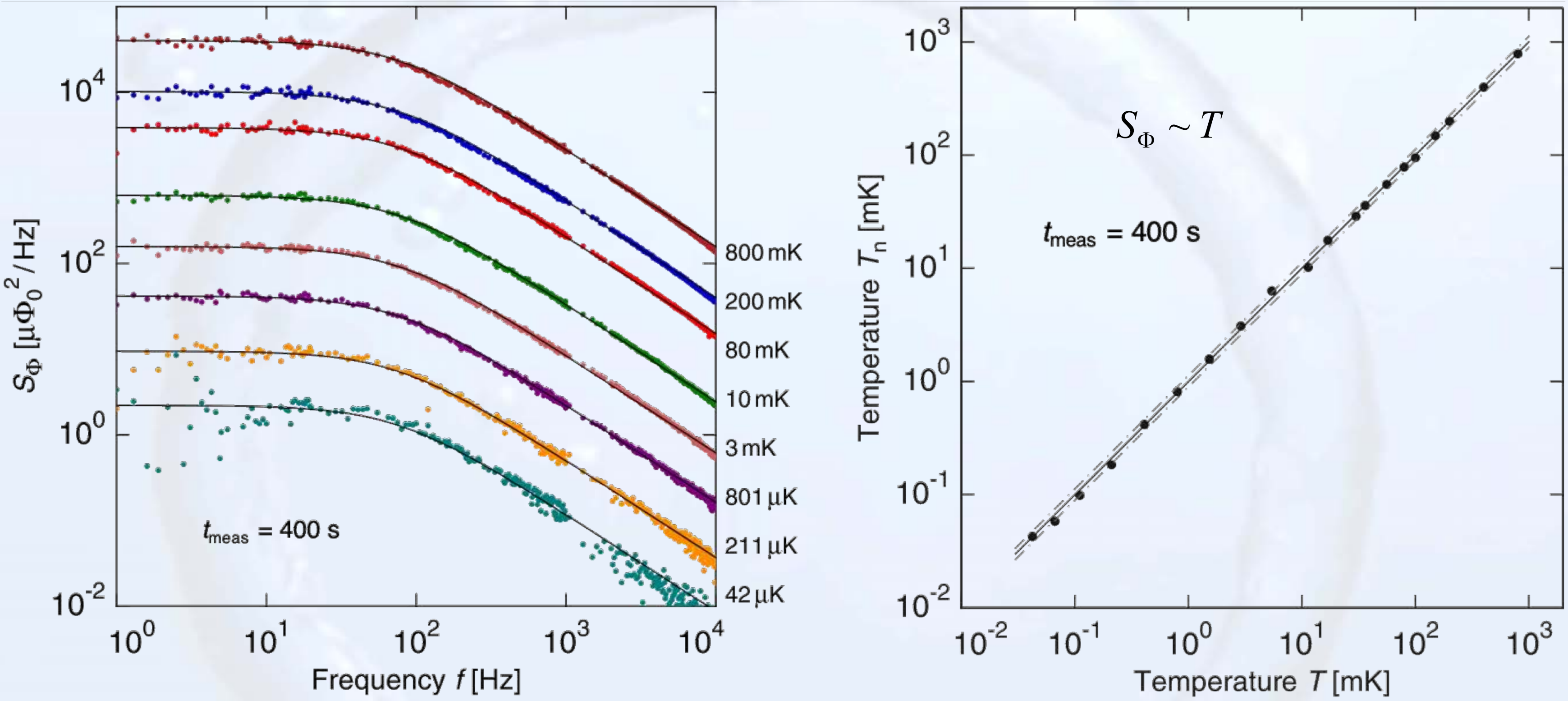
Auto correlation

Spectral power density via Wiener-Khinchin theorem

$$S(\omega) = 2 \int_{-\infty}^{\infty} R(t)\, \mathrm{e}^{-i\omega t} \mathrm{d}t$$

12. Thermometry at Low Temperatures

Spectral power density: after cross correlation



- spectral shape the same at all temperatures → R and L are constant
- spectral power density proportional to T over five orders of magnitude in temperature
- calibrated against fix point thermometer and ^{195}Pt

Videos of He-II Phonomena and Meißner Effect

superleak

fountain effect

Meissner effect