10.2 Microscopic Theory

Excitation of BCS ground state

ground state: $W_0 = -2\sum_{\boldsymbol{k}} E_{\boldsymbol{k}} v_{\boldsymbol{k}}^4$ — $E_{\boldsymbol{k}}^2 = \eta_{\boldsymbol{k}}^2 + \Delta_0^2$

breaking of one Cooper pair:

$(\boldsymbol{k}'\uparrow, -\boldsymbol{k}'\downarrow)$ →
- electron with $\boldsymbol{k}$ plus hole with $-\boldsymbol{k}'$
- electron with $-\boldsymbol{k}$ plus hole with $\boldsymbol{k}'$

} two quasi-particles

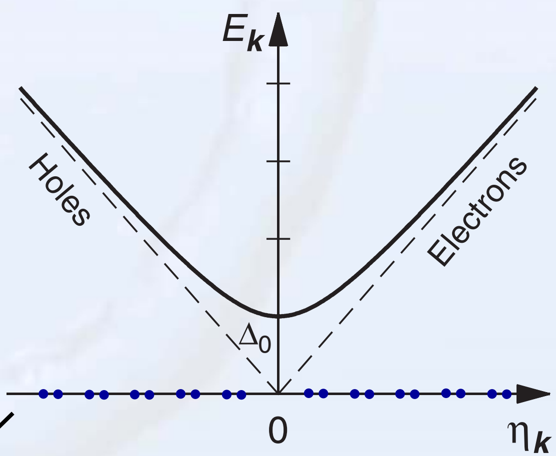
energy of remaining Cooper pairs $W_1 = -2\sum_{\boldsymbol{k}\neq\boldsymbol{k}'} E_{\boldsymbol{k}} v_{\boldsymbol{k}}^4$

energy difference: $\delta E = W_1 - W_0 = \boxed{2E_{\boldsymbol{k}'} = 2\sqrt{\eta_{\boldsymbol{k}'}^2 + \Delta_0^2}}$

dispersion of quasi-particles

→ even if unpaired electrons have no kinetic energy ($\eta_{\boldsymbol{k}'} = 0$) to break a Cooper pair one must invest $2\Delta_0$

→ energy gap: $\delta E_{\min} = 2\Delta_0$



ground state

two particle description

10.2 Microscopic Theory

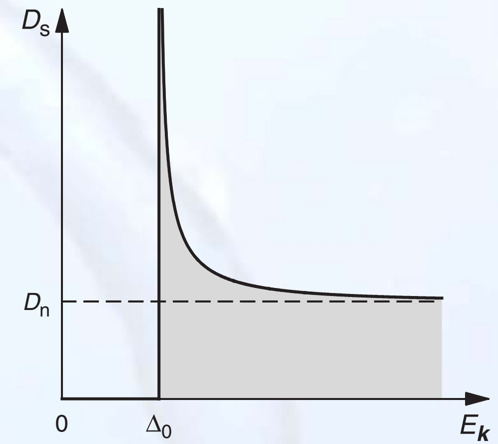
Density of states of quasi-particles

$D_n(\eta_k) \longleftrightarrow D_s(E_k)$ each state in normal conductor is uniquely connected with one in the superconductor

$$\Rightarrow \quad D_s(E_k)\,dE_k = D_n(\eta_k)\,d\eta_k$$

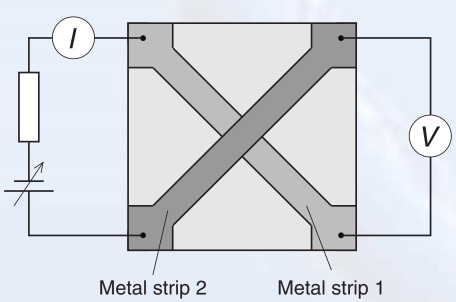
$$D_s(E_k) = D_n(\eta_k)\frac{d\eta_k}{dE_k} = \begin{cases} D_n(E_F)\dfrac{E_k}{\sqrt{E_k^2-\Delta_0^2}} & \text{for} \quad E_k > \Delta_0 \\ 0 & \text{for} \quad E_k < \Delta_0 \end{cases}$$

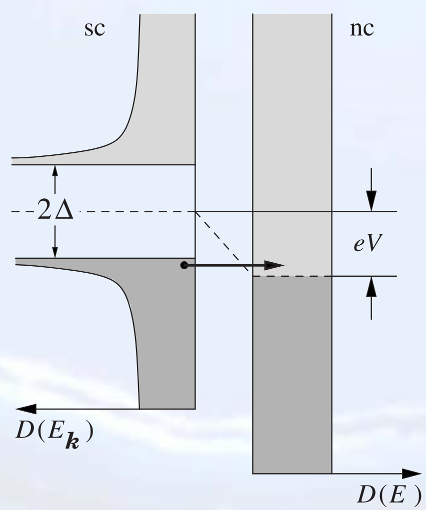
singularity at $E_k = \Delta_0$

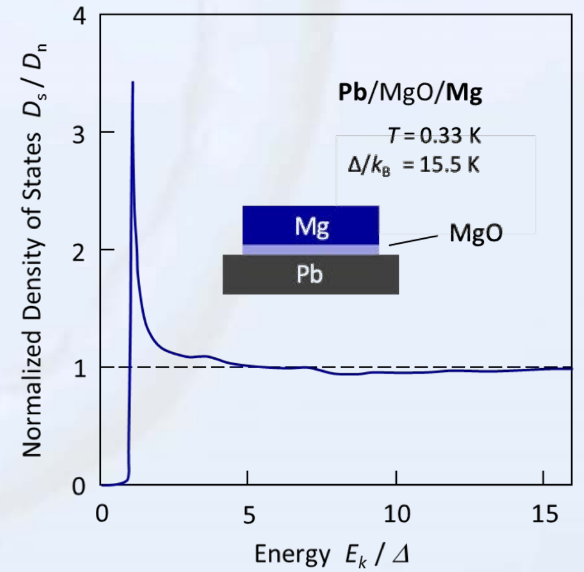


experimental observation using superconducting tunnel junctions

schematic setup







10.2 Microscopic Theory

BCS state at finite temperatures

Cooper pairs ⟶ quasi-particles ⟶ BCS state weakens ⟶ energy gap decreases

BCS theory in weak coupling limit

$$\Delta_0 = 2\,\hbar\omega_D\, e^{-2/\mathcal{V}_0\, D(E_F)}$$

$$k_B T_c = 1.14\,\hbar\omega_D\, e^{-2/\mathcal{V}_0 D(E_F)}$$

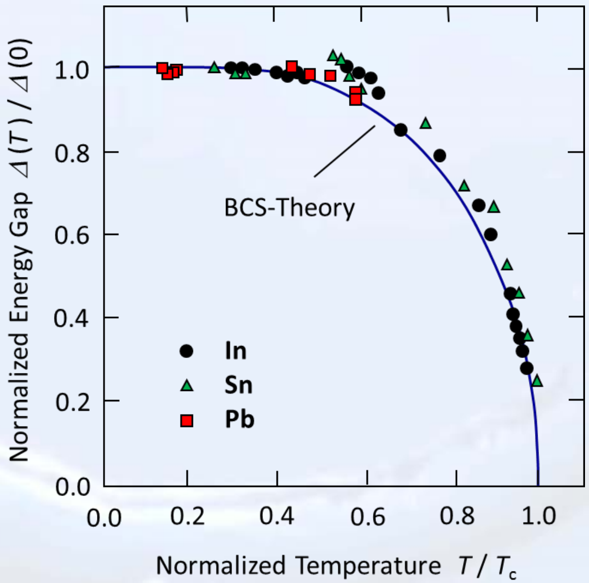
$$\Delta_0 = 1.76\, k_B T_c$$

	Al	Cd	Hg	In	Nb	Pb	Zn
$\Delta_0/(k_B T_c)$	1.7	1.6	2.3	1.8	1.9	2.15	1.6

weak coupling regime does not really apply

energy gap at finite temperatures

$$\frac{\Delta(T)}{\Delta_0} = 1.74\sqrt{1-\frac{T}{T_c}}$$



10.3 Macroscopic Quantum State

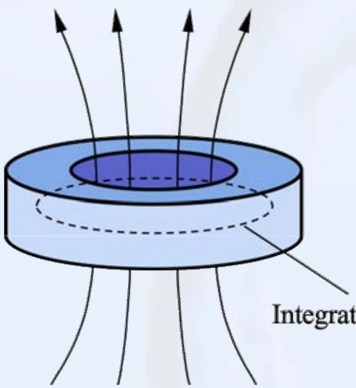
Macroscopic wave function $\Psi = \Psi_0 \, e^{i\varphi(\boldsymbol{r})}$ → flux quantization, Josephson effect

$|\Psi_0|^2 = n_s$

a) flux quantization

phase $\varphi(\boldsymbol{r})$ is well defined in entire superconducting system

consider superconducting ring in magnetic field



phase difference along a path $\Delta\varphi = \int_1^2 \operatorname{grad} \varphi(\boldsymbol{r}) \cdot d\boldsymbol{s}$

closed loop $\boldsymbol{r}_1 = \boldsymbol{r}_2 \longrightarrow \Delta\varphi = 2\pi p$

quantum mechanical current density $\boldsymbol{j} = i\frac{\hbar q}{2M}\left(\Psi^\star \nabla \Psi - \Psi \nabla \Psi^\star\right) - \frac{q^2}{M}\boldsymbol{A}\Psi^\star\Psi$

with $q = -2e$ and $M = 2m$ $\longrightarrow$ $\mu_0 \lambda_L^2 \boldsymbol{j} = \left(\frac{\hbar}{e}\nabla\varphi - 2\boldsymbol{A}\right)$

integration along closed contour line L

$$\mu_0 \lambda_L^2 \oint_L \boldsymbol{j} \cdot d\boldsymbol{s} = \frac{\hbar}{e}\oint_L \nabla\varphi \cdot d\boldsymbol{s} - 2\oint_L \boldsymbol{A} \cdot d\boldsymbol{s}$$

$\boldsymbol{j} = 0$

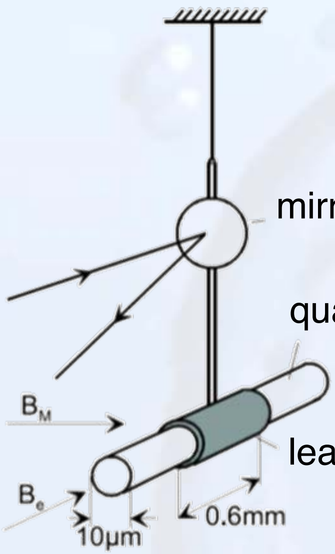
$\int_\Sigma \boldsymbol{B} \cdot d\boldsymbol{f} = \Phi$ Stokes theorem

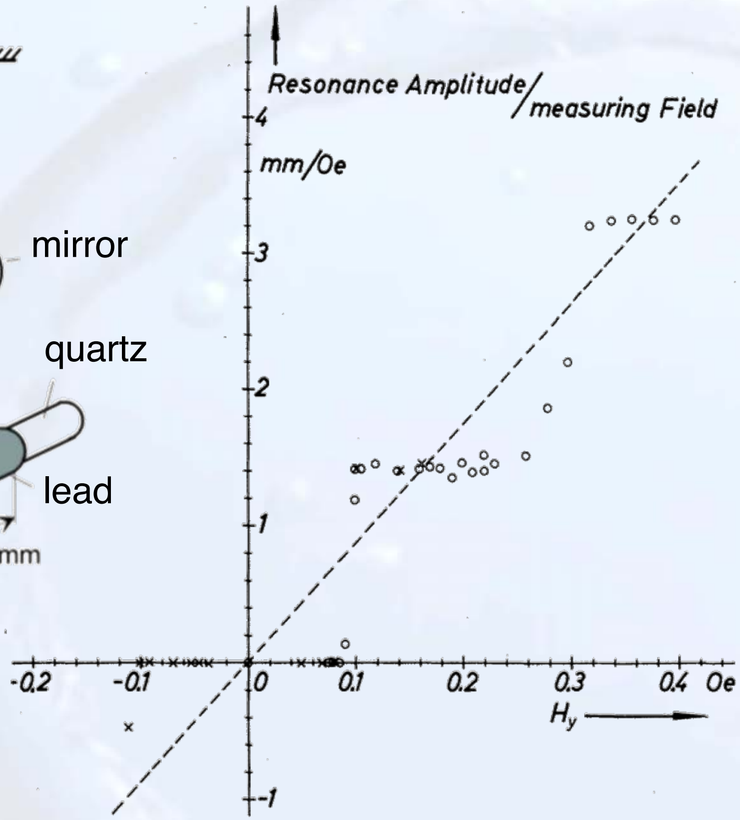
$$\Phi = p\,\frac{h}{2e} = p\,\Phi_0$$

magnetic flux enclosed by the ring is quantized

10.3 Macroscopic Quantum State

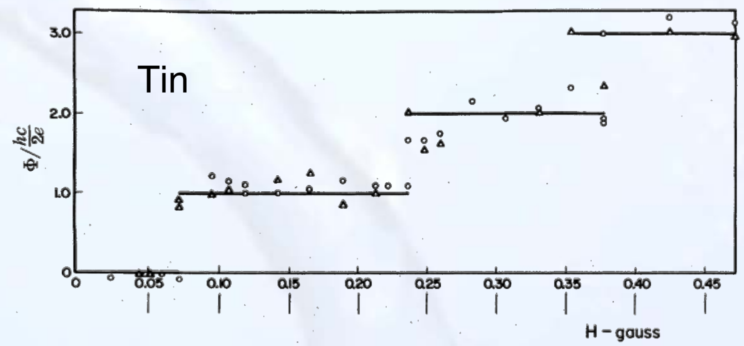
Experimental observation of flux quantization 1961



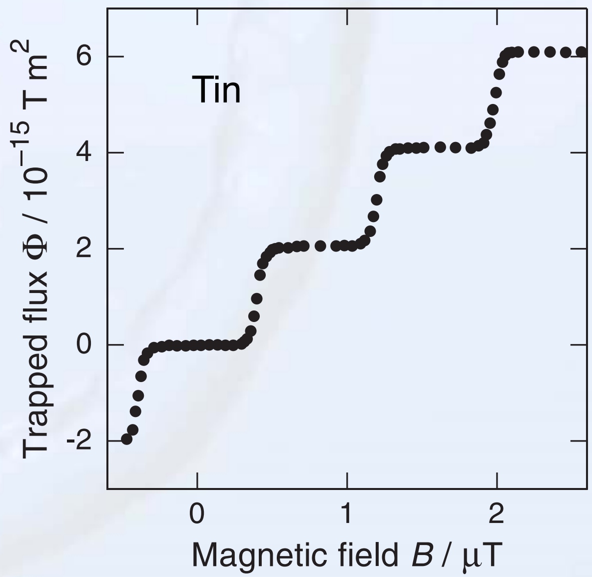


Doll and Näbauer

Deaver and Fairbank

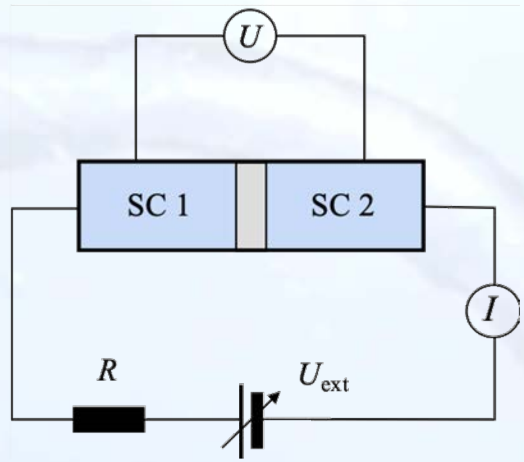


modern measurement



10.3 Macroscopic Quantum State

Josephson effects (1962)



Brain Josephson

Schrödinger equations

$$\mathrm{i}\hbar\dot{\Psi}_1 = \mu_1\Psi_1 + \mathcal{K}\Psi_2$$

$$\mathrm{i}\hbar\dot{\Psi}_2 = \mu_2\Psi_2 + \mathcal{K}\Psi_1$$

chemical potential (μ) coupling strength ($\mathcal{K}$)

ansatz $\Psi_1 = \sqrt{n_{s1}}\mathrm{e}^{\varphi_1}$ and $\Psi_2 = \sqrt{n_{s2}}\mathrm{e}^{\varphi_2}$

with $n_\mathrm{s} = n_{\mathrm{s}1} = n_{\mathrm{s}2}$

Josephson equations

$$\dot{n}_{\mathrm{s}1} = \frac{2\mathcal{K}}{\hbar}\, n_\mathrm{s} \sin(\varphi_2 - \varphi_1) = -\dot{n}_{\mathrm{s}2}$$

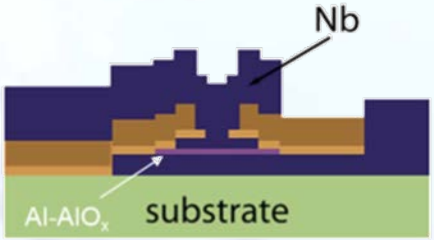
$$\hbar\,(\dot{\varphi}_2 - \dot{\varphi}_1) = -(\mu_2 - \mu_1) = 2eV$$

- $V = 0 \longrightarrow \mu_1 = \mu_2 \longrightarrow I_\mathrm{s} = I_\mathrm{c}\sin(\varphi_2 - \varphi_1)$ dc Josephson effect
- $V \neq 0 \longrightarrow \mu_2 - \mu_1 = -2eV \longrightarrow I_\mathrm{s} = I_\mathrm{c}\sin(\omega_\mathrm{J}\,t + \varphi_0)$ ac Josephson effect

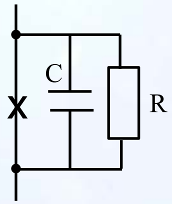
$$\omega_\mathrm{J} = 2eV/\hbar$$

10.3 Macroscopic Quantum State

Experimental observation of dc Josephson effect

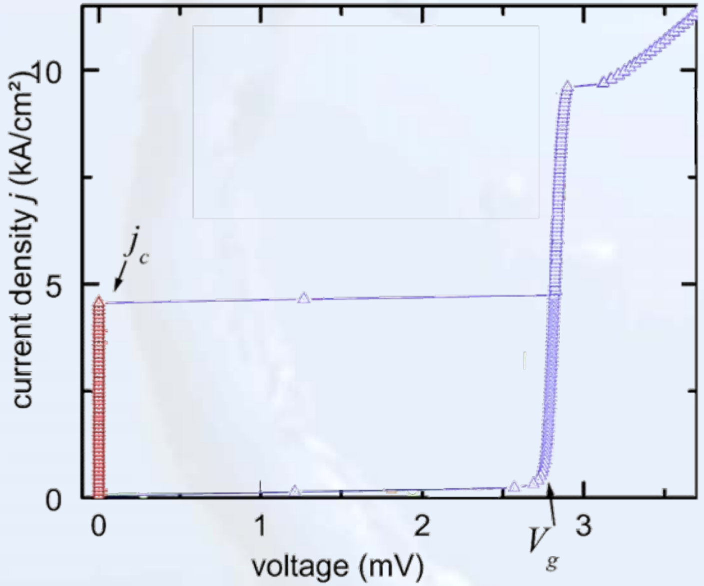


Josephson junction

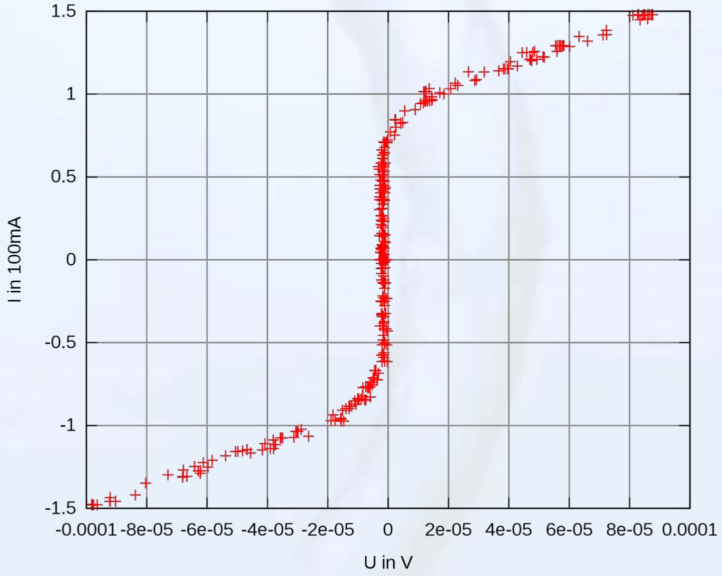


hysteresis parameter: $\beta_c = 2\pi I_c R^2 C / \Phi_0$

underdamped junction (large *R* and *C*)



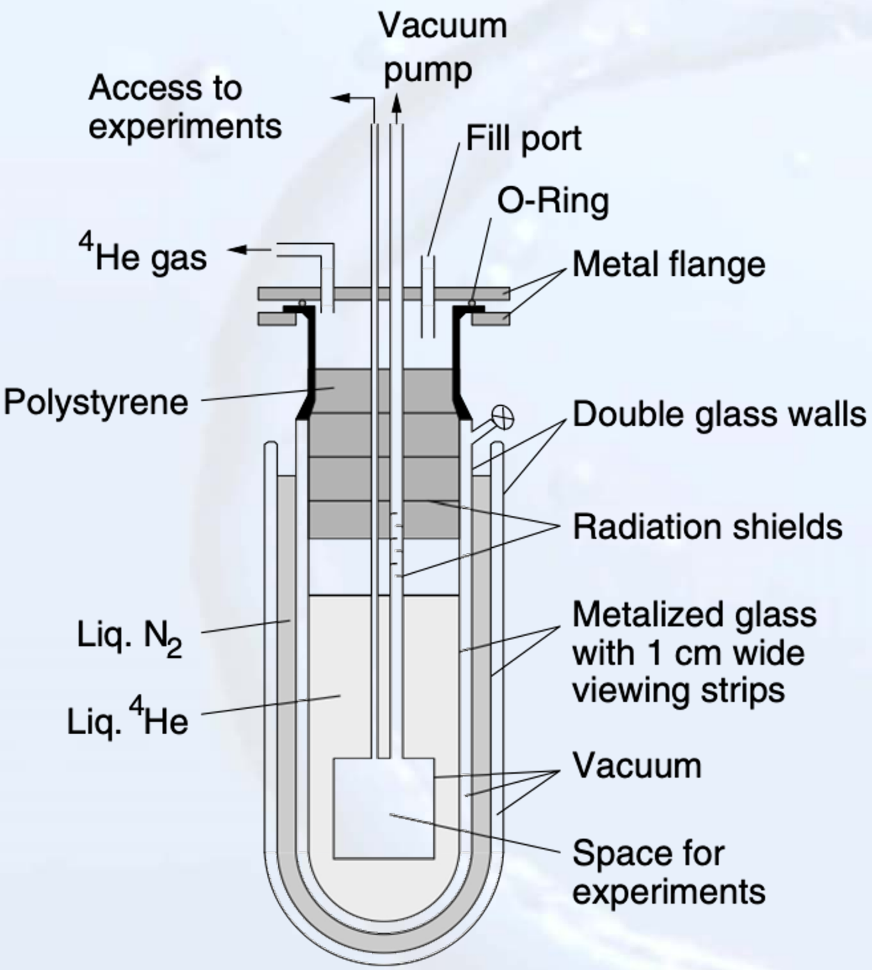
overdamped junction (small *R* and *C*)

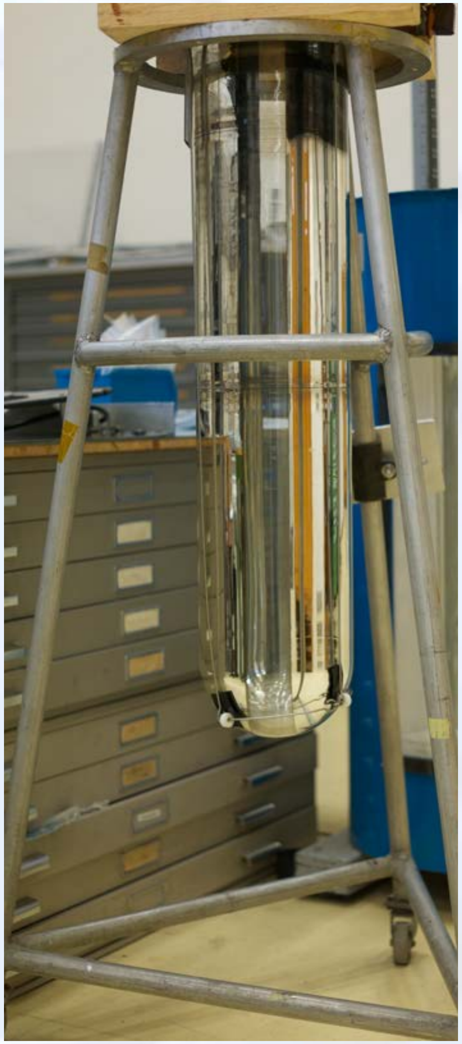


- hysteretic Josephson junction
- for $I < I_c$ current is determined by current source
- for $I > I_c$ super current breaks down

- non-hysteretic Josephson junction
- for $I > I_c$ super current breaks down

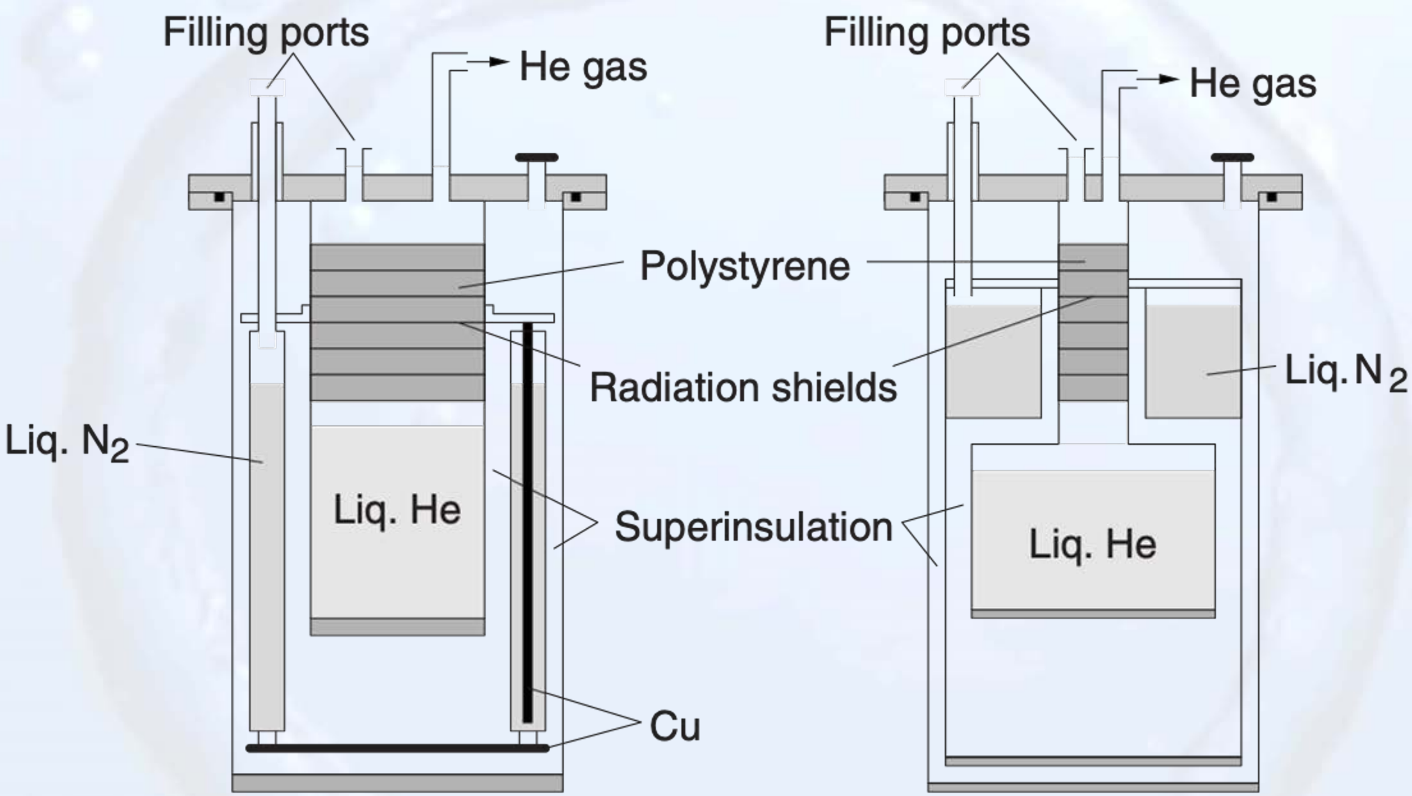
^{4}He bath cryostat: glass dewar





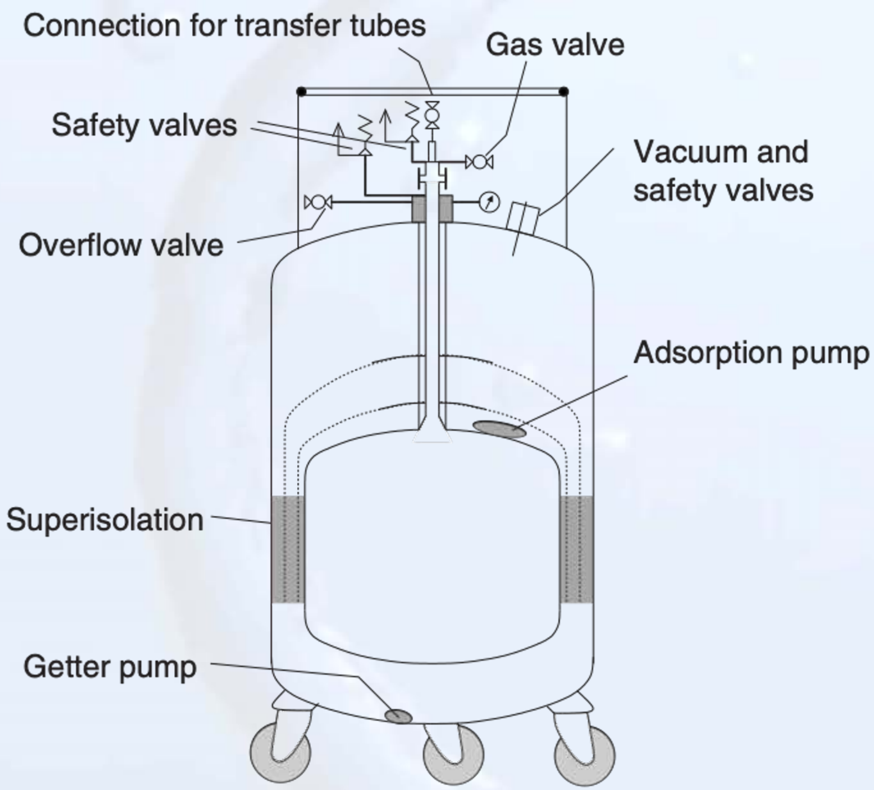
11.1 Bath Cryostats

^{4}He Bath cryostat: metal dewar

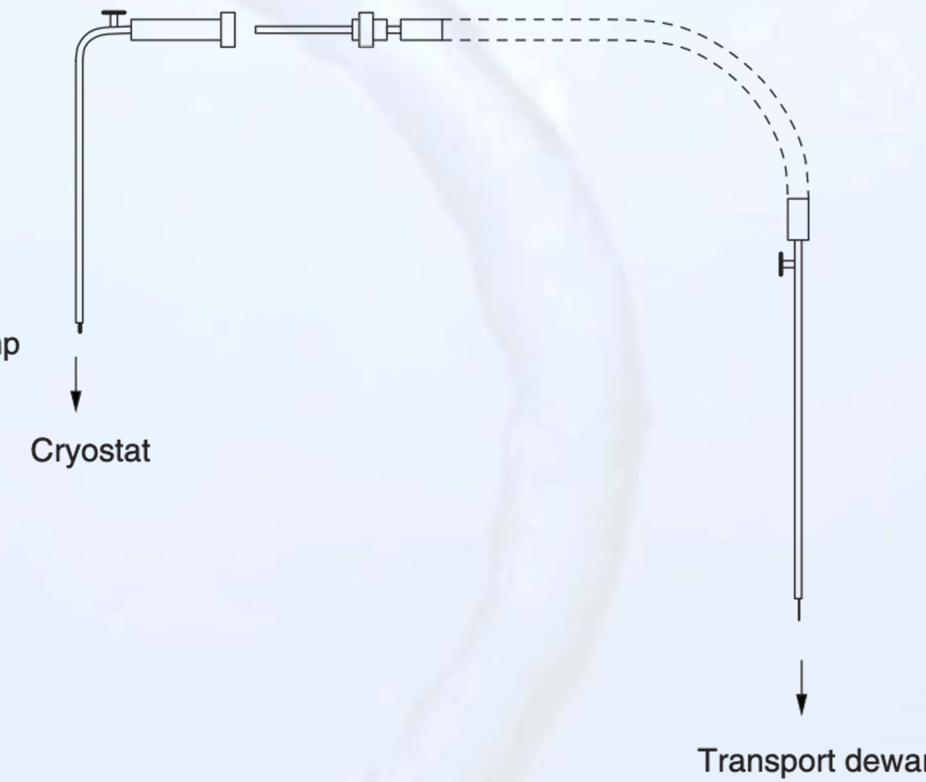


11.1 Bath Cryostats

helium transport vessel



helium transfer tube



11.1 Bath Cryostats

Radiation shields – super insulation

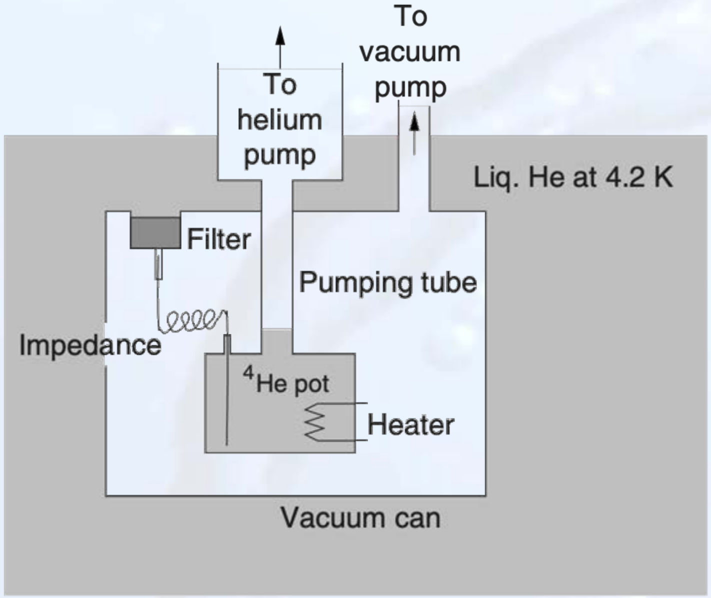
multiple radiation shields → smaller steps → reduction of heat flow

30 to 80 layers of low conductivity high reflection material → aluminized Mylar

apparent thermal conductivity
$\sim 10^{-4}$ to 10^{-5} W/(m K)

11.1 Bath Cryostats

Cryostats with 1-K-Pot



$$^4\mathrm{He} \quad L = 90\,\mathrm{J\,mol^{-1}}$$
$$^3\mathrm{He} \quad L = 40\,\mathrm{J\,mol^{-1}}$$

Vapor pressure curve of various cryogenic liquids

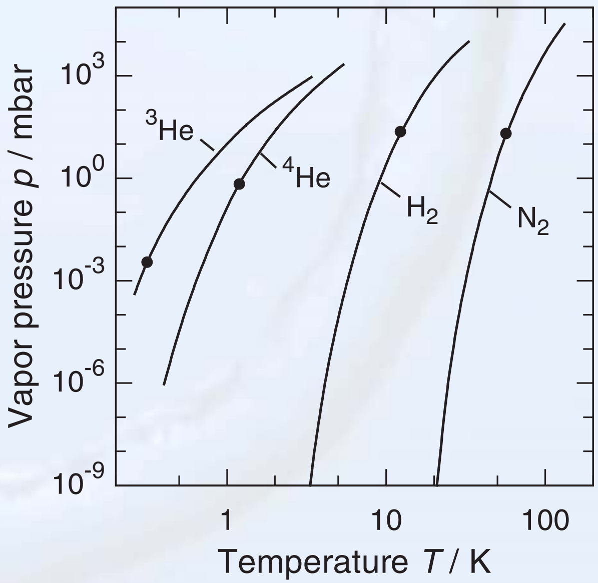
Clausius-Clapeyron equation

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{\Delta V\, T}$$

$$\Delta V = V_\mathrm{g} - V_\ell \approx V_\mathrm{g} \qquad pV_\mathrm{g} = RT$$

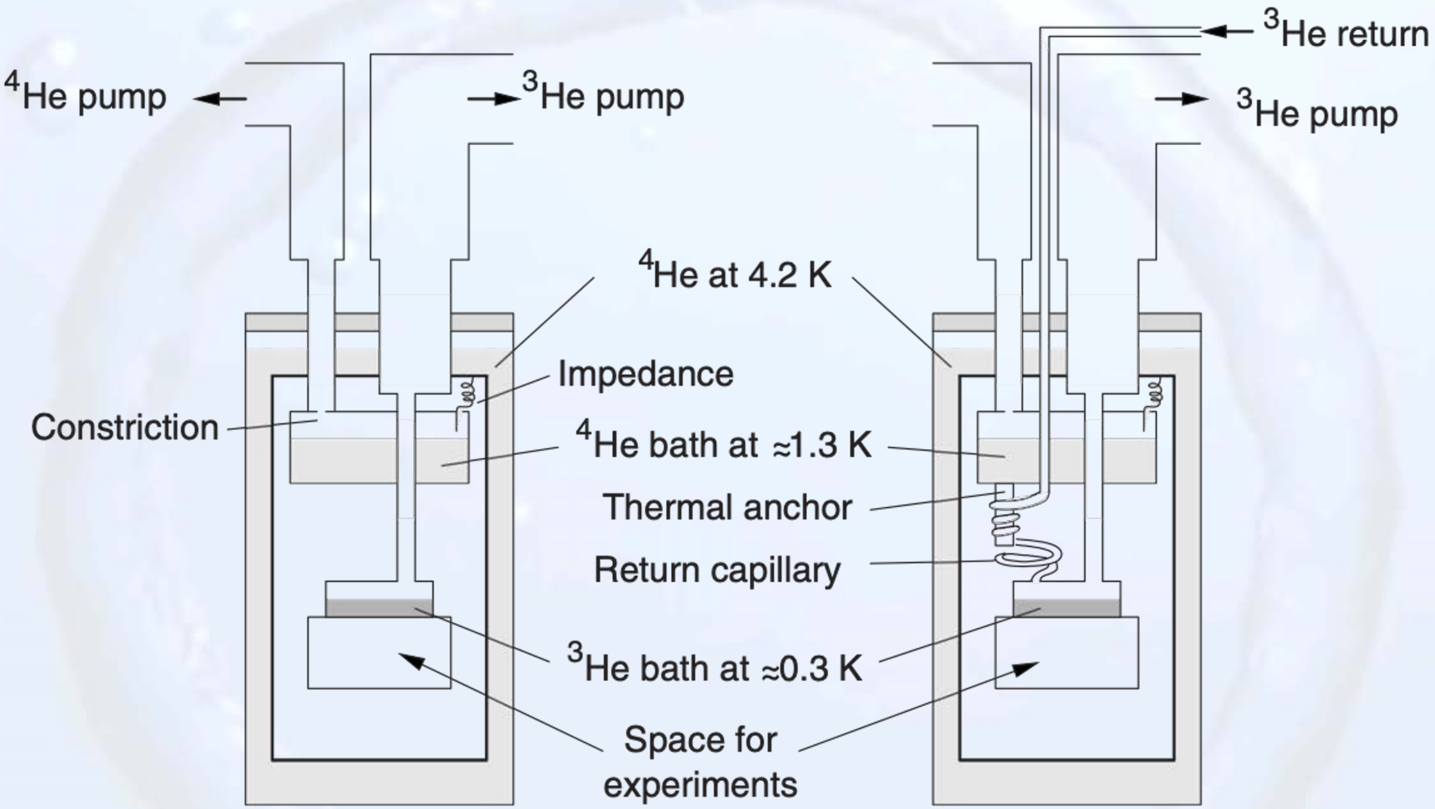
$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{R\,T^2}\,p \quad \rightarrow \quad p(T) = p_0\,\mathrm{e}^{-L/RT}$$

vapor pressure curve



11.1 Bath Cryostats

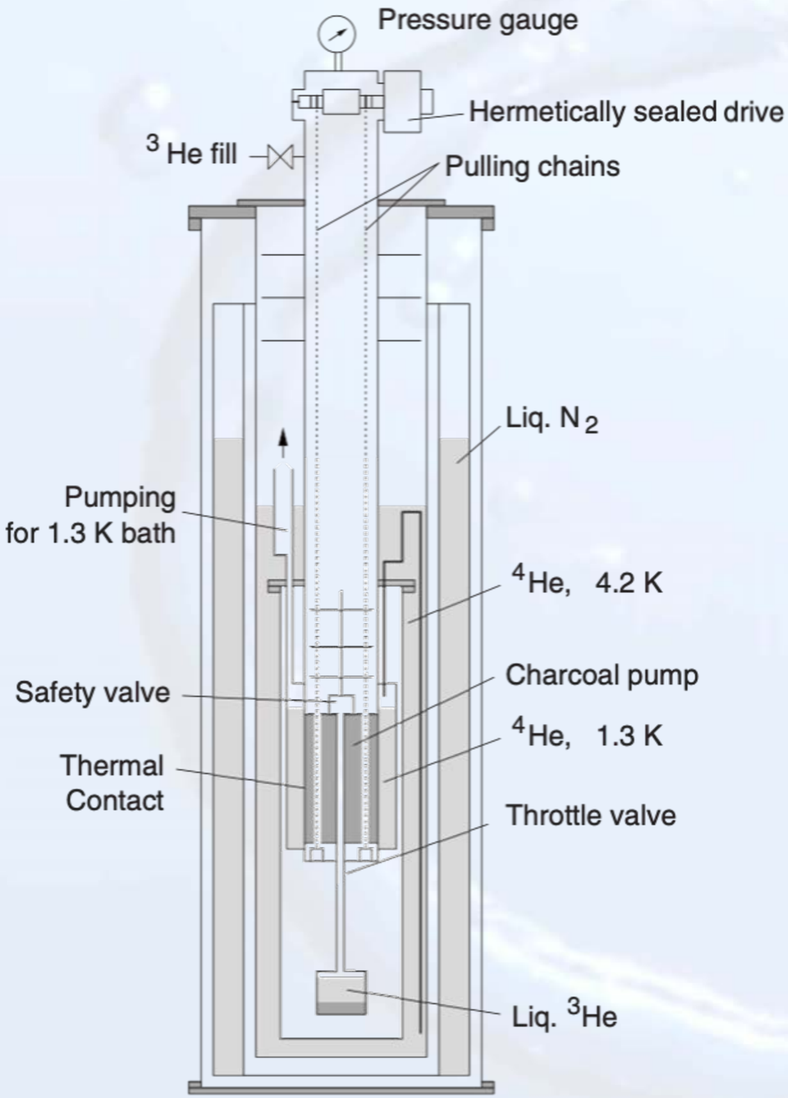
^{3}He cryostats

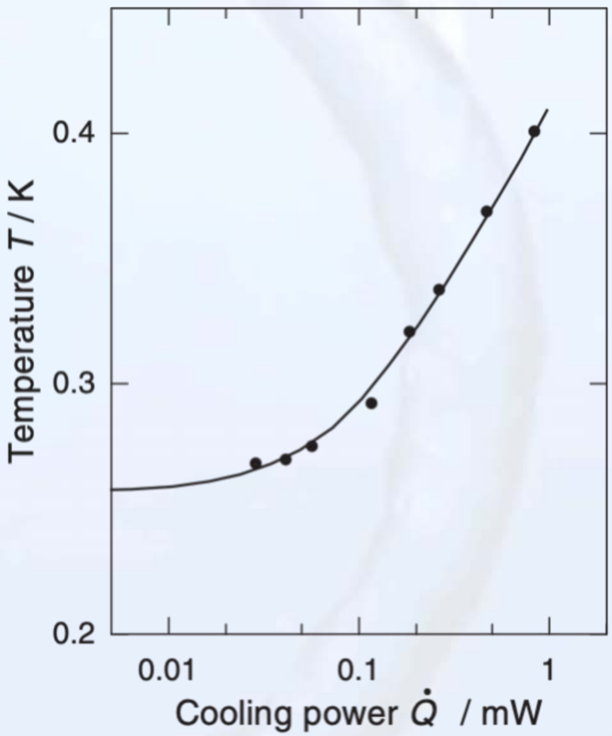


cooling power $\dot{Q} = \dot{n}_g \, L \propto p \propto e^{-L/RT}$

11.1 Bath Cryostats

Cooling power of a ^{3}He cryostat with charcoal absorption pump





Bardeen – Josephson Debate

The Nobel Laureate Versus the Graduate Student

In a recent note, Josephson uses a somewhat similar formulation to discuss the possibility of superfluid flow across the tunneling region, in which no quasi-particles are created. However, as pointed out by the author [Bardeen, in a previous publication], pairing does not extend into the barrier, so that there can be no such superfluid flow.

Physics Today **54,** 46-51 (2001)